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Geometry, Analysis, and the Baptism of Slaves: John West in Scotland and Jamaica

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The achievements of the little-known Scottish mathematician, John West (1756–1817), deserve recognition: his *Elements of Mathematics* (1784) shows him to be a skilled expositor and innovative geometer while his manuscript, *Mathematical Treatises,* unpublished until 1838, reveal him also to be an accomplished exponent of "continental" analysis, familiar with works of Lagrange, Laplace, and Arbogast then little studied in Britain.

First an assistant at St. Andrews University in Scotland, West then worked in isolation in Jamaica, combining mathematics with the duties of an Anglican rector. His life and his pastoral and mathematical works are here described. © 1998 Academic Press

L'oeuvre du mathématicien écossais méconnu John West (1756–1817) mérite une plus grande renommée. Dans ses *Elements of Mathematics* (1784), il apparaît comme un commentateur expérimenté et un géomètre original. Son manuscrit *Mathematical Treatises,* qui ne fut publié qu'en 1838, le montre comme un interprète accompli de l'analyse "continentale" prônée par Lagrange, Laplace et Arbogast, alors peu étudiés en Grande Bretane.

West fut d'abord assistant à l'Université de St. Andrews en Écosse puis travailla, seul, à la Jamaïque, joignant aux mathématiques les devoirs d'un pasteur anglican. Sa vie, ainsi que ses travaux pastoraux et mathématiques sont décrits ici. © 1998 Academic Press

Τα κατορθώματα του πολύ λίγο γνωστού Σκωτσέζου μαθηματικού John West (1756–1817) αξίζουν αναγνώριση. Το έργο του, *Elements of Mathematics* (1784) δείχνει ότι είναι επιδέξιος εκθέτης και καινότομος γεομέτρης, ενώ το χειρόγραφό του *Mathematical Treatises,* ανέκδοτο ως το 1838, του αποκαλύπτει δόκιμο εκθέτη της "ηπειρωτικής" αναλύσεως και οικείο των έργων του Lagrange, Laplace, και Arbogast πού, εκείνη την εποχή, δεν ήταν πολύ μελετημένοι στη Βρεταννία.

Ο West άρχισε σαν βοηθός στο Πανεπιστήμιο του St. Andrews στη Σκωτία, μετά εργάστηκε απομονωμένος στη Jamaica, συνδυάζων τα μαθηματικά με τα καθήκοντα του Αγγλικανού κληρικού. Στη παρούσα ανακοίνωση περιγράφονται η ζωή του ως και τα ποιμενικά και μαθηματικά του έργα. © 1998 Academic Press

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1. INTRODUCTION

Thanks to the rather myopic history of William Rouse Ball [90], deliberately restricted in scope to mathematics at Cambridge, the Cambridge Analytical Society has traditionally been credited with the belated popularisation in Britain of continental analysis based on the differential notation of Leibniz, rather than on the inferior fluxional notation of Newton. Rouse Ball expresses a similar view in his general *A Short Account of the History of Mathematics:* though giving some credit to Cam-

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bridge's Robert Woodhouse and the Dundee-born "English mathematician" James Ivory, he concluded that, due to the Analytical Society, "[t]he employment of analytical methods spread from Cambridge over the rest of Britain, and by 1830 these methods had come into general use there" [91, 443].

Certainly, during the brief period 1812–1814, the Analytical Society, through the efforts of the young Charles Babbage, John Herschel, and George Peacock, propounded French analytical achievements [27], and these three enthusiasts collaborated to translate into English Silvestre Lacroix's *Traité élémentaire.* Thus, Carl B. Boyer accurately but misleading wrote that "[i]n 1816, as a result of the Society's inspiration, an English translation of Lacroix's one-volume *Calculus* was published, and within a few years British mathematicians were in a position to vie with their contemporaries on the Continent" [49, 583]. Worse, John F. Scott, writing of Lacroix's four-volume *Traité* which he had confused with the one-volume *Traité élémentaire,* alleged that "[i]ts importance lies in the fact that it was translated into English and inspired three Cambridge mathematicians, Peacock, Babbage, Herschel, to introduce the continental methods and notation into England" [93, 199].

Yet in 1830 neither Babbage nor Herschel felt that the state of English mathematics, and of science generally, had improved. Babbage, in his polemical *Reflections on the Decline of Science in England . . .*, quotes from Herschel's *Treatise of Sound* printed in the *Encyclopaedia Metropolitana* that "[h]ere, whole branches of continental discovery are unstudied, and indeed almost unknown, even by name. It is in vain to conceal the melancholy truth. We are fast dropping behind. In mathematics we have long since drawn the rein, and given over a hopeless race" [44, viii]. But, as Niccolò Guicciardini has cogently demonstrated, the Analytical Society members were not alone in appreciating the importance of the works of Lagrange, Laplace, Lacroix, Legendre, and Arbogast [65, 95–142]. Before them, Robert Woodhouse in Cambridge, John Brinkley and Bartholomew Lloyd in Dublin, Charles Hutton at the Royal Military Academy at Woolwich, James Ivory and William Wallace at the Royal Military College at Marlow (later Sandhurst), John Playfair at Edinburgh, and the solitary William Spence in Greenock had all been advocates of the French analysts.

Of the above, Playfair, Ivory, Wallace, and Spence were Scots, and both Playfair and Ivory received their early education at the University of St. Andrews. This paper focusses on yet another St. Andrews-educated Scot, John West (1756–1817), whose early grasp of French analysis and its applications to astronomy has gone unrecognised owing to the unusual circumstances of his life and to the late, posthumous publication of his most important work, which did not appear until 1838. While Spence worked in isolation in Greenock, without a post or other form of recognition for his efforts, the situation of West was even more remote from the academic mainstream. From 1784 to his death, he lived in the colony of Jamaica, then a major centre for the slave trade and sugar plantations, and notable more for vice and corruption than for any intellectual or cultural life. For most of this time, he served as rector of an Anglican parish, and he pursued mathematics as a recreation. He read to such effect that he acquired an intimate knowledge of works by Laplace, Lagrange, and Arbogast at a time when few in Britain had reached such a level. His posthumous treatises [106] show him to be a skilled expositor of this analysis and of its applications, able to bring an order and clarity to his account that is sometimes absent from the French originals. A similar clarity and originality of exposition is evident in his more elementary and geometrically based *Elements of Mathematics* [103] published in 1784, the year in which he left St. Andrews for Jamaica. That he was able to make the transition from skilled geometer to expert analyst, without direct stimulus or motivation apart from his own reading and curiosity in disadvantageous circumstances, is a remarkable achievement.

In the following, I describe what I have been able to discover of the life and work of John West, as both priest and mathematician. I also give an account of the circumstances surrounding the long-delayed publication of his treatises and of what little recognition they eventually received.

West's own publications are *Elements of Mathematics, for the Use of Schools* [103] and the brief *A System of Shorthand with Plain and Easy Directions for Writing It* [105], both published in 1784; also *Mathematical Treatises, Containing I. The Theory of Analytic Functions. II. Spherical Trigonometry, with Practical and Nautical Astronomy.... Edited (after the Author's Death) from his Mss. by the Late Sir John Leslie ... Accompanied by a Memoir of the Life and Writings of the Author by Edward Sang, F.R.S.E.* [106], which did not appear until 1838 (Fig. 1).[1] Sang's biographical sketch appears to have been prepared with access to few documentary sources and is probably based on recollections of conversations with, and perhaps notes by, John Leslie. Certainly, Sang (1805–1890) would not have met West. Not surprisingly, there are factual errors; but there is also information not recorded elsewhere.[2] West's time in St. Andrews is authoritatively described in the unpublished thesis of Alonso D. Roberts [89], which also outlines his later career and mathematical writings. R. A. Minter [87] briefly discusses West's life and pastoral work in Jamaica, identifying several manuscript sources. Other biographical information has come to light, from printed and manuscript sources, in the course of this study.

2. THE ST. ANDREWS YEARS (1769–1784)

John West's father, Samuel West (1723–1766), was Church of Scotland minister to the parish of Logie, in Fife, from 1751 until his early death. The parish and village of Logie lie about seven miles northwest of St. Andrews. Samuel West and his wife Margaret (née Mein) had four sons and five daughters, one daughter dying in infancy. John was born at Logie on April 10, 1756, their fourth child and second son [92].

[1] The *National Union Catalogue of pre-1956 Imprints* also lists *Elements of Conic Sections for the Use of Students in the Universities* [104]: this is a (pirated?) 1820 reprint of part of *Elements of Mathematics.* All of these works are now rare. Cambridge University Library has no work by West, but one copy of each of the *Elements* and *Treatises* are in College libraries. *Shorthand* is exceedingly rare, with no copy in the British Library but two in the National Library of Scotland.

[2] In the following, where there is disagreement with Sang, references to other sources are given.

a

ELEMENTS
OF
MATHEMATICS.

COMPREHENDING

GEOMETRY. ‖ MENSURATION.
CONIC SECTIONS. ‖ SPHERICS.

ILLUSTRATED WITH 30 COPPER-PLATES.

FOR THE USE OF SCHOOLS.

BY
JOHN WEST,
ASSISTANT TEACHER OF MATHEMATICS IN THE
UNIVERSITY OF S^T. ANDREWS.

EDINBURGH:
PRINTED FOR WILLIAM CREECH;
AND SOLD IN LONDON BY
T. LONGMAN AND T. CADELL.

M,DCC,LXXXIV.

FIG. 1. Title pages of John West's (a) *Elements of Mathematics* and (b) *Mathematical Treatises.*

The widowed Margaret West struggled to provide for the education of her family, with financial help from the presbytery clerk, a Dr. Adamson, who donated his small salary from that office. John West's elder brother, Stewart, matriculated at St. Andrews University in 1766,[3] and John followed him there in 1769 [3; 42]. John

[3] Stewart West graduated M.A. in 1770. Later, in 1786 and having gone to Jamaica, he was awarded a doctorate in medicine (M.D.) on the recommendation of local St. Andrews medical practitioners, Drs. Gillespie and Flint. Though John's younger brothers, Maurice and Samuel, do not appear in the St.

b

MATHEMATICAL TREATISES,

CONTAINING

I. THE THEORY OF ANALYTICAL FUNCTIONS.

II. SPHERICAL TRIGONOMETRY, WITH PRACTICAL AND NAUTICAL ASTRONOMY.

BY THE REV. JOHN WEST,

FORMERLY ASSISTANT TEACHER OF MATHEMATICS IN THE UNIVERSITY OF ST ANDREWS; THEREAFTER RECTOR OF ST THOMAS'S IN THE EAST, MORANT BAY, JAMAICA; AUTHOR OF A TREATISE ON THE ELEMENTS OF MATHEMATICS.

EDITED (AFTER THE AUTHOR'S DEATH) FROM HIS MSS.

BY THE LATE

SIR JOHN LESLIE,

PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF EDINBURGH.

ACCOMPANIED BY A MEMOIR OF THE LIFE AND WRITINGS OF THE AUTHOR, BY EDWARD SANG, F.R.S.E.

EDINBURGH:
OLIVER & BOYD, TWEEDDALE COURT, HIGH STREET;
AND SIMPKIN, MARSHALL & CO. LONDON.

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FIG. 1—*Continued*

subsequently won prizes in the two mathematics classes and in natural philosophy, provided by the Chancellor, the Earl of Kinnoull [2; 89, 28]. John, like most students at that time, did not graduate on completion of his course, thereby avoiding a fee.

John and his brothers, Stewart and Maurice, studied mathematics under professor

Andrews University Matriculation Roll, Maurice attended classes at least during the session 1772–1773, when he, like John, won the prize in the second mathematics class. (Information provided by Dr. R. N. Smart, St. Andrews University Library, from University records.)

Nicolas Vilant (d. 1807). But Vilant's health was poor and, over many years from 1775, he employed a series of assistants to help discharge his duties [89, Ch. 2–6]. According to Sang, West assisted in the teaching of mathematics from 1775 to 1780 "along with James Rennie, the celebrated engineer, or with Dr M'Donald of Kembach" [106, vi–vii].[4] When his mother died in January 1777, the 21-year-old John West became responsible for his sisters, as his brothers Stewart and Maurice had gone to Jamaica. In 1780, when he took whole charge of the mathematical classes,

> his salary was raised from £20 to two-thirds of the fees paid by the students, a change, however, which scarcely added ten pounds to his income. For four years longer he continued his labours at St. Andrews, but finding some difficulty, no doubt, in supporting the family with such a slender income, although augmented by fees from private pupils, and disappointed in his hopes of college preferment, he came to the resolution of following his brother to Jamaica. [106, vii]

Yet John West was a popular and successful teacher: "The affections of the students he possessed in an eminent degree ... he delivered his lectures in a plain but perspicuous language, which, if it did not attract the ear by the fineness of its elocution, improved the judgment by its accuracy" [106, vii–viii]. John Hunter, Professor of Humanity, also referred to West's "distinguished character and abilities" during a dispute in 1796 over employment of assistants in mathematics [2, May 12, 1796 (deleted); 89, 17–18 and 29].

Among West's students were James Ivory, John Leslie, and James Brown. Ivory, already mentioned, was perhaps the leading British mathematician of his day; Leslie later became Professor of Mathematics and then of Natural Philosophy in Edinburgh University; Brown succeeded West as assistant in mathematics, became minister of Dunino near St. Andrews, and for a time held the chair of natural philosophy at Glasgow University [89; 66].[5] An anonymous obituarist wrote that James Ivory applied himself to mathematics "under the able instruction of the Rev. John West.... It reflects equal credit upon the pupil and the instructor, that for this gentleman Mr. Ivory ever after entertained the highest regard" [19]. Surprisingly, John Leslie's friend and obituarist, McVey Napier, does not mention West, describing Leslie as having been sent to St. Andrews "to study mathematics under Professor Vilant"

[4] Though Roberts could find no confirmatory evidence of the employment of "Dr. M'Donald," he notes that James Macdonald studied Arts during 1767–1771 and Divinity during 1773–1777 winning prizes in Mathematics and perhaps Natural Philosophy, before being presented by the United College to the nearby parish of Kemback in 1780. Roberts also observes that "Rennie" is a mistake for James Glenie or Glennie (1750–1817), who was certainly an assistant then [89, 23–26]: the Edinburgh-educated "celebrated engineer" Rennie seems to have had no St. Andrews connections [97].

[5] Unable, or unwilling, to fulfill the duties of the latter post, Brown retired on a pension to St. Andrews. Many letters to Brown from his close friends Leslie, Ivory, and Thomas Chalmers survive in Edinburgh University Library [1]. A few of those from Leslie refer briefly to West, as noted below.

[79]. But another anonymous, and at times critical, obituarist (possibly William Wallace) wrote more fully that Leslie

> had the advantage of receiving the instruction of Mr. West, the author of an excellent elementary course of mathematics, a man of original and inventive genius, and, after Dr. Matthew Stewart, one of the greatest masters of the ancient geometry, whom Scotland has produced. From this meritorious individual, who has never had justice rendered to his talents, and who, perhaps from that ignorance of the arts of advancement which is so frequently the lot of the secluded student, never succeeded in surmounting the obstacles of an unfavourable position, Leslie received an impulse to which he owed, in a great degree, all his future success. [18, 215]

Thomas Chalmers, who had acted for a time as Vilant's assistant, before achieving fame as preacher, professor, and leader of the Disruption of the Church of Scotland [66], recorded his own high opinion of West in the preface to a book of sermons by the minister of Brechin Cathedral, Robert Coutts, another assistant to Vilant.[6] Chalmers wrote that West's "Treatise on Geometry has long been admired, both for its structure as a whole, and for the exceeding beauty of many of its demonstrations" [53, v; 66, 12; 85, 46]. Further confirmation of West's high reputation comes from the writer Thomas Carlyle, as recollected by his friend David Masson. In Masson's words, "[a] greater hero ... than even Leslie ... was the now totally-forgotten John West ... [whom Carlyle] regarded as, after Robert Simson of Glasgow, the most original geometrical genius that there had been in Scotland ... [but who] had emigrated in despair for some chaplaincy or the like; [Carlyle] would avow his belief that Leslie had derived some of his best ideas from that poor man ..." [84, 236–237]. Carlyle, a competent mathematician, had been taught by Leslie; among his lesser-known works is a translation of Legendre's *Géométrie,* commissioned and published by David Brewster [77].

A remarkable record of West's industry at St. Andrews survives in the borrowing records of the University Library [4], where every book borrowed by him is signed for and dated. Between 1769 and 1784, he borrowed 375 items, 110 of these during the years 1769 to 1775. He borrowed only three mathematical works during his first two years, October 1769 to September 1771: Norwood's *Trigonometry* and two volumes of Dodson's *Mathematical Repository.* During the next session, he borrowed Maclaurin's *Algebra,* the works of Archimedes and Clavius, Muller's *Fluxions,* Dodson's *Mathematical Repository,* Matthew Stewart's *Propositiones geometriae,* Malcolm and Hutton's *Arithmetic,* Gordon's *Counting House,* and Fletcher's *Universal Measurer.* His other reading at this time included Sallust, Shakespeare, Addison, Swift, Smollett's *Roderick Random,* and works on chemistry and music. From 1772 to 1775, his only mathematical borrowings were Newton's *Principia* and Cocking's *Arithmetic;* but he also borrowed Linnaeus on *Plants,* Vergil, the physicomedical works of Hoffman, Adam Ferguson's *Essay* (presumably that on

[6] Sang's claim [106, x] that West taught Chalmers is incorrect, for Chalmers matriculated at St. Andrews in 1791 and studied under Brown after West had left; but he certainly knew of West and studied from his *Elements of Mathematics.*

the history of civil society), Locke's *On Understanding,* the works of Pope and of Addison, volumes on surgery and midwifery, and several works on moral philosophy and religion.

The proportion of mathematical works among his borrowings increased markedly after he became an assistant in 1776. From 1776 to 1779, these included David Gregory's *Euclid;* Apollonius's *Conic Sections;* Clavius; treatises on fluxions by Ditton, Muller, and Maclaurin; Newton's *Quadrature of Curves;* Emerson's *Projection of the Sphere;* and Dodson's *Mathematical Repository.* Yet he still found time for works by Demosthenes, Xenophon, Pliny, and Vergil; many theological works and sermons; three volumes on shorthand; and books on foreign voyages.

West's own *A System of Shorthand* [105], a slim volume printed at West's own expense in 1784, dates from these years. Though surely, and probably mistakenly, undertaken for commercial gain, the work is characterised by his typical clarity and lack of rhetoric.[7] Later mathematical and physical borrowings (1779–1784) comprised De Moivre's *Miscellanea analytica,* Emerson's *Mechanics, Geometry and Miscellanies,* volumes 3 and 4 of Boyle's *Lectures,* Wallis's "*Mathema. vol. 1*" (presumably *Opera mathematica*), Newton's *Philosophical Discoveries,* "Wolf's *Op. Math. v. 1, 2* (perhaps Christian Wolff's *Elementa matheseos universae*), Simson's *Posthumous Works* and *Conic Sections,* Clark's *Laws of Chance,* Saunderson's *Algebra,* Oughtred's *Mathematics,* Dechales's *Cursus seu mundus mathematicus* vol. 1, "*Mathem.* de Caille" (presumably Lacaille's *Leçons élémentaires de mathématiques*), Bion on *Mathematical Instruments,* works on mensuration, gauging, and navigation.[8]

This is an impressive, not to say formidable, reading list which few young men of his day could have claimed to match. In view of West's subsequent development, it is interesting to note the near-complete absence of continental works of mathematics, the only exceptions being Christian Wolff (1679–1754), Claude F. M. Dechales (1621–1678), Nicolas-Louis de Lacaille (1713–1762), and Nicolas Bion (c. 1652–1733). In particular, it seems that West had read not a word of Leibniz, the Bernoullis, l'Hopital, Euler, d'Alembert, or any later continental writer, but this was the norm in Britain at this time.

3. WEST'S *ELEMENTS OF MATHEMATICS*

West's reputation among his contemporaries was exclusively as a geometer in the strong Scottish tradition of Robert Simson, Colin Maclaurin, Matthew Stewart, and John Playfair. It was his *Elements of Mathematics* [103] that earned him this

[7] A brief introduction describes the system as "an improvement of Angel's." This is followed by 10 short sections or lessons. A curiosity is West's choice of examples for practice: all rather pious or "improving" exhortations, presumably selected from a religious tract. One example will suffice here: "Since the days that are past, are gone for ever, and those that are to come, may not come for thee, it behoveth thee, O man! to improve the present time, without regretting that which is past, or too much depending upon that which is to come" [105, 10].

[8] Non-mathematical borrowings in these years included Smollett's *History of England,* the *Memoirs* of Cardinal de Retz, Lesage's *Gil Blas* (which influenced Smollett's *Roderick Random*), and *l'Analyse des échecs.*

reputation, and this was his only mathematical publication during his lifetime. West's textbook was much used both in St. Andrews (as two well-worn copies testify) and in Edinburgh, but it did not become popular in the English universities. It belongs to the mainstream of classical Euclidean geometry, avoiding alternative analytical methods. Yet, as a didactic work, it combines originality of presentation with clarity and rigour of exposition. Indeed, the young West might be thought arrogant in his aim, set out in the Preface, to outdo Euclid:

> My original intention was not to include the First Elements of Geometry ... that the Elements of Euclid might continue to serve the purpose which they had done for many ages. My design was only to build upon the foundation which that illustrious author had laid, and, under the several heads of 'Conic Sections', 'Mensuration', and 'Spherics', to complete a system of Geometry for the instruction of youth....
>
> ... As I proposed to establish it entirely upon geometrical principles, I could derive no assistance from some of the best authors on the subjects I treat, who have introduced both Algebra and Fluxions into their demonstrations ... [but] considering the improved state of mathematics, Euclid's Geometry is now inadequate and defective, as an elementary work....
>
> The doctrine of Proportion is perhaps the most important in mathematics, and as treated by Euclid, discovers great penetration and sagacity ... [yet] his manner of treating this difficult subject, is so obscure, as to render it almost unintelligible to the reader.... [This] determined me to attempt a new theory of proportion, and to introduce a new system of the Elements of Geometry as the first part of my work.
>
> ... I have continually kept in view the celebrated Ancient. Indeed, I have never departed from him, unless I could give more easy demonstrations, or could substitute more useful or more general theorems.... I reflect, with pleasure, that notwithstanding considerable additions ... the number of propositions is so much abridged, as to render the study of it a task of much less labour and difficulty. [103, iii–vii]

West's innovative approach entailed a considerable reordering of the propositions of Euclid, substitution of many new proofs, and relegation of some theorems as examples for the reader to try. This iconoclasm contrasts with Simson's earlier painstaking attempts to reconstruct Euclid's works in their original form freed from the later accretions of Theon and others [96] and also with Playfair's later elaborations [88] on Simson's text. But West realised the pitfalls:

> I am aware of an objection, arising from the conciseness of this work. It will be said, perhaps, that many propositions are left undemonstrated, and annexed as corollaries to others.... But I have to observe, that such propositions serve to sharpen the genius, and to exercise the invention, of youth; and that, considering how much they are accustomed to exercises of memory ... it is of the utmost importance to engage them now to exert the powers of the understanding.... Many students estimate the difficulty of their task by its length; they wish to continue to lay the burden on their memory; and they imagine, that to repeat is the same thing as to comprehend. That they may be induced, therefore, to apply all their powers of reason ... I have thought it requisite ... to leave such propositions, as easily follow from those that are demonstrated, without further evidence of their truth: For, if the evidence ... be explained, when necessary, by the teacher, they will be much sooner apprehended than in the more formal demonstration. [103, vii–viii]

These progressive views on mathematical education are without obvious Scottish precedent for a work of such sophistication, but they may have proved unpopular with teachers unable to supply the missing proofs! However, the work succeeds in its objects: the structure is rigorously logical; many propositions are given elegant and apparently original proofs that differ from Euclid's; and the style is clear and free of rhetoric. The section on proportion, a deliberate simplification of Euclid's, avoids all questions of commensurability and incommensurability.

Later, John Leslie adopted a strategy rather similar to West's in his *Elements of Geometry, Geometrical Analysis and Plane Trigonometry* of 1809 [78]. He, too, gave a radical reordering of Euclid's propositions and simplified the treatment of proportion [78, 145–173]. But Leslie was roundly and justly criticised by his Edinburgh colleague Playfair [38] for lack of care: Leslie had even managed to do without the parallel postulate or any equivalent axiom! In 1789, Leslie had written to Brown, "Send me Mr J. West's address.—I believe it may be in my power to draw the attention of the public to his book" [1, f.172], yet he made no mention of West's work in any edition of his own, despite copious notes. Despite marked differences in style—West clear and concise, Leslie "strained and ornate" (to use McVey Napier's phrase [79, p. 7 of *Memoir*])—Leslie's book was surely influenced by West's.

After his rearrangement of the "Elements of Geometry" including solid geometry, West continues with "Conic Sections," the first three "books" of which are respectively devoted to properties of the parabola, ellipse, and hyperbola. West defines the parabola in terms of focus and directrix, and the ellipse and hyperbola in terms of the constant sum and difference of distances from two foci. In the fourth book, he relates these to the Apollonian sections of an oblique cone. This treatment of conics follows fairly closely that of Robert Simson's Latin work [95], which also adhered to traditional geometrical methods.

Three books on "Mensuration" follow, broadly divided into measurement of distances, areas, and volumes. Four more on "Spherics" comprise spherical geometry, spherical trigonometry, and stereographic and orthographic projection. The latter display marked similarities with the short tract on *Elements of Plane and Spherical Trigonometry* that Simson annexed to the 1781 and later editions of his *Elements of Euclid* [96]. Mention of the uses of stereographic and orthographic projections in cartography is notably absent.

Of these last seven books, the three on "Mensuration" are more novel. In addition to providing a comprehensive and useful digest of standard geometrical theorems on lines, areas, and volumes, they give early indications of West's interest in efficient algorithms for accurately computing numerical quantities: a preoccupation notable in his later *Treatises* (see section 7).

Consider, for example, his estimate, early in Book 1, of π to six figures. This uses the proposition that "[t]he supplemental chord of an arch is a mean proportional between the radius, and the sum of the diameter and supplemental chord of double the arch" [103, 274]. Introducing convenient notation, the chord of an arch c_1, its "supplemental chord" s_1, and a diameter together form a right-angled triangle inscribed in a circle with radius taken as unity (see Fig. 2). On halving the angle opposite c_1, the proposition gives the corresponding c_2 and s_2 for the new angle from $1 : s_2 :: s_2 : 2 + s_1$. Similarly, successive bisections of the angle give supplemental

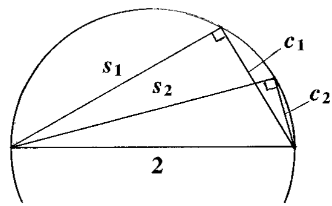


FIG. 2. Diagram for West's algorithm to calculate π.

chords that satisfy $s_{n+1} = (2 + s_n)^{1/2}$. Starting with an inscribed regular hexagon for which $c_1 = 1$ and $s_1 = 3^{1/2}$, seven more square-root extractions give $s_8 = 1.9999832669$. The chord c_9 of the angles of an inscribed polygon of $6.2^8 = 1536$ sides is thus $c_9 = (4 - (s_9)^2)^{1/2} = (2 - s_8)^{1/2} = 0.004090612$. The perimeters of inscribed and circumscribed polygons with 1536 sides are then easily found to be 6.283180032 and 6.283189248, respectively; these values bound 2π from below and above. Though the construction is just that of Archimedes' *Measurement of a Circle,* West's algorithm in terms of the "supplemental chords" is particularly compact and easy to implement (and better than one later used by Leslie [78, 228]).

Other examples follow. Starting with estimates of the sine and cosine of an angle of one minute, West gives procedures for finding "the sines of all arches from 1 minute to 90 degrees" [103, 278] and thence the tangents, secants, and versed sines (see section 9). Surprisingly, although he writes that "[o]n these principles may the trigonometrical canon be constructed" [103, 280] and gives a few theorems equivalent to trigonometric identities (such as $\sin 2x = 2 \sin x \cos x$), he nowhere states the standard identities involving trigonometric functions of the sums and differences of angles. Instead, he proceeds to practical examples on the use of chain, quadrant, square, and theodolite and to illustrations of the use of tables of logarithmic sines, secants, and tangents. He ends with a list of trigonometrical theorems (without proof) and practical problems.

The second book on mensuration, concerning areas, includes regular polygons up to the duodecagon inscribed in a circle, the ellipse, and segments and sectors of a parabola. There follows "an approximation to the area of any curve different from a Parabola" [103, 320], in effect Simpson's Rule for integration couched in geometrical language. Practical surveying and some numerical problems are also treated. The third book, on "solids and curve surfaces," treats the volumes of prisms, pyramids, the frustrum of a hemisphere, segments and sectors of a sphere, parabolic, elliptic, and hyperbolic conoids and their frustra. It also presents theorems on the surface area of segments and of frustra of a sphere. However, West does not give proofs of all these results.

West's reputation as a geometer was well deserved. His *Elements of Mathematics* is logically conceived and lucidly expressed. However, its strict geometrical focus would soon render it obsolete as a textbook for trigonometry and its applications. More powerful analytical tools were already available, and West himself would espouse them.

4. EMIGRATION

In 1784, the year in which both his *Elements of Mathematics* and *A System of Shorthand* were published in Edinburgh, West emigrated to Jamaica. His financial situation in St. Andrews was unbearable, and he must have wished to reunite his family.[9] Two sisters had already joined his brothers in Jamaica, and the two remaining sisters accompanied him, but one brother, Samuel, remains unaccounted for [106, xii]. This emigration of the West brothers is not at all surprising, given that their prospects of suitable professional employment in Scotland were poor. For centuries past, emigration, or temporary migration, had been a common option for enterprising Scots. In the aftermath of the unsuccessful 1745 Jacobite uprising, massive emigration, both willing and unwilling, took place to the colonies from both the highlands and the lowlands. In his detailed study of Scots' migration to Jamaica and the Chesapeake, Alan Karras [68] attempts to distinguish permanent emigrants from "sojourners." The former, he argues, were mainly agricultural labourers while the better-educated and sometimes richer sojourners went to the colonies in search of fortunes and with the intention of returning. The West brothers, well educated but without prospects, would surely have considered themselves in the latter category, but like many others they did not return to live in Britain.

The choice of Jamaica as their destination was almost certainly influenced by personal contacts, for large numbers of Scots were already established there. Karras [68, 10–11 and 123–130] estimates that in some parishes Scots comprised up to a third of the white population, many working as attorneys, estate managers, merchants, and physicians (see also [54, 40]). A few Scots were plantation owners (sometimes absentee), but "the surest way to wealth in Jamaica, at least according to Scottish belief, was to enter the island with a profession.... The Scottish practice of sending trained individuals abroad was almost certainly connected to the country's educational system and an economy that could not support everyone with a professional training" [68, 50]. Yet fortune could prove elusive and, even when gained, was difficult to transfer from the island; much notional wealth tied up in land, property, and slaves was hard to realise [68, 50–51].

Transport to and from Jamaica was certainly readily available. From the early 18th century, Greenock and Port Glasgow served as major ports for the mollasses trade and centres of sugar refining.[10] Figures for advertised sailings from Glasgow (i.e., Port Glasgow and Greenock) and Edinburgh (Leith) each year from 1746 to

[9] Sang recounts that in St. Andrews "he seems to have met with some sordid minds incapable of estimating his real worth," citing the case of a young nobleman, privately tutored by West, who offered so little that West chose "rather to be unpaid than to receive a pittance." Sang regrets "that in an evil hour for science, West buried his abilities in a society which, founded on the rupture of every moral principle, might check, but never could encourage, the development of physical truths" [106, x–xii].

[10] Impending closure of the last remaining refinery of this local industry was announced in 1996. Judith Grabiner emphasises the historic importance to the Scottish economy of the Jamaican molasses trade, in her description of the context and content of a memoir on the gauging of barrels written by Colin Maclaurin in 1735 for the Excise Commissioners at Port Glasgow [31].

1800 confirm the extent of the Jamaican connection [68, 32–35]. Moreover, although sailings to the American colonies virtually ceased during the Revolution, rising again only after independence was gained in 1783, regular sailings continued from Scotland to Jamaica and the other West Indies.

The West brothers' decisions to emigrate (or at least to "sojourn") in Jamaica seem quite natural. It was more unusual that their sisters accompanied them, for comparatively few women chose to go to Jamaica if it could be avoided.

5. LIFE IN JAMAICA

John West arrived in Jamaica at a bad time, for "loyalist refugees of all professions flocked to the island in the years after the American Revolution" [68, 56]. Together with a fellow Scot, he first tried, but failed, to establish a circulating library in Kingston. He then found employment as a teacher at Manning's Free School at Savannah-la-Mar in Westmoreland, but was soon recommended to the Governor of the Island as a suitable candidate for the Anglican priesthood [106; 87, 62]. The Governor, Alured Clarke, duly wrote to the Bishop of London on June 14, 1785, recommending John West as "a Gentleman of unblemished character and purity of manners" and requesting his admission into holy orders [7, XXIX, 180]. West sailed to London, receiving £20 to defray the cost of his return passage, for which he signed a bond [7, XXV, 215–216; 11]. He was ordained deacon on September 25, 1785, by the Archbishop of Canterbury, and was ordained priest on November 6 by the Bishop of Winchester [7, XXIX, 180; 7, XXXVIII, 66; 9, vol. 11; 56; 107]. A revealing testimonial for West, dated September 13, 1785, was received from James Gillespie, Principal of St. Mary's College, St. Andrews. His otherwise predictable commendation concludes, "... while he pursued his studies in this University, he preserved the most unblemished Moral character. His eminence is great in true literature, and not less in Real Virtue, which is hid but not diminished by an unassuming bashfulness which he possesses in an uncommon degree" [7, XXIX 182]. A "close retirement" mentioned by Sang [106, vii], West's "ignorance of the arts of advancement" recorded by Leslie's obituarist [18, 215], and an uncommon degree of "unassuming bashfulness" may help explain his inability to obtain in Scotland a post commensurate with his accomplishments. A prime requirement for advancement was (and to some extent still is!) powerful patronage, as the Brown Letters [1] make abundantly clear. The St. Andrews Colleges recommended many to a Church of Scotland parish (including James Brown himself), but West's "bashfulness" was probably thought too great an impediment for the pulpit, despite his acknowledged success as a teacher.

On his return, West resumed teaching at Manning's school in Savannah-la-Mar, and provisional appointment as headmaster was confirmed on April 8, 1788 [14; 78, 62; 10, CO 137 88]. Still, life in Jamaica was far from easy. Between 1770 and 1820, there were in the island "at least 17 earthquakes, 10 hurricanes, 4 droughts and famines and 9 great fires" [50, 4]. Moreover, recurrent outbreaks of yellow fever and cholera resulted in many deaths among both the free and slave populations. It was also a time of great political turmoil.[11] Savannah-la-Mar was among the hardest-hit by natural disaster: it was virtually destroyed by the hurricane of 1780 and had suffered five more by 1786. Its recovery was further threatened in 1789 by a decision of the Assembly to move the Law Court from there to Montego Bay. A petition from "the Inhabitants of the Parish of Westmoreland" was sent to the King, but without effect. Among the 300 or so signatories were John West and a Wm. West, who is perhaps a relative [10, CO 137 88].

West resigned from Manning's school in 1790 on appointment as rector of the parish of St. Mary's in the northeast of the island [87].[12] Around this time, he "married Anne Kelly, a young lady, with whom he had been acquainted in Westmoreland" [106, xiv]. It is possible that he briefly returned to England then, for John Leslie wrote to Brown on September 9, 1790, that he had seen "in a London Newspaper the name of Mr. John West in the list of passengers that have come home in the Jamaica pacquet—I have no doubt but it must be our old friend—I wish you could inform me of the reason; whether he only intends to visit London or to go to Scotland, how a letter would find him etc." [1, letter 133].

In 1791, West was appointed by the Governor, the Earl of Effingham, to the more desirable parish of St. Thomas in the East, with church and rectory at Morant Bay in the southeast corner of the island [15; 87, 166]. West remained there for the rest of his life, apart from one more brief visit to Britain. According to Sang, "[h]is clerical duties were regularly and strictly performed, yet he found sufficient time to prosecute his mathematical studies. Deprived, however, of the assistance of books, and wanting the companionship of men of science, his labours were often interrupted, to be relieved by the excitation of a game of chess, of which, as bearing some analogy to his favourite studies, he became very fond" [106, xiv].

West was a far from typical Jamaican rector, and the Church of England of that time hardly emerges with distinction.[13] The intellectual and moral quality of the clergy was generally low, some "detestable for their addiction to lewdness, drinking,

[11] From 1793 until 1805, Britain was at war with France, apart from the brief peace of 1802–1803, and this involved considerable action in the West Indian colonies. Jamaica had been threatened with a French–Spanish invasion in 1782, a French invasion was feared in 1802, and another was defeated in 1806. The collapse in 1792 of the neighbouring French colony of Saint Domingue (Haiti) and subsequent establishment of a free black state in 1804 added to the planters' anxiety over the likelihood of a major Jamaican slave rebellion (smaller uprisings, every 5 years or so, being brutally suppressed by strategically placed garrisons).

[12] His successor there, John Freeman, recorded that "[n]o register was kept by the late Rector Mr. West during his residence in this parish, viz from December 31 1789 to July 19 1791" [15]. West's lapse is perhaps excusable, for at that time the parish had no church building and no rectory [87].

[13] A substantial modern literature on the history, politics, sociology, and religion of the island cites many earlier sources: see particularly [50; 99; 67]. A comprehensive picture of the Anglican Church and its rectors in Jamaica during this period is given in [87; 57]. "[T]he Church itself was regarded as little more than a respectable and ornamental adjunct to the State, the survival of a harmless home institution which would cease to be tolerated if it showed any signs of energy or activity outside its own particular groove.... The most we can say is that the Church represented the religion of the white settlers and planters and officials; but it cannot claim to have been in any sense a missionary Church to the black labourers ..." [57, 41–42].

gambling and iniquity'' [57, 42]. Despite financial security, West encountered major challenges at St. Thomas in the East. His parish was larger than most, and travel was not easy. In his study of Creole society in Jamaica, Edward Braithwaite [50] estimates that, for every clergyman, there were about 1,500 white inhabitants and 15,000–18,000 total population, yet the size of most congregations on Sundays was typically very low. Prior to 1800, only 10 or 12 attended St. Thomas in the East [87, 183], and some churches failed to open at all!

From about 1789, missionary activity among the black and mixed-race population was undertaken by Wesleyan Methodists and other groups [99], a branch of the Methodist mission being established at Morant Bay by 1800. Unlike most of the rectors and the white population, West was not antagonistic to missionary endeavours on the island. He supported the (mostly unheeded) plea of the Bishop of London, Beilby Porteus, who in 1788 had asked that the clergy ''pay all the attention in their power to the conversion of slaves'' [87, 152].

This certainly brought West into direct conflict with Simon Taylor, a powerful and reactionary plantation owner and Assemblyman. One of the richest men in the Empire, he was custos, or chairman, of the Vestry of St. Thomas in the East and enforced the law in the parish. On December 17, 1802, the Jamaican Assembly passed an intolerant Act forbidding unlicensed persons from preaching to non-whites. Soon after, Daniel Campbell, Methodist missionary at Morant Bay, was imprisoned for one month and later denied permission to preach in St. Thomas in the East. This notorious case was publicised in Britain and led to suppression of the Act by the King in Privy Council. A letter from Campbell mentions that ''the Rev. Mr West, the clergyman at Morant Bay, had been obliged to leave the island on account of his health, and there was no licensed Minister, of any denomination of Christian whatsoever, at Morant Bay, nor within many miles thereof, when the Magistrates refused me permission to preach the Gospel ...'' [23]. Had he been there, West would surely have vigorously opposed these measures, for, in 1806, Campbell's successor Isaac Bradnack reported that

> [i]t gives me pleasure to say that the Rev^{d} John West Rector of this Parish and I are upon the most friendly terms. We have convers'd freely upon the necessity of plain sound Doctrin [sic], as being the only means of informing *the Ignorant, and reclaiming the wicked.* It hath been said by many he is turn'd a *Methodist Parson.* This no doubt *arises* from his great *familiarity with me.* I have the pleasure of hearing him often, and must say his Sermons are made up of *sound plain pointed Gospel language,* and his congregations have increased very much *of late.* [12, Bradnack's emphasis]

A further cause of friction was the strong prejudice against marriage between whites and free citizens of mixed race, graphically exemplified by the Rev. Richard Bickell, one-time naval chaplain and curate at Port Royal:

> In the parish of St. Thomas in the East also, a few years ago, a respectable man ... married a woman of colour, as privately as possible (by licence), for he and the rector (the late rector, Mr. West) well knew the deep-rooted prejudices of the Whites in the island ... and when it was ascertained that the marriage had taken place, he was not only shunned by his former friends and acquaintance, but was soon deprived of his situation, and as nearly ruined. The

> worthy rector also (for he was a very good man) incurred a good deal of odium, and they thwarted and injured him in all that they could.... [45, 224–227]

No letters survive from West to the Bishop of London, but there are several from 1818 onwards from his energetic successor, T. McCammon Trew, and from Trew's temporary replacement, Charles Cole. Some of these indicate the difficulties and opposition that West too had certainly encountered.[14] Regarding instruction of the slaves, Trew wondered, "what can a clergyman do? Paralysed in his efforts, he stands forth amidst a host of men who altho' they cannot be said to oppose, yet by their lukewarmness and dissent they disappoint his views, and hear him plead in vain for the instruction of his people" [8]. Nevertheless, there were some successes. A Vestry school in Morant Bay, attended by the children of poor whites and free coloureds, may well have been founded by West, and it thrived under Trew [87, 207; 10, CO 137 154; 45, 251]. By 1821, the parish had become the most progressive in the island. The eyewitness account of the Rev. Richard Bickell states that "more than a thousand Slaves regularly ... attend their church and chapels, and many of them, adults as well as children, are catechised every Sunday, a thing not regularly done, as I believe, in any other parish" [45, 76], and "[t]hey were certainly the best behaved, and most respectable in appearance of any slaves I saw in Jamaica" [45, 215].

From around 1810, rectors' incomes were grossly inflated by fees for marriages, baptisms, and funerals [98, 149–150]. The baptism and church membership of slaves was the subject of a circular from the Governor to all rectors in 1817, which resulted in perhaps the only extant letters from West, written not long before his death [10, CO 137 144]. The returns show that in most parishes huge numbers, increasing each year, were baptised; but most rectors are silent about membership of the Church. Some say that they refused the baptism fee or that it was paid by the owner, and others are (no doubt deliberately) vague on sums received. With a directness of style and content that contrasts with most other returns, West took the opportunity to air his views on the education of slaves, in a letter to the Governor's agent, William Bullock Esq., dated June 4, 1817. West acknowledges that

> I have received the Circular letter of the 26th Ult°, directed by his Grace the Governor to be sent to the several Clergymen of this Island, requesting Answers to the following Queries.
>
> What fee has been demanded for the baptism of Slaves? Whether has it been usually paid by the Master or by the Slave? And what numbers of Slaves have become Members of the Church of Eng.? To which I answer as follows.—
>
> the fee for the baptism of Slaves in this Parish (before the late Act of Assembly) was 6/8—But many paid less—1/8, 3/4, 5/- such a sum as they could easily afford: and from those whose appearance indicated poverty no fee was taken. It was seldom indeed that any one pleaded poverty, but when they did their plea was admitted.
>
> Masters have sometimes paid for their Slaves But in general the Slaves pay for themselves. Whenever I have been consulted respecting this point I have always advised that at least the

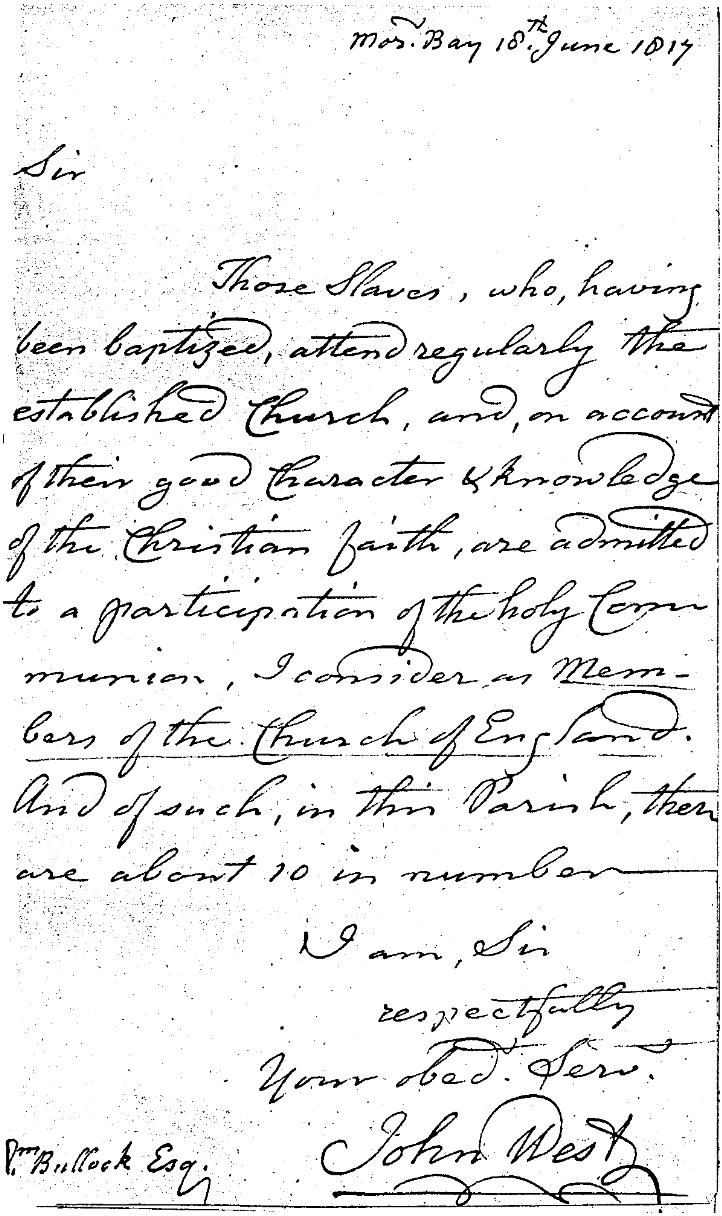
[14] The Records of the parish survive, in the Jamaica Archives, Spanish Town (and West recorded them, though he had not at St. Mary's) [15], but the Vestry Minutes were destroyed when the courthouse was burnt down during the Morant Bay Rebellion on October 11, 1865.

> *able* Adult Slaves should be allowed to pay for themselves and their children. Because I have not the least doubt of their ability, and because, tho' Negroes are ready to take whatever they can get from their Masters, yet they always value that *most* for which they pay their own money—
>
> About 200 black people attend the Church every Sunday—not exclusively however, for most of them attend also the Methodist Chapel in it's [sic] vicinity—some constantly, some transiently. And there are about 50 who have attained so much knowledge of Christianity as to be admitted to the holy Communion. But of these very few are slaves. The fact is, in respect to Slaves in general, that their knowledge of the English Language is so very limited, that they can derive little advantage from their attendance in Church. They are so conscious of their defect that when I go to Church for the express purpose of catechising them, very few will attend and not one of these will utter a word but what has been put in their mouths. How then it may be said are 26000 Slaves (the num. in this Parish) to be instructed? This subject has frequently engaged my thoughts And I cannot conceive any other mode than this. Let the young creole Slaves be taught to *speak* and to *read,* and at the same time be instructed in the first principles of the Christian Religion—in public schools established in different parts of the Parish—and let *them* communicate what instruction they have received in their own way to their African brethren, by whom it is impossible for white people to make themselves understood. [10, CO 137/144]

West wrote again on June 18, 1817: "Those Slaves, who, having been baptised, attend regularly the established Church, and, on account of their good Character and knowledge of the Christian faith, are admitted to a participation of the holy Communion, I consider as *Members of the Church of England.* And of such, in this Parish, there are about 10 in number" [10, CO 137/144] (Fig. 3). It is clear that, by insisting that the slaves themselves pay the fee, West was more interested in securing their real commitment to the Church than in maximising his income: this resulted in fewer baptisms than elsewhere, as was also true of marriages [87, 154].

John West died on October 17, 1817. In the Jamaica Archives, Spanish Town, there survives "An Inventory and appraisement of all and singular the goods and Chattels rights and Credits which were of John West late of the Parish of Saint Thomas in the East Rector Deceased" as shown to the appraisers by Dr. Stewart West, his administrator [16].[15] Total assets amounted to just over £10,000 Jamaican (equivalent to about two-thirds of that in pounds sterling), but included some bonds and "judgment debts" that were clearly unrealisable. Goods included "a set of Books £10.13.4," sadly unspecified, and a considerable quantity of rum and sherry. West's parishioners had a memorial plaque erected in the church; he was the first Jamaican rector so honoured. This church, at Church Corner, is now a ruin, having been abandoned about 1865. It was replaced by the present Morant Bay Parish Church, inside which West's plaque now hangs on the west wall (Fig. 4). This reads:

[15] Scanty information about West's brothers and their families is given in [21; 22; 26; 37; 112]. There may be more in Jamaican sources that I have not seen. Children of a John and Anne West of Jamaica are mentioned in [29; 30; 46; 86; 102]. An earlier draft of this paper, incorporating this and other peripheral material, is available from the author on request and has been deposited with the Manuscripts Department of St. Andrews University Library.


Mon. Bay 18th June 1817

Sir

Those Slaves, who, having been baptized, attend regularly the established Church, and, on account of their good Character & knowledge of the Christian faith, are admitted to a participation of the holy Communion, I consider as Members of the Church of England. And of such, in this Parish, there are about 10 in number —

I am, Sir
respectfully
Your obed.t Serv.t
John West

Wm Bullock Esq.

FIG. 3. Letter from John West, June 18, 1817. With kind permission of the Public Record Office, Kew.

Sacred to the Memory of
the Reverend JOHN WEST,
Upwards of twenty eight years Rector of this Parish;
His Parishioners
have caused this Monument to be erected,
As a Testimony
of their high sense of his Exemplary conduct
during the long period of his Ministry
And
of his many private Virtues:
Obiit 17th Octr. 1817, Aet. 61.

6. BOARD OF LONGITUDE MEMOIR

West's visit to Britain in 1803 was not only for the sake of his health. The Board of Longitude of the Admirality offered lucrative financial rewards for advances leading to the construction of improved astronomical tables and for the manufacture of more reliable timepieces, and West, like many other hopefuls, wrote a memoir for submission to the Board. Though he delivered it personally when in London, according to Sang "[h]e found the field, however, pre-occupied by Mendoza, and retired from his attempt with the indistinct feeling that all had not been conducted fairly,—a feeling which had been occasioned by some of those circumstances, which, from their very triflingness, are calculated to give the clearest view of the actions and intentions of man" [106, xv]. As John Leslie put it in a letter to James Brown, West "in his last visit to London ... did not get into the right channel" [1, letter 171].[16] The confirmed minutes of the Board for December 1, 1803, record

> A Memoir dated 30th Nov. and signed J. West. On the resolution of two problems in Nautical Astronomy. the one for determining the Latitude from two Altitudes of the Sun; and the other for clearing the distance of the Moon from the Sun or a Star, was read—The Board considered them meritorious altho' they nevertheless were of the opinion that they were inferior in point of conciseness to those of others; and therefore not the objects of their encouragement. [13][17]

The alleged lack of conciseness is surprising, for West's style is normally clear and plain. Perhaps the response was lukewarm because West's memoir did not directly address the calculation of longitude. The memoir does not survive; but sections of his second treatise, published in 1838, deal with similar topics (see Section 9 below).

[16] Yet, according to Sang, "In London, he enjoyed with high relish the society of scientific men, and became so fond of it, that he seriously contemplated his permanent removal thither ... [but] either found it impracticable or had forgot his resolution" [106, xv].

José de Mendoza y Rios (1762–1816) had recently been awarded a sum for completion and publication of extended tables for calculating longitude. His *A Complete Collection of Tables for Navigation and Nautical Astronomy* (London: Bentley, 1801), was extended and improved in editions of 1805 and 1809.

[17] Surprisingly, I have found no mention of West or his memoir in Sir Joseph Banks's papers, which include draft minutes of the Board's meetings [6], nor among the many detailed reports on such submissions drafted by the Astronomer Royal, Neville Maskelyne [13].

FIG. 4. (a) The ruined church at Church Corner, Morant Bay, Jamaica, where West preached; (b) plaque in memory of West, now situated in Morant Bay Parish Church. (Photographs taken by the author.)

7. THE POSTHUMOUS *MATHEMATICAL TREATISES* (1838)

West's first treatise is entitled *Introduction to the Theory of Analytical Functions,* a clear and no doubt deliberate echo of Lagrange's *Théorie des fonctions analytiques* [72] of 1797. Had this treatise been published promptly, it could have played a major role in popularising continental analysis in Britain. Its account of the differential and integral calculus stands comparison with the 1816 English translation of Lacroix's *Traité élémentaire* [71] made by Babbage, Herschel, and Peacock, and with Wallace's 1815 *Fluxions* article in the *Edinburgh Encyclopaedia* [40] (accurately described as "the first complete English treatise on the calculus written in differential notation"

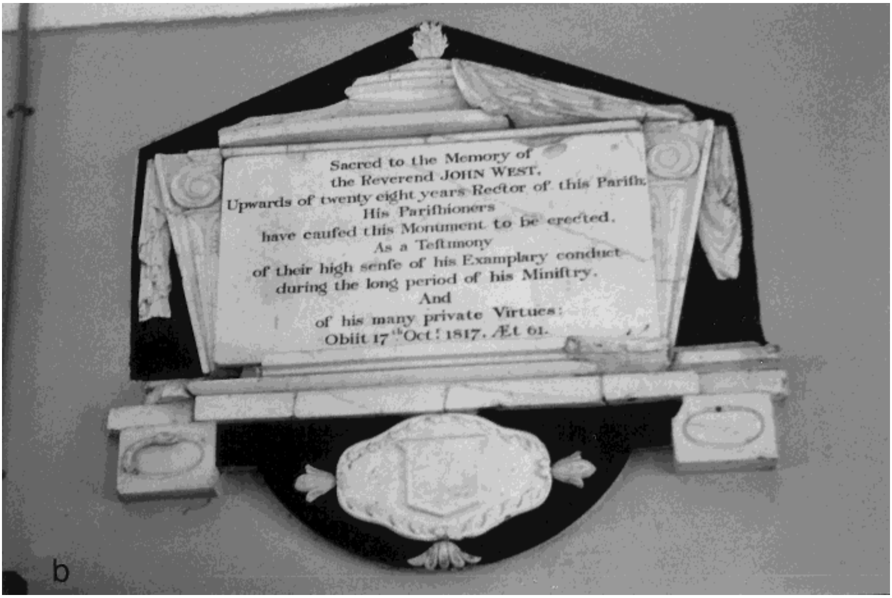


FIG. 4—*Continued*

[65, 120]). But West's first treatise is more advanced than both in giving a lucid account of Arbogast's 1800 *Calcul des dérivations* [43].

In contrast, the second treatise is more practical in slant, covering plane and spherical trigonometry, construction of trigonometric and logarithmic tables, and nautical and astronomical problems. Its approach is largely analytical, but with a few geometrical digressions. Particularly notable are the expert discussion of trigonometric series and their ingenious application to yield highly accurate numerical values. None of this is covered in Lagrange's *Théorie,* and much seems to derive directly from Euler. Though results in the first treatise are cited, it is likely that a version of the second treatise preceded the writing of the first, for not much depends on differential or integral calculus, nor on the methods of Arbogast. The sections on spherical trigonometry and on astronomical problems appear particularly early in date and are connected with West's lost Board of Longitude memoir of 1803.

We cannot know how West came by his books, for no evidence exists. It is plausible that he obtained copies of Lagrange, Laplace, and Arbogast when he visited London in 1803; Leslie or Ivory could have sent books to him; he may have ordered books from a London bookseller or commissioned a visitor to England or France. It is even possible, though unlikely, that despite French–British hostilities he received French books via Saint Domingue (Haiti): until the colony's collapse in 1792 this had an active scientific society, the *Cercle des Philadelphes,* supported by the French Government [81]. But the works of the key authors mentioned above were all published after that date. Even in Britain, French works were not always available: for instance, John Leslie wrote in 1811 from London that "[a]s for French publications, they absolutely cannot be got. Quantities are lying at Ostend ready to be imported by the first licence" [15].[18] West may have seen copies of Leybourn's *Mathematical Repository,* to which Ivory and Wallace were for a time regular contributors: certainly, he had read Dodson's earlier *Repository* when in St. Andrews, but again firm evidence is lacking. It is tempting to speculate that, by correspondence that has not survived, Ivory and West influenced each other at a time when few in Britain (and none in Jamaica) were abreast of either. But there is no evidence that West corresponded with *any* other mathematician.

What is sure is that West effected a remarkable transition from the skilled geometer of his youth to an equally proficient expositor of the most recent advances in analysis. If, as seems likely, he acquired this proficiency soon after his 1803 visit to London, stimulated by newly acquired books from France, then the analytical parts of West's treatises may well date from before 1810. If so, he would perhaps have been the most advanced British analyst of that time, ahead even of Ivory, with his first treatise the earliest work in English on the subject apart from Woodhouse's 1803 *The Principles of Analytical Calculation* [108] (likewise pioneering, but with a rather different focus). The second treatise displays a mastery of series of trigonometric functions and their use in numerical calculations that far surpasses in sophistication the corresponding trigonometric accounts of Woodhouse's influential *A Treatise on Plane and Spherical Trigonometry* [109] and the 1824 English translation of Legendre's *Éléments de géométrie* [77]. Only Euler's immortal *Introductio ad analysin infinitorum* [58] is of a similar level. Even if the more advanced analytical sections were not composed until shortly before West's death in 1817 (which I consider less likely), his treatises still rank among the first such studies in English. Whether he attempted to find a publisher is not known; but, given his situation, this would have been difficult without a strong supporter. Prompt publication after his death would surely have brought due, if posthumous, recognition of his considerable and isolated achievement.

Why was West so receptive to the "new analysis" when many able geometers and "fluxionists" were resistant to change? It is possible that his St. Andrews professor Nicolas Vilant—later maligned for teaching his classes by proxy—imparted to the young John West his own enthusiasm for "analysis." Vilant's own book, bearing the title *The Elements of Mathematical Analysis, Abridged for the Use of Students* [100], attempts an axiomatic approach to arithmetic and algebra.[19] It is certainly unsuitable for beginners and includes examples of binomial expansion, reversion of series in the style of Newton, and the use of series for numerical approximation. Vilant also shows a concern for limits, citing Landen's "Residual Analysis" and Newton's "ultimate ratios." His manuscript notebooks in St. Andrews

[18] Though Leslie mentions West in a few of his letters to Brown, none from Ivory does so, and it is uncertain whether either Leslie or Ivory corresponded with their former teacher.

[19] The St. Andrews copy is undated; N.U.C. gives just one entry, of 1798, but this seems to be a later edition.

University Library contain an expanded version of the same, with further sections on trigonometry and fluxionary calculus. Though fairly unoriginal, Vilant's writing, like West's, is notably clear and concise. West would have been likely to see a preliminary version, and it may well have struck a responsive chord and convinced him of the growing importance of "analysis." Nor does it seem mere coincidence that James Ivory and John Leslie, both taught by West and Vilant, were early advocates of continental analysis—though Leslie, unlike Ivory, was never to become an able practitioner—and that James Glenie, assistant to Vilant along with West, attempted to develop his own *Antecedental Calculus* [62; 65, 104].

8. THE FIRST TREATISE

The first treatise consists of two parts: "Derivation," made up of 21 sections [106, 1: 1–180], and "Integration," comprising 14 sections [106, 1: 181–318]. The first 87 pages concern differentiation and its applications, mostly expressed in a Lagrange-like notation and having obvious similarities with parts of Lagrange's *Théorie* [72]. For instance, much like Lagrange, West employs expansions

$$f(x+i) = fx + iP = fx + ip + i^2Q = \cdots = fx + ip + i^2q + i^3r + i^4s + \cdots$$

and connects $p, q, r, s, \ldots$ with the derivative functions $f', f'', f''', f^{iv}, \ldots$ as

$$p = f'x, \qquad q = (1/2)f''x, \qquad r = (1/(2\cdot 3))f'''x, \qquad s = (1/(2\cdot 3\cdot 4))f^{iv}x, \ldots,$$

like Lagrange, omitting brackets around the argument x of the functions $f(x)$, etc. (see also [64; 48]).

In his ninth section, West gives an interesting account of Taylor's series with remainder in Lagrangian form. In contrast, no mention is made of the remainder term in either Wallace's *Encyclopaedia* article [40] or Lacroix's *Traité élémentaire* [70; 71], while Lacroix's large *Traité* [69, 2nd ed., 1: 380–386] mentions it only briefly and inconspicuously. Lagrange's original version is rather tortuous [72, Ch. VI], involving expansions

$$fx = f(x-xz) + xP, \qquad fx = f(x-xz) + xzf'(x-xz) + x^2Q, \ldots,$$

where $P, Q, \ldots$ are functions of both x and z. After manipulation, he shows that the z-derivatives $P', Q', \ldots$ of $P, Q, \ldots$ are

$$P' = f'(x-xz), \qquad Q' = zf''(x-xz), \ldots,$$

and so $Q = (P - zP')/x$, along with similar results for the next-order functions R, S. Replacing x successively by the $n+1$ points $a, a+i, a+2i, \ldots,$ of an interval $[a, b]$, where $i = (b-a)/n$ is sufficiently small, he eventually obtains his "théorème nouveau et remarquable par sa simplicité et sa géneralité," [72, 2nd ed., 83] that

$$\begin{aligned} fx &= f. + xf'u \\ &= f. + xf'. + (x^2/2)f''u \\ &= f. + xf'. + (x^2/2)f''. + (x^3/2\cdot 3)f'''u, \text{etc.,} \end{aligned}$$

where u is some unknown value between 0 and x.

Lagrange's demonstration in Leçon IX of [73] is far clearer. There, he first sets

$$f(x+i) - f(x) = i[f'(x) + V],$$

choosing i sufficiently small that $-D < V < D$, where D is a given quantity as small as he wishes. Consideration of the intervals between the set of points x, $x + i, x + 2i, \ldots, x + ni$ leads to upper and lower bounds on $f(x + ni) - f(x)$ of

$$if'(x) + if'(x+i) + \cdots + if'[x + (n-1)i] \pm niD,$$

where all derivatives f' are assumed to have the same sign, and niD may be taken smaller than the modulus of the sum of all the preceding terms. It readily follows that $f(x + ni) - f(x)$ is bounded between 0 and $2niP$, where P is the greatest (positive or negative) value of the derivatives in the sum. Using this result, Lagrange goes on to establish his first-, second-, and third-derivative forms of the remainder. He had no doubt of the significance of his results: "La détermination de ces limites est surtout d'une grande importance dans l'application de la Théorie des fonctions à l'Analyse des courbes et à la Mécanique, pour pouvoir donner à cette application la rigueur de l'ancienne Géométrie ..." [73, *Oeuvres* **10:** 85–86].[20]

West's account is rather different from both. First restricting attention to functions with all derivatives positive, he compares the infinite Taylor expansion of $f(x + i)$ with the n-term Taylor expansion plus a postulated remainder $(i^n/n!)f^{(n)}(x + u)$ (i.e., Lagrange's form of the remainder), where the remainder is itself expanded as a Taylor series in u. Term-by-term comparison, for *any* finite positive increment i such that the series converge, shows that "u denotes some quantity greater than $i/(n + 1)$ and less than i" [106, 1: 57]. (A similar line is followed by Lacroix [69, 2nd ed., 1: 380–386]: it is possible, but far from certain, that West had seen this.) Moreover, when i is so small that each term of the Taylor expansion is greater than the sum of those succeeding it, u is bounded more closely by $i/(n + 1) < u < 2i/(n + 1)$, so that "in general, the nearest approximation is $i/(n + 1)$ when we make the series terminate at the nth derivative" [106, 1: 57].

West deals similarly with the expansion of $f(x - i)$ when the derivatives alternate in sign. He then treats functions with derivatives of arbitrary sign, but restricted (like Lagrange's versions) to sufficiently small increments i, deducing that u must lie between 0 and i. Unlike Lagrange, West does not take the trouble to consider sequences of such intervals between $x + i, x + 2i, \ldots$. West's bent for practical calculations is apparent in his concluding remark that "[i]n all cases,

$$Q = \frac{i^n}{1 \cdot 2 \cdots n} f^N\left(x + \frac{i}{n+1}\right)$$

gives a near approximation to the value of [the remainder] R; the first two terms in each being the same, and R being greater or less than Q, according as $i^{n+2}f^{N+2}x$

[20] On Lagrange's remainder, see also Judith V. Grabiner, The Origins of Cauchy's Theory of the Derivative, *Historia Mathematica* **5** (1978), 379–419.

is affirmative or negative'' [106, 1: 60]. I have not found any corresponding remark in Lagrange's writings: perhaps this result deserves to be called ''West's approximate form of the remainder''?

On differentiation, on limits using l'Hopital's rule, and on maxima and minima, West's notation may be Lagrange's, but many examples come from Euler's *Institutiones calculi differentialis* [59]. A notably complicated example of Euler's on limits, requiring four differentiations of numerator and denominator, is repeated by several later authors, including Lacroix [69; 70; 71]. West could also have consulted Charles Bossut's *Traités de calcul différential et de calcul intégral* [47]; but Bossut's work and that of Jacques Cousin [52], though covering some topics in greater detail than West's, seem rambling and unfocussed by comparison. There are no clear signs that West had read the works of Lacroix, Woodhouse [108], or any later writer.

The exposition of differentiation continues with a discussion based on Arbogast's *Calcul des dérivations* of 1800 [43] (though Arbogast's name is nowhere mentioned) [106, 1: 88–168]. This is the most novel part of the first treatise and is discussed in some detail below. West's account of ''derivation'' concludes with two brief sections [106, 1: 169–180]. The first concerns series solution of equations of the form $z = a + bf(z)$, as $z = a + Ab + Bb^2 + \cdots$, where $f(z)$ is a known function and A, B, ... are functions of a. Two consequent theorems are credited to Lagrange [72, Ch. XV], but West employs the method of Arbogast to derive them more quickly. Two of West's four examples are attributed to Laplace's *Mécanique céleste* [75, 1: 179–180]. The final section, on the expression of ''any function of a quantity by a series in terms of any other function of the same and of some arbitrary quantity'' [106, 1: 178–180], again employs Arbogast's method.

The second part of the first treatise [106, 1: 181–318], entitled ''Integration; or the inverse method of analysis,'' introduces integration as the inverse of differentiation; that is, it deals with ''finding the primitive of a given prime function.'' This, he says, ''is more difficult, as well as more extensively useful, than the direct'' method of finding the prime function from the primitive [106, 1: 183–184]. Though rules may be found by ''reversing the preceding operations ..., in most cases, recourse must be made to methods of approximation'' [106, 1: 184]. Throughout, West uses the notation $\int y' = y$ and $\int x^m x' = x^{m+1}/(m + 1) + c$, which resembles the hybrid notation common in Britain about this time, with its fluxion-like y' and an integral sign appearing together as $\int y' = y$. (The first use of ''pure'' Leibnizian notation in the Cambridge Tripos was not until 1817, due to George Peacock [55, 40; 27].) Though both $\dot{y}$ and West's y' or x' might be interpreted as differentials dy or dx, this conflicts with the definition of prime function (or fluxion) as a derivative, and not the differential. A more consistent interpretation is therefore to regard West's integral sign as equivalent to the operator $\int(\ )dt$, where t is an implicit Newtonian time-like variable such that y' is dy/dt, even if t may nowhere appear in the solution. This notation is certainly neither Euler's nor Lagrange's; indeed, Lagrange rarely uses the integral sign in his *Théorie* or *Leçons,* and when he does so it is always in the usual Leibnizian form $\int Ydx$. Integral signs are also absent, of course, from older British books on fluxions written in Newtonian notation. Even the late 1818 edition of Samuel Vince's *Fluxions* [101] has none except, apparently by accident, in one of his "miscellaneous problems."

West derives or states most standard integrals and then gives applications to "quadrature of curves, and cubature of solids" [106, 1: 193] and to the "rectification of curve lines, and quadrature of curve surfaces" [106, 1: 201]. There are again similarities with Lagrange's *Théorie,*[21] but Lagrange gives few examples, and West seems to have drawn many of his from Euler's *Institutiones calculi integralis* [60]. Digressions on partial fractions and complex numbers are followed by derivation of the series [106, 1: 221]

$$\tfrac{1}{2}z = \sin z - \tfrac{1}{2}\sin 2z + \tfrac{1}{3}\sin 3z - \tfrac{1}{4}\sin 4z + \cdots,$$

and solution of $\tan t/2 = \{(1 - c)/(1 + c)\}^{1/2} \tan x/2$ as the series [106, 1: 222–224]

$$t = x - \tfrac{2}{1}\lambda \sin x + \tfrac{2}{2}\lambda^2 \sin 2x - \tfrac{2}{3}\lambda^3 \sin 3x + \cdots,$$

where $\lambda = [1 - (1 - c^2)^{1/2}]/c$. West then factorises the trinomial $(x^{2n} - 2r^n x^n \cos z + r^{2n})$, the binomial $(x^n \pm r^n)$, and related expressions [106, 1: 224–229]. These examples show West's confidence in handling complex quantities at a time when some still denied their validity in analysis. But his derivation of the sine-series for $z/2$ is unacceptable, even allowing for the absence of any statement of the range of z-values for which it holds. For, to derive it, he substitutes $v = \exp[z(-1)^{1/2}]$ into the series [106, 1: 222]

$$\log v = (v - v^{-1}) - \tfrac{1}{2}(v^2 - v^{-2}) + \tfrac{1}{3}(v^3 - v^{-3}) - \tfrac{1}{4}(v^4 - v^{-4}) + \cdots,$$

earlier obtained from the difference of the logarithmic series for $\log(1 + v)$ and $\log(1 + v^{-1})$. Such a lack of regard for convergence is characteristic of this and earlier times. With all his numerical expertise, West must surely have realized that this series for $\log v$ is impossible for all real v but ± 1, yet he was content to perform formal manipulations, as Euler had often done before, and in this case it gives a correct result.

The method of Arbogast is next combined with (the unnamed) de Moivre's theorem to find the expansion of $(1 - \beta \cos z)^{-2}$ in terms of the form $a_n \cos nz$. Similar expansions in trigonometric functions occur frequently in the researches of Laplace and later of Ivory and, of course, Joseph Fourier. Other series expansions are extensively treated in the second treatise, discussed below.

Further examples of integration follow, the most difficult [106, 1: 245–247] occurring also in Euler's treatise on integration [60]. Integration by parts is fully discussed, including generation of series expansions by repeated application [106, 1: 249–257]. Then "the general method of approximation by infinite series" is treated, whereby the integrand is expanded and integrated term by term [106, 1: 258–273]. Here again there are marked similarities with Euler's work, as well as connections with Maclaurin's 1742 *Treatise of Fluxions* [80, 2: Ch. 2]. One example

[21] Compare, for example, West's [106, 1: 194, 202] and Lagrange's [72, 155–156, 158–159].

well illustrates West's fluency with series expansion: the integral of $(1 - x^2)^{-1/2}$ from $x = 0$ to $x = 1$ (i.e., $\pi/2$), yields by direct expansion "the well-known series"

$$1 + \frac{1}{2 \cdot 3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} + \cdots,$$

which is "not sufficiently convergent for use" [106, 1: 279]. Instead, repeated integration by parts, based upon the general result derived for the integral of $(a - bx^n)^p x^m$ gives a series with much improved convergence, correct to five decimal places after only four terms [106, 1: 280]. The section concludes with a discussion of summation of series when the general term is known.

The first treatise closes with a discussion of first-order differential equations, and this has an unfinished look [106, 1: 295–318]. West presents 16 worked examples of equations of different sorts, without any other comment. In particular, he does not describe the now standard classification of equations that can be integrated in closed form, though several of these are represented. Some examples are fairly elementary, requiring only a change of variable; others are less so. His sixth example contains a rare mathematical lapse (though there are quite a few typographical errors). There, an equation of Clairaut type (but not identified as such and expressed in disguised form) is solved in the standard manner by first differentiating.[22] But West misses the general solution and includes a spurious arbitrary constant in his singular solution [106, 1: 298–299]. The same example, though with different nomenclature, appears in Vince's *Treatise of Fluxions* [101, 5th ed., 225] and Vince, too, omits the general solution.

Examples 9, 10, and 11 [106, 1: 300–303] resemble those of Lagrange's *Théorie* [72, Ch. XI] and are treated similarly, among them

$$x'/F(x) = y'/F(y), \qquad F(s) = (a + bs + cs^2 + ds^3 + es^4)^{1/2}, \text{ and}$$

$$x'(m + ny + cy^2)^{1/2} - y'(a + bx + cx^2)^{1/2} = 0,$$

which are not best solved by separation of variables and integration. Example 12 outlines the method of solution for linear first-order equations and applies it to $ay' - y = cx^2$ (a, c being constants); then example 13 solves this same equation by assuming $y(x)$ is an infinite series, finding the coefficients, and summing the series. The next example also employs series solution, obtaining the answer in two different forms [106, 1: 304–307].

The last three examples are applications to astronomy, a topic explored in more detail in the second treatise. One involves finding the "true anomaly" $t(x)$ in terms of the "eccentric" x, from the equation

$$t' + (c \cos x)t' - (1 - c^2)^{1/2} = 0 \qquad (c \text{ constant}),$$

the solution being the same series expansion for t in terms of x and $\sin nx$ as that

[22] Clairaut's form is $y = x(dy/dx) + F(dy/dx)$. West's example 6 may be put in this form, with $F(u) = -au^2(1 + u)^{-1}$.

given above. In the final example, which is far from trivial, West finds the "mean or true anomaly" m from the equation

$$(1 - c\cos t)^2 m' = (1 - c^2)^{3/2} t',$$

where t' may be taken as unity since m is regarded as a function of t. Eventually, m is found as a power series in t; then this is reversed, giving t as a power series in m [106, 1: 308–311].

West's desire for solutions in power series and in series of sine functions at first appears strange, since both the above equations are separable and can be integrated, by substitution, in terms of inverse tangents. But West was well aware of this, having earlier calculated the same series for t from the trigonometric relation $\tan t/2 = \{(1 - c)/(1 + c)\}^{1/2} \tan x/2$ as described above. Clearly, he did not consider a solution in terms of inverse tangents as satisfactory, since sufficiently accurate tables did not exist; his series expansions, on the other hand, yield results to any desired accuracy. West's familiarity with real astronomical calculations, rather than mere manipulations, is also clear, for he adds that "[t]he former series converges quickly when the planet is near the aphelion; and the latter, when it is near the perihelion" [106, 1: 311]. Citing "La Lande's Astronomy" [74; 106, 1: 312], he gives a practical illustration—that of finding the true anomaly of Mercury when the mean anomaly is 45°—calculated to seven significant figures. He then concludes the treatise with six pages of discussion of how this approximation can be made as accurate as desired, by employing Arbogast's techniques.

9. ARBOGAST'S THEORY AND WEST'S FIRST TREATISE

As noted above, Arbogast's *Calcul des dérivations* of 1800 [43] provided the basis for the most novel and interesting section of West's first treatise [106, 1: 88–168]. From Alsace, Arbogast was not part of the Parisian intellectual mainstream.[23] His book deals mainly with procedures for deriving the polynomial expansion of functions of polynomials, a subject described in Lacroix's large *Traité* as one "dont on s'occupe encore fort peu en France" [69, 2nd ed., 1: 315]. Arbogast's account is long, highly technical, and at times repetitious. Lacroix commented on "des nombreuses notations qu'Arbogast a employées" [69, 2nd ed., 1: 247–248], a feature also deplored by Leslie [79, 601–602] (see below). But Arbogast's aim was ambitious; he viewed his work as a major extension of the calculus to embrace functions of polynomials of a variable x, rather than merely functions of x itself. As he put it in his preface,

> On s'est proposé dans cet ouvrage d'offrir un genre de calcul qui embrasse la théorie des suites, et dont le calcul différential n'est qu'un cas particulier: il fournit des procédés qui abrègent des opérations laborieuses . . .: appliqué à une assez grande variété d'objects, il m'a conduit directement, et le plus souvent sans peine, à des résultats, dont plusieurs me paroissent nouveaux, et d'autres présentés sous un aspect nouveau. . . .

[23] See [64, 211–217] on Arbogast and his later French followers, especially Jacques Frédéric Français and François Joseph Servois.

> Le calcul différentiel donne avec tant de facilité le développement en série des fonctions de binomes, qu'il est naturel de désirer une méthode qui s'étende avec la même facilité à des fonctions de polynomes d'un nombre de termes quelconque. [43, i–ii]

Arbogast proceeds to derive various rules to facilitate the construction of polynomial expansions, in both one and several independent variables. For this, he introduces his own notations for the various coefficients, including an operator D to denote differentiation and another operator $D.$ for what may be termed the "polynomial derivative." Thus, a function ϕ of a single polynomial $\alpha + \beta x + \gamma x^2 + \cdots$ has a series expansion of the form

$$\phi(\alpha + \beta x + \gamma x^2 + \delta x^3 + \cdots) = \phi(\alpha) + (D.\phi\alpha)x + (1/2)(D^2.\phi\alpha)x^2 + (1/2\cdot 3)(D^3.\phi\alpha)x^3 + \cdots,$$

where the "polynomial derivatives" $D^r.\phi\alpha$ in the coefficients of x^r are ever more complicated functions of β, γ, δ, ... and the various (ordinary) derivatives $D^j\phi\alpha$ with $j \leq r$. Since evaluation of the coefficients by application of Taylor's theorem and multiplication of the various powers of the polynomial $\beta x + \gamma x^2 + \cdots$ would quickly become impractical, Arbogast aims to find a better way. For convenience, he incorporates the factorial coefficients into a new set of symbols, $D^r_c.\phi\alpha$ (his c is placed directly below the D), defined as $(D^r.\phi\alpha)/r!$ and sets about formulating a sort of calculus of coefficients. This is achieved by first expressing the rth polynomial derivative as a sum of terms of the form $D^{r-j}_c\phi\alpha.D^j_c.\beta^{r-j}$, where j takes the values 0 to $r - 1$. For example,

$$D^7_c.\phi\alpha = D^7_c\phi\alpha.\beta^7 + D^6_c\phi\alpha.D.\beta^6 + D^5_c\phi\alpha.D^2_c.\beta^5 + \cdots + D^2_c\phi\alpha.D^5_c.\beta^2 + D\phi\alpha.D^6_c.\beta.$$

Here, β is shorthand for the polynomial $\beta + \gamma x + \delta x^2 + \cdots$ obtained by deleting α from the original polynomial and dividing by x, and care must be taken to distinguish the operators D^r_c and $D^r_c.$ and also dots $\cdot$ that signify multiplication. All operations on $\phi\alpha$ and β are, of course, evaluated at $x = 0$. Clearly, $D^r_c.\beta$ is just the rth Greek letter occurring after β; but the polynomial derivatives of higher powers of β take some effort to calculate. Arbogast evolves a set of rules for their efficient determination, whereby the next can readily be calculated from its predecessors. Key results useful in numerical calculations are summarised in a table [43, 28–29] (see below). He later goes on to investigate the extension of his results to include operators $D^r_c.\beta^j$ with negative index r corresponding to integrations; to elaborate the expansion of functions and products of functions of more than one polynomial; and to develop the expansion of functions of a polynomial in two or more variables. Though very original, Arbogast's work suffers from repetition, lack of clarity, and poor organisation. Despite his claims of great utility, there is a shortage of numerical illustrations of the power of his procedures.

West's account of Arbogast's methods is restricted to expansions of functions of polynomials in a single independent variable and it employs a very different notation. West denotes his "polynomial derivatives" of a function $\phi(\alpha + \beta z + \gamma z^2 + \delta z^3 + \cdots)$ by the notation $|\phi\alpha/1|'$, $|\phi\alpha/1|''$, $|\phi\alpha/1|'''$, ... in place of Arbogast's $D.\phi\alpha$, $D^2.\phi\alpha$, $D^3.\phi\alpha$, ... To avoid writing the factorial denominators that regularly appear, he uses $|\phi\alpha/.|'$, $|\phi\alpha/.|''$, $|\phi\alpha/.|'''$, ... for $|\phi\alpha/1|'$, $|\phi\alpha/1\cdot 2|''$, $|\phi\alpha/1\cdot 2\cdot 3|'''$, ..., instead of Arbogast's $D_c.\phi\alpha$, $D_c^2.\phi\alpha$, $D_c^3.\phi\alpha$, ... Like both Arbogast and Lagrange in his *Théorie,* West as usual omits brackets from $\phi(\alpha)$, writing it as $\phi\alpha$. Thus,

$$\phi(\alpha + \beta z + \gamma z^2 + \delta z^3 + \cdots) = \phi\alpha + \left|\frac{\phi\alpha}{\cdot}\right|' z + \left|\frac{\phi\alpha}{\cdot}\right|'' z^2 + \left|\frac{\phi\alpha}{\cdot}\right|''' z^3 + \cdots.$$

Essentially, West follows Arbogast, but he is both clearer and more concise, using notation that eliminates possible confusion between Arbogast's D and $D.$ operators. West's table [106, 1: 105–106], from which the first nine coefficients $|\phi\alpha/.|^{(n)}$ may be found, is reproduced in Fig. 5. This shows the constants that multiply the various derivative terms $\phi^{(r)}(\alpha)/r!$ (i.e., Arbogast's $D_c^r\phi\alpha$), where r runs from 1 to n, in the expansions of each $|\phi\alpha/.|^{(n)}$ (i.e., Arbogast's $D_c^n.\phi\alpha$: see the above expansion for $D_c^7.\phi\alpha$, for example). For instance, using the table, one obtains

$$|\phi\alpha/.|^{\text{iv}} = \phi'\alpha.\varepsilon + (1/2)\phi''\alpha.(2\beta\delta + \gamma^2) + (1/2\cdot 3)\phi'''\alpha.3\beta^2\gamma + (1/2\cdot 3\cdot 4)\phi^{\text{iv}}\alpha.\beta^4.$$

West's table is rather more compact than Arbogast's, but Arbogast gives one more term. This calculus of polynomial derivatives, though complicated and unfamiliar at first sight, nevertheless supplies rules which reduce tedious manipulations and so speed the calculation of high-order approximations in numerical examples.

Unlike Arbogast, West regularly motivates the reader by practical illustrations. Thus, after only five pages of his account, he follows his derivation of the expansion of $\log(\alpha + \beta z + \gamma z^2 + \delta z^3 + \varepsilon z^4 + \cdots)$ with numerical examples. He first uses the fact that $\log(10/3) = \log(3 + 3/10 + 3/100 + 3/1000 + \cdots)$, for which the expansion immediately gives $2 \log 3 = \log 10 - 1/10 - 1/200 - 1/3000 - 1/40{,}000 - \cdots$ on taking $z = 1/10$ and $\alpha = \beta = \gamma = \cdots = 3$ [106, 1: 93]. (Confirmation of this result using the table is left as an instructive exercise for the reader.)

Arbogast's methods may also be applied to sum "recurring series" [106, 1: 121–134]. Let A_m denote the $(m + 1)$th term of such a series, where a series of, say, third order has a recurrence relation

$$\alpha A_m + \beta A_{m+1} + \gamma A_{m+2} + \delta A_{m+3} = 0$$

(with obvious generalisation to series of nth order). If the A_m are identified with $|A/.|^{(m)}$, the mth polynomial derivative of the given series A, then the above relation is equivalent to $|\alpha A/.|^{(m)}$, where α abbreviates the four-term polynomial $\alpha + \beta x + \gamma x^2 + \delta x^3$. Consequently, $|\alpha A/.|^{(m)}$ denotes the coefficient of x^m in the expansion of the product αA of this polynomial and the given series. Moreover, because of the recurrence relation, only the first three terms of this product are nonzero, as $a + bx + cx^2$, where a, b, and c are easily found. It follows that the given series equals $(a + bx + cx^2)/(\alpha + \beta x + \gamma x^2 + \delta x^3)$. Series of nth order can be summed

TABLE II.

$$\left|\frac{\varphi\alpha}{\cdot}\right|' \ldots \beta.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|'' \ldots \gamma,\ \beta^2.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|''' \ldots \delta,\ 2\beta\gamma,\ \beta^3.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|^{\text{IV}} \ldots \varepsilon,\ (2\beta\delta+\gamma^2),\ 3\beta^2\gamma,\ \beta^4.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|^{\text{V}} \ldots \zeta,\ (2\beta\varepsilon+2\gamma\delta),\ (3\beta^2\delta+3\beta\gamma^2),\ 4\beta^3\gamma,\ \beta^5.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|^{\text{VI}} \ldots \eta,\ (2\beta\zeta+2\gamma\varepsilon+\delta^2),\ (3\beta^2\varepsilon+6\beta\gamma\delta+\gamma^3),\ (4\beta^3\delta+6\beta^2\gamma^2),\ 5\beta^4\gamma,\ \beta^6.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|^{\text{VII}} \ldots \theta,\ (2\beta\eta+2\gamma\zeta+2\delta\varepsilon),\ (3\beta^2\zeta+6\beta\gamma\varepsilon+3\beta\delta^2+3\gamma^2\delta),\ (4\beta^3\varepsilon+12\beta^2\gamma\delta+4\beta\gamma^3),(5\beta^4\delta+10\beta^3\gamma^2),6\beta^5\gamma,\ \beta^7.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|^{\text{VIII}} \ldots \iota,\ (2\beta\theta+2\gamma\eta+2\delta\zeta+\varepsilon^2),\ (3\beta^2\eta+6\beta\gamma\zeta+6\beta\delta\varepsilon+3\gamma^2\varepsilon+3\gamma\delta^2),\ (4\beta^3\zeta+12\beta^2\gamma\varepsilon+6\beta^2\delta^2+12\beta\gamma^2\delta+\gamma^4),\ (5\beta^4\varepsilon+20\beta^3\gamma\delta+10\beta^2\gamma^3),\ (6\beta^5\delta+15\beta^4\gamma^2),\ 7\beta^6\gamma,\ \beta^8.$$

$$\left|\frac{\varphi\alpha}{\cdot}\right|^{\text{IX}} \ldots \varkappa,\ (2\beta\iota+2\gamma\theta+2\delta\eta+2\varepsilon\zeta),\ (3\beta^2\theta+6\beta\gamma\eta+6\beta\delta\zeta+3\beta\varepsilon^2+3\gamma^2\zeta+6\gamma\delta\varepsilon+\delta^3),\ (4\beta^3\eta+12\beta^2\gamma\zeta+12\beta^2\delta\varepsilon+12\beta\gamma^2\varepsilon+12\beta\gamma\delta^2+4\gamma^3\delta),\ (5\beta^4\zeta+20\beta^3\gamma\varepsilon+10\beta^3\delta^2+30\beta^2\gamma\delta^2+5\beta\gamma^4),\ (6\beta^5\varepsilon+30\beta^4\gamma\delta+20\beta^3\gamma^3),\ (7\beta^6\delta+21\beta^5\gamma^2),\ 8\beta^7\gamma,\ \beta^9.$$

&c. &c.

FIG. 5. West's table [106, 1: 105–106], giving coefficients of terms for polynomial derivatives.

similarly. West gives several worked examples of this process, including for example the series with constant fourth differences [106, 1: 131–132],

$$1 - 15x^2 + 70x^4 - 210x^6 + 495x^8 - 1001x^{10} + \cdots = \frac{1 - 10x^2 + 5x^4}{(1 + x^2)^5}.$$

Another useful application is to the reversion of series. If $z = \alpha x + \beta x^2 + \gamma x^3 + \delta x^4 + \cdots$, then it can be shown that

$$x = \alpha^{-1} z + \frac{1}{2}\left|\frac{\alpha^{-2}}{.}\right|' z^2 + \frac{1}{3}\left|\frac{\alpha^{-3}}{.}\right|'' z^3 + \frac{1}{4}\left|\frac{\alpha^{-4}}{.}\right|''' z^4 + \cdots,$$

where the α^{-r} within the polynomial derivatives are short for $(\alpha + \beta x + \gamma x^2 + \cdots)^{-r}$ [106, 1: 157–159]. This may be generalised to find the z-expansion of a function $f(a + x)$, given $z(x)$; for example, $z = x(\alpha + \beta x + \gamma x^2 + \cdots)^m$. West gives several worked examples of these procedures, only a few of which are in Arbogast's account [106, 1: 160–168].

Though there are naturally very many similarities, West's version is no mere recapitulation of Arbogast's; it is a thoughtful and well-ordered reworking, restricted to functions of a single variable, of the rather rambling original. Both authors also derive many technical results connecting the various polynomial derivatives; some of West's, especially [106, 1: 135–141], I have been unable to trace in Arbogast.

Though few at this time in Britain seem to have known Arbogast's work, one exception is Thomas Knight of Papcastle, a regular contributor to Leybourn's *Mathematical Repository.* In two letters of 1809 [32, 67–70], Knight dealt with "the expansion of any Function of a Multinomial," mentioned Arbogast's "very learned work," but pursued "a method ... quite different from his" [32, 67]. Two years later, his long paper on this theme appeared in the *Philosophical Transactions of the Royal Society,* supposedly containing "many new and remarkable theorems ... which could not, I imagine, be very easily found from M. Arbogast's methods" [33, 49]. But Knight's work is marred by a singularly horrible notation of his own devising which makes it virtually unreadable. It is unlikely that West saw it and even less likely that he would have learned much from it.

10. THE SECOND TREATISE

West's second treatise comprises three distinct sections. The first deals with the properties of trigonometric functions and logarithms, with much about finite and infinite series and the construction of accurate numerical tables [106, 2: 1–90]. The second concerns spherical trigonometry, including numerical illustrations to high accuracy [106, 2: 91–161]. Finally, the third is devoted to 28 astronomical problems, employing the preceding theory [106, 2: 163–256]. Since the first section is the most noteworthy, discussion of it will be reserved for last.

The section "Spherical Trigonometry" begins with definitions, many of which agree verbatim with those of West's *Elements of Mathematics* [103]. However, the geometric language of the latter work is here mostly abandoned and replaced by a long list of trigonometrical theorems for general spherical triangles. Only the first of these receives detailed proof; for the remainder, reference numbers to helpful previous results are given, several corresponding to propositions proved in *Elements of Mathematics.* West discusses "Napier's practical rule"; right-angled spherical triangles; the surface area of spherical triangles; and estimation of errors, or small quantities, such as when "four parts of a spherical triangle ... vary in magnitude, while the other two remain fixed" [106, 2: 119]. Stereographic projection and Legendre's theorem relating spherical and plane triangles are also treated. Typically, numerical examples involve calculations to seven decimal places and a discussion of accuracy.

Though this section displays West's mastery of spherical trigonometry, it looks old-fashioned when compared with the corresponding part of Robert Woodhouse's 1809 *A Treatise of Plane and Spherical Trigonometry* [109]. In particular, West displays most of his results in "proportional" notation, with only a few intrusive equality signs, suggesting an early date of composition. The reverse is true of West's account of plane trigonometry, and particularly of series involving trigonometric functions, which is far more sophisticated than Woodhouse's.

The final section, "Astronomical Problems," also largely employs "proportional" notation. These most probably include versions of the Board of Longitude problems which West submitted in 1803. Being merely a collection of 26 worked problems on various aspects of practical astronomy, this account naturally does not compare, as a didactic work, with Woodhouse's *An Elementary Treatise on Astronomy* [110]. Yet the problems are a good representative sample of the various types of calculation required of a practising astronomer. The first 10 problems are direct applications of spherical trigonometry to astronomical observations; 11 concern the various corrections that must be made to clear observations from the effects of parallax and refraction, and to take into account aberration of light, precession, and nutation of the Earth; and the last 5 are calculations involving eclipses.

The section begins with astronomical "Definitions," inserted by the Editor (presumably Leslie), as West gives none [106, 2: 163–167]. There is a tantalising connection between Problem V, "To find the day and duration of the shortest twilight at a given place" [106, 2: 176], and the article "Problems Relating to the Twilight of Shortest Duration," by "Astronomicus," which appeared in the first volume of Leybourn's *Mathematical Repository* [17].

The problems submitted by West to the Board of Longitude in 1803 are described in the Board minutes (see Section 6) as "one for determining the Latitude from two Altitudes of the Sun; and the other for clearing the distance of the Moon from the Sun or a Star" [13]. In West's second treatise, both his problems I and X consider the former, with three separate methods given in the 6-page solution to Problem X [106, 2: 187–193]. Problem XI and the 8-page problem XII treat the influence of parallax on the moon's apparent position, while the 10-page problem XIII and problem XIV consider the effect of parallax on the apparent (angular) distance between sun and moon and the moon and a star [106, 2: 193–214]. Problems XV and XVI consider the influence of the Earth's obliquity on the apparent position of stars [106, 2: 214–221]. There seems little doubt that these are versions of West's Board of Longitude submission. Finally, problems XXIII–XXVII on the computation of solar and lunar eclipses [106, 2: 234–256] are noteworthy in view of Sang's later interest in such calculations (see Section 11).

The first, and most interesting, section of West's second treatise quickly disposes of the standard trigonometric identities (notably absent from his earlier *Elements of Mathematics*) and some results on series illustrating the binomial coefficients. Expressions for sin na, cos na, and tan na in series of powers of sin a, cos a, and tan a are followed by those for integer powers of sin a and cos a as series of sines and cosines of the "multiple arcs" na. West then demonstrates the usefulness of these results by obtaining $(1 - 2\varepsilon \cos z)^n$ as an expansion in $\cos mz$ ($m = 1, 2, \ldots$) [106, 2: 26]. Much of this is in Euler's *Introductio* [58], but little is in either Lagrange's *Théorie* [72] or his *Leçons* [73]. A few results not in Euler's *Introductio* include

$$\tfrac{1}{2}\log\cot(a/2) = \cos a + \tfrac{1}{3}\cos 3a + \tfrac{1}{5}\cos 5a + \tfrac{1}{7}\cos 7a + \cdots,$$

$$\frac{4\sin a}{5 + 4\cos a} = \sin a - \tfrac{1}{2}\sin 2a + \tfrac{1}{4}\sin 3a - \tfrac{1}{8}\sin 4a + \cdots.$$

West does not derive the second, rightly saying that it may be deduced "without having recourse to any other principles than such as are here explained" [106, 2: 35, note]. They can also readily be derived from results in Euler's *Institutiones calculi integralis* [60, 1: *Opera* 181], the latter by differentiating the special case $n = 4/5$ of Euler's series for $\log(1 + n\cos\phi)$. Surprisingly, this series is not listed in the standard modern compilation by Gradshteyn and Ryzhik [63]. A less-convincing remark by West [106, 2: 34, note] explains an apparent paradox by invoking the "result" (also perpetrated by Euler) that $1/2 = 1 - 1 + 1 - 1 + 1 - \cdots$.

West moves on ingeniously to use his result that

$$\sin na = 2\sin a\,[\tfrac{1}{2} + \cos 2a + \cos 4a + \cos 6a + \cdots + \cos(n-1)a],$$

where n is odd [106, 2: 29], to show that "the seventeenth part of the circumference of a circle can be determined geometrically" [106, 2: 36–38]. This, of course, relates to the famous proof in Gauss's *Disquisitiones arithmeticae* of 1801 that the only regular polygons with a prime number of sides p that can be inscribed within a circle are those with $p - 1$ equal to 2^{2^r}, r being a positive integer. Gauss's own demonstration of the case $p = 17$ [61, sect. VII: art. 354] does not greatly resemble West's, but a very similar proof is given in a volume of Leybourn's *Mathematical Repository* to which James Ivory contributed shortly after he moved to Marlow. Its "Notices Relating to Mathematics" mentions the discovery of "Mr. Gauss of Brunswick" and adds that "Mr. Legendre has given to the French National Institute a demonstration of this very curious proposition, in the case when the number of sides is 17. It is founded on these two lemmas ..." [17, 77–78]. Though this volume is dated 1806, the first number seems to have been issued in late 1803 (since solutions to posed problems were requested by February 1, 1804), which is about the time when West was in London.

With $n = 17$ and a equal to "the seventeenth part of the circumference to the radius 1," $\sin na$ is zero and so West's series of cosines above equals $-\frac{1}{2}$. This sum of cosines is divided into two parts.

$$x = \cos 2a + \cos 4a + \cos 8a + \cos 16a,$$

$$y = \cos 6a + \cos 10a + \cos 12a + \cos 14a,$$

such that $x + y = -\frac{1}{2}$, and it is further shown that $xy = -1$. Hence both x and y can be found geometrically. Further partitions of x, y and their parts can similarly be found geometrically, ultimately yielding $\cos 16a$, which is just $\cos a$, from which the requisite arc follows. Based on this idea, West also gives the full geometrical construction, in just a few lines [106, 2: 38].

West follows this with the power series for sines, cosines, tangents, cotangents, secants, and versed sines. Then, by invoking general formulae for reversion of series, he expresses the arcs (or angles) as series involving sines, tangents, and versed sines [106, 2: 38–47]. Such series are then employed, with great fluency, to obtain accurate estimates of π and to illustrate the construction of a table of sines.

To evaluate π, arcs are first decomposed into smaller arcs using the result that "if $c^2 + 1 = mn$, then $1/(c + m)$ and $1/(c + n)$ are the tangents of two arcs, of which $1/c$ is the tangent of their sum" [106, 2: 49–50]. This form of the familiar result for the tangent of a sum of angles is particularly suited to finding angle pairs with simple numerical values for their tangents. West states several useful results for 45° obtained in this way. For instance,

$$
\begin{aligned}
45^\circ &= 3\arctan(1/7) + 2\arctan(2/11) && \text{(i)}\\
&= 8\arctan(1/10) - 4\arctan(1/515) - \arctan(1/239) && \text{(ii)}\\
&= 3\arctan(1/4) + \arctan(1/20) + \arctan(1/1985), && \text{(iii)}
\end{aligned}
$$

but West uses the notation ${}^{n}\overline{|a/b}$ to denote $n \arctan a/b$ [106, 2: 50–52]. The smaller arcs may then be obtained from the series (in fact Gregory's formula, unattributed)

$$a = t - \tfrac{1}{3}t^3 + \tfrac{1}{5}t^5 - \tfrac{1}{7}t^7 + \tfrac{1}{9}t^9 - \cdots,$$

where a is the angle (in radians) and t its known tangent. West shows that this converges quite rapidly, giving π to 20 decimal places from result (ii); but he then observes that the series

$$a = \frac{t}{1 + t^2} + \frac{\frac{2}{3}t^3}{(1 + t^2)^2} + \frac{\frac{2\cdot 4}{3\cdot 5}t^5}{(1 + t^2)^3} + \cdots$$

converges even more rapidly, and he illustrates it for result (i) [106, 2: 52].

There had already been a long history of such calculations. This is exhaustively and exhaustingly set out in the six volumes of Baron Francis Maseres's *Scriptores logarithmici* [83] published over the period 1791–1807. For example, John Machin's formula, known before 1706, is just $\pi/4 = 4\arctan(1/5) - \arctan(1/239)$ (a result also noted by West), allied with Gregory's formula for arctan. This had been used by many to calculate π. In 1741, for example, Abraham Sharp gave it to 73 places. Sharp's tract, first published with Henry Sherwin's Mathematical Tables [94], was reprinted by Maseres [83, 3: 99–154]. Euler, too [58, arts. 140–141], had recommended the arctan series for finding angles, but had stated only that for $\pi/4 = \arctan(1/2) + \arctan(1/3)$, which does not converge particularly rapidly. Euler nevertheless gave π to 126 decimal places. Other such arctan formulae are noted by Maseres [83, 3: 185–204], including five sent to him by the Rev. John Hellins. None of Hellins's results is stated by West, and the most useful of West's results are not to be found in Maseres's volumes. Though it is probable that West had seen Sharp's by then old-fashioned tract in an edition of Sherwin's Tables, it seems unlikely that he had seen any of Maseres's thick tomes. If he had, he surely would have been repelled by their eccentric and excessive verbosity. On the other hand, West certainly possessed Jean Callet's tables of logarithms to 20 places [51] (see below). Callet provided a substantial introduction on the compilation of logarithmic and trigonometric tables and on their applications, on which West may have drawn for ideas: but again their are no great similarities of detail.

The later history of π calculations includes William Shanks's 1853 calculation to 530 decimal places by Machin's formula and the 1940s extension to 810 decimals by D. F. Ferguson [28], independently checked by John Wrench and Levi Smith [41]. Ferguson wrote that "[i]n May, 1944, a colleague of mine ..., Mr. R. W. Morris, produced a series for the evaluation of π which we believe to be original"; Ferguson then used this in his calculations. Unknown to them, this "original" series is none other than one of West's results, that labelled (iii) above!

West's numerical fluency is also apparent in his description of how to construct an accurate table of sines [106, 2: 52–60]. West's brevity contrasts with the ludicrous prolixity of a tract by Dr. Andrew Mackay of Aberdeen, which, with later additions but excluding Maseres's own lengthy summary, occupies over 150 pages of the sixth volume of Maseres's work. In this, Mackay tediously computes "the length of the Tangent of one Minute of a Degree" [83, 6: 763], starting from 45° and using (not unlike West) trigonometric formulae to obtain the tangents of ever smaller angles. Maseres follows this with a short note from James Ivory, pointing out that, for such small angles, the standard trigonometric power series converge very rapidly. Mackay finally admits that this might have been a better way to proceed and shows agreement with his hard-won result [83, 6: 763 ff].

Turning to logarithms, West quickly derives the standard logarithmic series expansions, for logarithms with "modulus" m, noting that "hyperbolic logarithms" (i.e., with base e) have $m = 1$ and those with base 10 have $m^{-1} = \text{hyplog } 10$. This number must be found accurately and to do so requires rapidly converging logarithmic series with arguments close to unity. Useful series are

$$\tfrac{1}{2}\,\text{hyp log}\,\frac{a+d}{a} = r + \frac{r^3}{3} + \frac{r^5}{5} + \frac{r^7}{7} + \cdots, \tag{iv}$$

$$\text{hyp log}\,\frac{2a+d}{4a(a+d)} = r^2 + \frac{r^4}{2} + \frac{r^6}{3} + \cdots, \tag{v}$$

where $r = d/(2a + d)$ in both, for, when $\log a$ is known, they together yield $\log(a + d)$ and $\log(2a + d)$ [106, 2: 63]. Introducing the abbreviation ${}^{p}\overline{|r}$ to denote p times the series (iv) with factor r, West observes that, when $a^2 - 1 = mn$, then

$$\overline{\left|\frac{1}{a}\right.} = \overline{\left|\frac{1}{a+m}\right.} + \overline{\left|\frac{1}{a+n}\right.} = \overline{\left|\frac{1}{a-m}\right.} - \overline{\left|\frac{1}{n-a}\right.}.$$

Starting with $(1/2)\log 10 = {}^{3}\overline{|1/3} + \overline{|1/9}$ (dropping "hyp"), West shows that

$$\log 10 = {}^{20}\overline{|1/9} + {}^{12}\overline{|1/191} + {}^{6}\overline{|1/721},$$

and the respective series for such small r-values converge rapidly. Retaining eight terms from the first series, four from the second, and three from the third gives log 10 to seventeen significant figures [106, 2: 68].

West proceeds similarly to calculate the common logarithms (i.e., those to base 10) of various prime numbers, and then outlines an improved method. He describes how a table of logarithms of the first 1000 numbers can be constructed, adding that "the Table may be easily filled up to 100,000" using his rules [106, 2: 79]. When lesser accuracy is required, he gives a third, quicker method. He next deals with the computation of logarithms to 20 decimals, of numbers "consisting of many figures" [106, 2: 81–87]. This employs as starting point the 20-figure tables of Callet (presumably the *Tables portatives* of 1795 [51]) and essentially interpolates between the eight-figure arguments listed by Callet. As examples, West calculates $\log_{10}\pi$ and $\log_{10}m$, where $m = \log_{10}e = 1/\text{hyplog } 10$, in each case taking the argument to 21 figures. West's examples are not those of Callet's introduction. He concludes the section with the series expansions of logarithmic sines, cosines, and other trigonometric functions.

A series equivalent to (iv) above was also used by Archibald Sharp to construct his logarithms. Sharp thought it necessary also to give his rule in words: "Add an unit to twice the denominator, the sum (which is the sum of the numerator and denominator) shall be the divisor …" and so on [94; 83, 3: 102]. However, his numerical examples are not West's.

Notably absent from West's account of trigonometric and logarithmic series are the 1808 results of William Wallace [38] involving series expansions with successively halved angles, such as

$$\frac{1}{a} - \frac{1}{\tan a} = \tfrac{1}{2}\tan\tfrac{1}{2}a + \tfrac{1}{4}\tan\tfrac{1}{4}a + \tfrac{1}{8}\tan\tfrac{1}{8}a + \cdots$$

and also certain series for $(\log x)^{-1}$ and $(\log x)^{-2}$, all of which have good convergence properties. West surely would have been interested in them had he known them, and their absence may suggest that he corresponded neither with Wallace nor with his colleague Ivory.

The main limitation of West's treatises is the deliberate restriction to functions of a single variable, yet even the most elementary parts could have served as a useful calculus textbook in English, almost certainly written before Wallace's

Encyclopaedia article [40] and the Babbage–Herschel–Peacock translation of Lacroix [71]. The latter works are far surpassed by West's skillful account of infinite series, his clear exposition of Arbogast's theory, and his illustration of efficient methods for numerical calculation. However, West's treatises, his finest work, languished unpublished for 21 years after his death.

11. DELAYED PUBLICATION OF THE *TREATISES*

In 1825, *Blackwood's Edinburgh Magazine* announced, under "Deaths":

> —[no date] In Jamaica, the Rev. John West, Rector of St. Thomas in the East, a man of superior genius and worth. He was one of the most ingeneous [sic] and accurate teachers of mathematics which Scotland has produced. He was for some years, before he went to Jamaica, assistant to Professor Vitant [sic], in the University of St Andrews, and when in that capacity, published, about 40 years ago, "Elements of Mathematics", a work which, like the Diaries in England, has, since that time, had more effect in stimulating mathematical study and geometric invention in this country than any other performance extant. A valuable collection of his other mathematical papers are preparing for the press, and may perhaps be accompanied by a new edition of his Elements, now out of print. In that department of science, in which Leslie and Ivory have acquired so great and well-merited distinction, Mr West was their earliest teacher and patron; and to the same master they and others will never forget how deeply they are indebted for their elementary lessons in Mathematics. [20]

Interestingly, West's treatises are here described as "preparing for the press" in 1825. When Leslie received the manuscript treatises is unknown, but it may safely be assumed that they were in his hands by then. It is not known who placed the above entry in *Blackwood's Magazine.* Was it Leslie, at that time intending to proceed? Or was it someone else, perhaps a relative of John West visiting Edinburgh, anxious to spur Leslie on? Certainly, relatives were in Edinburgh around this time: another Stewart West, son of Maurice, was married there in 1828 [26]. Most importantly, what delayed publication of the Treatises for another 13 years?

Several facts may be relevant here. The publishers, Constable, went bankrupt in 1826. Moreover, relations between Leslie and the firm of Blackwood were cool, following a libel action by Leslie in 1822 against *Blackwood's Edinburgh Magazine.* (The *Magazine* had suggested that Leslie's acclaimed new method for making ice was not original, and it also (wrongly) alleged that he had reviewed one of his own books in the *Edinburgh Review.*) As for Leslie himself, he was no workaholic but an indefatigable traveller who regularly visited the continent. Also, the fact that in 1828 he undertook—for the then huge fee of £400—to prepare the *Fourth Dissertation on the Progress of Mathematical and Physical Sciences Chiefly during the Eighteenth Century* for *Encyclopaedia Britannica,* edited by his friend McVey Napier, left him even less time for West's treatises. Finally, Leslie was unlikely to benefit financially from publishing West's treatises and, as Napier delightfully put it, "his care of his fortune went somewhat beyond what is seemly in a philosopher" [79, 45].

In fact, there is little evidence of any editorial involvement in the treatises, as eventually published under Sang's direction, despite the assertion on the title page that they were edited by Leslie. The equations of the treatises are carefully numbered, with detailed cross-references that could only have been made with a full understanding of the contents. This strongly suggests West's, and certainly not Leslie's work. Only at one point are editor's additions identified: the introductory astronomical "Definitions" mentioned above. Though it is "probable that the Author had intended some introduction" [106, 1: 163] to the treatise as a whole, as well as to the astronomical examples, none is given, and no editorial comments are made elsewhere. The fact that much of the first treatise derives from Arbogast and Lagrange is nowhere mentioned, which also represents a remarkable editorial omission.

It seems unlikely that Leslie had much understanding of the part based on Arbogast's highly technical "Method of Derivations." In his *Fourth Dissertation,* Leslie rather vaguely wrote:

> But though the method of Derivations should not possess that logical superiority over the Fluctionary or Differential Calculus which its author so fondly supposes, yet is the invention entitled to the highest praise for its beautiful perspicuity and its ready and most extensive application. We have only to regret that it has required a new system of characters, when the ordinary notation has become so familiar, and attained so great perfection. Such mutations, like the diversity of languages, may be deemed a serious evil, since they divert the attention to the mere accesaries [sic] of learning, and retard or obstruct the acquisition of real knowledge. [79, 601–602]

As further examples of Leslie's superficial judgment, he writes that "[t]he large work of Lacroix is valuable for its contents, but deficient in clearness and elegance. His Abridgment seems very obscure and unsatisfactory" [79, 602], the latter a curiously damning verdict on the work selected for translation by Babbage, Herschel, and Peacock [71] to propagate the gospel of the Cambridge Analytical Society!

Nothing in any other of Leslie's writings seems to indicate that he was abreast of recent advances in analysis, though, following Playfair, he certainly supported its introduction to Britain. This view is shared by his obituarist in the *Mathematical Repository,* who wrote that "[a]s a mathematician Sir John Leslie did not rank high. He possessed, doubtlessly, an extensive knowledge of elementary mathematics; but with the higher departments of the new analysis he had a very slender acquaintance, and he was altogether unable to wield it successfully as an instrument of investigation." The writer, perhaps William Wallace, further suggests that Leslie had "an impatience of prolonged labour, and a strong desire of popular applause" [18, 217].

Following Leslie's death on November 5, 1832, his extensive library was sold over 14 afternoons, in February 1833, one more afternoon being devoted to the sale of his scientific apparatus, paintings, and "sundries." The 2072 catalogue items are mainly, but not exclusively, on mathematics and natural philosophy. Among this treasure trove, there appear West's *Elements of Mathematics,* Lagrange's *Théorie des fonctions analytiques,* and Arbogast's *Théorie des dérivations.* There is no mention of West's manuscripts, but, tantalisingly, lot 2067 was a "Lot of MS. Volumes on Mathematical subjects, by Mnaseres [sic] and others" [24].

Many books were bought by Edward Sang and by James D. Forbes. The former's were eventually bequeathed to the Royal Society of Edinburgh, but then mostly sold in 1983 [25]. Forbes's impressive personal collection is now in the University of St. Andrews, and he too acquired many books (now mainly dispersed) for the Royal Society of Edinburgh. Did Sang acquire West's manuscript treatises at this sale, with the other books, as part of lot 2067, or had Leslie passed them on to him earlier? In either case, the young Sang was better equipped to understand and appreciate their contents than his late mentor, and the regrettably delayed publication was finally accomplished in 1838.

12. INFLUENCE OF THE *TREATISES*

But what influence, if any, did West's treatises actually have? The short answer appears to be "not much." However, some belated recognition did come his way from Augustus De Morgan, who was one of very few British mathematicians to take an interest in Arbogast's treatise. In a paper entitled "On Arbogast's Formulae of Expansion," De Morgan begins:

> The theory of Arbogast has received so little attention in this country, that no excuse is necessary for an attempt to exhibit its rules in a short and comparatively easy manner. Arbogast himself was more occupied in proving the ease with which his method could be applied to very complicated cases, than in illustrating the connexion of its principles with those of other parts of analysis.
>
> The first attempt of which I know, to write on this subject in English, is contained in the posthumous work of Mr. West, which is a very complete attempt, as far as series of one variable are concerned. The next is that which I made myself in my work on the Differential Calculus (at which time I did not know of Mr. West's work): and I am not aware of any other. [35, 238]

(Thomas Knight's work, mentioned above, was presumably unknown to De Morgan.)

West's treatises must have had a more direct influence on Edward Sang, as he oversaw their publication. Certainly, his remarkably wide-ranging work on engineering and "practical" mathematics includes topics emphasised by West, most notably the calculation of accurate tables of logarithms and other functions, and the prediction of eclipses. Sang left Edinburgh in 1841, where he had worked as surveyor, civil engineer, and mathematics teacher, to become the first mathematics professor at Manchester New College. Soon after, he went to Constantinople to help establish engineering schools and plan railroads and an ironworks in Turkey. Later, he lectured (in Turkish) and collaborated with his pupils in compiling treatises in that language on mathematical and mechanical topics. In 1847, with great attendant publicity, he predicted an imminent solar eclipse. This was hailed as "a triumph of science which had a lasting effect on the population ... for it not only removed in a great measure the superstition of the people, but also excited and stimulated the students ..." [36, xxiii]. Sang left Constantinople via Russia, where he had gone in 1851 to observe another eclipse, and against the wishes of the Sultan. He returned to Edinburgh as a private tutor, actuary, and researcher. His many papers on wide-ranging applications of mathematics are listed in [36]. These include optical and astronomical instruments, machines and measuring devices, harbours, arches, railways, and life assurance. He published several tables of logarithms and other functions. But his greatest and most accurate compilation of tables remains, unpublished and now unpublishable, within 47 manuscript volumes (plus duplicates) in Edinburgh University Library and the National Library of Scotland [34].

13. CONCLUSION

It was not for his mathematics that his parishioners in Jamaica honoured John West, but for his "Exemplary conduct ... and many private Virtues" during his long ministry. His Christian faith and commitment to his ministry proved strong in the face of considerable opposition and adversity. The evidence of his life suggests that West's diffident external manner concealed an inward self-assurance and strength of character (a not atypical Scottish phenomenon). This combination of "bashfulness" and confidence perhaps suited him for the role of isolated scholar. His solitary mathematics (and his interest in chess) no doubt provided intellectual respite from the brutal society to which he ministered. The contrasting facets of his character are well caught in two of the maxims that he chose to illustrate his *System of Shorthand:* "In all thy undertakings, let a reasonable assurance animate thy endeavors: if thou despairest of success, thou shalt not succeed," and "The speech of a modest man giveth lustre to truth, and the diffidence of his words absolveth his error" [105, 10–11].

Within Scotland, however, John West was remembered as an outstanding geometer; as the author of the *Elements of Mathematics;* and as an effective teacher, most notably of James Ivory and John Leslie, yet he was forced to emigrate through lack of prospects. Even more impressively, his posthumously published *Mathematical Treatises* establish him as one of very few British mathematicians who, in the early years of the 19th century, mastered analytical works of Lagrange, Arbogast, and Laplace. Though he cannot be said to have carried out research of startling originality, his exposition and improvement of Arbogast's "Theory of Derivations" is exemplary in its lucidity. He displayed considerable virtuosity in the use of infinite series to obtain extremely accurate numerical approximations. These were then applied both to illustrate the construction of logarithmic and other tables and to solve practical astronomical problems.

Though he was isolated from other scholars for most of his life, his early education as student and teacher at St. Andrews provided a firm base from which he could explore the analytical works of Euler and the later French mathematicians. The influence on West of the ailing St. Andrews professor Nicolas Vilant was probably significant. But the St. Andrews establishment did not value West sufficiently to appoint him as a permanent, and adequately paid, assistant with right of succession to Vilant's chair (as sometimes happened in Scotland at this time). Had they done so, the development of mathematics in St. Andrews and Scotland might have been very different. Instead, Vilant was helped by a succession of less illustrious assistants, and Vilant's eventual successor was a worthy divine, preferred over James Ivory [89].

After his emigration to Jamaica, West may have retained some contact with Ivory, but I have found no evidence of this, nor of links with any other mathematician. Though John Leslie may have maintained tenuous contact with him, he could not appreciate West's analytical work and did him the disservice of failing to publish the manuscript treatises promptly. As described above, these treatises give perhaps the earliest full account in English of the "continental" style of calculus and include much advanced work on series. Surely, West's contribution to British mathematics quickly would have been recognised had his treatises been published when written. However, the potential audience in Scotland for West's treatises was probably far smaller than that which the Analytical Society was later to exploit in Cambridge, and it seems unlikely that Leslie's failure to publish West's treatises greatly delayed the reform of British mathematics.

West's isolation was not necessarily a hindrance to his mathematical development, given that he had access to recent French texts. Scottish professors (unlike those at Cambridge, then and now) carried a heavy teaching load and some, such as Leslie and Wallace, were drawn into textbook writing at the expense of scholarship, as a means of supplementing their incomes. West, despite his pastoral duties, had sufficient time to write what he chose, and he was at last financially secure. It is noteworthy that most of the major names in British mathematics around this time—Woodhouse, Ivory (after his early retirement), Babbage, Herschel, Airy—likewise had few teaching commitments. Yet in West's case it is astonishing that, while working as a Jamaican parish priest and without any outside recognition, he achieved so much.

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