International Conference on Industrial Engineering, ICIE 2016

Development of Mathematical Model for Detection of Conditional Sizes Primordial Cracks in LEFM and Its Implementation in Russian Cax System

S.A. Renev\textsuperscript{a,*,b}, V.S. Prokopov\textsuperscript{b}

\textsuperscript{a} Bauman Moscow State Technical University, st. Bauman\textsuperscript{a}ya 2-ya, 5, Moscow 105005, Russia
\textsuperscript{b} Research and Software Development Centre APM, Oktyabrs\textsuperscript{y}sky Blvd 14, office 6, Moscow region, Korolev 141070, Russia

Abstract

This paper addresses the definition of conventional primordial crack size by solving the inverse problem of traditional LEFM and implementation of this tool in the Russian CAx system APM WinMachine. The condition of the primordial origin of the crack with the principal stresses and classical criteria of strength are shown. A mathematical model for finding the conditional size of the crack is shown. The critical value of the parameter of fracture mechanics – the stress intensity factor (SIF) and the value of the stress concentrator are used for this purpose. The critical value of SIF was defined by experiment. A mathematical model has been implemented in the Russian CAx system APM WinMachine which uses a numerical finite element method (FEM). Output data (crack dimensions) of the mathematical model are used in solving the analytical problem to determine the SIF. The validity of the numerical and analytical calculation of the data is checked by comparing the SIF critical value obtained experimentally, with the analytical one. The calculations were performed on the example of solving the static problem with a different loading step.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Peer-review under responsibility of the organizing committee of ICIE 2016

Keywords: Lineal Elastic Fracture Mechanics; crack; critical stress intensity factor; finite element method; computer-aided technologies.

* Corresponding author. Tel.: +7-905-770-6210.
E-mail address: chevrole59@mail.ru
1. Introduction

In this paper, we consider the linear elastic fracture mechanics (LEFM). LEFM valid for quasi brittle solids, characterized by a small region at the crack tip plasticity (plasticity field is not more than 20% of the size of the crack). The main parameter in LEFM characterizing the state of the material near the crack tip is the stress intensity factor (SIF). By comparing the calculated SIF at the crack tip with the critical SIF, obtained experimentally [2], one can say whether the crack propagation. If yes, what is the character (stable or unstable):

- There is no mathematical model for finding conditional sizes primordial cracks (In the article «Development of an algorithm for solving problems of fracture mechanics» of the journal «Materials Physics and Mechanics» in 2016 year mentioned algorithm for finding the conditional size of the primordial crack. The disadvantage of this algorithm is that it does not depend on the stress concentrator and the hub only considers critical SIF, obtained experimentally);
- In Russian CAx system LEFM no tools to determine the contingent dimensions of primordial cracks.

The purpose of this work is to develop a mathematical model to find the size of conventional primordial crack by solving the inverse problem using LEFM SIF and stress concentrator, as well as the introduction of the mathematical model in the Russia CAx system APM WinMachine.

2. The condition of crack initiation

The FE model loaded without macroscopic defects there are dangerous fields, which are characterized by the presence of stress higher than the yield stress or the strength of the material. A dangerous field is determined by finding the maximum Von Mises stress $S_{VM}$.

To evaluate the strength of the dangerous field is used maximum strain theory, which includes the principal stresses $\sigma_1, \sigma_2, \sigma_3$.

$$S_{solve} = S_1 - \mu(S_2 + S_3) \quad (1)$$

where $S_1, S_2, S_3$ – principal stresses, $\mu$ - Poisson's ratio.

For LEFM true condition of the classical criterial of strength.

$$S_{solve} \geq S_{critical} \quad (2)$$

If (2) is performed, the field of the concentrator there is a crack.

3. The definition of conditional dimensions crack

According to ASTM E399-08 (GOST 25.506-85) one of the requirements to obtain the critical value of SIF, obtained experimentally:

$$t, L \geq 2.5\left(\frac{K_{IC}}{\sigma_{yield}}\right)^2 \quad (3)$$

where $t$ - the thickness of the specimen, $L$ - crack length, $\sigma_{yield}$ - yield stress of the material, $K_{IC}$ - critical SIF, obtained experimentally.

In (3) replace $\sigma_{yield}$ on $\sigma_{SYM}$. This makes it possible to take into account not only the critical value SIF [5] of the material, but also the amount of stress concentrator ($\sigma_{SYM}$) obtained by Von Mises [6] to determine the size of conventional primordial cracks.
Conditional dimensions of opening and expansion of the crack [6]:

$$
\Delta = \frac{4K_{IC}}{E} \sqrt{\frac{2L}{\pi}}
$$

(5)

where $E$ - the modulus of elasticity for the plain strain condition.

$$
\alpha = \frac{L}{\beta [0.125...1]}
$$

(6)

4. Analytical model

Consider a plain strain condition of a rectangular strip with symmetric surface semi-elliptical crack loaded by uniformly tensile load (see Fig. 1). Parametric dimensions semi-elliptical crack is shown in Fig. 2.

Input data: the sizes of a crack ($L$, $\Delta$, $a$), nominal tensile stress ($\sigma$). Output data: the value of SIF at point A ($K_{I,A}$) (see Fig. 2).

Fig.1. Test problem dimensions.

Fig.2. The geometric shape and parametric dimensions of crack.

Analytical calculation of SIF at point A (see Fig. 2) by Murakami [7], Subpart 1.16:

$$
K_{I,A} = \sigma \sqrt{\pi L (M / F)}
$$

(7)

where $\sigma$ - nominal tensile stress.
\[ M = (1.13 - 0.09\alpha) + (-0.54 + \frac{0.89}{0.2 + \alpha})\beta^{2} + (0.5 - \frac{1}{0.65 + \alpha} + 14.0(1 - \alpha)^{2})\beta^{4} \]  

(8)

where \( \alpha = \frac{L}{a}, \beta = \frac{L}{t} \)

\[ F = \sqrt{1 + 1.464\alpha^{1.65}} \]  

(9)

5. FE model

The numerical model is built by finite element method (FEM) [8] in CAx system APM WinMachine. The numerical model is FE model. As object of research used a rectangular strip with a stress concentrator (without cracks). The geometric dimensions and loading conditions of the numerical model are identical to the analytical. The material is structural steel [5]. Mode of deformation shown in Fig.3.

FEM is applicable for the case [7]: \( L \leq a \) and \( L \leq 0.8t \)

FE model: number of nodes – 2893, number of elements – 10929, 4 node tetrahedral elements.

Fig. 3. Mode of deformation of plate in CAx system APM WinMachine.
Input data: nominal tensile stress ($\sigma$), the critical value SIF ($K_c$). Output data: the sizes of a crack (L, $\Delta$, a), the value of stress concentrator ($\sigma_{VSM}$). FE model with primordial crack is shown in Fig. 4.

Fig. 4. The primordial crack in the FE model in Russian CAx system APM WinMachine.

6. Perform calculations and analysis of results

Analytical calculations and numerical model are carried out step by step (each step is responsible for solving the static problem). At each step sets the nominal tensile stress (see tab.1). Structural steel has a yield strength of $\sigma_{yield} = 500$ MPa [5]. A critical value of SIF, obtained experimentally, $K_c = 1581$ MPa$\cdot$mm [5]. The parameter $t$ for the analytical and numerical models is a constant with the condition that $L \leq 0,8t$ and (3). According to (3):

$$t \geq 2,5(\frac{1581}{500})^\frac{3}{2} = 24,99 mm$$  \hfill (10)$$

The maximum crack length L for the task in solving numerical method is 24.64 mm (see. Table 1). Conditions $L \leq 0,8t$ at $t = 24,99$ mm is not performed. According to [9] the precision is the higher, the more $t$ in the plane of the crack relative to $L$. The calculation is performed at $t = 550$ mm. Conditions $L \leq 0,8t$ and (3) are performed.

Input and output data for analytical and numerical model are presented in Table.1.

The decrease in the length $L$ and the extension $a$ of a crack with increasing nominal tensile stress $\sigma$ (in $K_c$ = const) related to (3). For the crack length $L$ assumes a value which is obtained when the condition (2).

At each step of loading was obtained five the value of SIF at the point A (see Fig.2). For the true value of SIF is accepted critical value, obtained experimentally [5].

In statics, for analytical stress-strain state of the error differences of the analytical method with the numerical calculation is less than 3%. This value is recommended international practice of application of FEM analysis in the field of strength [10]. Error in finding the critical value SIF to GOST 25.506.95 is about 9%. Then the summary allowable error is 12%. The resulting relative error SIF measurement for each step of the loading is within the permissible range (<12%).
Table 1. Input and output data of the analytical and numerical model

<table>
<thead>
<tr>
<th>The step of loading</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal tensile stress, $\sigma$ (MPa)</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>Stress concentrator, $\sigma_{SVM}$ (MPa)</td>
<td>687,4</td>
<td>1374,8</td>
<td>2062,2</td>
<td>2749,6</td>
<td>3437,0</td>
</tr>
<tr>
<td>The length of the crack (2), L (mm)</td>
<td>13,22</td>
<td>3,30</td>
<td>1,46</td>
<td>0,82</td>
<td>0,529</td>
</tr>
<tr>
<td>The expansion of the crack (4), a (mm)</td>
<td>90,52</td>
<td>25,34</td>
<td>15,92</td>
<td>10,52</td>
<td>7,03</td>
</tr>
<tr>
<td>SIF at point A $K_{I_c}$ (MPa $\sqrt{mm}$)</td>
<td>1400</td>
<td>1406</td>
<td>1421</td>
<td>1427</td>
<td>1435</td>
</tr>
<tr>
<td>The critical value of SIF was found experimentally $K_c$ (MPa $\sqrt{mm}$)</td>
<td>1581</td>
<td>1581</td>
<td>1581</td>
<td>1581</td>
<td>1581</td>
</tr>
<tr>
<td>The relative error SIF, %</td>
<td>11,44</td>
<td>11,06</td>
<td>10,12</td>
<td>9,74</td>
<td>9,23</td>
</tr>
</tbody>
</table>

7. Results

The result of this research confirmed the validity of (4) to determine the conditional length of the primordial cracks in LEFM. This is supported by numerical and analytical calculation in solving the problem of strength. The resulting mathematical model makes it possible to move away from traditional methods LEFM and solve the inverse problem (when there is a stress concentrator, but there is no crack).

The mathematical model can be successfully integrated into the Russian CAx system APM WinMachine and used to solve the problems of strength. This will allow to design of secure technical systems, and significantly reduce material costs and shorten the "design - production" cycle.

Acknowledgements

This work on the grant agreement № 14.574.21.0117 with the Department of Education and Science. Unique identifier for Applied Scientific Research (Project) - RFMEFI57414X0117.

References