



Back-door fine-tuning in supersymmetric low scale inflation

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Abstract

Low scale inflation has many virtues and it has been claimed that its natural realisation in the supersymmetric Standard Model can be achieved rather easily. In this Letter we demonstrate that the dynamics of the hidden sector responsible for supersymmetry breaking and the structure of the soft terms affects significantly, and in fact often spoils, the would-be inflationary dynamics. We also point out that the issue of the cosmological constant cancellation in the post-inflationary vacuum strongly affects supersymmetric inflation. It is important to note the crucial difference between the freezing of the modulus and actually stabilising it—the first approach misses parts of the scalar potential which turn out to be relevant for inflation. We argue that it is more likely that low scale supersymmetric inflation occurs at a critical point at the origin of the field space than at an inflection point away from the origin, as the necessary fine-tuning in the second case is typically larger.

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1. Introduction

Implementation of the inflationary paradigm in most promising extensions of the Standard Model, inspired by such ideas as supersymmetry or extra dimensions, has proven to be a non-trivial and highly demanding task. A particular problem of supersymmetric models is related to the existence of the moduli sector, consisting of scalar fields which mix with matter-like scalars and acquire a nontrivial potential in the course of supersymmetry breakdown. As pointed out in [1,2] this complicates the inflationary dynamics, and may easily invalidate the expectations for successful inflation based on a single-field approximation in the presence of frozen moduli. Since this problem has often been overlooked, in the present Letter we shall discuss it again, in the context of low scale supersymmetric inflation [3], which clearly exposes its significance. Among various interesting proposals in this area, there is the proposal of generating inflation by using flat directions of the minimal supersymmetric Standard Model (MSSM) as candidate inflatons [4], which

we shall use as a specific example. The necessary curvature of the inflationary potential arises there due to non-renormalisable corrections to the superpotential and due to the soft terms which are borne in the underlying microscopic theory. The problem appears in the form of a particular relation between the soft terms which must be fulfilled to a very high accuracy in order to produce phenomenologically acceptable inflation. The solution put forward in [5] attempts to explain the troublesome relation via the functional form of the Kähler potential of the modulus controlling the soft terms at the level of the microscopic theory—four-dimensional supergravity. Also, the form of the Kähler potential coupling of the modulus to the inflaton needs to be prepared accordingly. This would be indeed an elegant resolution of the problem, as the choice of the form of the Kähler potential and the superpotential automatically gives relations between individual couplings in the Lagrangian, and supersymmetry could be hoped to control these relations beyond the tree-level expressions.

Unfortunately, proper embedding of the inflationary version of the MSSM in supergravity moderates these expectations. The point is that the modulus sector of the theory does not really decouple from the inflationary dynamics. In fact, one needs to minimise and evolve the whole system consisting of the inflaton and the modulus on the same footing, and this may easily

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modify the MSSM-level predictions as we demonstrate in this Letter. We also point out that the issue of the cancellation of the cosmological constant in the post-inflationary vacuum affects supersymmetric inflation. It is important to note the crucial difference between freezing the modulus and actually stabilising it—the first approach misses parts of the scalar potential which turn out to be relevant for inflation. Supergravity corrections and the dynamics of the modulus result in a situation where the single choice of the Kähler potential and superpotential becomes replaced by the whole series of tunings between Lagrangian couplings and the usual fine-tuning problem gets re-created in its full severity.

We argue that it is more likely that the low scale supersymmetric inflation occurs at a critical point at the origin in the field space than at an inflection point away from the origin, as the necessary fine-tuning in the second case is typically larger.

2. Small-scale inflation

2.1. General remarks

When one attempts to find an inflationary model in supergravity, one often finds that the typical scale of the scalar potential is $V \sim m_{3/2}^2 M_P^2$, which for gravitino masses close to the electroweak scale is about $\sim 10^{-32} M_P^4$. This is to be compared with the magnitude of the spectrum of primordial curvature perturbations:

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{24\pi^2 \epsilon} \frac{V}{M_P^4} \quad (1)$$

and its spectral index:

$$n_s = 1 - 6\epsilon + 2\eta \quad (2)$$

where $\epsilon = (M_P^2/2)(V'/V)^2$ and $\eta = M_P^2(V''/V)$ are the usual slow-roll parameters. It follows from observations that $\mathcal{P}_{\mathcal{R}} \sim 2 \times 10^{-9}$ while n_s is smaller than unity by a few per cent [6]. At first sight, inflation with a low scale suggested by supergravity appears to fit well into this picture: one can envision looking for models with very small ϵ to provide a correct normalisation of the spectrum and with a negative η is of the order $\mathcal{O}(10^{-2})$ for a correct value of the spectral index. Therefore, we start our analysis by discussing to what extent this assumption is realistic in reasonably simple inflationary models.

When constructing inflationary models, one often looks for the saddle points of the potential. If such a saddle point exists and there is only one unstable direction at this point, a possibility of successful inflation arises. In the vicinity of a saddle point, we can approximate the inflationary potential V as a function of a real scalar field φ corresponding to the unstable direction by:

$$V(\varphi) = V_0 \left(1 - \frac{y}{2q} \left(\frac{\varphi}{M_P} \right)^q \right)^2. \quad (3)$$

Let us define a dimensionless field $\sigma \equiv \varphi/M_P$. Inflation ends at some field value σ_f for which ϵ becomes large. Usually this condition is taken as $\epsilon = 1$, but it will not introduce a large

error if we identify σ_f with the field value for which ϵ actually diverges, $\sigma_f^q = 2q/y$. The earliest moment of inflation we shall be interested in is when the characteristic scales of CMB left the Hubble radius, $N \sim 50$ e-folds before the end of inflation when the field value was σ_i . The number of e-folds N is given by:

$$N = \frac{1}{M_P} \int_{\sigma_f}^{\sigma_i} \frac{V}{V'} d\sigma = \frac{1}{4q} (\sigma_i^2 - \sigma_f^2) + \begin{cases} \frac{1}{y} \ln \frac{\sigma_f}{\sigma_i}, & q = 2, \\ \left[\frac{1}{(q-2)y} \left(\frac{1}{\sigma_i^{q-2}} - \frac{1}{\sigma_f^{q-2}} \right) \right], & q > 2. \end{cases} \quad (4)$$

In the following, we shall try to estimate crudely $\mathcal{P}_{\mathcal{R}}$ in terms of the quantities entering directly in the inflationary observables, assuming that inflation takes place close to the saddle point at $\sigma = 0$.

If $q = 2$ and $1/N \lesssim y \ll 1$, the second term dominates in (4) and then $\sigma_i = \sqrt{2q/y} e^{-Ny}$. One then finds $\eta \approx -y$ and $\epsilon \approx 2ye^{-2Ny}$. In this regime, ϵ is bounded from below and it is impossible to reconcile (1) with a small scale of the potential.¹ For $y \ll 1/N$, we find that $\eta \approx \epsilon \approx 1/(2N)$ and a similar conclusion holds. We also note that for $y \gtrsim 1$ the potential (3) cannot support slow-roll inflation.

For $q > 2$, it follows from (4) that $\sigma_i^{q-2} \approx \frac{1}{N(q-1)y}$. If $Ny \ll 1$, one finds that $\eta = 2\epsilon$ which is inconsistent with a low-scale inflation predicting a realistic scale dependence of the perturbations. For $Ny \gtrsim 1$, one gets

$$\epsilon \approx (N(q-2))^{-(2q-2)/(q-2)} y^{-2/(q-2)}, \quad \eta \approx -\frac{q-1}{q-2} \frac{1}{N}. \quad (5)$$

A striking feature of this regime is that η automatically has a realistic value which does not depend on y . One can then make ϵ arbitrarily small by taking a sufficiently large y , which allows reconciling the predictions (1) and (2) of low-scale inflation with observations.

A lesson from this analysis is that single-field low-scale inflation can be realised if the potential is sufficiently flat. It is, however, not enough that only $V'(\varphi_0)$ vanishes at some point φ_0 , since then the sizes of both slow-roll parameters are similar, contrary to what is required for a realistic spectral index. If the potential obeys $V'(\varphi_0) = V''(\varphi_0) = 0$, then one can introduce an arbitrary hierarchy between the slow-roll parameters and satisfy the observational constraints.

2.2. Examples of models of low-scale supersymmetric inflation

One of the models aiming at lowering the scale of inflation in supergravity was that of Ref. [7]. It employed a single chiral superfield with a minimal Kähler potential and the superpotential $W(\phi) = \Delta^{-1}(\phi - M_P)^2$. This choice ensures that all contributions to terms quadratic in ϕ in the potential cancel out and

¹ If the potential (3) ceases to be an accurate approximation for $\sigma \sim \sigma_f$ and inflation actually ends at a smaller value σ_f than discussed above, it leads to smaller σ_i and improves the quality of the rough approximations used above.

the model can be effectively described by the potential (3) with $q = 3$ and $y = 12$. Of course, such a small value of y did not allow for the inflationary scale to be significantly smaller than the GUT scale.

Another model of small-scale inflation was proposed in Ref. [3]. The potential used in that analysis was basically (3) with an additional term quadratic in φ . It was shown that satisfactory predictions for the spectrum of primordial perturbations can be achieved in such a model for $q > 2$ and that the presence of a subleading quadratic term can quantitatively affect n_s , but the normalisation of the power spectrum is basically left intact. The author also tried to embed this model in supergravity, in the context of D -term inflation.

The recent model of MSSM inflation [4] identifies the inflaton with a flat direction in the unbroken MSSM. Such a flat direction φ can be lifted by a nonrenormalisable superpotential $W = \lambda\varphi^n/M_P^{n-3}$. In the presence of gravitationally mediated supersymmetry breaking, its potential generically reads:

$$V_{\text{MSSM}}(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{\lambda A}{nM_P^{n-3}}\varphi^n + \frac{\lambda^2}{M_P^{2(n-3)}}\varphi^{2(n-1)}. \quad (6)$$

Given that the parameters of the potential obey

$$A^2 = 8(n-1)m^2, \quad (7)$$

we have $V'(\varphi_0) = V''(\varphi_0) = 0$ at a certain field value φ_0 . When expanded in terms of $(\varphi - \varphi_0)$, the model can be well approximated by (3) around the inflection point. As demonstrated in the preceding section, a sufficient number of e-folds of inflation may occur, giving rise to primordial density perturbations of the right magnitude. In particular, with the flat directions $\varphi^3 = udd$ or $\varphi = LLe$ we have $n = 6$, which allows for the correct normalization of the spectrum of the curvature perturbations with suggestive values $m, A \sim \mathcal{O}(1 \text{ TeV})$ as expected for a supersymmetric model solving the hierarchy problem. It is instructive to compare various energy scales in this setup. With $A \sim 1 \text{ TeV}$ we have $\varphi_0^{n-2}/M_P^{n-2} \sim A/M_P \sim 10^{-16}$, $V(\varphi_0)^{1/4}/M_P \sim (A/M_P)^{1/2}(\varphi_0/M_P)^{1/2} \sim 10^{-10}$, $V'''(\varphi_0)/M_P \sim (A/M_P)^2(\varphi_0/M_P)^{-1} \sim 10^{-28}$ and $H/M_P \sim (V(\varphi_0)/M_P)^{1/2} \sim 10^{-19}$. As written in (6) and (7), the model is severely fine-tuned: relation (7) must be satisfied with the accuracy of ~ 20 digits to keep the potential sufficiently flat for inflation, though a minuscule de-tuning of this order might also improve the prediction $n_s = 1 - 4/N$, following from (5) for $q = 3$, which is more than 2σ off from the WMAP3 measurements [8]. It has, however, been argued that relation (7) might reflect the dynamical properties of the embedding into four-dimension supergravity with a reasonably motivated ansatz about the Kähler potential and the superpotential [5] and that supergravity corrections might actually reconcile the prediction for the spectral index with WMAP3 results [9].

3. Supergravity embedding

Although in a realistic model the hidden sector may consist of a plethora of fields with complicated interactions, we take for

simplicity only one hidden sector field T and a Kähler potential of the form (from now on we set $M_P = 1$, unless specified otherwise):

$$K = \beta \log(T + \bar{T}) + \sum_{l=1}^{\infty} Z_{2l}(\phi\bar{\phi})^l. \quad (8)$$

We further simplify the form of the Kähler potential by taking $Z_2 = \kappa/(T + \bar{T})^\alpha$ and $Z_{2l} = 0$ for $l > 1$. We also assume that the superpotential is separable, $W(T, \phi) = F(T) + G(\phi)$, and we assume that the potential has an extremum at $(T, \phi) = (T_0, 0)$. In addition, we shall work with the inflaton superpotential which can be written as $G(\phi) = \phi\tilde{G}(\phi)$, with $\tilde{G}(0) = 0$.

3.1. The cosmological constant problem

For the rest of this Letter we shall relegate the possible D -term contributions to the supergravity scalar potential to a separate sector of the model which is responsible for cancellation of the cosmological constant, which may contain in addition an O’Raifeartaigh F -term described below or nonsupersymmetric contributions. The remaining part, containing F -terms of the inflaton and of the active modulus as well as the suitable negative gravitational contribution, we shall simply call (with some abuse of precision) the F -term potential.

Let us suppose that this F -term potential vanishes at the extremum at $(T_0, 0)$. This and the vanishing of $\partial V/\partial T$ at this point allows expressing F_T and F_{TT} in terms of $W_0 \equiv W(T_0, 0)$, T_0 and β . In particular, a solution to $V(T_0, 0) = 0$ reads:

$$F_T = \frac{W_0}{T_0 + \bar{T}_0} (e^{i\theta} \sqrt{-3\beta} - \beta), \quad (9)$$

where θ is an unknown angle. Substituting these results to the matrix of the second derivatives of the scalar potential $\frac{\partial^2 V}{\partial x_i \partial \bar{x}_j}$, where $x_i = (T, \bar{T}, \phi, \bar{\phi})$, we obtain a block-diagonal form:

$$\frac{\partial^2 V}{\partial x_i \partial \bar{x}_j} \Big|_{(T_0, 0)} = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}, \quad (10)$$

where D_1 and D_2 are 2×2 matrices and, in particular,

$$(D_1)_{T\bar{T}}|_{(T_0, 0)} = -\frac{2(\beta + 3)W_0W_0^*}{(T_0 + \bar{T}_0)^{2-\beta}}, \quad (11)$$

$$(D_1)_{TT}|_{(T_0, 0)} = -\frac{\sqrt{-3\beta}W_0W_0^*}{(T_0 + \bar{T}_0)^{2-\beta}} \left(e^{-i\theta} \left(\beta^2 + 3\beta + 1 + \frac{X_T}{\beta} \right) - 18e^{i\theta} + \frac{9\beta^2 + 36 - 2\beta e^{2i\theta}}{\sqrt{-3\beta}} \right), \quad (12)$$

where $X_T \equiv F_{TTT}(T_0 + \bar{T}_0)^3/W_0$. If $\beta \neq -3$ and the superpotential is a nontrivial function of T , it can be shown that one of the eigenvalues of D_1 is negative; otherwise, the smaller eigenvalue vanishes. For $\beta = -3$ and $\alpha = 1$, corresponding to no-scale supergravity, this conclusion is valid for an arbitrary form of G . There are, therefore, two possibilities: (i) the hidden sector field T is stabilised, but the point $(T_0, 0)$ at which the potential vanishes cannot be a local minimum of the potential (hence, at the true minimum the F -term potential assumes

a negative value and we are faced with the usual problem of the cosmological constant) or (ii) the cosmological constant vanishes, but the hidden sector field is not stabilised. In the following we assume that the case (i) holds and, somewhat optimistically, that the cosmological constant problem is solved by an additional mechanism which does not affect form of the ϕ -dependent F -term contributions to the potential.

To be somewhat more specific let us discuss briefly a supersymmetric uplifting with an additional chiral (O’Raifeartaigh) sector in the superpotential, which does not depend on the T modulus and fixes the expectation values of its scalar components at values much smaller than the Planck scale, $\langle \psi \rangle \ll M_P$, in such a way that the scale of the O’Raifeartaigh F -terms in the globally supersymmetric limit is $|F_\psi|^2 \approx \mu^4 < M_P^4$. Then one can expand the supergravity scalar potential in powers of ψ/M_P and leading terms in this expansion can be written as, see [10–12],

$$V(T, \psi) = \frac{\mu^4}{8t^3} + \frac{\Sigma^4}{2}(t - t_0)^2 - \Sigma_0, \quad (13)$$

where $t = \text{Re}(T)$, $\Sigma_0 > 0$, and the dependence of the first term on T comes from the factor e^K with $K(T) \supset -3 \log(T + \bar{T})$. With the O’Raifeartaigh sector switched off the scalar potential has a minimum at $t = t_0$ with a negative vacuum energy $-\Sigma_0$. However, when one takes $\mu \neq 0$ the minimum becomes shifted to $\langle t \rangle = t_0 + \delta$ with $\delta \approx \frac{3\mu^4}{8\Sigma^4 t_0^3}$. In addition to this, a single tuning $\mu^4 \approx 8t_0^3 \Sigma_0$ gives a vanishing vacuum energy density at this minimum. Requesting $\delta \ll t_0$ one obtains a condition on Σ , μ and t_0 : $\Sigma > \frac{\mu}{t_0}$ which is equivalent to the condition $\Sigma^4 > \frac{3\Sigma_0}{t_0}$. One can check that if the above condition holds, the gravitino mass before and after uplifting changes at the order $\mathcal{O}(\delta)$, hence $m_{3/2}^2$ shifts by a negligible amount.

In what follows we shall assume a mechanism of uplifting which leaves mixed inflationary/modular sector of the model unaffected to any practically required accuracy. As discussed above, this can be achieved, though at the expense of further tuning.

3.2. Supergravity corrections

For $G(\phi) = (\tilde{\lambda}/\nu)\phi^\nu$ with $\nu \geq 3$ we can expand the scalar potential as:

$$\begin{aligned} V_F(T, \phi) = & -V_s(T) + \left[-V_s(T) \left(1 + \frac{\alpha}{\beta} \right) \right. \\ & \left. + (T_0 + \bar{T}_0)^\beta \left(1 + \frac{3\alpha}{\beta} \right) W_0 W_0^* \right] \\ & \times \frac{\kappa}{(T + \bar{T})^\alpha} \phi \bar{\phi} + \mathcal{O}((\phi \bar{\phi})^{3/2}), \end{aligned} \quad (14)$$

where

$$\begin{aligned} -V_s(T) = & \frac{(T_0 + \bar{T}_0)^2}{-\beta} F_T^* F_T - (T_0 + \bar{T}_0)(F_T W_0^* + F_T^* W_0) \\ & - (\beta + 3)W_0 W_0^*. \end{aligned} \quad (15)$$

3.2.1. Saddle point at $\phi = 0$

As argued above, there is a negative contribution to the vacuum energy from the F -term potential (14), which yields a negative contribution to the mass squared of ϕ . Without specifying the superpotential $F(T)$ it cannot be guaranteed that such a negative contribution can be compensated for by the second term in the square brackets in (14). If the direction of $|\phi|$ is unstable, we may try to identify the requirements necessary for such a saddle point to support inflation consistent with observations. In this case, we denote by V_0 the sum of the vacuum energy at $\phi = 0$ and the uplifting needed to set the cosmological constant to zero at the true minimum of the potential at $\phi \neq 0$. We first proceed by calculating higher order terms in the expansion in powers of ϕ , i.e., $V = V_0 + V_2|\phi|^2 + V_3|\phi|^3 + \dots$, and then comparing them to the coefficients of the effective potential reminiscent of (6), $V = V_0 + m^2 \varphi^2/2 + A\lambda \varphi^\nu / (\nu M_P^{\nu-3}) + \lambda^2 \varphi^{2(\nu-1)} / M_P^{2(n-3)}$, where $\varphi/M_P = (\sqrt{\kappa}/(T_0 + \bar{T}_0)^{\alpha/2})|\phi|$ is the almost canonically normalised field.² At the lowest order in κ , we then find:

$$\begin{aligned} V_\nu = & (T_0 + \bar{T}_0)^\beta (-1)^k \frac{|\tilde{\lambda}|}{|W_0|} \left[- \left| \frac{1}{\nu} + \frac{\alpha}{\beta} \right| (T_0 + \bar{T}_0) \right. \\ & \left. \times (W_0^* F_T + F_T^* W_0) + 2 \left(1 - \frac{\beta+3}{\nu} - \alpha \right) W_0 W_0^* \right], \end{aligned} \quad (16)$$

$$V_{2(\nu-1)} = (T_0 + \bar{T}_0)^\beta |\tilde{\lambda}|^2 \frac{(T_0 + \bar{T}_0)^\alpha}{\kappa}. \quad (17)$$

In (16), k accounts for minimization with respect to $\arg(\phi)$ and is an even (odd) number if the expression in the square brackets is negative (positive). The relation between the coupling constants $\tilde{\lambda}$ and λ is given by $\lambda = |\tilde{\lambda}|(T_0 + \bar{T}_0)^{\beta+\alpha\nu/2}/\kappa^{\nu/2}$, and we assume that $\lambda \sim \mathcal{O}(1)$. Then we can read the coupling A from (16):

$$\begin{aligned} \frac{A}{M_P} = & \frac{\nu(-1)^k (T + \bar{T})^{\beta/2}}{|W_0|} \left[- \left| \frac{1}{\nu} + \frac{\alpha}{\beta} \right| (T_0 + \bar{T}_0) \right. \\ & \left. \times (W_0^* F_T + F_T^* W_0) + 2 \left(1 - \frac{\beta+3}{\nu} - \alpha \right) W_0 W_0^* \right]. \end{aligned} \quad (18)$$

Although the overall form of the scalar potential is somewhat different from the one discussed in Section 2, a successful inflation would be supported mainly by the constant term and the A -term, identical to the one studied there. We can, therefore, follow the analysis presented in Section 2 in studying conditions that have to be satisfied for a successful inflation at the saddle point.

Firstly, we need $|m^2|M_P^2/V_0 \lesssim \mathcal{O}(10^{-2})$ in order not to spoil the predictions for the spectral index. Secondly, during inflation the A -term contribution to the potential must be much larger than the mass term contribution, $1 \ll A\lambda \varphi^{n-2}/(n|m^2|M_P^{n-3})$, which gives $|m^2|M_P^2/V_0 \ll 1/(n(n-1)N)$. Finally, we need

² As in Ref. [5] we assume that the kinetic term is $(\partial_\mu \varphi)(\partial^\mu \varphi)$; rescaling to canonical normalization $\varphi \rightarrow \varphi/\sqrt{2}$ can be compensated by $\lambda \rightarrow 2^{(\nu-1)/2}\lambda$, $A \rightarrow 2^{1/2}A$ and $m^2 \rightarrow 2m^2$, which, in particular, does not change relation (7).

to be in the large y regime of Section 2.1, which gives $1 \ll AM_P^3/V_0$. With $A^2 \sim |m^2|$ this can be satisfied rather easily.

In addition to various constraints from the tree-level potential, there may also be potentially dangerous contributions from quantum corrections, accounted for by the $(32\pi^2)^{-1}V''(\varphi)\Lambda^2$ term in the Coleman–Weinberg potential. A rough condition that the size of these corrections is much smaller than the size of the tree-level terms can be written as $1 \gtrsim \Lambda^2/\varphi_i^2$ (assuming that $(32\pi^2)^{-1}$ gives a satisfactory suppression). Using the findings of Section 2, we obtain the condition $\Lambda^2/M_P^2 \lesssim (q-2)^2 N^2 \epsilon$ or, using the normalisation of the spectrum of the scalar perturbations, $\Lambda \lesssim 10^5 V_0^{1/2}/M_P \gg \sqrt{|m^2|}$, A . One expects Λ to be of the order of the visible supersymmetry breaking scale, that is close to m to within a few orders of magnitude. This is consistent with our condition if $V_0^{1/2}/M_P$ is significantly larger than m . Fortunately, this is indeed the case for the examples discussed in Section 2. However, one should note that if $V_0 \approx m$, the above consistency condition would imply a very large scale of m , and hence also a high scale of the inflationary Hubble parameter. Thus the condition of negligibility of quantum corrections to the inflationary potential implies certain model dependent degree of fine-tuning in low-scale models.

This discussion shows that it is possible to accommodate a low-scale inflation in supergravity given that a number of conditions are satisfied and a certain degree of fine-tuning is arranged for, as we schematically depict in Fig. 1. Firstly, the supergravity potential must be uplifted to solve the cosmological constant problem (the enormous associated fine-tuning should be perhaps treated as a separate problem); the uplifting should also leave the ϕ dependence of the F -term potential qualitatively unchanged. Secondly, the condition $|m^2|M_P^2 \ll V_0$ tells us that we need V_0 to be a few orders of magnitude larger than the scale of the natural un-tuned vacuum value of the scalar potential, usually negative, $m_{3/2}^2 M^2$, as $m^2 \approx M_{3/2}^2$. This tells us also that there is a hierarchy between the value of the scalar potential at the saddle point before uplifting and its value at the true minimum (un-lifted) ($-V_s$ and $-V_0 - V_s$, respectively), which is needed to alleviate the η problem of supergravity [3] and to provide sufficiently long lasting period of inflation. Of course, accommodating these general guidelines in a concrete model is by itself a challenging task, which we leave for a future study.

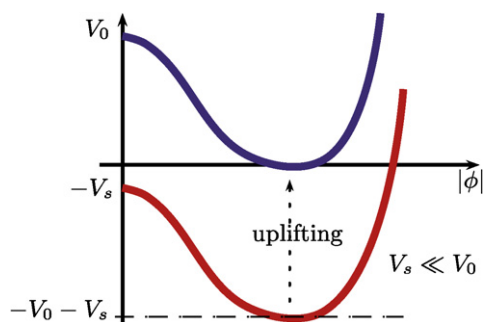


Fig. 1. A schematic representation of a shape of the $T = T_0$ section of the potential necessary for successful inflation.

3.2.2. Minimum at $\phi = 0$: The case of the MSSM inflation

In addition to the situation discussed in Section 3.2.1, there is also an alternative possibility that the mass squared of $|\phi|$ is positive despite the negative contributions related to the supersymmetric contributions to the vacuum energy. All formulae presented above are then also valid in this case, with an obvious difference that now $V_0 = 0$, and we might try to realise MSSM inflation in such a setup. In order to achieve inflection point inflation, we have to arrange the parameters of the theory in such a way that the fine-tuning condition (7) is satisfied. The relevant relation can be obtained by comparing (14) and (18). In the simplest case $W(T) = W_0$, it reduces to a very simple equation relating two numbers α and β :

$$(3 - \beta - 6\alpha)^2 = -20(\beta + \alpha + 2). \tag{19}$$

This has rational solutions for α and β , as already found in Ref. [5]. However, with $W_T \neq 0$, this simple algebraic relation between two numbers no longer holds—the solution for α depends on the details of the interactions of the hidden sector, namely, the values of $T_0 + \bar{T}_0$ and F_T/W_0 at the minimum of the potential. It is rather difficult to envision a natural origin of the particular alignment of the parameters in the Kähler potential and the superpotential, satisfying (7) in this more general case

3.2.3. Dynamical supergravity corrections

So far we have discussed the supergravity corrections following merely from the statics of the theory. There may be also truly dynamical corrections resulting from slight changes in the values of the hidden sector fields, as inflation proceeds. In order to illustrate this, we shall again consider the case with $W(T) = W_0$. Let us also assume that $\beta \neq -3$ and the cosmological constant is driven to small values by some unknown nonsupersymmetric mechanism. For nonzero values of φ , we can calculate perturbatively small shifts δT from the value of T at the minimum with $\varphi = 0$. The extremum condition then reads $(\partial^2 V(\varphi = 0)/\partial T \partial T)\delta T + (\partial V_2/\partial T)\phi\bar{\phi} = 0$. As follows from (14), a straightforward estimate from the supersymmetric part of the potential is that $\delta T/(T + \bar{T}) \sim \mathcal{O}(\varphi^2/M_P^2)$. The contribution from the vacuum energy uplifting sector is unknown, but it is unlikely to cancel the former one. Since the canonically normalised hidden sector field is $\sim \ln(T + \bar{T})$, we can expect that the canonically normalised hidden sector field can move slightly during inflation but with a velocity much smaller than that of φ , thereby giving negligible contributions to actual slow-roll parameters, even though the slow-roll parameters in the directions of the hidden sector fields are generically large.

3.3. Higher-order corrections

In Section 3.2, we calculated the coefficients V_0 , V_v and $V_{2(v-1)}$ of the potential and discussed the viability of inflation, assuming that these are the most important contributions to the inflationary potential. Let us now verify this assumption by calculating higher order corrections.

We start with a calculation of the quartic term in the potential, of the form $\Delta V(\varphi) = \frac{\Delta\kappa}{8}\phi^4$. To make things simple, let us

again assume that only Z_2 is nonzero in the expansion of the Kähler potential (8) and that $W(T) = W_0$. Then, expanding the full scalar potential to the fourth order in the inflaton field, we obtain:

$$V_4 = e^{\hat{K}} |W_0|^2 Z_2^2 \left[\frac{1}{2} \hat{K}^T \hat{K}_T - \frac{1}{2} + \hat{K}^T \hat{K}^{\bar{T}} Y_{T\bar{T}} + (\hat{K}^T \hat{K}_T)^{-1} (\hat{K}^T \hat{K}^{\bar{T}} Y_{T\bar{T}})^2 \right], \quad (20)$$

where $Y_{T\bar{T}} = Z_2^{-2} Z_{2T} Z_{2\bar{T}} - Z_2^{-1} Z_{2T\bar{T}}$ and $\hat{K} = K|_{\phi=0}$. In principle, this coupling is of the same order of magnitude as A^2 (in units of M_P), that is $\sim 10^{-32}$, as argued above. Note that the canonically normalised inflaton field is $\varphi = Z_2^{1/2} |\phi|$, hence the powers of Z_2 in this expansion are irrelevant and, in particular, $\Delta\kappa = Z_2^{-2} V_4$.

For saddle point inflation, the ratio of the quartic term (20) to the A -term (16) is $\sim (\Delta\kappa M_P/A)(\varphi/M_P)^{4-\nu} \sim (A/M_P)^{2/(\nu-2)}$, so this contribution does not spoil the form of the potential (3).

In the case of MSSM inflation it suffices to ensure that the quartic correction does not spoil the flatness of the potential near the inflection point [9]. For $\Delta\kappa < 0$ such a correction will result in shifting the point at which $V' = 0$ from φ_0 to $\varphi_0 + \Delta\varphi$, where $\Delta\varphi$ is given by (at the lowest order in $\Delta\varphi$ and $\Delta\kappa$):

$$0 = \frac{1}{2} V'''(\varphi_0) \Delta\varphi^2 + \frac{1}{2} \Delta\kappa \varphi_0^3. \quad (21)$$

Demanding that $1 \gg \Delta\varphi/\varphi_0$, we obtain the condition $1 \gg |\Delta\kappa|\varphi_0/V'''(\varphi_0) \sim |\Delta\kappa|\varphi_0^2/A^2 \sim |\Delta\kappa|10^{-24}$. This is easy to satisfy, as a natural size for the parameters in the potential is $(A/M_P)^2 \sim 10^{-32}$. However, in the inflationary context we must also make sure that after adding ΔV the potential remains sufficiently flat. The curvature of the potential at the new extremum $\varphi_0 + \Delta\varphi$ is described by:

$$\eta = M_P^2 \frac{V'''(\varphi_0) \Delta\varphi}{V(\varphi_0)} \sim \frac{M_P^2}{\varphi_0 A} \sqrt{|\Delta\kappa|} \sim 10^{20} \sqrt{|\Delta\kappa|}. \quad (22)$$

From this we infer that $|\Delta\kappa|$ cannot exceed 10^{-40} or it will spoil the flatness of the inflationary potential. As emphasized in Ref. [5], the flatness of the inflationary potential can be preserved if we admit higher order corrections to the Kähler potential, such as $Z_4(\phi\bar{\phi})^2$. This requires a precise cancellation between contributions from Z_2 and Z_4 , which can be achieved for $Z_4 = \mu Z_2^2$, where μ is a number determined from the lower order terms. In particular, it follows from the considerations above that the parameter μ must be adjusted at a level of $\sim 10^{-8}$. While it is conceivable that both Z_2 and Z_4 can be simple power functions of $T + \bar{T}$, the required accuracy of the relative normalization appears a bit artificial.

4. Summary

Low scale inflation has many virtues and it has been claimed that its natural realisation in the supersymmetric Standard Model can be achieved rather easily. In this Letter we have demonstrated that also in this case the dynamics of the hidden sector responsible for supersymmetry breakdown and the

structure of the soft terms may affect significantly, and in fact spoil, the would-be inflationary dynamics. Moreover, a significant enhancement is expected, as additional hierarchy between the slow-roll parameters has to develop in this type of inflationary models. The parameter ϵ has to be very much suppressed due to the COBE normalisation and at the same time η has to be of the order 0.01 to account for the WMAP3 preference for the spectral index smaller than unity. And indeed, we have shown that a severe back-door fine tuning appears via the necessary arrangements of the hidden sector parameters.

As a working example we have analysed in some detail the model of Ref. [5], whose authors aim to explain a severe fine-tuning (7) of the parameters in the inflationary potential (6) by embedding the model in supergravity so that the soft supersymmetry breaking parameters A and m^2 are simple and suggestive functions of the hidden sector field, and the relation (7) reflects the structure of the underlying supergravity model. In this note, we have identified two difficulties of such a proposal.

The first problem is that Ref. [5] neglects the dynamics of the hidden sector fields. If the superpotential of the theory depends on the hidden sector fields, the mass of the inflaton may be different than in that analysis, or the configuration with $\phi = 0$ may not even be a minimum of the potential. Although this does not preclude the scalar potential from having an inflection point suitable for inflation, the explanation of the relation (7) by the structure of the underlying supergravity theory no longer holds in this case.

Secondly, although the relation (7) can be neatly explained at the leading order in the inflaton field by assuming a reasonably well-motivated form of the Kähler potential, higher order corrections can easily destroy this result unless one fine-tunes the coefficients of the higher order terms in the Kähler potential. Hence, the solution of the fine-tuning (7) relies on shifting the fine-tuning to a different sector of the theory.

However, an analysis of the above difficulties shows that they should be easier to overcome in models where low-scale supersymmetric inflation occurs near a critical point near the origin in the field space. In this case the main problem lies in finding a suitable mechanism to cancel the cosmological constant in such a way that the resulting scale of the scalar potential at the origin is much higher than $m_{3/2}^2 M_P^2$, but the soft breaking parameters m^2 and A do not have to be precisely tuned to each other, which is the case if the inflection point needs to be created away from the origin. One may also hope that the origin, that is the point of enhanced symmetry, is the more likely place to find naturally small derivatives needed for successful inflation. Another lesson which may be drawn from our discussion is that the hidden sector with more than a single active modulus controlling the soft terms and vacuum energy would be more suitable to fulfill all the constraints, in particular to create a supersymmetric minimum with vanishing cosmological constant.

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