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AdS_2/CFT_1, canonical transformations and superconformal mechanics

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Abstract

We propose a simple conformal mechanics model which is classically equivalent to a charged massive particle propagating near the $AdS_2 \times S^2$ horizon of an extreme Reissner–Nordström black hole. The equivalence holds for any finite value of the black hole mass and with both the radial and angular degrees of freedom of the particle taken into account. It is ensured by the existence of a canonical transformation in the Hamiltonian formalism. Using this transformation, we construct the Hamiltonian of a $N = 4$ superparticle on $AdS_2 \times S^2$ background.

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1. Introduction

In the web of AdS/CFT dualities the AdS_2/CFT_1 case has a distinguished status and still remains to be fully understood [1]. One of its peculiarities is that in $d = 1$ one encounters superconformal algebras which cannot be obtained by a dimensional reduction from higher dimensions (see, e.g. [2] for a review). Using this type of the AdS/CFT correspondence one can hope to get insights into quantum properties of supergravity black holes studying simple (super)conformal mechanics as the relevant boundary theory [3–5].

An interesting application of the AdS_2/CFT_1 correspondence is provided by a massive charged particle propagating near the horizon of an extreme Reissner–Nordström black hole [3]. The geometry characterizing this case is $AdS_2 \times S^2$ and in the limit of large black hole mass M one recovers[1] the conformal mechanics of [6].

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[1] To be more precise, one considers a specific limit when the black hole mass M is large and the difference between the particle mass and the absolute value of its charge $(\mu - q)$ tends to zero, with $M^2(\mu - q)$ being kept fixed.

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This relationship [3] suggested an elegant resolution for the problem of an infinite number of quantum states of a particle probe localized near the horizon of a black hole (see the relevant discussion in Ref. [2]). It was traced to the absence of a ground state in the conformal mechanics and the necessity of redefining the Hamiltonian [6].

It is important to notice, however, that it is the radial coordinate of $AdS_2 \times S^2$ which is identified with the degree of freedom described by the conformal mechanics. The angular variables effectively decouple in the large M limit and show up only in an indirect way via the effective coupling constant. The latter point recently received attention [7], where a particular case of the general transformation constructed in [8] was considered. It was shown that the radial part of the particle on $AdS_2 \times S^2$ background is classically equivalent to the conformal mechanics for any *finite* value of the black hole mass, i.e., without taking any specific limit.

In order to get further insights into quantum properties of a test particle near the horizon of a black hole, a proper accounting of the angular degrees of freedom is necessary. It is the purpose of this Letter to construct a simple conformal mechanics, which is classically equivalent to a particle moving on $AdS_2 \times S^2$ background, with both radial and angular variables being retained. Specifically, we take the advantage of the Hamiltonian formalism and demonstrate that the two theories are connected by a *canonical* transformation. The clue to finding such a transformation is offered by the symmetry group. Requiring the conserved charges to coincide in both theories, one reveals the desired canonical transformation.

The outline of Letter is as follows. In the next section we compare the radial part of the particle on $AdS_2 \times S^2$ with the conformal mechanics of Ref. [6]. Equating the conformal currents (which involve the Hamiltonian!) inherent both theories we find a canonical transformation which establishes the equivalence relation between them. In Section 3 we extend the analysis to include the angular variables into our consideration. The symmetry underlying this case is $so(1,2) \oplus su(2)$ and we expose an appropriate extension of the model of Ref. [6] which supports this symmetry and is canonically equivalent to the particle on $AdS_2 \times S^2$. Section 4 is devoted to possible applications of the canonical transformation we found. In particular, we construct a Hamiltonian of a $N = 4$ superparticle on $AdS_2 \times S^2$ by firstly supersymmetrizing our simple conformal model and then applying the canonical transformation to the resulting system. Some open questions and further developments are discussed in the concluding Section 5.

2. AdS_2 background as a canonical transformation of conformal mechanics

The motion of a charged massive particle near the horizon of an extreme Reissner–Nordström black hole is governed by the (static gauge) action functional

$$S = \int dt\, (2R/r)^2 \left[q - \mu \sqrt{1 - (r/2R)^2 \dot{r}^2 - R^2 (r/2R)^4 \left(\dot{\theta}^2 + \sin^2\theta \dot{\varphi}^2\right)} \right]. \tag{1}$$

Here μ and q stand for the mass and electric charge of the particle and R is the radius of the sphere in the underlying $AdS_2 \times S^2$ geometry (which is equal to that of the AdS_2 space and coincides with the black hole 'mass' in units for which $G = 1$). As has been argued in Ref. [3], in the limit $R \to \infty$, $(\mu - q) \to 0$, with $R^2(\mu - q)$ fixed, the corresponding quantum mechanical description reduces to that of the 'old' (or 'non-relativistic') conformal mechanics [6]

$$S = \frac{1}{2} \int dt \left(\dot{x}^2 - \frac{\hat{g}}{x^2} \right), \quad x = \sqrt{\mu}\, r, \tag{2}$$

provided $\hat{g} = 8R^2\mu(\mu - q) + 4l(l + 1)$. Here l stands for the orbital angular momentum of the particle. This relation between the two models has been recognized to be a manifestation of the AdS_2/CFT_1 correspondence.

Since in the aforementioned limit the angular variables effectively decouple and show up in an indirect way only in the coupling constant $\hat{g}$, it seems interesting to discuss a connection between the radial part of the model (1) and conformal mechanics (2) in more detail. According to a recent analysis [7], for a finite non-zero value of the radius R and $l = 0$ the systems are equivalent and correspond to two different non-linear realizations of the conformal group $SO(1,2)$. In particular, the actions (1) and (2) at $\theta = \varphi = \text{const}$ and $\hat{g} = 8R^2\mu(\mu - q) \equiv g$ are connected by a specific field redefinition involving coordinates along with their time derivatives.

It turns out that a similar conclusion can be reached in a simpler and suggestive way if one switches to the Hamiltonian framework. The former case is characterized by the Hamiltonian

$$H_{\text{AdS}} = (2R/r)^2\Big[\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\big(p_\theta^2 + \sin^{-2}\theta p_\varphi^2\big)} - q\Big], \tag{3}$$

and for our subsequent discussion in this section we will need only the radial part

$$H = (2R/r)^2\Big[\sqrt{\mu^2 + (r/2R)^2 p_r^2} - q\Big]. \tag{4}$$

Apart from time translations generated by this Hamiltonian one reveals two more conserved charges corresponding to dilatations and special conformal transformations

$$D = tH - \frac{1}{2} r p_r, \qquad K = t^2 H - t(r p_r) + \frac{1}{4} r^2 \Big(\sqrt{\mu^2 + (r/2R)^2 p_r^2} + q\Big). \tag{5}$$

Altogether these form a $so(1,2)$ algebra

$$\{H, D\} = H, \qquad \{H, K\} = 2D, \qquad \{D, K\} = K, \tag{6}$$

under the standard Poisson bracket $\{r, p_r\} = 1$, which is the conformal algebra in $d = 1$. In the conformal mechanics case (2) (with $\hat{g} = g$) a representation of the algebra reads [6]

$$H = \frac{1}{2}\left(p^2 + \frac{g}{x^2}\right), \qquad D = tH - \frac{1}{2} x p, \qquad K = t^2 H - t(xp) + \frac{1}{2} x^2. \tag{7}$$

Searching for a classical correspondence between the two models, we wonder if there exists a transformation from the phase space coordinates (x, p) to (r, p_r) which brings the Hamiltonian in Eq. (7) to the form (4). Furthermore, since the Hamiltonian makes part of the conformal algebra it seems reasonable to strengthen the condition and demand *all* the conformal generators to coincide. Comparing the charges corresponding to dilatations one immediately finds

$$xp = r p_r, \tag{8}$$

while requiring the identity of the charges generating special conformal transformations leads one to set

$$x = \frac{1}{\sqrt{2}} r \Big[\sqrt{\mu^2 + (r/2R)^2 p_r^2} + q\Big]^{1/2}, \qquad p = \sqrt{2}\, p_r \Big[\sqrt{\mu^2 + (r/2R)^2 p_r^2} + q\Big]^{-1/2}. \tag{9}$$

It is straightforward to verify that, being performed in the Hamiltonian (7), this substitution does produce Eq. (4), provided the identification $g = (2R)^2(\mu^2 - q^2)$. Notice that this correlates well with the coupling constant appearing in the aforementioned limit

$$g = (2R)^2\big(\mu^2 - q^2\big) \to 8R^2\mu(\mu - q), \tag{10}$$

if one suppresses the angular variables. Besides, the transformation (9) is *canonical* with the unit Jacobian.

We thus demonstrated that at the classical level the radial part of a charged massive particle moving near the horizon of an extreme Reissner–Nordström black hole is canonically equivalent to the old conformal mechanics. This equivalence is implicit in the Hamiltonian analysis of Ref. [9]. In the above, we established this connection in an explicit way. Moreover, the method by which we have reached this conclusion, i.e., the principle of identifying the symmetry generators, is new and allows one to treat more complicated cases (see next sections).

It is worth mentioning that according to the analysis of Ref. [9] (see also references therein) the system (2) in the Hamiltonian approach exhibits a larger symmetry than one could expect to find. In particular, it was shown that the $so(1,2)$ algebra formed by the conserved charges H, D, K can be extended to w_∞ algebra of area-preserving symplectic diffeomorphisms, the latter including the Virasoro algebra as a subalgebra. It was subsequently realized [10], however, that the charges are functionally dependent which matches with the fact that the system (2) involves only a finite number of degrees of freedom. Due to the existence of the equivalence transformation (9) the same symmetries should also persist in the model with the Hamiltonian (3). In what follows we shall concentrate only on finite-dimensional subalgebras.

3. Adding angular variables

Guided by the observation made in the preceding section it seems natural to inquire whether it is possible to extend the conformal mechanics (2) by angular variables so as to construct a model canonically equivalent to the particle moving on the $AdS_2 \times S^2$ background. A reasonably good starting point is offered by the Hamiltonian

$$H = \frac{1}{2}\left(p^2 + \frac{g}{x^2}\right) + \frac{2}{x^2}\left(p_\Theta^2 + \sin^{-2}\Theta p_\Phi^2\right), \tag{11}$$

which exhibits conformal symmetry (the generators of dilatations and special conformal transformations maintain their form (7) with H defined by Eq. (11)) along with the rotation $SO(3)$ invariance. The Hamiltonian (11) arises from (3) in the same limit $R \to \infty$, $(\mu - q) \to 0$, and $R^2(\mu - q)$ fixed, with the full angular part being taken into account. Now we are going to demonstrate that it produces (3) after performing a proper canonical transformation (with $g = (2R)^2(\mu^2 - q^2)$).

For the model at hand a representation of the $su(2)$ algebra is realized in the standard way ($\epsilon_{123} = 1$)

$$\mathcal{J}_1 = -p_\Phi \cot\Theta \cos\Phi - p_\Theta \sin\Phi, \qquad \mathcal{J}_2 = -p_\Phi \cot\Theta \sin\Phi + p_\Theta \cos\Phi,$$
$$\mathcal{J}_3 = p_\Phi, \qquad \{\mathcal{J}_i, \mathcal{J}_j\} = \epsilon_{ijk}\mathcal{J}_k, \tag{12}$$

and it is noteworthy that the angular part of the Hamiltonian is provided by the Casimir operator of the $su(2)$ algebra $\mathcal{J}^2 = \mathcal{J}_i\mathcal{J}_i = {p_\Theta}^2 + \sin^{-2}\Theta\, {p_\Phi}^2$.

Much alike the preceding case a transformation $(x, \Theta, \Phi, p, p_\Theta, p_\Phi) \to (r, \theta, \varphi, p_r, p_\theta, p_\varphi)$ which brings the test Hamiltonian (11) to that associated with the model (1) (see Eq. (3) above) is relatively easy to deduce for the radial variables by comparing the relevant expressions for the conformal generators

$$x = \frac{1}{\sqrt{2}} r\left[\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\left(p_\theta^2 + \sin^{-2}\theta p_\varphi^2\right)} + q\right]^{1/2},$$
$$p = \sqrt{2}\, p_r\left[\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\left(p_\theta^2 + \sin^{-2}\theta p_\varphi^2\right)} + q\right]^{-1/2}. \tag{13}$$

Besides, one has to make the identification $q^2 = \mu^2 - g/(2R)^2$ and require the Casimir operator to remain invariant ${p_\Theta}^2 + \sin^{-2}\Theta\, {p_\Phi}^2 = {p_\theta}^2 + \sin^{-2}\theta\, {p_\varphi}^2$. The latter requirement, however, does not fix the canonical transformations for the rest of the involved variables. Clearly, the reason lies in the additional $SO(3)$ symmetry characterizing the case under consideration. A sensible way out is to require *all* the symmetry generators in both pictures to coincide. In particular, we put $\mathcal{J}_i = J_i$, where the transformed generators J_i have the same form as in Eq. (12) but involve $(\theta, \varphi, p_\theta, p_\varphi)$ instead of $(\Theta, \Phi, p_\Theta, p_\Phi)$. Being algebraic equations, these allow one to express three variables

$$p_\Theta = J_2 \cos\Phi - J_1 \sin\Phi, \qquad \cot\Theta = -\frac{1}{J_3}(J_1 \cos\Phi + J_2 \sin\Phi), \qquad p_\Phi = p_\varphi, \tag{14}$$

in terms of Φ. Besides, we demand the change to be canonical. Let us discuss the latter point in more detail.

The dependence of Φ on the radial coordinates (r, p_r) is dictated by the requirement that it commutes with the pair (x, p) from Eq. (13). Given the transformation (13), the equality $xp = rp_r$ holds and one immediately faces the restriction

$$\{\Phi, rp_r\} = 0. \tag{15}$$

It means, in particular, that Φ is a function of (rp_r). Making use of Eq. (15) one can verify that only one of the two equations $\{\Phi, x\} = \{\Phi, p\} = 0$ is independent and amounts to

$$\frac{\partial \Phi}{\partial p_r} + \frac{r\{\mathbf{J}^2, \Phi\}}{(2R)^2\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\mathbf{J}^2}\,(\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\mathbf{J}^2} + q)} = 0, \tag{16}$$

where $\mathbf{J}^2 = J_i J_i$. Taking into account that the Casimir operator maintains its form, the explicit expression for $\{\mathbf{J}^2, \Phi\}$ can be easily computed. Then, introducing a specific subsidiary function

$$\alpha = A + \frac{\sqrt{\mathbf{J}^2}}{R\sqrt{\mu^2 + (1/R)^2\mathbf{J}^2 - q^2}}\left[\arctan\left(\frac{rp_r}{2R\sqrt{\mu^2 + (1/R)^2\mathbf{J}^2 - q^2}}\right) - \arctan\left(\frac{qrp_r}{2R\sqrt{\mu^2 + (1/R)^2\mathbf{J}^2 - q^2}}\frac{1}{\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\mathbf{J}^2}}\right)\right], \tag{17}$$

where A depends on the angular variables $(\theta, \varphi, p_\theta, p_\varphi)$ only, one can readily integrate the radial equation (16)

$$\tan\Phi = \frac{J_3\sqrt{\mathbf{J}^2}}{J_2^2 + J_3^2}\tan\alpha - \frac{J_1 J_2}{J_2^2 + J_3^2}. \tag{18}$$

Here we made use of Eq. (14) and assumed the conditions $\{\Phi, \Theta\} = \{\Phi, p_\Theta\} = 0$, $\{\Phi, p_\Phi\} = 1$ to hold. Obviously, the last three equations are designed to fix the explicit form of A which enters the subsidiary function. A straightforward calculation reveals the following restrictions

$$\{A, J_1\} = 0, \qquad \{A, J_3\} = \frac{J_3\sqrt{\mathbf{J}^2}}{J_2^2 + J_3^2}, \qquad \{A, J_2\} = \frac{J_2\sqrt{\mathbf{J}^2}}{J_2^2 + J_3^2}. \tag{19}$$

Beautifully enough, the following solution to the first equation:

$$A = \arctan\left(\frac{p_\varphi \sin^{-2}\theta \tan\varphi - p_\theta \cot\theta}{\sqrt{\mathbf{J}^2}}\right), \tag{20}$$

solves the others as well.

Having specified the explicit form of Φ, one has to verify yet that the whole change is canonical. It proves to be the case. In particular, the conjugate momentum p_Φ commutes with (x, p, Θ, p_Θ) while the pair (Θ, p_Θ) is canonical $\{\Theta, p_\Theta\} = 1$. Besides, as Φ has the vanishing bracket with the pair (x, p), so do (Θ, p_Θ).

To summarize, the canonical change of the variables exposed above in Eqs. (13), (14), (18) establishes the equivalence relation between the charged massive particle moving near the horizon of an extreme Reissner–Nordström black hole (see Eq. (3) above) and conformal mechanics (11). Although the transformation looks pretty bulky when applied to the angular variables, the quantities of physical interest like the angular momentum or the angular contribution to the Hamiltonian remain invariant and are easily handled. It is noteworthy that the equivalence holds for any fixed value of the black hole mass and is not bound to any specific limit.

4. A $N = 4$ superparticle on $AdS_2 \times S^2$ background

Among possible applications of the model (11) which we briefly outline in the concluding section there is one which can be addressed immediately. It has been known for a long time that conformal mechanics (2) admits supersymmetric generalizations [11–13]. It is interesting to find analogous superextensions of the particle on $AdS_2 \times S^2$. Since the full superisometry of the $AdS_2 \times S^2$ background is known to be $SU(1,1|2)$, the corresponding $N=4$ superconformal mechanics should possess this symmetry. In this context the $SU(2)$ symmetry underlying the bosonic case comes out as the R-symmetry contained in the superconformal group.

In order to construct a $N=4$ superconformal mechanics in AdS space one could either use the non-linear realizations [13,14], or properly fix the gauge with respect to κ-symmetry in the 0-brane Green–Schwarz action on $AdS_2 \times S^2$ [15] or, working in a more general geometric setting, analyse the conditions for a particle moving in an arbitrary curved background to admit a $N=4$ superconformal symmetry (see, e.g., Refs. [16–18]). Observe now that our consideration in the preceding section suggests quite new and interesting possibility to construct a $su(1,1|2)$-invariant superconformal mechanics in $AdS_2 \times S^2$ space by making use of the Hamiltonian approach. Indeed, it suffices to extend the simple model (11) by fermions in a manner which complements the $so(1,2) \oplus su(2)$-symmetry algebra of the bosonic case to the entire $su(1,1|2)$ and then apply to the resulting theory the canonical transformation found above with the fermions kept untouched.

The construction turns out to be mostly algebraic. One introduces a pair of complex fermions $(\psi^i)^* = \bar{\psi}_i$, $i=1,2$, obeying the bracket $\{\psi^i, \bar{\psi}_j\} = -i\delta^i{}_j$, and modifies the $su(2)$ generators (12) by adding the appropriate fermionic bilinears (without spoiling the algebra!)

$$\widetilde{\mathcal{J}}_1 = \mathcal{J}_1 + \frac{i}{2}\big(\psi^2\bar{\psi}_1 - \psi^1\bar{\psi}_2\big), \qquad \widetilde{\mathcal{J}}_2 = \mathcal{J}_2 - \frac{1}{2}\big(\psi^2\bar{\psi}_1 + \psi^1\bar{\psi}_2\big), \qquad \widetilde{\mathcal{J}}_3 = \mathcal{J}_3 + \frac{1}{2}\big(\psi^1\bar{\psi}_1 - \psi^2\bar{\psi}_2\big). \tag{21}$$

Requiring them to obey proper Poisson brackets with the Poincaré supersymmetry generators $G^i, \overline{G}_i$, one severely restricts the form of the latter. Observing further that the bracket $\{G^i, \overline{G}_j\} = -2iH\delta^i{}_j$, $i=1,2$, makes part of the $su(1,1|2)$ superalgebra, it suffices to find fermionic generators G^i and $\overline{G}_i$ whose Poisson bracket yields a Hamiltonian which reduces to Eq. (11) in the bosonic limit. Besides, one has to make sure that the conditions $\{G^i, G^j\} = \{\overline{G}_i, \overline{G}_j\} = 0$ hold which, by Jacobi identities, provide the conservation of the supercharges. It should be also mentioned that, in order to guarantee the stability of the vacuum (see the discussion in Refs. [3,14]) one is forced to set $\mu = q$. We thus put $g=0$ in our subsequent consideration.

It turns out that all these restrictions are met by the following representation for the supersymmetry charges

$$\begin{aligned} G^1 &= \left(p - \frac{2i}{x}\mathcal{J}_3\right)\psi^1 + \frac{2}{x}(\mathcal{J}_1 + i\mathcal{J}_2)\psi^2 + \frac{i}{x}\psi^1\psi^2\bar{\psi}_2, \\ G^2 &= -\left(p + \frac{2i}{x}\mathcal{J}_3\right)\psi^2 + \frac{2}{x}(\mathcal{J}_1 - i\mathcal{J}_2)\psi^1 - \frac{i}{x}\psi^1\bar{\psi}_1\bar{\psi}^2, \end{aligned} \tag{22}$$

which yield the Hamiltonian

$$\begin{aligned} H = {} & \frac{1}{2}\left[p^2 + \frac{4}{x^2}\big(p_\Theta^2 + \sin^{-2}\Theta p_\Phi^2\big)\right] + \frac{2i}{x^2}(\mathcal{J}_1 - i\mathcal{J}_2)\psi^1\bar{\psi}_2 - \frac{2i}{x^2}(\mathcal{J}_1 + i\mathcal{J}_2)\psi^2\bar{\psi}_1 \\ & - \frac{2}{x^2}\mathcal{J}_3\big(\psi^1\bar{\psi}_1 - \psi^2\bar{\psi}_2\big) + \frac{1}{x^2}\psi^1\bar{\psi}_1\psi^2\bar{\psi}_2. \end{aligned} \tag{23}$$

Given the Hamiltonian, one can readily verify that the generators of dilatations and special conformal transformations maintain their previous form (7) (with the Hamiltonian taken from the previous line). Finally, evaluating the Poisson brackets of the supersymmetry charges with the generators of special conformal transformations one finds a representation for the superconformal generators

$$S^1 = tG^1 - x\psi^1, \qquad S^2 = tG^2 + x\psi^2. \tag{24}$$

Having formulated the model in the conformal basis, we now proceed to construct its $AdS_2 \times S^2$ equivalent. To this end we apply the transformation (13) (with $\mu = q$) to the Hamiltonian (23) which yields

$$\begin{aligned} H_{N=4} = {} & (2R/r)^2\left[\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2 \mathbf{J}^2} - \mu\right] \\ & + \left[(J_1 - iJ_2)\psi^1\bar{\psi}_2 - (J_1 + iJ_2)\psi^2\bar{\psi}_1 + iJ_3\left(\psi^1\bar{\psi}_1 - \psi^2\bar{\psi}_2\right) - \frac{i}{2}\psi^1\bar{\psi}_1\psi^2\bar{\psi}_2\right] \\ & \times \frac{4i}{r^2\left(\sqrt{\mu^2 + (r/2R)^2 p_r^2 + (1/R)^2\mathbf{J}^2} + \mu\right)}. \end{aligned} \tag{25}$$

Because the bosonic limit of this theory does coincide with the Hamiltonian (3) one ends up with a $SU(1,1|2)$ supersymmetric generalization of the model (1). It is interesting to compare the result with the $SU(1,1|2)$ superparticle in the Green–Schwarz approach [15]. This requires the construction of a Lagrangian formulation which will be given elsewhere.

5. Conclusion

To summarize, in the present Letter we took advantage of the Hamiltonian formalism, in order to establish a precise classical correspondence between a massive charged particle moving near the horizon of an extreme Reissner–Nordström black hole and conformal mechanics (11). Since our construction does not rely upon a specific limit, it becomes possible to investigate in full generality quantum properties of the former model (at any finite value of the black hole mass) working with the latter theory. It is then tempting to study the quantum spectrum and the transition amplitude for the theory (11). Although we have a little hope to literally transform into the *AdS* basis the results of the operator quantization because of the complexity of the transformations (13), (14), (18), the path integral quantization is still quite feasible.

In constructing a $N = 4$ supersymmetric generalization of the model (11) we assumed the stability of the vacuum and set $g = 0$. The case $g \neq 0$ can also be considered. It also remains to explore how the equivalence in the Hamiltonian approach is translated into the Lagrangian language and how it is linked to the off-shell map of Refs. [7,8].

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