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Mechanisms of perceptual attention in precuing of location

Barbara Anne Dosher [a,*], Zhong-Lin Lu [b]

[a] *Department of Cognitive Sciences, Institute of Mathematical Behavioral Science, 3151 SSP, University of California, Irvine, CA 92697-5100, USA*

[b] *Department of Psychology, SGM 501, University of Southern California, Los Angeles, CA 90089-1061, USA*

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Abstract

What are the mechanisms of spatial attention underlying precue validity effects? We answer this question within the framework of a perceptual template model (PTM) [Lu & Dosher (1998). External noise distinguishes attention mechanisms. *Vision Research*, *38*, 1183–1198; Dosher & Lu (1999). Mechanisms of perceptual learning. *Vision Research*, *39*, 3197–3221] and an external noise plus attention paradigm for orientation judgments in two- to eight-location displays. Attentional mechanisms correspond to behavioral signatures: External noise exclusion produces cuing effects in high external noise and stimulus enhancement produces cuing effects in noiseless displays. We found that external noise exclusion was the primary mechanism of cue validity effects, with large effects in high-noise displays. Stimulus enhancement coexisted as a secondary mechanism in noiseless displays for a subset of observers and display conditions. Contrast threshold ratio tests ruled out attentionally mediated changes in gain control. The ratio rules were also shown to hold for a stochastic PTM model. Effects were equivalent for four-alternative (Experiment 1) and two-alternative (Experiment 2) orientation identification. Precues allow observers to reduce noise and focus on the target in the precued location. External noise exclusion was more important in larger displays. Previous results are reclassified and understood within the PTM framework.

Keywords: Spatial attention; Stimulus enhancement; External noise exclusion; Internal noise suppression; Additive internal noise; Multiplicative internal noise; Equivalent internal noise; Perceptual template model; Cuing paradigm

1. Mechanisms of perceptual attention

1.1. Attention and perceptual clarity

In Wundt's (1924 [1912]) classical demonstration of attention, observers fixated the center of a letter array while attending to a letter in the periphery. Wundt observed that the attended letter and those in the immediate area appeared perceptually clear while other letters 'retreated into the darker field of consciousness'. Wundt's claim that attention results in improved perceptual clarity — if improved perceptual clarity is associated with improvements in discriminability or sensitivity — has proven complicated to document. In relatively sparse and clear displays, several cases of improved discriminability have now been reported (Bashinski & Bacharach, 1980; Lu & Dosher, 1998a; Lu, Liu & Dosher, 2000; but see e.g. Shiffrin, Mckay & Shaffer, 1976), but the range of circumstances reliably producing those effects on discriminability still require investigation. In more cluttered multi-element displays, especially in masked displays, a range of effects on accuracy have been documented (Henderson, 1991; Shiu & Pashler, 1994; Cheal & Gregory, 1997), but the mechanisms of attention which may underlie these effects are actively debated.

In this paper, we consider the roles of *external noise exclusion* and *stimulus enhancement* as mechanisms of attention in multi-element displays. Specifically, we consider the impact of multi-element load and of decision complexity on attentional effects. These factors may be functionally important in determining the nature and size of attention effects. The mechanisms of attention are documented within the framework of an external noise plus attention paradigm and a formal

* Corresponding author. Tel.: +1-949-8247373; fax: +1-949-8242517.
E-mail addresses: bdosher@uci.edu (B.A. Dosher), zhonglin@rcf.usc.edu (Z.-L. Lu)

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model of the observer, the *perceptual template model* (PTM) (Dosher & Lu, 1997; Lu & Dosher, 1998a). This model is reviewed, and a stochastic form of the model is also developed and shown to be generally consistent with the original formulations. First, however, we briefly introduce the issue of attention in multi-location paradigms.

1.2. Spatial cuing in multi-location paradigms

Spatial precues have been consistently shown to affect accuracy in masked multilocation detection and discrimination tasks. Observers are more accurate in identifying a target in a precued location when all possible target locations are masked with a delayed high-contrast pattern mask (Henderson, 1991; Shiu & Pashler, 1994; Cheal & Gregory, 1997). In a typical example (Henderson, 1991), a target stimulus, an X or O, could appear briefly in one of eight possible locations at approximately 9° eccentricity, followed by a masking pattern in all locations. Observers judge whether the target was an X or O. A precue marking an attended location improved 2AFC accuracy by approximately 10% for eight-location displays.

A number of possible mechanisms of attention have been posited to underlie spatial precuing effects. First, spatial uncertainty effects may underlie decrements in performance in some uncued versus cued multi-location experiments. Spatial uncertainty decision effects, in which samples from multiple non-target locations increase the possibility of false alarms, have been shown to account for performance in a range of unmasked search tasks (Sperling, 1984; Shaw, 1984; Sperling & Dosher, 1986; Palmer, Ames & Lindsey, 1993; Shiu & Pashler, 1994). Second, original attention theories identified spatial precuing with facilitation in perceptual processing (Posner, Nissen & Ogden, 1978; Cheal, Lyon & Gottlob, 1994), a claim supported by some investigations of neural responses to precued stimuli (Corbetta, Miezin, Shulman & Petersen, 1993; Mangun, Hillyard & Luck, 1993). However, the claim that attention to a location facilitates perceptual processing in that location predicts that spatial precues should affect performance even in the absence of masks in multi-location paradigms, a claim that has been debated (Lyon, 1990; Cheal et al., 1994; Henderson, 1996). Third, several investigators have proposed that spatially cued attention is mediated by redistribution of limited capacity processes (Henderson, 1996); one extreme example is sample size sharing (Shaw, 1980; Palmer et al., 1993). Finally, some investigators have proposed that spatially cued attention may allow the suppression of the mask in cued locations (Enns & Di Lollo, 1997).

The proposed mechanisms are neither exhaustive nor mutually exclusive. With the exception of uncertainty and sample size sharing models (Shaw, 1980, 1984; Palmer et al., 1993), the proposed mechanisms are described verbally. The external noise plus attention paradigm provides an empirical and theoretical structure for identifying three attentional mechanisms: stimulus enhancement, external noise exclusion, and internal (multiplicative) noise reduction. These three mechanisms map partially, but not directly, onto the verbally defined mechanisms of the prior literature.

This paper reports several experiments which ask the question: what are the mechanisms of attention associated with spatial precuing in multi-location displays? Previous studies of cued attention, including several using the external noise paradigm, have identified two attentional mechanisms. The current study uses the framework of the external noise paradigm to investigate the conditions associated with particular attentional mechanisms through the manipulation of display size and task difficulty. Although display size effects have been exhaustively studied in the context of visual search tasks (e.g. Palmer et al., 1993; Morgan, Ward & Castet, 1998), the current experiments investigate the impact of display size in a task in which the observer reports the contents of a single specified location. The importance of external noise exclusion in large display sizes provides an explanation for contrasting outcomes of previous studies. First, however, the external noise plus attention paradigm must be introduced.

2. External noise plus attention paradigm

The external noise plus attention paradigm (Dosher & Lu, 1997; Lu & Dosher, 1998a) embeds a perceptual target stimulus in systematically varying amounts of external noise (see Fig. 1a).

The external noise in the current studies is random Gaussian pixel noise, in which each pixel takes on a randomly sampled value from a Gaussian distribution with mean zero and some variance (contrast power) N^2_{ext}. A target stimulus, an oriented Gabor patch in Fig. 1a, is embedded in the external noise. The dependent measure is the target or 'signal' contrast necessary to support a criterion level of discrimination or identification performance. For example, observers might be asked to decide whether an oriented Gabor is tilted top to the right or left.

The contrast threshold measures are strongly dependent upon external noise in the stimulus (*stimulus noise*) (Fig. 1b). In the region of high external noise, contrast threshold is directly related to external noise level — external noise (along with associated multiplicative noise) is the limiting factor in performance. In the low external noise region, external noise is no longer a controlling factor in performance. Instead, contrast threshold reflects internal processing limits or processing inefficiencies, quantified as equivalent internal

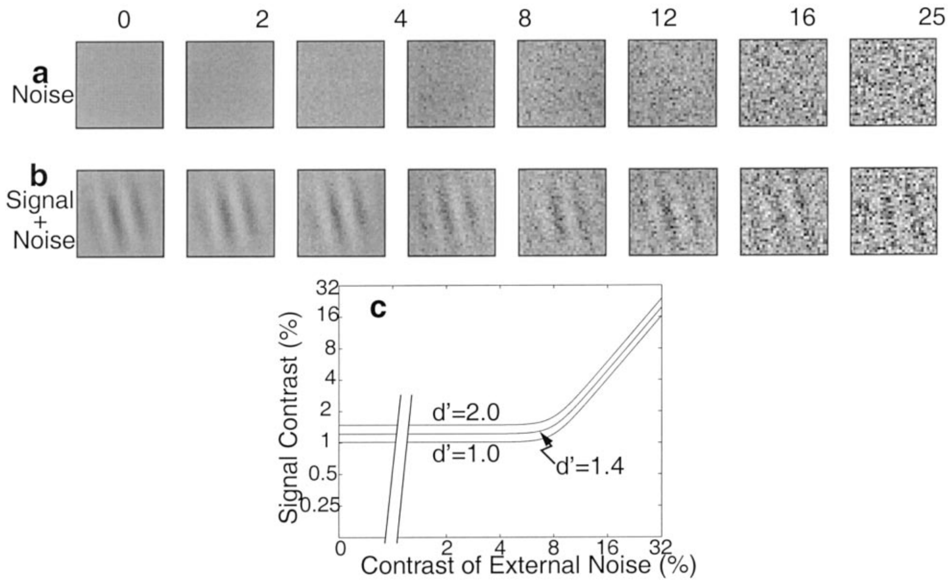

Fig. 1. The external noise paradigm embeds target stimuli in increasing amounts of external noise and measures contrast thresholds. (a) Samples of random pixel noise with increasing variance (contrast power) for noise only and signal plus noise displays. The signal is an oriented Gabor. (b) Contrast thresholds as a function of external noise (TVC functions) for three d' criteria. Thresholds are controlled by external noise in the high noise region, and by internal limitations in the low noise region.

noise. These characteristic functions are sometimes called threshold versus contrast (TVC) functions.

In the external noise plus attention paradigm, the experimenter manipulates the stimulus by addition of external noise and the observer's attentional state by instruction, dual task load, probability manipulations, or, in this case, by precuing. Comparisons of the threshold versus contrast functions under different attentional states are the basis of distinguishing attentional mechanisms. For example, an attentional mechanism of external noise exclusion should be especially important when there are high levels of external noise in the stimulus. These intuitive identifications of attentional mechanism are more clearly detailed by a consideration of a formal model of the observer, the perceptual template model (PTM).

3. Noisy perceptual template model

3.1. Components of the model

The perceptual template model considers the performance of the observer in terms of the fundamental properties of signal and limiting noise. The PTM model has been formally developed in several prior papers (Lu & Dosher, 1998a, 1999a; Dosher & Lu, 1999). There are five components in the observer model (Fig. 2): (1) A perceptual template appropriate for the experimental task; (2) optional transduction nonlinearities; (3) an independent multiplicative internal noise source; (4) an independent additive internal noise source; and (5) a task appropriate decision structure. Components (2) and (3) jointly accommodate gain control properties that are sometimes described instead in terms of divisive inhibition (e.g. Legge & Foley, 1980; Carlson & Klopfenstein, 1985; Sperling, 1989).[1]

The PTM model is an extension and development of a well-studied observer model, the linear amplifier model (LAM) (Burgess, Wagner, Jennings & Barlow, 1981; Pelli, 1981). The LAM is a special case of the PTM without nonlinearity (2) or multiplicative noise

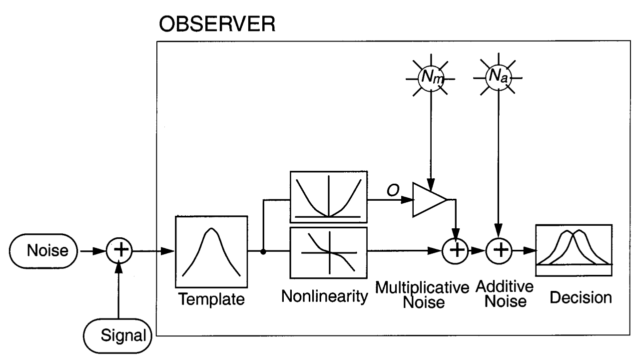

Fig. 2. The perceptual template model (PTM) of the observer consists of five components: a perceptual template tuned to a signal stimulus; optional transduction nonlinearities; multiplicative internal noise; additive internal noise; and a decision process. The PTM generated the sample TVC functions of Fig. 1.

[1] Variants of the PTM model may place additive internal noise before the filter, after the filter but before nonlinearity and multiplicative noise, or after nonlinearity and multiplicative noise. Variants with 'early' additive noise may be rewritten in terms of the 'late' noise variant shown here; see Dosher and Lu (1999) for a discussion.

(3); or, to say it differently, the LAM model provided the framework for the expanded PTM model. The simpler LAM generally is rejected in favor of the PTM model (Lu & Dosher, 1999a). Without nonlinearity (and possibly multiplicative noise), the simpler LAM observer models cannot account simultaneously for data from more than a single criterion level on the psychometric function (see description of ratio tests in Section 6 for further details).

3.2. Limiting signal and noise relationships

The basic signal and noise relationship in the PTM model is:

$$d' = \frac{(\beta c)^{\gamma}}{\sqrt{N_{\mathrm{ext}}^{2\gamma} + N_{\mathrm{m}}^{2}(\beta^{2\gamma}c^{2\gamma} + N_{\mathrm{ext}}^{2\gamma}) + N_{\mathrm{a}}^{2}}}. \quad (1)$$

The value d' measures bias-free accuracy; the parameter c is the contrast of the signal or target; β is the gain on a signal-valued stimulus (and is related to the cross product of the signal with the filter); N_{ext} reflects the contrast of the external noise; N_{m} estimates equivalent internal multiplicative noise; N_{a} estimates equivalent internal additive noise; γ reflects nonlinear transduction in the signal path and in the multiplicative noise path.

The PTM model provides an account of the threshold versus contrast functions. The log form of the TVC (Fig. 1b) is derived by choosing a threshold d' and solving for a threshold level of contrast, c_{τ}:[2]

$$\log(c_{\tau}) = \frac{1}{2\gamma}\log((1 + N_{\mathrm{m}}^{2})N_{\mathrm{ext}}^{2\gamma} + N_{\mathrm{a}}^{2}) - \frac{1}{2\gamma}\log(1/d'^{2} - N_{\mathrm{m}}^{2}) - \log(\beta). \quad (2)$$

The PTM model with $\gamma = 1$ and $N_{\mathrm{m}} = 0$ reduces to the LAM: $d' = \beta c/\sqrt{N_{\mathrm{ext}}^{2} + N_{\mathrm{a}}^{2}}$. In the LAM, β traditionally measures 'efficiency', or the match between the signal and the template. In the PTM form, β is related to the concept of efficiency, but is a generalization to situations with nonlinear transduction.[3]

The LAM form is consistent with the Gaussian distribution assumptions of signal detection theory. So long as the extreme tails are avoided, the Gaussian assumption provides a good approximation to the output of the PTM model even with nonlinearity y in observed ranges (1–3). For example, when evaluated by simulation for typical PTM parameter values, the match of the q–q plot of the distribution of outputs from the PTM model and the Gaussian yielded an r^2 of approximately 0.999 for percentiles from 0.05–0.95.

The PTM model described in this d' relation is an analytical approximation of a stochastic PTM considered in Appendix A. This analytical PTM model is a simplification in two ways: random variables are replaced by their expectations, and cross products are eliminated. For example, the form of multiplicative noise, $N_{\mathrm{m}}^{2}(\beta^{2\gamma}c^{2\gamma} + N_{\mathrm{ext}}^{2\gamma})$ is simplified in cross-product terms.[4] In a new model development, the stochastic form of the PTM model is considered in relation to key signatures and ratio rules described in the next sections. As shown in Appendix A, the two forms are fully consistent.

[2] In several previous cases (Lu & Dosher, 1998a; Dosher & Lu, 1999), two separate nonlinearity factors were considered, γ_1 for nonlinear transduction in the signal path and γ_2 for nonlinear transduction in the multiplicative noise path. In Dosher and Lu (1999), the two-γ models were explicitly tested and rejected in favor of the more parsimonious one-γ models. Derivations here and in Appendix A can easily be generalized to the γ_1–γ_2 form.

[3] In the analytic PTM, β also compensates for missing cross-product terms.

4. Three mechanisms of attention

4.1. Behavioral signatures of attention mechanisms

Three attentional mechanisms are associated with characteristic behavioral signatures in the external noise plus attention paradigm. The behavioral signatures in the analytic PTM and the stochastic PTM have equivalent properties (see Appendix A). Each of the three attentional mechanisms reflects a distinct change in the PTM model (Fig. 3). In *stimulus enhancement*, attention increases the gain on the stimulus, which is formally equivalent to reducing internal additive noise. This can improve performance only in low external noise stimuli, since external noise is the limiting factor in high external noise stimuli (Fig. 3a–b). In *external noise exclusion*, attention operates by changing the filter or template so as to reduce external noise. This improves performance for high external noise stimuli, where there is external noise to exclude (Fig. 3c–d). In *internal (multiplicative) noise reduction*, attention operates by reducing multiplicative noise (Fig. 3e–f). This affects performance in all external noise conditions, with a slightly larger effect in high external noise. Stimulus enhancement is most similar to the verbal idea of sensory perceptual facilitation and external noise exclusion is related to ideas of noise filtering and distractor exclusion.

[4] Explicitly comparisons of model fits with the simplified form of multiplicative noise and a form of multiplicative noise including cross products (solved by an iterative computational method) found no significant difference in the quality of fit to the data (Dosher & Lu, 1999).

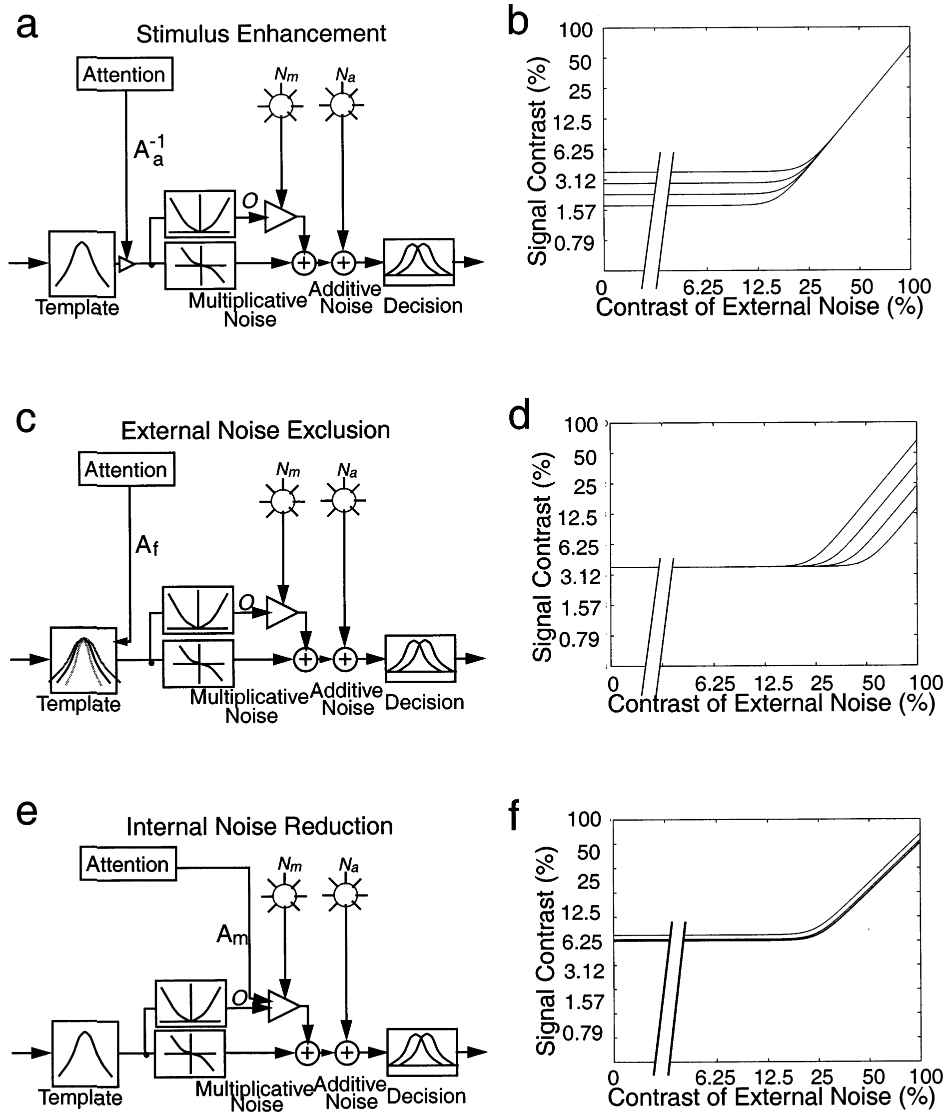

Fig. 3. Three mechanisms of attention in the perceptual template model and their behavioral signatures. (a–b) Stimulus enhancement produces attentional effects in the low external noise region of the threshold curves; this is formally equivalent to reducing internal additive noise. (c–d) External noise exclusion (tuning the perceptual template) produces attentional effects in the high noise region of the threshold curves. (e–f) Internal multiplicative noise reduction produces attentional effects in both low and high external noise.

4.2. Attentional mechanisms in the PTM model

Attentional effects are implemented within the PTM model as a reduction of external noise by a proportional factor A_f, reduction of additive internal noise by a proportional factor A_a (equivalent to increasing the stimulus gain by a factor A_a^{-1}), or reduction of multiplicative internal noise by a proportional factor A_m.[5] Incorporating all three mechanisms of attentional improvement in the analytic PTM model, the threshold versus contrast equation, in log form is:

$$\log(c_\tau) = \frac{1}{2\gamma}\log((1 + A_m^2 N_m^2)\, A_f^{2\gamma} N_{ext}^{2\gamma} + A_a^2 N_a^2) - \frac{1}{2\gamma}\log(1/d'^2 - A_m^2 N_m^2) - \log(\beta). \quad (3)$$

5. Empirical demonstrations of attentional signatures

5.1. Stimulus enhancement

Two examples of pure stimulus enhancement have been reported in an external noise paradigm. Empirical demonstrations of pure mechanisms are important in

[5] Changes in response to a signal valued stimulus, such as those due to an attentional shift in the filter (Yeshurun & Carrasco, 1998) relative to the signal might require changes in parameter β as well, see Dosher and Lu (1999) for a discussion.

validating both the general approach and the formal structure as a model of attentional mechanisms. In the first example, observers were asked to perform an orientation discrimination task on relatively small Gabor patches appearing in the periphery to the right or left of fixation (Dosher & Lu, 1997; Lu & Dosher, 1998a). A combined central and peripheral precue indicated that the stimulus either on the right or left should receive more attention, or a neutral cue indicated equal attention to both stimuli. The attended and unattended locations differed in contrast at threshold in the zero and low noise conditions only, with the neutral condition intermediate between attended and unattended location performance. Attentional state had no effect on performance in high external noise conditions in this display with two stimulus locations. These data clearly demonstrate the pattern for stimulus enhancement, or equivalently, additive internal noise reduction. Another example of an attention effect restricted to zero or low external noise conditions was documented for widely separated patches of second order motion, but not for patches of first order motion (Lu et al., 2000). The conditions leading to stimulus enhancement in these two cases may not be the same (see Section 9). Even so, these two cases provide empirical existence demonstrations of the signature patterns of pure stimulus enhancement (internal additive noise reduction).

5.2. External noise exclusion

A very different performance signature has been demonstrated in two immediately relevant cases of attentional precuing advantages in masked multiple location discrimination. In experiments closely modeled on experiments by Lyon (1990) and Cheal and Lyon (1991a,b), potential target items (upright or rotated Ts) appeared briefly in four locations arranged on the corners of a square array (Lu & Dosher, 1998b). A single report location was designated either by a cue appearing shortly before (precue) or simultaneous with (simultaneous cue) the stimulus array. This is not a search task, but rather a cued report task. Observers identified the orientation (up, down, right, left) of the T in the report location. In this external noise experiment, the amount of external stimulus noise was parametrically varied from zero to high noise, although in previous experiments (e.g. Lyon, 1990), target items were always followed by high contrast pattern masks. For central precuing, the substantial precuing advantage occurred in high external noise only. The pattern corresponds exactly with the signature for external noise exclusion — an effect of attention in high external noise. The second example, described in Section 8.2, involves the comparison of validly and invalidly cued locations on contrast thresholds in a discrimination task with central precues. It is a direct precursor to the experiments reported here. These two cases provide empirical existence demonstrations of the signature pattern of pure external noise exclusion.

5.3. Multiplicative internal noise reduction and gain control

Changes in multiplicative internal noise have *not* been found in previous applications of the external noise paradigm to attentional tasks. Multiplicative internal noise and nonlinear transduction together account for properties of nonlinear contrast gain control. Data from the current experiments are consistent with these observations. We speculate that attention rarely modulates or alters system properties of gain control captured by multiplicative noise and nonlinear transduction factors.[6] Specific tests for changes in multiplicative noise and/or transducer nonlinearities based on multiple criterion levels are described in Section 6.

5.4. Mechanisms and associated conditions

The experiments described in Sections 5.1 and 5.2 demonstrate stimulus enhancement without external noise exclusion in one case of spatial precuing and external noise exclusion without stimulus enhancement in another case of spatial precuing. In particular, external noise exclusion has been demonstrated as a mechanism of spatially precued attention in a four locations and a four-stimulus orientation identification task (Dosher & Lu, 2000), but not in a two-location two-stimulus orientation identification task (Lu & Dosher, 1998a). The current experiments directly address the preconditions leading to external noise exclusion as a component of the attentional response. Section 8 considers the mechanisms of attentional precuing in multi-location paradigms.

6. Discriminating mechanism mixtures

6.1. Multiple criterion tests for mechanism mixtures

The behavioral existence of pure signatures for mechanisms of attention identified by the PTM model provides a strong empirical basis for the model beyond the mathematical predictions of distinct attentional mechanisms. However, the differences in attentional state are unlikely to correspond with a pure signature (one mechanism) in all tasks. More than one mechanism may play a role in any particular task, which raises the

[6] In contrast, Lee, Itti, Koch and Braun (1999) have argued that peripheral cuing may alter nonlinear gain control, although changes in spatial uncertainty may provide an alternative interpretation of their results, see also footnote 5.

issue of discriminating task mixtures. In particular, multiplicative noise reduction must be discriminated from a mixture of stimulus enhancement and external noise exclusion. The key to discriminating mixtures resides in measurement of contrast thresholds at several different criterion levels. Multiple criterion levels provide additional information critical to the estimation of multiplicative noise and nonlinear transduction.

The importance of considering multiple criteria (d') for discriminating mixtures is easily seen by examining Eq. (3) (log form). The only term which depends upon d' is $1/2\gamma\ \log(1/d'^2 - A_m^2 N_m^2)$. The choice of criterion level (d') interacts (in the log) only with multiplicative noise (A_m) and nonlinearity (γ). This makes a very strong set of predictions: stimulus enhancement (internal additive noise reduction) and external noise exclusion effects will have an effect size which is independent (in the log) of the choice of criterion level. In contrast, a multiplicative internal noise reduction effect will interact (in the log) with choice of criterion level. These effects are illustrated in Fig. 4 and further considered in Sections 6.2 and 6.3. Related tests were critical in identifying mechanism mixtures in perceptual learning (Dosher & Lu, 1998, 1999).

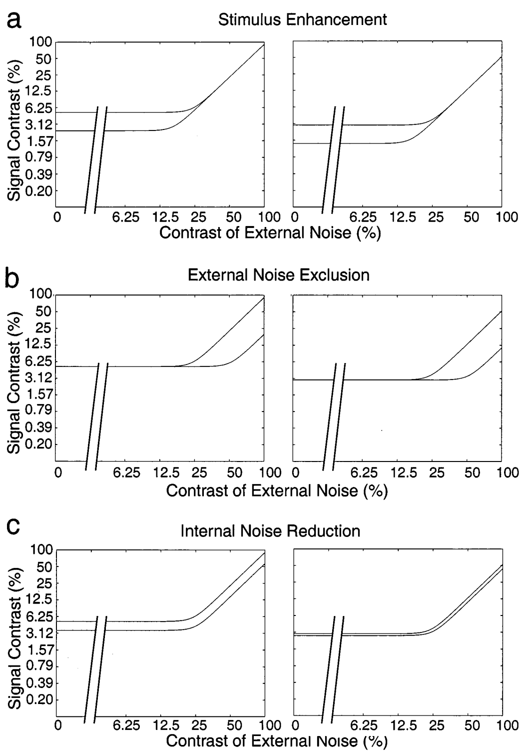

Fig. 4. Contrast thresholds at several spaced criteria are necessary to distinguish a mixture of stimulus enhancement and external noise exclusion from changes in multiplicative internal noise. Left panels depict higher criteria and hence higher thresholds, while right panels depict lower criteria and thresholds. Stimulus enhancement and external noise exclusion produce a shift relationship on log axes, corresponding to a unchanging contrast threshold ratios.

6.2. Ratio tests for nonlinearity parameters

Testing for effect size constancy in the log is equivalent to testing for equality for different conditions of the ratios of contrast thresholds at different criteria. Beginning with Eq. (1) of the PTM model and solving for c_τ for a d' criterion (in linear form):

$$c_\tau = \frac{1}{\beta}\left[\frac{(1+N_m^2)\,N_{ext}^{2\gamma} + N_a^2}{1/d' - N_m^2}\right]^{1/2\gamma}.$$

Taking the ratio of two criterion thresholds at the same level of external noise,

$$\frac{c_1}{c_2} = \left[\frac{1/d_2'^2 - N_m^2}{1/d_1'^2 - N_m^2}\right]^{1/2\gamma}.$$

This ratio, which does not depend on N_{ext}, should be identical for all external noise conditions. Taking two such pairwise ratios from three criterion levels d'_1, d'_2, and d'_3 yields two ratio equations in two unknowns, nonlinearity γ and multiplicative internal noise N_m. To estimate γ and N_m from a single condition, three reasonably widely spaced criterion levels will constrain model estimates. If nonlinearity or multiplicative noise differ between two conditions, then these ratios will differ for the two conditions.

As described earlier, the PTM model was designed to reduce to the linear amplifier model (LAM) when there is no nonlinear transduction ($\gamma = 1$) or multiplicative internal noise ($N_m = 0$). For the LAM, the ratio equation reduces to the prediction that $(c_1/c_2) = d'_1/d'_2$, a strong prediction of the LAM which generally fails. On the other hand, the more general PTM form is generally successful.[7] For further discussion of these issues, see Lu and Dosher (1999a) and Dosher and Lu (1999).

6.3. Ratio tests for attention mechanisms

Considering performance at several criterion levels is essential for providing strong constraints which will unambiguously reveal mechanisms of attention. For example, for two attention conditions, comparison of the attention effect at two criterion levels will identify whether attention altered multiplicative internal noise (or nonlinearity) from a mixture of stimulus enhancement and external noise exclusion due to attention. The basic contrast threshold equation including attentional factors A_a, A_f, and A_m is:

$$c_\tau = \frac{1}{\beta}\left[\frac{(1+A_m^2 N_m^2)\,A_f^{2\gamma} N_{ext}^{2\gamma} + A_a^2 N_a^2}{1/d'^2 - A_m^2 N_m^2}\right]^{1/2\gamma}.$$

[7] Nonlinearity and multiplicative noise in the PTM model is interpreted in certain other models, such as that of Eckstein, Ahumada and Watson (1997) as the result of uncertainty coupled with an LAM form. Indeed, changes in uncertainty properties between two conditions are likely to be attributed to condition-dependent nonlinearity or multiplicative noise.

Suppose that another attention condition is characterized by the attentional factors A'_a, A'_f, and A'_m. Then the ratio of contrast thresholds for the two attention conditions for each d' takes the form:

$$\frac{c'_\tau}{c_\tau}=\left[\frac{(1+A'^2_m N^2_m)\, A'^{2\gamma}_f N^{2\gamma}_{ext}+A'^2_m N^2_a}{(1+A^2_m N^2_m)\, A^{2\gamma}_f N^{2\gamma}_{ext}+A^2_m N^2_a}\right]^{1/2\gamma} \times\left[\frac{1/d'^2-A^2_m N^2_m}{1/d'^2-A'^2_m N^2_m}\right]^{1/2\gamma}.$$

Only the rightmost term of this equation depends on d'. If $A_m = A'_m$ — two attention conditions do not differ in respect to multiplicative internal noise — and, additionally, γ is unchanged, then this factor cancels and the remaining terms are independent of d'. Hence, the ratio of contrast thresholds for two attention conditions not differing in internal multiplicative noise — but possibly differing in either stimulus enhancement, or external noise exclusion or both — will not depend on the criterion d' (Dosher & Lu, 1999). Of course, this ratio in general will be different for different external noise conditions: for stimulus enhancement, the ratio approaches one in high external noise but differs from one in low external noise, and for external noise exclusion, the ratio approaches one in low external noise but differs from one in high external noise. In the log form, only in the case of altered multiplicative noise does the difference between log contrast threshold in two attention conditions depend upon criterion d', as illustrated in Fig. 4. The two forms of ratio test (Sections 6.2 and 6.3) both provide a test for changes in multiplicative noise or nonlinearity between conditions. The nonlinearity ratio tests (Section 6.2) are stronger than the attention ratio tests (Section 6.3); if the nonlinearity tests indicate unchanging multiplicative noise and nonlinearity, it will rule out changes in multiplicative noise. The same relationships are shown to hold asymptotically for the stochastic PTM in Appendix A. Derivations for the analytic PTM model directly follow those of Appendix A.

7. The endpoint method

This paper reports two experiments using an abbreviated 'endpoint' protocol to evaluate mechanisms of attention. Previous applications of the external noise paradigm and PTM model have measured full TVC functions (performance for six to eight external noise conditions). This allows direct comparison to the behavioral signatures, and further supports estimation of model parameters. However, the measurement of full TVC functions requires extensive data collection, often 5000 experimental trials per attention condition, which may be prohibitive for within observer comparisons of several attention conditions.

In the current experiments, only the two endpoints of the TVC function — zero external noise and high external noise — are measured in order to allow a within observer comparison of several display conditions. For pure external noise exclusion, attention improves performance only in the presence of high external noise in the stimulus, corresponding to an attentional effect only in the high external noise condition but not in the zero noise condition of the endpoint protocol. For pure stimulus enhancement, attention improves performance only in low external noise conditions, corresponding to an attentional effect only in the zero external noise condition but not the high external noise condition of the endpoint protocol.

When attention improves performance in both zero and high external noise conditions, then additional analysis is necessary. A mixture of stimulus enhancement and external noise exclusion is discriminated from changes in multiplicative internal noise by consideration of performance at several criterion levels. In the endpoint method, the key constraints on multiplicative internal noise from thresholds at different criterion levels may be evaluated by the contrast threshold ratio tests described in Sections 6.2 and 6.3. When full TVC functions are measured, this occurs automatically during fitting of the PTM model so long as measurements are included for two or more criterion levels. The endpoint method allows a qualitative analysis of a larger number of conditions.

8. The effects of attentional precuing

8.1. Comparing valid and invalid cuing

The experiments in this paper investigate the attentional mechanisms underlying central precuing effects in multi-location displays. The goal is to understand the conditions under which attention improves performance by exclusion of external noise. The focus is on comparisons of valid and invalid location cues. The experiments are most similar to studies of the effect of precuing on the accuracy of target detection or target discrimination in multi-location displays (Grindley & Townsend, 1968; Shaw, 1984; Downing, 1988; Lyon, 1990; Henderson, 1991; Cheal et al., 1994; Shiu & Pashler, 1994; Henderson, 1996). Precuing effects on discrimination accuracy, while often small for clear or unmasked displays (Grindley & Townsend, 1968) were shown by Lyon (1990) to be substantial for masked multi-location displays. This precuing effect has been identified with elimination of noise from masks or other locations by Shiu and Pashler (1994). Henderson (1991, 1996) argued that precuing had a facilitatory effect apart from noise or distractor exclusion. He demonstrated that precuing effects occurred in what he called

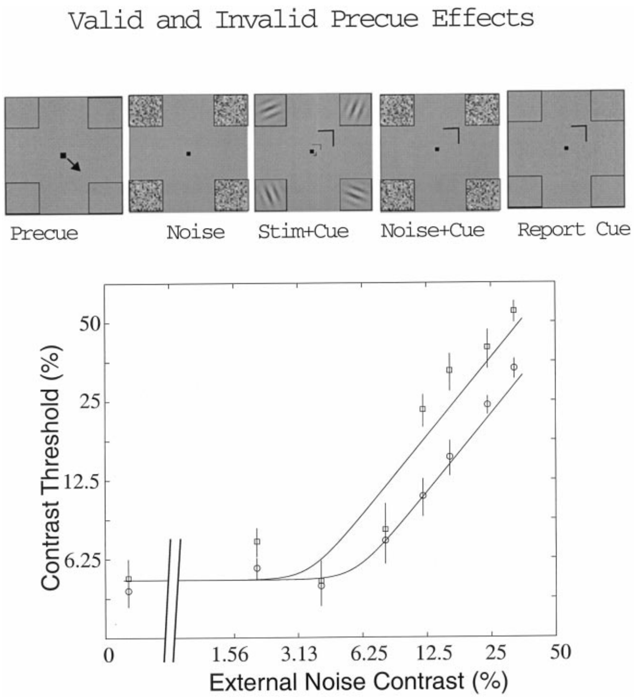

Fig. 5. Paradigm and sample results from the precued attention experiment of Dosher and Lu (1999). (a) The display sequence for the precued attention paradigm, including an arrow precue which precedes the signal by 150 ms and the stimulus sequence including a 'caret' report cue. Noise is added to signal by temporal integration by rapidly sandwiching noise and signal frames. (b) Sample contrast threshold data for one observer of Dosher and Lu (1999) exhibiting a pure signature for external noise exclusion — an effect of cue validity specifically in high external noise conditions.

'clear displays' — displays where stimulus and mask occur only in a single validly or invalidly cued location. These 'clear displays', however, included a high intensity mask, and we interpret these conditions as including substantial external noise. Cheal and Gregory (1997), although emphasizing masked displays, argued for independent contributions of facilitation and noise reduction.

8.2. The multi-location paradigm

In a previous study (Dosher & Lu, 2000), we documented the importance of external noise exclusion associated with precuing in a four-location display for a four-alternative orientation identification task. Fig. 5a illustrates the four-location central precuing paradigm used in that study. An oriented Gabor appeared at each location. Observers identified the orientation of the Gabor at a single report location, indicated by the 'caret' which appeared simultaneously with the targets. An arrow precue occurred 150 ms prior to target presentation. The precue was valid (consistent with the report cue) on five out of eight of the trials, and was invalid (inconsistent with the report cue) on three out of eight of the trials.

This application of the external noise plus attention paradigm differed from previous work in several ways: First, performance was evaluated both in the absence of external noise and in the presence of external noise using a full range of noise levels. Second, contrast thresholds were measured while previous work chose a single contrast (and noise) condition and measured the resulting accuracy. Contrast thresholds equate for difficulty level, constraining arguments that the results simply reflect differential difficulty rather than noise exclusion. Third, a target occurs in all locations and the observer was instructed to report only a single report location. Prior experiments presented a single target and the report reflected the identity (or presence/absence) of a target in an unspecified location. Thus, many previous multi-location paradigms were essentially visual search tasks that have known performance decrements in larger displays associated with location uncertainty; the altered paradigm eliminates structural uncertainty.

8.3. Eliminating structural uncertainty

The use of a report cue to specify the relevant location is an important aspect of the modified paradigm. Shiu and Pashler (1994) clearly showed that observer errors followed closely on errors of localizing the target, suggesting that location uncertainty was a major component in that and prior experiments. We elected to use a simultaneous report cue to identify the to-be-reported location in order to eliminate *structural location uncertainty*. The report cue remained available until response to ensure correct localization. Structural location uncertainty refers to purely statistical decision losses in multiple location or multiple sample experiments (Shaw, 1984; Palmer et al., 1993). If a sample is taken from each location in a multiple location experiment, even if those multiple samples can be acquired without capacity limitations, there are losses in accuracy reflecting the incorporation of more noise samples into the decision. In visual search for a target among distractors, display size effects have been shown to largely reflect structural decision losses for simple target/distractor contrasts such as luminance or length increments (Palmer et al., 1993).

Although our paradigm cannot eliminate genuine capacity limitations, it does eliminate purely structural location uncertainty in decision. An ideal observer which had sampled the stimulus in each location without loss could respond purely on the basis of the report location.

8.4. Cued discrimination in external noise

Attentional effects of a central precue were measured using threshold versus contrast (TVC) functions for four observers in a four-location display with targets at 5° eccentricity in four-alternative orientation judgments (Dosher & Lu, 2000). For all observers, attended (validly cued) conditions yielded substantially lower contrast thresholds than unattended (invalidly cued) conditions in high external noise levels.[8] Typical data from one observer at 62.5% threshold are shown in Fig. 5b. An analysis of the data at three criterion levels indicated that there were no changes in multiplicative noise associated with the attentional manipulation. The data from this multi-location cuing paradigm clearly identified external noise exclusion as an isolable and important mechanism underlying the precue advantage.

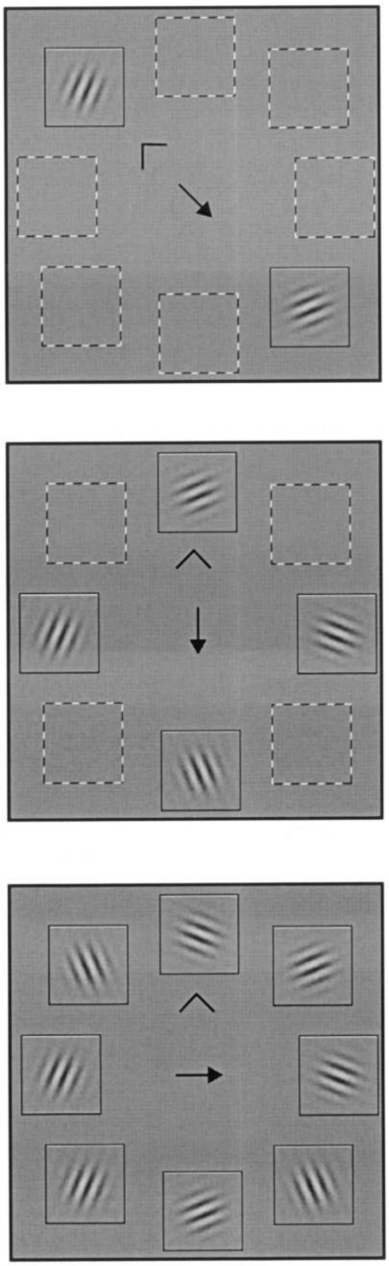

Fig. 6. Example layouts for display sizes of 2, 4 and 8.

8.5. Experiments

External noise exclusion was the key attentional mechanism revealed in this centrally cued attention task with stimuli of four possible orientations in each of four display locations (Dosher & Lu, 2000). Yet in an apparently similar attentionally cued task (Lu & Dosher, 1998a) with stimuli of two possible orientation in each of two display locations, there was no evidence of external noise exclusion as a mechanism of attention.

The current experiments were designed to investigate the range of display and task conditions under which the noise exclusion mechanism is observed. The experiments investigate the importance of multiple stimulus locations by examining conditions with three display sizes (two, four and eight locations) using a report cue to eliminate the structural uncertainty typical of search tasks (see Fig. 6). The endpoint method is used in these experiments. Measuring only two noise conditions made it possible to evaluate performance for three different location loads within single observers.

In Shiu and Pashler (1994), the size of the valid precue advantage depended on the number of locations, with essentially no effect for one location, and a substantial effect for eight locations. However, in the Shiu and Pashler experiments only one target appeared, targets were always masked, and masks appeared in all locations-essentially masked visual search task. Critically, errors were often associated with mislocalization of the target, a symptom of location uncertainty in a search task. Our paradigm eliminates search and structural uncertainty by the use of report cues. Cuing effects in the presence of high external noise should reflect the external noise exclusion seen in the original cuing experiment (Dosher & Lu, 2000) for four-location displays, but may not exhibit external noise exclusion in two-location displays (Lu & Dosher, 1998a).

The previous research differed not just in the number of display locations, but also in the complexity or difficulty of the task. It has been argued that the size of attentional effects may depend on task complexity (Shaw, 1984; Bonnel, Stein & Bertucci, 1992; Bonnel & Hafter, 1998). Experiment 1 used a four-orientation identification task. Experiment 2 used a two-orientation judgment, identifying a Gabor as tilted top to the right or left of vertical. In both experiments, we chose oriented stimuli such that each stimulus differed by 45° or more from any other. Hence, each stimulus effectively stimulated only a single channel (Sekuler, 1965; Campbell & Kulikowshi, 1966; Blakemore & Nachmias, 1971; DeValois, Albrecht & Thorell, 1982).[9]

[8] These data do not rule out a possible effect of stimulus enhancement which may occur in a low noise region, an additional and separable effect which coexisted with external noise exclusion in one observer.

[9] An oriented Gabor stimulus correlated less than 0.03 with the perfectly matched template for any other stimulus in these experiments.

8.6. Method

8.6.1. Stimulus and display

The patterns to be identified by the observer were Gabor patches tilted $\theta°$ relative to vertical.

$$l(x,y) = l_0\left(1.0 + c\,\sin(2\pi f(x\cos\theta \pm y\sin\theta))\right.$$
$$\left.\times \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right).$$

In Experiment 1, θ took on one of four values of ± 22.5 and $\pm 67.5°$. In Experiment 2, θ took on one of two values of $\pm 22.5°$. Each Gabor was rendered on a 72×72 pixel array, and extended 3.9×3.9 deg^2, with a center frequency of 1.15 cyc/deg, and standard deviation of 0.65°. The four Gabor stimuli are shown in Fig. 5a. The contrasts c of the Gabor pattern were determined for each condition based on pilot data for each observer.

External noise frames were also 72×72 pixels, with 3×3 pixel noise elements with contrast values chosen from a Gaussian distribution with mean 0 and standard deviation of 0 for the no noise condition or of 33% of the maximum achievable contrast for the high noise condition. Two signal frames were sandwiched between three noise (or null) frames, combining signal and noise via temporal integration (e.g. NSNSN, for two refreshes or 0.0167 s per frame).

The layout of displays is shown in Fig. 6. There were eight locations equally spaced on an annulus around fixation; the annulus appears at 6.84° eccentricity at a viewing distance of approximately 62 cm. In the eight-location condition, all eight locations were highlighted along with the fixation mark, and Gabors occurred in all eight locations. In the four-location condition, one of two sets of four equally spaced locations were highlighted along with the fixation mark, and Gabors occurred in the four highlighted locations. In the two-location condition, one of four possible pairs of locations on opposite sides of fixation were highlighted along with the fixation mark, and Gabors occurred in the two highlighted locations.

8.6.2. Apparatus

Signal and noise frames were computed online by a Power Macintosh 7300/200 on a Nanao Technology monitor with a P4 phosphor and a refresh rate of 120 Hz driven by the internal video graphics controller and the Video Toolbox (Pelli & Zhang, 1991). A special circuit combined two output channels to yield 6144 grayscale levels (12.6 bits). The luminance of the monitor with all pixels set to the minimum value was 0.52 cd/m^2; the luminance with all pixels set to the maximum value was 49.50 cd/m^2; the background level was 25.01 cd/m^2. Calibration linearized the full luminance range of the monitor.

8.6.3. Design

The no external noise and high external noise conditions of the two, four and eight location displays were intermixed randomly throughout each session. However, the observer was informed at the beginning of each trial of the relevant locations by highlighting their positions. In Experiment 1, each Gabor was selected randomly and independently from the four possible Gabor orientations, while in Experiment 2, each Gabor was selected randomly and independently from the two possible Gabor orientations. The precued location was chosen at random on each trial. The report cue matched the precue on five out of eight of the trials, and the report cue indicated one of the remaining relevant locations on three out of eight of the trials. Nine signal contrasts were chosen for each external noise and location load condition to span a psychometric function for each observer. There were 864 trials per session, yielding five trials per valid condition and three per invalid condition per session.

8.6.4. Procedure

The fixation and highlighted location boxes indicating the relevant locations (two, four or eight) appeared for 675 ms. A precue appeared 133 ms prior to the first noise frame. Hence, the precue preceded the first stimulus frame by 150 ms, precluding eye movements. A report cue appeared simultaneously with the first signal frame. The observer entered the identity of the Gabor at the location indicated by the report cue (‘d’, ‘f’, ‘j’ and ‘k’, respectively for the Gabor tilted far to the left, near to the left, near to the right, and far to the right for Experiment 1, or ‘f’ and ‘j’, respectively for near to the right and left for Experiment 2). A beep sounded as feedback when the response was accurate.

8.6.5. Observers

Three observers participated in these experiments. All had normal or corrected to normal vision. They were paid for their participation. Observers participated in four to six practice sessions, followed by nine to ten experimental sessions in Experiment 1 and in one practice session and seven to nine experimental sessions in Experiment 2.

8.7. Experiment 1 results

8.7.1. Psychometric functions

The 12 psychometric functions measured for the three observers are shown in Fig. 7a–c. The no external noise conditions appear on the left while the high external noise conditions appear on the right, shown separately for the two, four and eight location displays. Smooth curves represent Weibull functions (% correct $= 1.0 - 0.25 \times 2^{-(c/\alpha)^{\eta}}$) fitted to the psychometric function data by maximum likelihood methods.

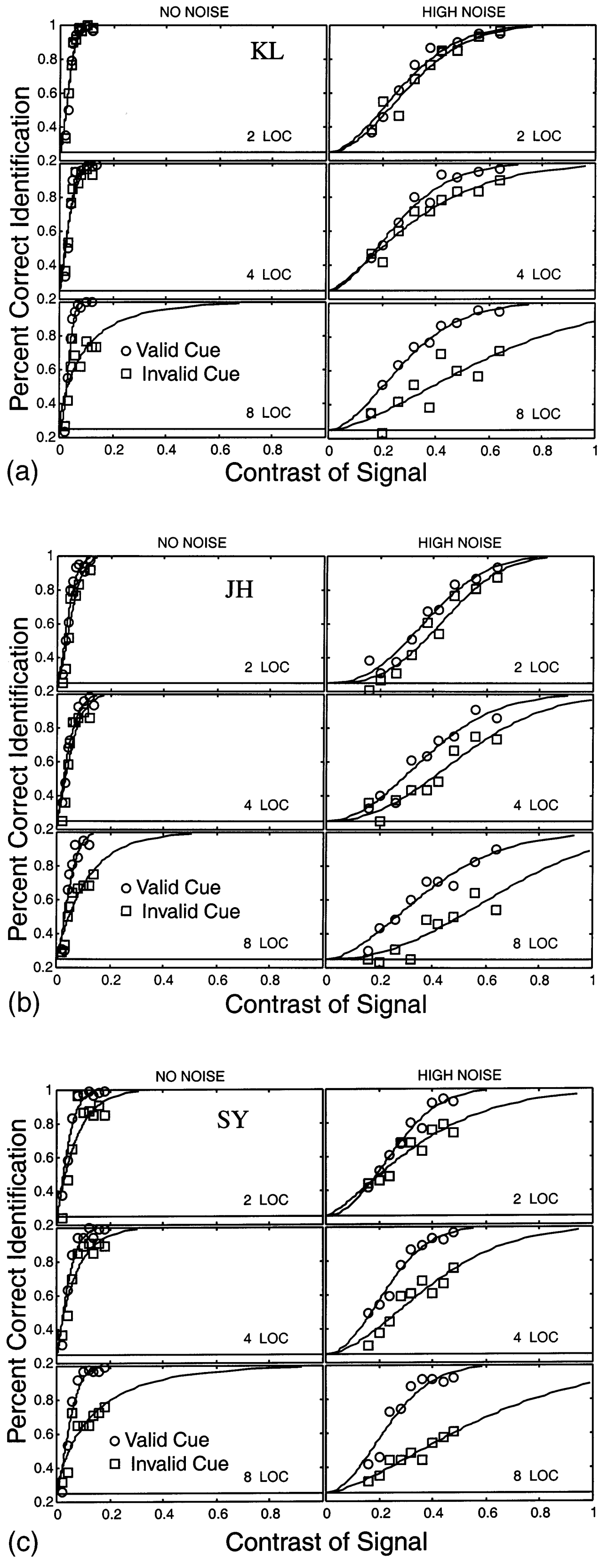

Fig. 7. Psychometric functions for validly and invalidly precued trials in no noise and in high noise conditions for display sizes 2, 4, and 8 for three observers in Experiment 1. Cue validity has an increasingly large effect for larger display sizes.

For all three observers, the difference between validly and invalidly cued trials in the high external noise conditions depends directly on the number of locations in the display. The size of the attention effect was very small to small in high noise for display sizes of 2, intermediate for display sizes of 4, and very large for display sizes of 8. The validly cued condition for all three display sizes appear to yield very similar psychometric functions. The pattern of significance tests followed the patterns visually evident in the data. Significance tests were based on nested model χ^2 tests on the maximum likelihood Weibull fits of the psychometric functions. Tests for the equivalence of the psychometric function may be more sensitive than the comparable tests for differences at a contrast threshold. Comparing psychometric functions for valid and invalid cuing in high noise yielded significant differences for all observers for display sizes 4 and 8 [$\chi^2(2) = 20.4$, $P < 0.001$ and 121.6, $P < 0.001$ for KL, 23.7, $P < 0.001$ and 63.8, $P < 0.001$ for JH, and 4.5, $P \approx 0.10$ and 64.6, $P < 0.001$ for SY for display sizes 4 and 8 respectively] and mixed significance for display size 2 [$\chi^2(2) = 2.6$, ns, for KL, 8.9, $P < 0.05$ for JH, and 35.8, $P < 0.001$ for SY].

For two of the three observers (KL, JH), validly cued and invalidly cued trials differed slightly in the absence of external noise in the two and four location conditions; for the remaining observer (SY), this difference was moderate [$\chi^2(2) = 0.4$, ns, and 4.1, ns, for KL, 12.6, $P < 0.001$ and 7.0, $P < 0.05$ for JH, and 59.6, $P < 0.001$ and 46.3, $P < 0.001$ for SY]. However, there were significant reductions in performance for invalidly cued trials in the noiseless eight-location condition for all observers [$\chi^2(2) = 218.9$, $P < 0.001$ for KL, 72.9, $P < 0.001$ for JH, and 162.1, $P < 0.001$ for SY]. As in the presence of high noise, performance in the validly cued trials appeared to be relatively unaffected by display size, even in the eight-location condition.

8.7.2. Discriminating mechanism mixtures: ratio tests

The impact of attention (cue validity) on identification performance in high noise conditions is consistent with the pattern observed in the previous investigation of four-location displays (Dosher & Lu, 2000). In that experiment, the primary mechanism was identified as external noise exclusion, sometimes accompanied by stimulus enhancement. Contributions of multiplicative internal noise reduction were ruled out by analysis of thresholds at three spaced criterion levels for a range of external noise conditions. Performance at three criterion levels were chosen to test for attentionally mediated changes in multiplicative internal noise based on the constraints described above in Sections 6.2 and 6.3 and in Appendix A.

Multiplicative internal noise reduction was evaluated in the current experiment by examining the ratios of performance at three criterion levels: 50, 62.5 and 75%

correct. These values are estimated from the psychometric functions using the Weibull as the interpolation function. The threshold contrast ratios for 62.5/50% (R1) and for 75/62.5% (R2) criteria were similar for both the validly cued and invalidly cued attentional conditions. The ratios were calculated for each observer for each display size by noise by attention condition (12 ratios). The standard deviation associated with each ratio was estimated based on the variability associated with the two contrast thresholds contributing to that ratio. The standard deviations of the ratios were approximately 0.1–0.2. Over observers, R1 averaged 1.37 and 1.42, and R2 averaged 1.31 and 1.34 for attended and unattended conditions, respectively (excluding one outlier condition, see below). In no case did the threshold contrast ratios differ significantly in comparisons of validly and invalidly cued conditions [all six t's (9) ranging from −0.85 to 1.66, all ns]. Individual ratios for the validly cued and invalidly cued conditions generally differed by less than 0.1. The ratios for the noiseless, eight-location invalid condition were an exception; the estimated ratios for this condition were somewhat higher than other ratios for all observers, but this single condition also had unusually high standard deviations; and any differences were not significant. The data in this condition apparently do not sufficiently constrain the Weibull estimates.

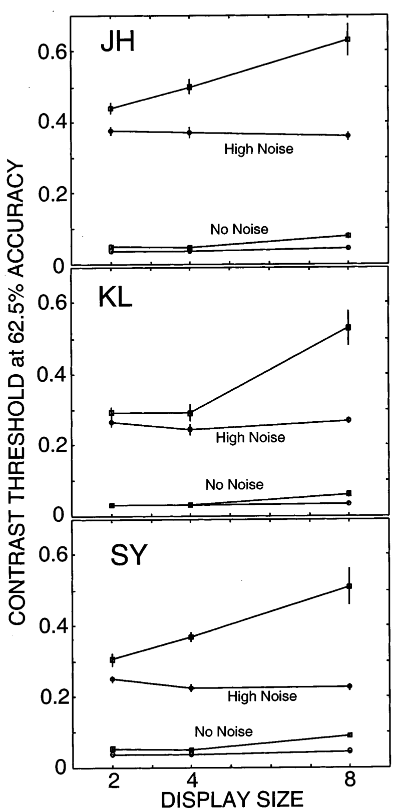

Fig. 8. Contrast thresholds (at 62.5% accuracy) for validly (circles) and invalidly (squares) trials as a function of display size and external noise condition in Experiment 1.

If attention had been mediated by changes in multiplicative internal noise, then these ratios should have systematically differed for the two cuing conditions. The ratio tests do not support changes in multiplicative internal noise reduction as a significant contribution to the attentional effects in this Experiment. This conclusion is consistent with previous results for the multi-location cue validity paradigm and related results for pre- versus simultaneous-cues (Lu & Dosher, 1999b; Dosher & Lu, 2000).

To provide collateral tests supporting this conclusion, we also carried out a lattice of nested model fits to the data of Experiment 1. We do not present these model fits in detail because they are based on only two levels of external noise, zero and high noise, which provide no constraints on the transition region of the TVC functions. Models incorporate between four free parameters for a model with no attention effects (β, γ, N_m, N_a) and seven free parameters for a model with all three attention mechanisms (the four above plus A_f for external noise exclusion, A_a for stimulus enhancement, and A_m for multiplicative internal noise reduction). Although the models do not support significance testing, they provide supporting evidence for the analysis based on ratio tests. The lattice considers eight models including the base with no attention effects, any single attention mechanism, all pairwise comparisons, and the full model with all three attentional mechanisms. Considering three observers and three display size manipulations, nine full model lattices were evaluated. In every case with a significant cuing effect, a model with external noise exclusion and/or stimulus enhancement yielded a best fit to the data (measured in r^2).

8.7.3. Display size effects

The magnitudes of the valid versus misleading cuing effects for display sizes 2, 4, and 8, are shown in Fig. 8. The contrast thresholds at the 62.5% criterion are shown for all three observers for both the no noise and high noise conditions. The standard error bars for these contrast thresholds are shown as twigs through the symbols. (In some cases, these are smaller than the symbols.) The standard errors are estimated by a resampling method based on assuming binomial error around the percent correct for each point of the psychometric functions (Maloney, 1990; Dosher & Lu, 1999; Lu & Dosher, 1999a).

Thresholds for validly cued locations (circles) are essentially unaffected by display size, as seen by the flat functions of display size for validly cued conditions. A similar pattern of results was reported in Morgan et al. (1998) in a very different cued search task. In contrast, the invalidly cued trials (squares) show increasing threshold contrasts with larger display sizes. This result holds despite the use of report cues which, unlike visual search, specify the relevant target location. This implies

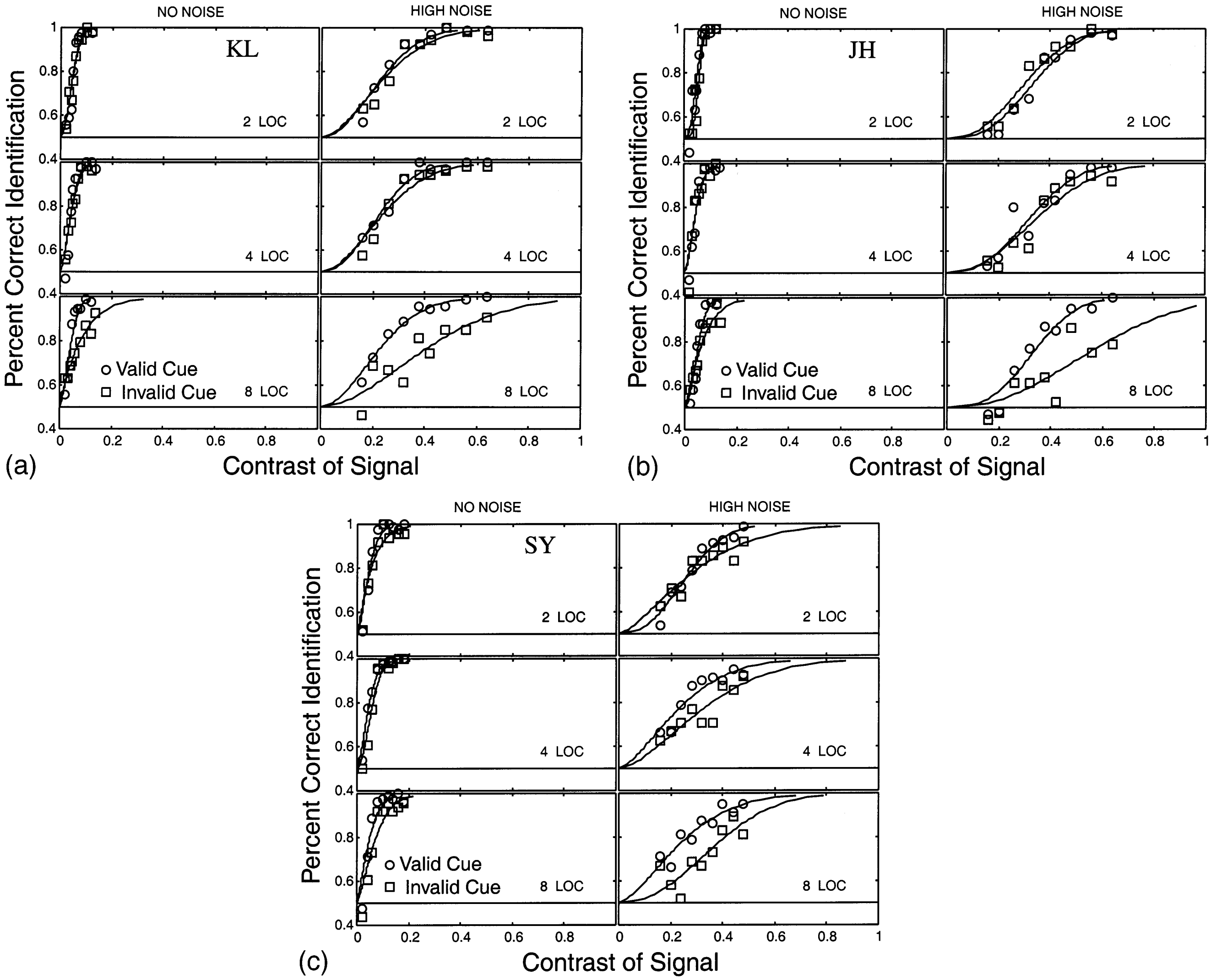

Fig. 9. Psychometric functions for validly and invalidly precued trials in no noise and in high noise conditions for display sizes 2, 4, and 8 for three observers in Experiment 2. Cue validity has an increasingly large effect for large display sizes.

that attention to the validly cued location yields a performance that is independent of display size, while observers are increasingly unable to exclude external noise in processing the report location following an invalid cue, or misdirection.

8.8. Experiment 2 results

8.8.1. Psychometric functions

Experiment 2 was the same as Experiment 1 except that the task was simplified to require the identification of the Gabor target as one of two orientations rather than four. Twelve psychometric functions were measured for the three observers (Fig. 9a–c), shown separately for noiseless and high noise conditions and the two-, four- and eight-location displays. Smooth curves were best fitting Weibull functions.

The accuracy range is smaller in these data since the guessing rate is 50% rather than 25%. Otherwise, the pattern of data are remarkably similar to those of Experiment 1. (See Section 8.9 for a full treatment.) In high external noise conditions (right panels), the effect of cue validity increases with the number of locations in the display, with the largest effects in high noise for display size 8. The validly cued and invalidly cued psychometric functions in high noise generally differed significantly for display size 4 and 8 [$\chi^2(2) = 4.5$, $P \approx 0.10$ and 64.6, $P < 0.001$ for KL, 3.0, ns, and 48.5, $P < 0.001$ for JH, and 15.0, $P < 0.001$, and 29.3, $P < 0.001$ for SY] but not for display size 2 [$\chi^2(2) = 2.3$, ns, for KL, 1.3, ns, for JH, but 9.3, $P < 0.01$ for SY].

As in Experiment 1, there were some effects of cue validity in the noiseless conditions (left panels). This is especially evident in the eight-location data, where it is significant for all observers [$\chi^2(2) = 49.3$, $P < 0.001$ for KL, 13.5, $P < 0.01$ for JH, and 13.9, $P < 0.001$ for SY]. As before, the effects are relatively small of nonexistent for the two- and four-location data, yielding mixed significance [$\chi^2(2) = 0$, ns and 0.7, ns for KL, 4.1, ns, and 0.3, ns, for JH, and 10.1, $P < 0.01$, and 3.9, $P \approx 0.10$ for SY, for two and four location displays, respectively]. We conclude that cuing advantages in noiseless conditions reflecting stimulus enhancement are zero or small under these conditions, with the exception of the eight-location displays (which may reflect lateral interactions or crowding).

8.8.2. Discriminating mechanism mixtures: ratio tests

Contrast threshold ratios were examined to rule out significant contributions of multiplicative internal noise reduction. The contrast threshold ratios R1 (75%:65%) and R2 (85%:75%) were calculated for each observer for each psychometric function. There was no evidence of a significant violation of ratio constancy in these data. Averaged over observers, R1 was 1.43 and 1.43 and R2 was 1.34 and 1.35 for attended and unattended conditions, respectively [six t(9)'s range from -0.03 to 1.5, all ns]. Attentionally mediated changes in multiplicative internal noise would have been associated with different ratios for validly and invalidly cued conditions. The ratio tests do not provide evidence for systematic changes in multiplicative noise. As before, a lattice of nested model fits were carried out despite the lack of intermediate external noise conditions, and the results were supportive of attentional mechanisms of external noise exclusion and of stimulus enhancement. This finding is consistent with the analysis of Experiment 1, and with the observations of Dosher and Lu (2000) for multi-location cuing.

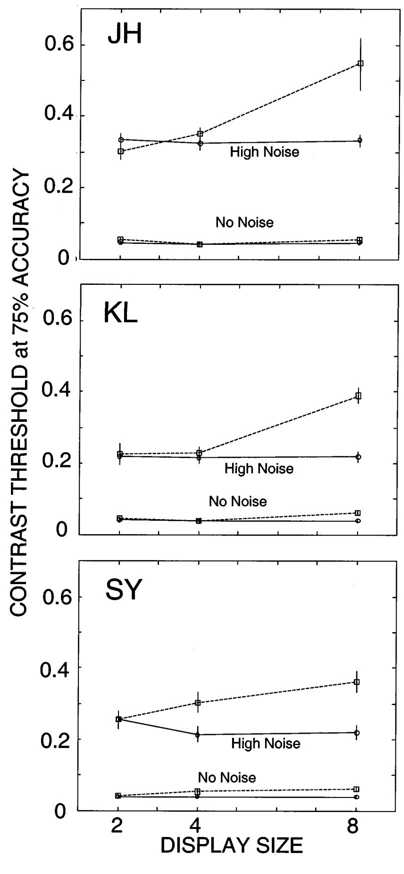

Fig. 10. Contrast thresholds (at 62.5% accuracy) for validly (circles) and invalidly (squares) trials as a function of display size and external noise condition in Experiment 2.

8.8.3. Display size effects

The magnitudes of the valid versus misleading cuing effects for display sizes 2, 4, and 8, are shown in Fig. 10. The contrast threshold at the 75% criterion are shown for all three observers for both the no noise and high noise conditions. These data are similar to those of Experiment 1. Again, the thresholds for validly cued locations (circles) are essentially unaffected by display size, while the invalidly cued trials (squares) show increasing threshold contrasts with larger display sizes. Attention to the validly cued location yields a performance that is independent of display size, while observers are increasingly unable to exclude external noise in processing the report location following an invalid cue, or misdirection.

8.9. Relationship of four and two alternative identification performance

In Experiment 1, observers identified a Gabor patch of one of four possible orientations (4AFC identification, with $\theta = \pm 22.5$ or $\pm 67.5°$ from vertical), while in Experiment 2 observers identified a Gabor patch of one of two possible orientations (2AFC identification, with $\theta = \pm 22.5°$ from vertical). The orientations were chosen so that the stimuli differed by at least 45°. Each stimulus stimulated only a single channel based on psychophysical estimates (Sekuler, 1965; Campbell & Kulikowshi, 1966; Blakemore & Nachmias, 1971; De-Valois et al., 1982). The correlation of each Gabor image with the perfectly matched template was 1.0, and the correlation with any unmatched template was approximately zero (≤ 0.03). Furthermore, the outputs of each template applied to the same image of random noise were essentially uncorrelated (verified by simulation). The stimulus matches one template, so under the signal detection model, one sample is taken from a signal distribution and the remaining $N-1$ samples are drawn from a noise distribution with mean zero. The template with the highest (max) value determines the response. This model is the same as that used in the standard NAFC tables (assuming equal variance of signal and noise) for multi-interval paradigms (Macmillan & Creelman, 1991).

The statistical tests reported previously for Experiment 1 (4AFC) and Experiment 2 (2AFC) were independent of a d' model since they involved direct comparisons between psychometric functions. How-

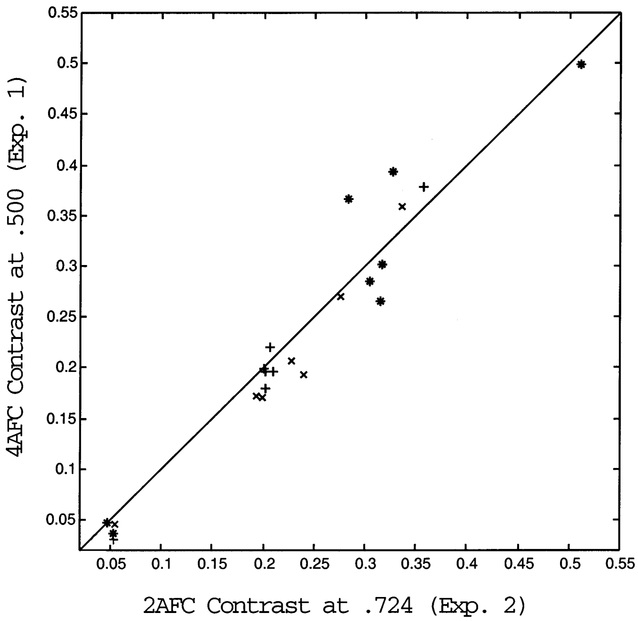

Fig. 11. A standard signal detection model of NAFC accounts for the relationship of performance in Experiments 1 and 2. Contrast thresholds at 72.4% accuracy for four-orientation identification (4AFC) (Experiment 1) and at 50% accuracy for two-orientation (2AFC) (Experiment 2). These accuracy levels both equate to a d' of 0.84 under the signal detection model. The contrast levels should be the same (on the major diagonal). Different symbols represent the subjects (*, JH; +, KL; x, SY).

ever, a d' analysis is implicit in any fit of the PTM model, and also is useful for relating the data of Experiments 1 and 2.

The signal detection model was used to evaluate the relationship between the performance in Experiments 1 and 2. A d' of 0.84, for example, is equivalent to a 4AFC accuracy of 50% and a 2AFC accuracy of 72.4%. If the signal detection model accounts entirely for the performance differences in the two experiments, then the contrast yielding 50% accuracy in 4AFC should equal the contrast yielding 72.4% in 2AFC. Fig. 11 graphs contrast at 50% accuracy in 4AFC versus contrast at 72.4% accuracy in 2AFC. Ideally, the data from all conditions should fall on a line of unit slope and zero intercept. The signal detection model provides quite a good account of the data from the two experiments, although the 4AFC contrasts are biased slightly low in the low noise conditions (points close to the origin).[10] The quality of the relationship is quantified by regression; the estimated regression intercepts and slopes (4AFC on 2AFC) over all 12 conditions were 0.00 and 1.04, −0.02 and 1.09, and −0.01 and 1.00 for observers JH, KL, and SY, respectively. The ideal values of 0 for intercept and 1 for slope are easily within the 90% confidence intervals on the observed values. The consistency with the signal detection model was remarkable especially since the 4AFC and 2AFC tasks were performed in successive experiments that differ in levels of practice or fatigue. We conclude that there appear to be no major differences between the two tasks which would require separate notions of task difficulty. In both experiments, display size is the critical manipulation.

9. General discussion

9.1. External noise paradigm

The external noise plus attention paradigm and the PTM model provide a theoretical framework within which mechanisms of spatial attention could be evaluated in a range of display conditions using a simplified endpoint method. The endpoint method evaluates attention in a zero external noise condition to assess stimulus enhancement and in a high external noise condition to assess external noise exclusion as a mechanism of attention. A contrast threshold ratio test was used to rule out attentionally mediated changes in multiplicative external noise. Although the endpoint method does not support full quantitative model testing and estimation, it does, however, provide a practical method for qualitative identification of attention mechanisms in several stimulus conditions within a single observer.

9.2. Attentional mechanisms in multi-location cuing

9.2.1. Alternative mechanisms of cuing effects

The mechanisms of visual attention underlying improvements in detection or discrimination accuracy with precuing in multi-location paradigms have been actively debated. Improvements for precuing versus (simultaneous cuing) of a report location have been especially large in paradigms involving high contrast masks (Lyon, 1990; Cheal & Lyon, 1991a,b; Henderson, 1991, 1996; Shiu & Pashler, 1994). This led some researchers to focus on noise exclusion (Shiu & Pashler, 1994) or mask elimination (Cheal & Gregory, 1997; Enns & Di Lollo, 1997) as primary mechanisms of attention in these tasks. In contrast, some researchers (e.g. Henderson, 1996) have argued strongly that, although noise exclusion may play some role, attention cuing effects exist even in the absence of multiple locations or multiple stimuli, and that large effects with multiple locations and masks may simply reflect larger effects of sensory facilitation under more difficult task conditions. The external noise plus attention paradigm provides an excellent structure to test these theoretical

[10] The data may more closely approximate the signal detection model in high noise conditions because the external noise dominates variability and hence guarantees that the equal variance assumption is correct.

claims within the context of the PTM model or its stochastic form.

9.2.2. External noise exclusion

The current experiments investigated central precues, generally associated with endogenous attentional orienting. External noise exclusion was the primary mechanism of central precuing in a four-location paradigm which eliminated structural uncertainty (Dosher & Lu, 2000), but not in a two-location paradigm (Lu & Dosher, 1998a). In the current experiments, we found that the magnitude of the attentional effect in high external noise — associated with external noise exclusion — depended on the number of stimuli in the display. There was a substantial effect of precuing in high noise for display size 4, but very small or no effect for display size 2. This was consistent with both previous observations, and provides an explanation of the original inconsistency. For eight-location displays, the difference between validly and invalidly cued trials is especially substantial — percent correct identification at a given contrast in the high noise condition differs by as much as 40–55%. Our results are related to earlier reports of substantial cuing benefits for masked displays, although previous demonstrations were contaminated by structural uncertainty and functional localization errors (Shiu & Pashler, 1994).[11]

9.2.3. Stimulus enhancement

Consistent with previous reports, there was only weak evidence for stimulus enhancement associated with central precuing. Stimulus enhancement, a separate mechanism of attention that operates in noiseless displays, was exhibited by only one of four observers in Dosher and Lu (2000) and none of the five observers in Lu and Dosher (1998b). The current experiments are consistent with these previous observations for display sizes of 2 and 4. In contrast, a reliable cuing effect existed in the noiseless condition for display size 8 (interpretation of this effect should be tempered by the fact that the Weibull fits for the invalid condition were not optimally constrained by the selection of contrasts). However, we suspect that in the size 8 displays, other factors such as stimulus crowding or lateral interactions (Palmer et al., 1993) may be contributing to the apparent stimulus enhancement in the absence of external noise.

Simulus enhancement may play a more consistent and prominent role in experiments involving peripheral cuing of spatial location. Peripheral cues reliably induced stimulus enhancement effects in comparing precues to simultaneous cues in low noise conditions in an external noise paradigm (Lu & Dosher, 1999a). The example of pure stimulus enhancement due to precuing cited in Section 5.1 used simultaneous peripheral and central cuing in two-location displays; the peripheral cuing apparently induced stimulus enhancement, while external noise exclusion did not appear due to the small display load.

9.2.4. Multiplicative noise reduction

Multiplicative noise and nonlinearity were unaffected by attentional precuing in the current experiments. These results were consistent with previous observations, using performance from multiple criteria (Dosher & Lu, 1998, 1999, 2000; Lu & Dosher, 1999a; Lu et al., 2000). Multiplicative noise and nonlinearity may generally be unchanged by attentional and perceptual learning manipulations.

9.2.5. Identification task effects

An analysis of the relative performance in the 4AFC orientation identification task of Experiment 1 and the 2AFC orientation identification task of Experiment 2 indicated that performance in the two tasks was compatible with a signal detection model. Task difficulty effects associated with increasing the number of orientation templates from two to four were essentially entirely accounted for by the statistical properties of the decision task.

9.2.6. The function of noise exclusion

In these experiments, external noise exclusion is the primary mechanism underlying attentional cuing, yielding substantial effects of valid precuing in four-location displays and larger effects in eight-location displays. However, external noise exclusion plays little role in two-location displays. The fact that the size of the external noise effect increases with display size rules out the simplest model of precuing in which observers focus exclusively on the precued location and then, in invalidly cued trials, switch attention to the report cue location. In this simple model, only the precued location would be relevant to performance on valid trials, and only the precued and report cued locations would be relevant to performance on invalid trials. Hence, the attention effect should be of exactly the same magnitude for all display sizes of two or larger. The interaction of external noise exclusion and display size rules out this simple switching model.

[11] Henderson (1996) reports modest cue validity effects (approximately 5% accuracy differences) in a variant in which only a single masked target appears either in the single validly cued location, or in one of seven invalidly cued locations. He describes this as evidence for facilitation in clear displays (e.g. displays without competing stimulus locations); we interpret his result as a cue validity effect in high noise because of the presence of a poststimulus mask. Although this might appear to be a significant external noise exclusion effect in a size 1 display, this interpretation is complicated by the fact that the observer does not know until stimulus (or perhaps mask) onset in which of seven possible other locations an invalid stimulus might appear. Hence, this condition does not map precisely onto any of our display size manipulations.

The pattern of results is also incompatible with a simple statistical uncertainty model usually applied to set size effects in visual search (Palmer et al., 1993). In this model, modest decrements in performance with set size are accommodated by a signal detection model incorporating additional noise samples for additional locations. This kind of model is not expected to apply to the current experiments because the observer is only asked about a single report location and, additionally, there are potential targets in all locations. Even if the current paradigm met the conditions of standard search, a simple (functional) uncertainty model would not account fully for the data. This is because many conditions exhibit a dissociation between the effect of attention in the absence of external noise, which may be small to nonexistent, and the effect of attention in high noise, which may be quite significant. Under an uncertainty model, attention would have comparable effects in both low and high noise, which are equated relative to a common threshold. The interaction of attention with external noise condition rules out a simple functional uncertainty model similar to those applied to simple search tasks (Palmer et al., 1993; Morgan et al., 1999). The pattern of data indicates a special function or mechanism of attention in noise exclusion.

The equivalence of the validly cued trials in high noise regardless of display size suggests that a perceptual template (filter) is tuned primarily to exclude external noise in the precued location. The increasing decrements for invalid trials depending on display size may be mediated by either of the following processes: (1) A limited capacity process may, in addition to focusing on the precued location, possess residual capacity to orient toward one or more additional locations which might be tested on invalid trials. Limitations in capacity must be critical only in displays of three or more. (2) Alternatively, competition from multiple display locations might make it more difficult to reorient the perceptual template toward the report-cue location on invalid trials. If the report cue is less effective in displays with more stimuli, then attentional reorientation may be delayed.

In either case, the external noise exclusion mechanism serves to focus or tune an effective perceptual template at the precued location — enabling the observer to optimize when, where, and in which spatial frequency range to 'look' for the target. The perceptual templates in other locations are less successful in excluding external noise.

10. Conclusions

Several conclusion are supported by the current experiments: (1) Central precues are associated with the attentional mechanism of external noise exclusion in multilocation displays. External noise exclusion is effective in high noise conditions. (2) The noise exclusion mechanism due to central precuing is larger and more important for larger display sizes. The ability to tune a perceptual template to exclude external noise is capacity limited, especially for display sizes exceeding two. (3) A secondary mechanism of attention in precued multi-location displays is stimulus enhancement which may, but need not, occur in noiseless display conditions. Conditions which reliably lead to the expression of stimulus enhancement require further investigation. Based on other work (Lu & Dosher, 1998a,b; Lu et al., 2000), stimulus enhancement may occur primarily in response to peripheral precues. However, we speculate that difficult visual conditions (e.g. size for eccentricity or crowding in larger displays) may also lead to stimulus enhancement. (4) The external noise plus attention paradigm and the PTM model provide a useful framework for identification of mechanisms of visual attention. The framework also provides an alternative organization and interpretation of the previous literature. (5) Finally, a stochastic version of the PTM model (Appendix A) is shown to be consistent with the key properties of the analytic PTM model.

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Appendix A. Properties of a stochastic PTM model

At the request of an anonymous referee, we briefly describe the properties of a stochastic version of the PTM model, of which the PTM model (Lu & Dosher, 1998a, 1999a; Dosher & Lu, 1999) is an analytical approximation. As shown below, the analytic PTM model captures all the essential properties of the stochastic model, including the three signature patterns and the endpoint ratio properties.

A.1. The stochastic PTM model

The diagram in Fig. 2a can be implemented as a stochastic model. In this development, we consider the stochastic model as applied to a 2AFC identification task. A noisy stimulus is presented to the model and the model decides which of the two possible signal stimuli is embeded in the noisy stimulus. The current implementation follows the signal detection structure for 2AFC tasks outlined in Section 8.9. This involves submitting the noisy stimulus to two templates for matching, and then choosing the identity corresponding to the template with the maximum response, implemented by the difference rule.

In what follows, we will use bold characters, e.g. $\mathbf{N}_{\mathrm{ext}}$, to denote random variables; normal characters, e.g. N_{ext}, to denote the expectations of random variables.

The noisy stimulus consists of two parts, a signal stimulus $I_S(x, y, t)$ at signal contrast c, and a random image $\mathbf{I_N}(x, y, t)$ in which all the pixels are independent, identically distributed Gaussian random variables with mean 0 and standard deviation of N_{ext}. This corresponds to the definition of external noise used in the current and related applications of the PTM model. The stochastic PTM model consists of two fixed templates, $T_1(x, y, t)$ and $T_2(x, y, t)$. The two templates are both normalized such that $\int\int\int T_1^2(x, y, t)\,dx\,dy\,dt = 1.0$, $\int\int\int T_2^2(x, y, t)\,dx\,dy\,dt = 1.0$. One natural consequence of the normalization is that: both $\int\int\int T_1(x, y, t)\mathbf{I_N}(x, y, t)\,dx\,dy\,dt$ and $\int\int\int T_2(x, y, t)\mathbf{I_N}(x, y, t)\,dx\,dy\,dt$ are Gaussian random variables with mean 0 and standard deviations αN_{ext}. The proportional constant α depends on the pixel size and the number of frames of the external noise processed by the observer; we simplify (scale) by setting $\alpha = 1.0$ in subsequent equations. The outputs of the templates given the noise images as input are random variables denoted as $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$, respectively. Depending on the relationship between the two templates, $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$ may or may not be correlated.

We define $\int\int\int T_1(x, y, t)I_S(x, y, t)\,dx\,dy\,dt = \beta_1 c$, $\int\int\int T_2(x, y, t)I_S(x, y, t)\,dx\,dy\,dt = \beta_2 c$. The parameter β is related to the match between the signal stimulus and the template. (The function of β in the model is similar to efficiency in a linear amplifier model, and a simplified PTM model without nonlinearity or multiplicative noise will be identical to the PTM model.)

For the combination of the signal stimulus and the noise stimulus, the template matching stage (leftmost in Fig. 2a) produces two outputs, one for each template:

$$\mathbf{S}_1 = \beta_1 c + \mathbf{N_{ext}}, \qquad \text{(A1a)}$$

$$\mathbf{S}_2 = \beta_2 c + \mathbf{N'_{ext}}. \qquad \text{(A1b)}$$

Nonlinearity in the signal path, $\|\cdot\|^{\gamma}$ preserves the sign of its input. The outputs from the two templates become:

$$\mathbf{U}_1 = \mathrm{sign}(\mathbf{S}_1)|\mathbf{S}_1|^{\gamma}, \qquad \text{(A2a)}$$

$$\mathbf{U}_2 = \mathrm{sign}(\mathbf{S}_2)|\mathbf{S}_2|^{\gamma}. \qquad \text{(A2b)}$$

The multiplicative noises in the two templates are random variables $\mathbf{N_{1,mul}}$ and $\mathbf{N_{2,mul}}$. By assumption, these are two Gaussian random variables with mean 0 and standard deviations $\boldsymbol{\sigma}_{\mathbf{1,mul}}$ and $\boldsymbol{\sigma}_{\mathbf{2,mul}}$ proportional to the ‘energy’ in the path with proportional constant N_{m}:

$$\boldsymbol{\sigma}_{\mathbf{1,mul}} = N_{\mathrm{m}}|\mathbf{S}_1|^{\gamma}, \qquad \text{(A3a)}$$

$$\boldsymbol{\sigma}_{\mathbf{2,mul}} = N_{\mathrm{m}}|\mathbf{S}_2|^{\gamma}. \qquad \text{(A3b)}$$

The additive internal noises in the two templates are random variables $\mathbf{N_{1,add}}$ and $\mathbf{N_{2,add}}$. By assumption, these are both Gaussian random variables with mean 0 and standard deviations $\sqrt{2}/2N_{\mathrm{a}}$. (The standard deviations are scaled in this way to yield a final additive internal noise term of N_{a}.)

In the final, decision stage, the model is confronted with two random variables, $\mathbf{V}_1$ and $\mathbf{V}_2$, reflecting the output of the two templates applied to the stimulus:

$$\mathbf{V}_1 = \mathbf{U}_1 + \mathbf{N_{1,mul}} + \mathbf{N_{1,add}}, \qquad \text{(A4a)}$$

$$\mathbf{V}_2 = \mathbf{U}_2 + \mathbf{N_{2,mul}} + \mathbf{N_{2,add}}. \qquad \text{(A4b)}$$

The decision is based on the random variable $\mathbf{D}$, which is the difference of $\mathbf{V}_1$ and $\mathbf{V}_2$:

$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = (\mathbf{U}_1 - \mathbf{U}_2) + (\mathbf{N_{1,mul}} - \mathbf{N_{2,mul}}) + (\mathbf{N_{1,add}} - \mathbf{N_{2,add}}). \qquad \text{(A5)}$$

In a particular trial, if $\mathbf{D} \geq 0$, the model decides that signal one is presented; if $\mathbf{D} < 0$, the model decides that signal two is presented.

The stochastic PTM model described here has been simulated on a computer to check the key model properties. The signatures and ratio tests of the PTM model were derived for the stochastic model. First, however, we further develop the stochastic model by computing the expected mean and standard deviations of the decision variable $\mathbf{D}$. To reiterate, plain characters, e.g. D, denote the expectations of random variables.

The mean of $\mathbf{D}$ is simply:

$$D = U_1 - U_2, \qquad \text{(A6)}$$

since by hypothesis $\mathbf{N_{1,mul}}$, $\mathbf{N_{2,mul}}$, $\mathbf{N_{1,add}}$ and $\mathbf{N_{2,add}}$ have mean 0.

The variance of $\mathbf{D}$ consists of the summation of the variances of all the six terms in Eq. (A5). Because $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$ could in principal be correlated (e.g. for two very similar templates), the variance of the first two terms is considered together:

$$\sigma^2_{total} = \mathrm{var}(\mathbf{U}_1 - \mathbf{U}_2) + N_{\mathrm{m}}^2((\mathrm{S}_1^2)^{\gamma} + (\mathrm{S}_2^2)^{\gamma}) + N_{\mathrm{a}}^2. \qquad \text{(A7)}$$

In this equation, $(\mathrm{S}_1^2)^{\gamma} = E(\mathbf{S}_1^{2\gamma})$ and $(\mathrm{S}_2^2)^{\gamma} = E(\mathbf{S}_2^{2\gamma})$

The estimated signal detectibility, the d' of signal detection theory, can be derived:

$$d' = D/\sigma_{\mathrm{total}} = \frac{U_1 - U_2}{\sqrt{\mathrm{var}(\mathbf{U}_1 - \mathbf{U}_2) + N_{\mathrm{m}}^2((S_1^2)^{\gamma} + (S_2^2)^{\gamma}) + N_{\mathrm{a}}^2}}. \qquad \text{(A8a)}$$

By assumption, the internal multiplicative noise and internal additive noise follow a Gaussian distribution. The output from template matching to external noise is also Gaussian prior to nonlinearity given independent Gaussian pixel noise in the stimulus. Then, the distribution of the noise at the decision stage is essentially

Gaussian[12], and standard signal detection calculations apply.

It is useful to develop the relationship between Eq. (A8a) and the classical linear amplifier model (LAM) (see main text). In the special case in which $\gamma = 1$, $U_1 - U_2$ can be simplified to $(\beta_1 - \beta_2)c$. Since $E(\mathbf{N_{ext}}) = E(\mathbf{N'_{ext}}) = 0$, Eq. (A8a) can then be reduced to:

$$d' = \frac{(\beta_1 - \beta_2)c}{\sqrt{\mathrm{var}(\mathbf{N_{ext}} - \mathbf{N'_{ext}}) + N_m^2((\beta_1 c)^2 + (\beta_2 c)^2 + 2N_{ext}^2) + N_{add}^2}}. \quad \text{(A8b)}$$

Eq. (A8b) can be further reduced to the same d' expression as obtained from the LAM model if $N_m = 0$, but possibly with some correlated noise in the two templates:

$$d' = \frac{(\beta_1 - \beta_2)c}{\sqrt{\mathrm{var}(\mathbf{N_{ext}} - \mathbf{N'_{ext}}) + N_{add}^2}}. \quad \text{(A8c)}$$

(Note that the LAM equation is often further simplified in terms of a single β, either reflecting the difference or assuming that $\beta_2 = 0$.)

In general, however, γ may be any real number greater than 0, and typically γ is not 1. In this general case, it is impossible to analytically develop all the terms in Eq. (A8a). Nor is it possible to solve Eq. (A8a) analytically to predict threshold contrast levels for a given specified model with known parameters (N_m, N_{add}, β_1, β_2, γ, and known correlation between $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$) confronted with signals in various levels of noise, i.e. TVC functions. We were able, however, to combine Monte Carlo simulation with numerical methods to generate TVC functions for a specified stochastic PTM model. Fig. A1 shows such a TVC function for a stochastic PTM model in which, $N_m = 0.32$, $N_{add} = 0.008$, $\beta_1 = 8$, $\beta_2 = 0$, $\gamma = 2$, and zero correlation between $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$) These combined simulation and numerical methods were used to fit certain sets of data, as described later in the Appendix.

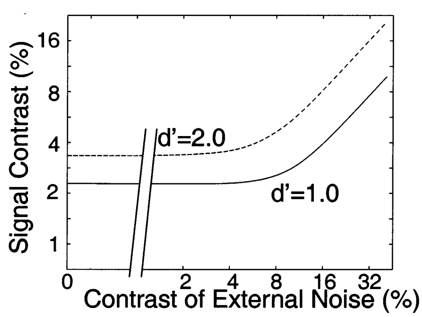

Fig. A1. Contrast thresholds as a function of external noise (TVC functions) for two d' criteria for the stochastic form of the perceptual template model.

[12] This is correct for the range of nonlinearities ($\gamma \approx 2 \pm 1$) that we have encountered, so long as we restrict ourselves to performance ranges less than 95% correct.

A.2. Signature performance patterns of the processing model

As discussed in Section 4, within the PTM model structure, attention improves human performance via three distinctive mechanisms: stimulus enhancement, external noise exclusion, and multiplicative noise reduction. Paralleling the approach with the analytic PTM model, in the stochastic PTM model stimulus enhancement is implemented by multiplying N_a by a factor $A_a \leq 1.0$; external noise exclusion is implemented by multiplying both $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$ by a factor $A_f < 1.0$; and multiplicative noise reduction is implemented by multiplying N_m by a factor $A_m \leq 1.0$. The joint impact of all the three mechanisms can be summarized:

$$d' = \frac{U_{1A} - U_{2A}}{\sqrt{\mathrm{var}\,(\mathbf{U_{1A}} - \mathbf{U_{2A}}) + (A_m N_m)^2((\mathbf{S}_{1A}^2)^\gamma + (\mathbf{S}_{2A}^2)^\gamma) + (A_a N_a)^2}}, \quad \text{(A9a)}$$

where:

$$\mathbf{S_{1A}} = \beta_1 c + A_f \mathbf{N_{ext}}, \quad \text{(A9b)}$$

$$\mathbf{S_{2A}} = \beta_2 c + A_f \mathbf{N'_{ext}}, \quad \text{(A9c)}$$

$$\mathbf{U_{1A}} = \mathrm{sign}(\mathbf{S_{1A}})|\mathbf{S_{1A}}|^\gamma, \quad \text{(A9d)}$$

$$\mathbf{U_{2A}} = \mathrm{sign}(\mathbf{S_{2A}})|\mathbf{S_{2A}}|^\gamma. \quad \text{(A9e)}$$

We generate the signature pattern for stimulus enhancement for a given model by setting $A_f = A_m = 1.0$ for both the attended and the unattended conditions, $A_a = 1.0$ for the unattended condition and $A_a < 1.0$ for the attended condition. Fig. A2a shows a performance pattern for the stochastic model specified in Fig. A1 at two d' performance levels for $A_a = 0.5$. Similarly, we can generate the signature performance patterns for external noise exclusion and multiplicative noise reduction. These are shown in Fig. A2b and c, for $A_f = 0.5$ or $A_m = 0.5$ in the attended condition.

The performance signatures derived from the Monte Carlo simulations of the processing model are very similar to those derived from the PTM model (Fig. 3). In the next sections we derive ratio rules that demonstrate the equivalence in the endpoints of the signature patterns in the analytic and stochastic PTM models.

A.3. Ratio tests for attention mechanisms in the stochastic model

The endpoint method proposed in this article is based on the following ratio rules in the PTM model: the size of the attention effect (in terms of log threshold reduction) will not depend on the criterion d' level for either stimulus enhancement or external noise reduction; it will depend critically on the criterion d' level for multiplicative noise reduction. In this section, we prove that the ratio rules hold asymptotically in the stochastic

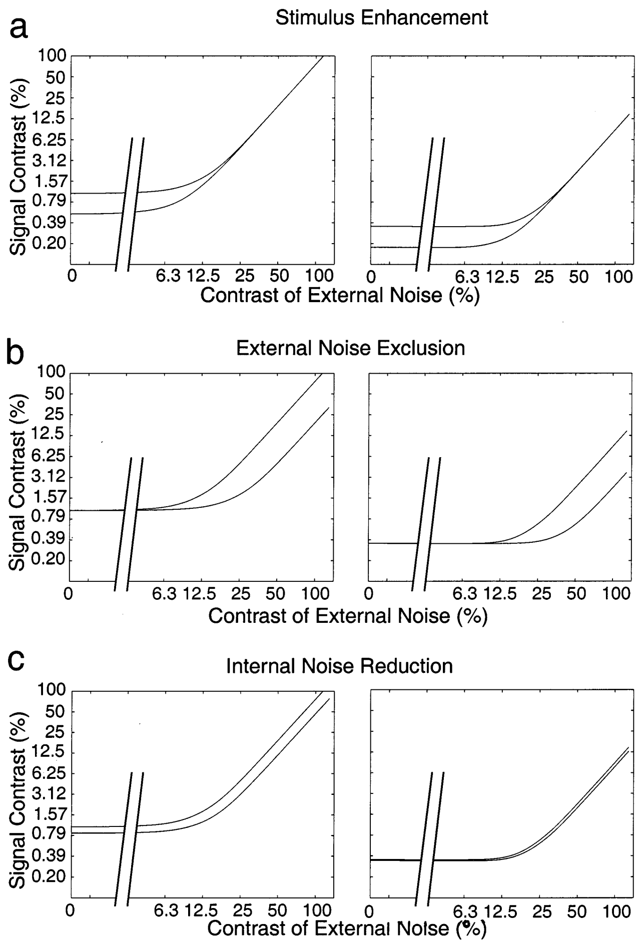

Fig. A2. Contrast thresholds for a higher (left) and lower (right) criterion for three mechanisms of attention predicted by the stochastic perceptual template model. These predictions show the same ratio relations as the analytic PTM, as shown in Fig. 4. See Fig. 4 for an explanation.

PTM model for the end regions (where $N_{\text{ext}} \ll N_a$ or $N_{\text{ext}} \gg N_{\text{a}}$).

A.3.1. Stimulus enhancement

Consider the case in which stimulus enhancement is the only mechanism of attention. Stimulus enhancement has its largest effects where internal additive noise dominates ($N_{\text{ext}} \ll N_{\text{a}}$). In this region (and as $N_{\text{ext}} \to 0$), we can ignore all the terms associated with $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$. Combining Eqs. (A9a, b, c, d, e) we have:

$$d' = \frac{(\beta_1 c)^\gamma - (\beta_2 c)^\gamma}{\sqrt{N_{\text{m}}^2((\beta_1 c^2)^\gamma + (\beta_2 c^2)^\gamma) + (A_{\text{a}} N_{\text{a}})^2}}. \quad \text{(A10)}$$

Set $A_{\text{a}} = 1.0$ for the unattended condition and $A_{\text{a}} \leq 1.0$ for the attended condition. For a given d' criterion level, suppose the threshold in the unattended condition is c_{uA}. From Eq. (A10) it follows that the threshold in the attended condition is $c_{\text{A}} = A_\alpha^{1/\gamma} c_{\text{uA}}$.Thus,

$$\frac{c_{\text{A}}}{c_{\text{uA}}} = (A_{\text{a}})^{1/\gamma}. \quad \text{(A11)}$$

The ratio between the thresholds in the attended and unattended conditions is determined completely by A_{a} and γ, independent of the particular d' level, in the region where internal additive noise dominates — the region where stimulus enhancement has its largest effects.

On the other hand, in the region where external noise dominates ($N_{\text{ext}} \gg N_{\text{a}}$), we can ignore N_{a}, and hence any impact of A_{a}. Thus, the ratio between thresholds in the attended and unattended regions in a model differing only in A_{a}, would be 1. This demonstrates the signature pattern of stimulus enhancement: an effect in low external noise but no effect in high external noise.

A.3.2. External noise exclusion

Consider the case in which external noise exclusion is the only mechanism of attention. External noise exclusion has its largest effects where external noise dominates internal additive noise ($N_{\text{ext}} \gg N_{\text{a}}$). (Because $\mathbf{N_{ext}}$ cannot be eliminated, this formulation cannot be written directly in terms of βs.) In this region, we can ignore N_{a} in Eq. (A9a):

$$d' = \frac{U_{1\text{A}} - U_{2\text{A}}}{\sqrt{\text{var}(\mathbf{U_{1A}} - \mathbf{U_{2A}}) + N_{\text{mul}}^2((S_{1\text{A}}^2)^\gamma + (S_{2\text{A}}^2)^\gamma)}}, \quad \text{(A12)}$$

where:

$$\mathbf{S_{1A}} = \beta_1 c + A_{\text{f}} \mathbf{N_{ext}},$$

$$\mathbf{S_{2A}} = \beta_2 c + A_{\text{f}} \mathbf{N'_{ext}}.$$

Set $A_{\text{f}} = 1.0$ for the unattended condition and $A_{\text{f}} \leq 1.0$ for the attended condition. For a given d' criterion level, suppose the threshold in the unattended condition is c_{uA}. It follows from Eqs. (A8b, c, e) that the threshold in the attended condition is $c_{\text{A}} = A_{\text{f}} c_{\text{uA}}$. Thus,

$$\frac{c_{\text{A}}}{c_{\text{uA}}} = A_{\text{f}}. \quad \text{(A13)}$$

The ratio between the thresholds in the attended and unattended conditions is determined by A_{f}, independent of the particular d' level in the region where external noise dominates and where external noise exclusion has its largest effects. In the region where additive internal noise dominates ($N_{\text{ext}} \ll N_{\text{a}}$), the factor A_{f} has no impact, and the ratio of performance does not depend on attention. This demonstrates the signature for external noise exclusion: an effect in high external noise but no effect in low or zero external noise.

A.3.3. Multiplicative noise reduction

Finally we consider the case in which multiplicative noise is the only mechanism of attention. We divide our discussion in two regions: the region where internal additive noise dominates external noise, and the region where external noise dominates internal noise.

When internal additive noise dominates ($N_a \gg N_{ext}$), we can ignore all the terms associated with $\mathbf{N_{ext}}$ and $\mathbf{N'_{ext}}$ in Eq. (A9a):

$$d' = \frac{(\beta_1^\gamma - \beta_2^\gamma)c^\gamma}{\sqrt{(A_m N_m)^2((\beta_1 c)^{2\gamma} + (\beta_2 C)^{2\gamma}) + N_a^2}}. \quad \text{(A14)}$$

Thus, for a given d' level, the threshold contrast c_τ can be solved from Eq. (A14):

$$c_\tau = \left(\frac{N_a^2}{(\beta_1^\gamma - \beta_2^\gamma)^2/d'^2 - (A_m N_m)^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})} \right)^{1/2\gamma}. \quad \text{(A15)}$$

Set $A_m = 1.0$ for the unattended condition and $A_m \leq 1.0$ for the attended condition. For a given d' criterion level, the threshold ratio between the attended and the unattended conditions is:

$$\frac{c_A}{c_{uA}} = \left(\frac{(\beta_1^\gamma - \beta_2^\gamma)^2/d'^2 - (\beta_1^{2\gamma} + \beta_2^{2\gamma})(N_m^2)}{(\beta_1^\gamma - \beta_2^\gamma)^2/d'^2 - (\beta_1^{2\gamma} + \beta_2^{2\gamma})(A_m N_m)^2} \right)^{1/2\gamma}. \quad \text{(A16)}$$

The threshold ratio in Eq. (A16) is a function of d'. In fact, the ratio is quite sensitive to d' in the range of parameters we have encountered (see Fig. A2c).

When external noise dominates internal additive noise ($N_a \ll N_{ext}$) we can ignore N_a in Eq. (A9a). To make the proof easier, we also ignore all the cross terms (if both $\beta_1 c$ and $\beta_2 c$ are much greater or smaller than N_{ext} and N'_{ext}, a condition that is normally met by our data, ignoring cross terms introduces very small errors). We thus have:

$$d' = \frac{(\beta_1^\gamma - \beta_2^\gamma)c^\gamma}{\sqrt{\mathrm{var}(\mathbf{N}_{ext}^\gamma - \mathbf{N}_{ext}'^\gamma) + (A_m N_m)^2((\beta_1 c)^{2\gamma} + (\beta_2 c)^{2\gamma} + 2N_{ext}^{2\gamma})}}. \quad \text{(A17)}$$

For a given d' level, the threshold contrast c_τ can be solved from Eq. (8f):

$$c_\tau = \left(\frac{N_a^2 + \mathrm{var}(\mathbf{N}_{ext}^\gamma - \mathbf{N}_{ext}'^\gamma) + 2(A_m N_m)^2 N_{ext}^{2\gamma}}{(\beta_1^\gamma - \beta_2^\gamma)^2/d'^2 - (\beta_1^{2\gamma} + \beta_2^{2\gamma})\,(A_m N_m)^2} \right)^{1/2\gamma}. \quad \text{(A18)}$$

Again, we can derive the threshold ratio between the attended condition and the unattended condition from Eq. (A18). The ratio is clearly a complicated function of d'. In fact, our simulation investigations of the stochastic PTM model showed that the ratio is very sensitive to d'.

A.3.4. Ratio tests for nonlinearity in the stochastic model

When additive internal noise dominates ($N_a \gg N_{ext}$), we can re-write Eq. (A15) to express threshold contrast levels at two d' criteria within the same attention condition:

$$c_1 = \left(\frac{N_a^2}{(\beta_1^\gamma - \beta_2^\gamma)^2/d_2'^2 - N_m^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})} \right)^{1/2\gamma}. \quad \text{(A15a)}$$

$$c_2 = \left(\frac{N_a^2}{(\beta_1^\gamma - \beta_2^\gamma)^2/d_2'^2 - N_m^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})} \right)^{1/2\gamma}. \quad \text{(A15b)}$$

Thus, within the same attention condition, when internal additive noise dominates, the ratio between thresholds at two criterion levels can be expressed as:

$$\frac{c_1}{c_2} = \left(\frac{(\beta_1^\gamma - \beta_2^\gamma)^2/d_2'^2 - N_m^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})}{(\beta_1^\gamma - \beta_2^\gamma)^2/d_1'^2 - N_m^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})} \right)^{1/2\gamma}. \quad \text{(A19)}$$

Similarly, from Eq. (A18), we can derive the threshold ratio between two criterion levels within one attention condition at high external noise levels:

$$\frac{c_1}{c_2} = \left(\frac{(\beta_1^\gamma - \beta_2^\gamma)^2/d_2'^2 - N_m^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})}{(\beta_1^\gamma - \beta_2^\gamma)^2/d_1'^2 - N_m^2(\beta_1^{2\gamma} + \beta_2^{2\gamma})} \right)^{1/2\gamma}. \quad \text{(A20)}$$

It is easy to notice that the right side of Eqs. (A19) and (A20) are identical. In other words, the ratio between thresholds at two d' criterion levels within a single attention condition depends on nonlinearity in the model and is the same at the two ends of the TVC functions.

A.3.5. Summary

To summarize, the ratio rules hold asymptotically in the end regions of the TVC functions in the stochastic model: the size of the attention effect (in terms of log threshold reduction) does not depend on the criterion d' level for stimulus enhancement in the region where internal additive noise dominates and stimulus enhancement has its largest effect; it does not depend on the criterion d' level for external noise exclusion when external noise dominates and external noise exclusion has its largest effect; it depends critically on the criterion d' level for multiplicative noise reduction in both the low and the high external noise regions. This corresponds to a replication of the key ratio tests for changes in nonlinearity and multiplicative noise. Furthermore, an analysis of these ratios shows that the ratio of attended to unattended conditions differs from one only in the low noise regions for stimulus enhancement, only in the high noise regions for external noise exclusion, and in a complex way in both regions for internal multiplicative noise reduction. This corresponds to a ratio formulation of the three performance signatures of the PTM model.

A.4. Relationship to the PTM model

As stated earlier, it is possible to combine Monte Carlo simulation with numerical methods to generate TVC functions from a given stochastic PTM model. Unfortunately, the extremely high computational load makes it impractical to fit the stochastic PTM model in its many nested model variants (e.g. various combinations of attention mechanisms) to experimental data.

The original PTM model was an analytical simplification of the stochastic PTM model. In approximating the stochastic PTM model with the analytic PTM model, we made two simplifications: (1) using the expectations of the random variables in place of the random variables; and (2) ignoring all the cross products.

The previous sections demonstrate that the stochastic PTM model exhibits all the key characteristics derived for the (analytic) PTM model. In general, the analytic PTM model is a close approximation to the stochastic PTM model, and provides a good approach to model testing: (1) The (analytic) PTM model fits all the data we have collected very well. (2) In the special case when $\gamma = 1.0$, the (analytic) PTM model is identical to the stochastic PTM model (up to a re-interpretation of β). (3) In the two extreme regions of the external noise manipulation, i.e. when internal additive noise dominates or when external noise dominates, the (analytic) PTM model approaches the stochastic model asymptotically.

In order to further validate these statements, we have undertaken (very computationally intensive) model fits of the stochastic PTM model to two data sets, one from Lu and Dosher (1999a) and the other from Dosher and Lu (2000) and compared these fits to the original fits of the analytic PTM model. The fits of the stochastic PTM model are less stable than those of the analytic PTM model for a number of reasons including the fact that the stochastic PTM invokes simulated variances attributed to external noise as one step in the nonlinear minimization process. Nonetheless, the two models fit the data approximately equally well. Although certain parameters such as β are simply different in the two forms (e.g. β absorbs dropped cross-products in the analytic PTM), key attentional parameter estimates are essentially identical in the stochastic and analytic form. For example, the key estimate of A_f for the data of Dosher and Lu (2000) on attentional precuing in a multi-location paradigm yielded identical estimates for the stochastic and the analytic PTM.

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