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V–A hadronic tau decays: A laboratory for the QCD vacuum

Stephan Narison

Laboratoire de Physique Théorique et d'Astrophysique, CNRS and Université de Montpellier 2, Place Eugène Bataillon, 34095 Montpellier Cedex 05, France

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Abstract

ALEPH/OPAL data on the V–A spectral functions from hadronic τ decays are used in connection with a set of Laplace transform sum rules (LSR) for fixing the size of the QCD vacuum condensates up to dimension 18. Our results favor the ones from large-N_c QCD within the minimal hadronic approximation (MHA) and show a violation of about a factor 2–5 of the vacuum saturation estimate of the dimension-six to -ten condensates. We scrutinize the different determinations of the QCD vacuum condensates using τ-decays data. After revisiting some of the existing results, we present coherent values of the condensates from different methods.

1. Introduction

Hadronic tau decays have been demonstrated [1] to be an efficient laboratory for testing perturbative and non-perturbative QCD. That is due both to the exceptional value of the tau mass situated at a frontier regime between perturbative and non-perturbative QCD and to the excellent quality of the ALEPH/OPAL [2,3] data. On the other, it is also known before the advent of QCD, that the Weinberg [4] and DMO [5] sum rules are important tools for controlling the chiral and flavor symmetry realizations of QCD, which are broken by light quark mass terms to higher order [6] and by higher-dimensions QCD condensates [7] within the SVZ expansion [8].[1] For completing our program in the vector and $V+A$ channel [10–14], we probe, in this Letter, the structure of the QCD vacuum using the ALEPH/OPAL data on the V–A spectral functions in connection with a set of Laplace sum rules (LSR). We have already initiated the analysis of the V–A channel in previous papers [15,16]. However, our main motivation here is due to the recent interests on the V–A hadronic correlator, which can serve as an order parameter of spontaneous chiral symmetry breaking in the chiral limit $m_q=0$. This correlator also governs the dynamics of the weak matrix elements of the elec-

E-mail address: snarison@yahoo.fr (S. Narison).

[1] For a review, see e.g. [9].

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troweak penguin-like operators [17,18]. These important properties require a detailed structure of the related QCD vacuum which can be parametrized by the sum of power corrections [8]. A large number of papers on the estimates of these power corrections using different methods exist in the literature, but with conflicting results in [2,3,19–23] and in [24–26,28,29]. In the following we propose a set of Laplace transform sum rules (LSR) which can help to clarify such discrepancies. We shall be concerned here with the V–A two-point correlator:

$$\Pi^{LR}_{\mu\nu}(q) \equiv i\int d^4x\, e^{iqx}\langle 0|\mathcal{T} J^L_\mu(x)\big(J^R_\nu(0)\big)^\dagger|0\rangle \stackrel{m_q\to 0}{=} -\big(g_{\mu\nu}q^2-q_\mu q_\nu\big)\Pi^{LR}\big(q^2\big), \quad (1)$$

built from the left- and right-handed components of the local weak current:

$$J^L_\mu=\bar u\gamma_\mu(1-\gamma_5)d, \qquad J^R_\mu=\bar u\gamma^\mu(1+\gamma_5)d. \quad (2)$$

Following SVZ [8], the correlator can be approximated by:

$$\Pi^{LR}\big(Q^2\big)\simeq\sum_{d\geqslant 2}\frac{\mathcal{O}_{2d}}{(Q^2)^d}, \quad (3)$$

where $\mathcal{O}_{2d}\equiv C_{2d}\langle\mathcal{O}_{2d}\rangle$ is the short-hand notation of the QCD non-perturbative condensates $\langle\mathcal{O}_{2d}\rangle$ of dimension $D\equiv 2d$ and its associated perturbative Wilson coefficient C_{2d}; $q^2\equiv-(Q^2>0)$ is the momentum transfer. In the chiral limit $m_{u,d}=0$, there is no $D=2$ term as unflavored contribution of the renormalon-type [10,11,30] vanishes. The spectral function $(v-a)$:

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi^{LR}\equiv\frac{1}{2\pi^2}(v-a) \quad (4)$$

has been measured by ALEPH and OPAL [2,3] using τ-decay data. Within a such normalization, the original “sacrosante” first and second Weinberg sum rules [4,5] read, in the chiral limit $m_{u,d}=0$:

$$\mathcal{S}_0\equiv\int_0^\infty dt\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{LR}-2f_\pi^2=0,$$
$$\mathcal{S}_1\equiv\int_0^\infty dt\,t\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{LR}=0, \quad (5)$$

where $f_\pi=(92.4\pm0.26)$ MeV is the experimental pion decay constant.

2. The Laplace sum rules (LSR)

In order to exploit the ALEPH/OPAL [2,3] data on the spectral function $v-a$ from hadronic tau decays, we shall work with the LSR version of the 1st Weinberg sum rule, in the chiral limit $m_{u,d}=0$:

$$\mathcal{L}_0(\tau)=\int_0^\infty dt\,e^{-t\tau}\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{LR}-2f_\pi^2 \simeq\sum_{d\geqslant 2}\frac{\tau^{(d-1)}}{(d-1)!}\mathcal{O}_{2d}, \quad (6)$$

from which on can obtain, by taking successive derivatives in τ, the set of LSR:

$$\mathcal{L}_n\equiv(-1)^n\frac{d^n\mathcal{L}_0}{d\tau^n}\simeq\int_0^\infty dt\,t^n e^{-t\tau}\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{LR} \simeq(-1)^n\sum_{d\geqslant(n+1)}\frac{\tau^{(d-n-1)}}{(d-n-1)!}\mathcal{O}_{2d}. \quad (7)$$

For our purpose, we shall truncate (in order to have a much better comparison with the existing results) the series at $2d=18$-dimension condensates,[2] assuming that this approximation provides a good description of the exact expression of the two-point correlator Π^{LR}. Then, from our previous general formula, one can write the set of sum rules:

$$\mathcal{L}_8\simeq+\mathcal{O}_{18},$$
$$\mathcal{L}_7\simeq-\mathcal{O}_{16}-\mathcal{O}_{18}\tau,$$
$$\mathcal{L}_6\simeq+\mathcal{O}_{14}+\mathcal{O}_{16}\tau+\mathcal{O}_{18}\frac{\tau^2}{2},$$
$$\vdots \quad (8)$$

Therefore, we can extract iteratively the vacuum condensates beginning from $\mathcal{L}_8$. The value of $\mathcal{O}_{18}$ obtained in this way will be inserted into $\mathcal{L}_7$ for determining $\mathcal{O}_{16}$ and so on.

We parametrize the spectral function by using the ALEPH/OPAL [2,3] data on the spectral function $v-a$ from hadronic tau decays below t_c. Above t_c, we

[2] The result will not depend crucially on the choice of the truncation of the series. We shall see that at the region where the condensates are estimated the OPE presents a good convergence.

use a QCD continuum coming from the discontinuity of the QCD diagrams. In the particular case of $\mathrm{Im}\,\Pi^{LR}$ studied here, a such contribution vanishes identically, which is equivalent to cut the integral in Eq. (8) at t_c. The appropriate values of t_c has been studied in [15, 20,24,31] by requiring that the 1st and the 2nd Weinberg sum rules vanishes in the chiral limit to leading order of the OPE. Two solutions have been found:

$$t_c \simeq (1.4\text{–}1.5)\ \mathrm{GeV}^2 \quad \text{and} \quad (2.5\text{–}2.6)\ \mathrm{GeV}^2. \tag{9}$$

In [15] the lowest value of t_c has been favored due to the inaccuracy of the ALEPH/OPAL data which affects the highest value, though intuitively, one tends to favor this highest value of t_c where pQCD is expected to work better. Exluding the high-t_c value solution, and requiring simultaneous zeros of the 1st and 2nd Weinberg sum rules, Ref. [15] deduces the accurate number:

$$t_c = (1.475 \pm 0.015)\ \mathrm{GeV}^2. \tag{10}$$

This choice, as emphasized in [20] coïncides with the t_c-value obtained for the MHA in the large N_c-limit, which follows from the duality relation [31]:

$$t_c \simeq 8\pi^2 f_\pi^2 \frac{1}{1-g_A} \simeq (1.2 \pm 0.2)\ \mathrm{GeV}^2, \tag{11}$$

with $g_A \simeq 0.5 \pm 0.06$. Instead in [24–26], the higher value of t_c around 2.5 GeV^2 has been favored.

In order to avoid results which strongly depend on these choices of t_c, we only consider the above values of t_c as a guideline of our analysis. Indeed, it is unlikely to take $t_c \leqslant 1.4\ \mathrm{GeV}^2$ as we will loose part of the ρ meson tails, and then most of the lowest ground state dynamics. Taking $t_c \geqslant 2.6\ \mathrm{GeV}^2$, the kinematic region is small and the data become very inaccurate. Then, they cannot provide useful information to the spectral function. Indeed in this region, the spectral function does not have a definite sign, for a given data point, due to the large error bars.

For an illustration, we show the analysis of $\mathcal{O}_{18,16}$ and $\mathcal{O}_{6,4}$ in Fig. 1 for different choices of the t_c-cut until which we use the ALEPH/OPAL data, and beyond which the pQCD diagram is expected to describe the two-point correlator. The analysis of the other condensates present similar features.

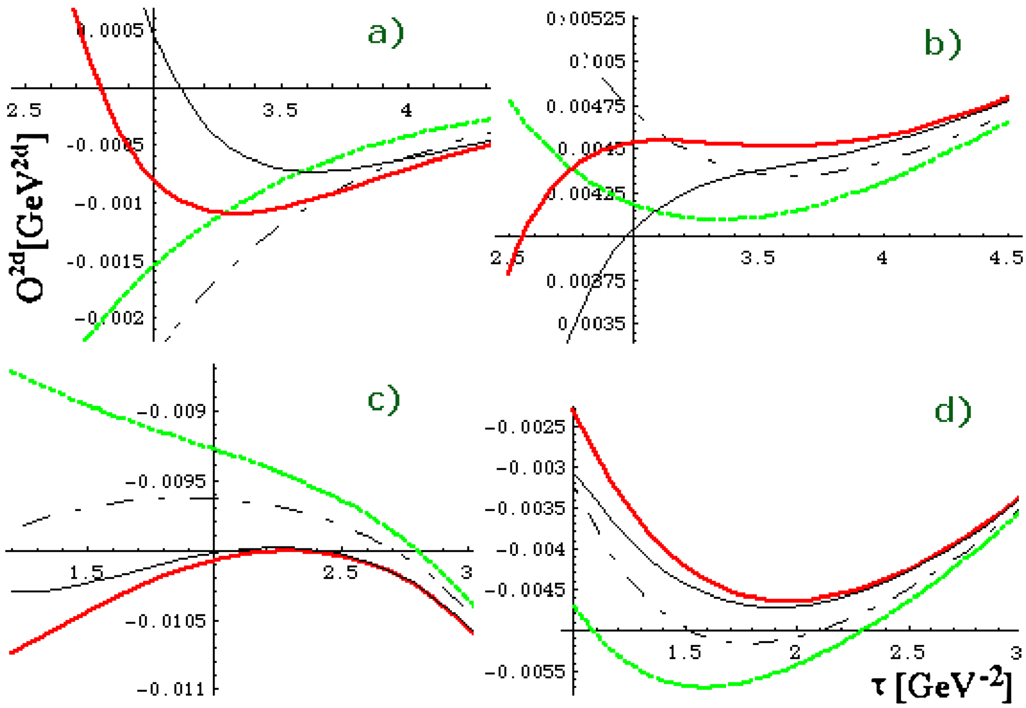

Fig. 1. τ in GeV^{-2}-behaviour of the condensates $\mathcal{O}_{2d}$ in units of GeV^{2d} for different values of t_c in GeV^2: 1.4 (dot-dashed), 1.5 (dashed bold), 2.5 (continuous bold), 2.6 (continuous): (a) $\mathcal{O}_{18}$, (b) $\mathcal{O}_{16}$, (c) $\mathcal{O}_6$ and (d) $\mathcal{O}_4$.

Table 1
Estimated values of the $D \equiv 2d \leqslant 18$-dimension $\langle \mathcal{O}_D \rangle$ condensates in units of 10^{-3} GeV^D. We have ordered the condensates from $\mathcal{O}_{18}$ to $\mathcal{O}_4$ according to their chronological estimate

Authors	$\mathcal{O}_{18}$	$\mathcal{O}_{16}$	$\mathcal{O}_{14}$	$\mathcal{O}_{12}$	$\mathcal{O}_{10}$	$\mathcal{O}_{8}$	$\mathcal{O}_{6}$	$\mathcal{O}_{4}$
This work								
LSR								
Eq. (8)	-1 ± 0.6	$+4.3\pm1.9$	-9.6 ± 3.1	$+14.7\pm3.7$	-17.1 ± 4.4	$+15.4\pm4.8$	-9.7 ± 4.1	-0.5 ± 0.1
Eqs. (14), (15)						$+15.8\pm3.2$	-8.4 ± 1.6	
Eq. (16)							-8.0 ± 1.1	
Average						$\mathbf{+15.6\pm4.0}$	$\mathbf{-8.7\pm2.3}$	
wFESR								
Rev.[a]		$+30\pm10$	-28 ± 8	$+25\pm5$	-22 ± 3	$+16.8\pm2.0$	-10.2 ± 0.4	
Orig. [28][b]		-946 ± 147	$+390\pm65$	-146 ± 7	$+43.5\pm10.5$	-4.4 ± 3.8	-4.8 ± 0.9	
FESR[c]								
BG Rev.[d]						$+12\pm1.5$	-6.6 ± 0.2	
BG Orig. [24]						-12.4 ± 9.0	-3.2 ± 2.0	
DS Rev.[e]						$+10\pm2$	-8.0 ± 2.0	
DS Orig. [25]						-2 ± 12	-8.0 ± 2.0	
LR Rev.[f]				$+53\pm16$	-39 ± 12	$+26.0\pm8.4$	-14.7 ± 4.8	
LR Orig. [26]				-260 ± 80	$+78\pm24$	-120^{+7}_{-11}	-4 ± 2	
Others								
MHA $+\rho'$ [19]		$+11.5\pm3.5$	-12.5 ± 3.4	$+13.2\pm3.3$	-13.1 ± 3.0	$+11.7\pm2.6$	-7.9 ± 1.6	
MHA [19]		$+11.9\pm3.9$	-12.8 ± 3.9	$+13.3\pm3.9$	-13.2 ± 3.6	$+11.7\pm3.1$	-7.9 ± 2.0	
ZYA [22]					-4.5 ± 3.4	$+7.8\pm3.0$	-7.1 ± 1.5	
IZ [23]						$+7.0\pm4.0$	-6.4 ± 1.6	
ALEPH [2]						$+11.0\pm1.0$	-7.7 ± 0.8	
OPAL [3]						$+7.5\pm1.3$	-6.0 ± 0.6	
DGHS [21]						$+8.7\pm2.4$	-6.0 ± 0.6	
CS3 [29]							-4.0 ± 2.8	
Average[g]	-1 ± 0.6	$+14.4\pm4.6$	-15.7 ± 3.7	$+23.8\pm6.4$	-18.2 ± 5.9	12.2 ± 2.9	-7.8 ± 1.6	

[a] We have redone the analysis of [28] using t_c-stability criterion.
[b] We use the mean value of the results from the ALEPH and OPAL data.
[c] The revised (Rev.) FESR results have been obtained at $t_c \approx 1.5$ GeV^2; the original (Orig.) ones at $t_c \approx 2.5$ GeV^2.
[d] These results have been obtained by [24] at 1.5 GeV^2.
[e] We have corrected the value of $\mathcal{O}_8$ (see Section 4) and rescaled the results of [25].
[f] The central values come from [27]. Inspired from the results of [26] at 2.5 GeV^2, we have roughly estimated the systematic errors to be about 30%.
[g] Numbers in the lines *Orig.* are not considered into the average.

The optimal results given in Table 1 correspond to the one at the minimum or inflexion point of τ for different t_c-values inside the range in Eq. (9). The τ-stability criterion has been often used in the Laplace sum rules analysis as it signals the *compromise region* where the OPE is reliable (smaller τ-values) and where the information from the data still remains optimal (larger τ-values). It is also unlikely if the result is strongly dependent on the choice of t_c-values as this signals a strong model dependence of the result on the form of the QCD continuum. Then, in the following, we shall use in connection these two stabilities criteria for extracting the optimal results.[3]

The error takes into account the one of the data and the systematics of the method due to the range of t_c-values given in Eq. (9) and to the propagation of errors induced by the ones of the input condensates. We do not include some eventual statistical errors.

It is important to notice from our analysis that in the range of t_c given in Eq. (9), the extracted values of the condensates do not flip sign contrary to the case

[3] For more complete discussions, see e.g. [9].

of FESR's results given in the existing literature. One can attribute this feature to the role of the exponential weight in LSR which enhances the contribution of the low-energy region to the sum rule.

One can also notice that, for high-dimension condensates, the optimal values are obtained at large τ-values like also in the least square fit analysis of [22, 23]. However, we have checked that, during the analysis of each sum rule, the high-dimension condensates remain corrections to the low-dimension contributions and do not break the OPE. One can also notice that the position of the minimum shifts to lower values of τ for decreasing dimension condensates, as one can see in Fig. 1 for the $D = 18$ to the $D = 4$ condensates. These features are re-assuring for the reliability of the result.

In order to test the accuracy of our estimate, we have extracted from $\mathcal{L}_0$ the known tiny value of the $\mathcal{O}_4$ quark condensate contribution using as input all higher-dimension condensates. Including radiative corrections, this contribution reads [8,32]:

$$\mathcal{O}_4^{\rm th} = 2(m_u + m_d)\langle\bar{u}u\rangle\left[1 + \frac{4}{3}\frac{\alpha_s}{\pi} + \frac{59}{6}\left(\frac{\alpha_s}{\pi}\right)^2\right], \tag{12}$$

where $(m_u + m_d)\langle\bar{u}u + \bar{d}d\rangle = -2f_\pi^2 m_\pi^2$. The size of the radiative corrections is about 35% at the τ-scale where the optimal results are extracted. This gives in units of 10^{-3} GeV^4:

$$\mathcal{O}_4^{\rm th} \simeq -0.44, \tag{13}$$

in excellent agreement with our fit -0.5 ± 0.1 given in Table 1 from Fig. 1(d). This test increases the confidence on our other predictions in Table 1 obtained in the same way.

3. Alternative estimates of $\mathcal{O}_6$ and $\mathcal{O}_8$

Using the previous method, we have obtained from Eq. (8) the results in Table 1 in units of 10^{-3} GeV^D (D being the dimension of the condensates). Here, we present alternative estimates based on some combinations of LSR in the chiral limit $m_{u,d} = 0$.

The first sum rule is chosen in such a way that $\mathcal{O}_8$ disappears to leading order while higher dimensions: $D = 10, 12$ have smaller coefficients than in the individual sum rules:

$$3\mathcal{L}_0 + \tau\mathcal{L}_1 = 2\mathcal{O}_4\tau + \mathcal{O}_6\frac{\tau^2}{2} - \mathcal{O}_{10}\frac{\tau^4}{24} - \mathcal{O}_{12}\frac{\tau^5}{60} - \mathcal{O}_{14}\frac{\tau^6}{240} - \mathcal{O}_{16}\frac{\tau^7}{1260} - \mathcal{O}_{18}\frac{\tau^8}{8064}. \tag{14}$$

In the second sum rule, $\mathcal{O}_6$ disappears and then $\mathcal{O}_8$ will dominate the LSR:

$$\mathcal{L}_0 + \frac{\tau}{2}\mathcal{L}_1 = \mathcal{O}_4\frac{\tau}{2} - \mathcal{O}_8\frac{\tau^3}{12} - \mathcal{O}_{10}\frac{\tau^4}{24} - \mathcal{O}_{12}\frac{\tau^5}{80} - \mathcal{O}_{14}\frac{\tau^6}{360} - \mathcal{O}_{16}\frac{\tau^7}{2016} - \mathcal{O}_{18}\frac{\tau^8}{13440}. \tag{15}$$

Therefore, we use the sum rule in Eq. (14) (respectively Eq. (15)) for extracting $\mathcal{O}_6$ (respectively $\mathcal{O}_8$). We use the known tiny value of $D = 4$ quark condensate contribution given in Eq. (13). The analysis is shown in Fig. 2 and the results are given in Table 1.

Finally, we analyze the τ-like decay sum rule, which has the advantage to be kinematically suppressed near the real axis:

$$\mathcal{L}_{01} \equiv \int_0^{t_c} dt\left(1 - \frac{t}{t_c}\right)e^{-t\tau}\frac{1}{\pi}\,{\rm Im}\,\Pi_{\rm LR} = \sum_{n\geqslant 2}\mathcal{O}_{2n}\frac{\tau^{(n-1)}}{(n-1)!}\left[1 + \frac{(n-1)}{t_c\tau}\right], \tag{16}$$

from which we deduce $\mathcal{O}_6$ using as input $\mathcal{O}_4$ and the higher-dimension condensates.

Our different results are summarize in Table 1.

4. Comparison with existing estimates

4.1. Large-N_c and minimal hadronic approximation (MHA)

Our results agree in signs and in magnitude until the $D = 14$-dimension condensates with the ones in [19] obtained using large N_c and the minimal hadronic approximation (MHA) and with its improved version including the next radial vector meson ρ'.

Our result for the $D = 16$ condensate still agrees in sign with the one in [19] but our absolute value is lower than the one in [19] by about 2σ.

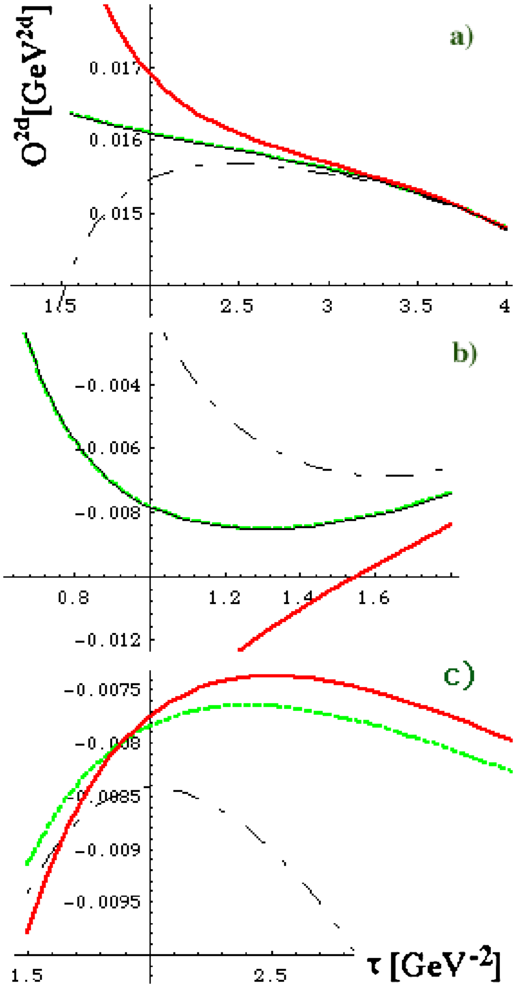

Fig. 2. Same as in Fig. 1 but for the improved analysis in Section 3: (a) $\mathcal{O}_8$ from Eq. (15), (b) $\mathcal{O}_6$ from Eq. (14) and (c) $\mathcal{O}_6$ from Eq. (16).

4.2. ALEPH and OPAL estimates from τ-decay

Our results for the low $D = 6, 8$ condensates agree also quite well with the ALEPH/OPAL estimate of the separate V and A channels [2,3], [21], from which one can deduce the $V-A$ difference.

4.3. Exponential-like sum rules

The value of the $D = 6$ condensate also agrees within the errors with the results in [22], but the values of $\mathcal{O}_{8,10}$ obtained in the present Letter are about two times higher. However, our analysis differs from [22] who use a least square fitting procedure with some other forms of LSR with a different kernel. Due to the alternate sign of the condensate contributions to the OPE, the fitting procedure can be inaccurate as we shall see explicitly in a forthcoming example.

4.4. Finite energy sum rules (FESR)

In [24–26], FESR:

$$\mathcal{M}_n \equiv \int_0^{t_c} dt\, t^n \frac{1}{\pi} \operatorname{Im} \Pi_{LR} = (-1)^n \mathcal{O}_{2n+2} \qquad (17)$$

($n = 0, 1, 2, 3$), and its slight variants have been used for determining $\mathcal{O}_{6,8}$. However, unlike the case of LSR analyzed in previous sections, the results depend crucially on the choice of t_c at which one extracts the optimal results. The two sets of t_c-values corresponding to the zeros of the 1st or/and 2nd Weinberg sum rules are given in Eq. (9). The results from LSR are consistent with the ones corresponding to value of $t_c \approx 1.5\ \mathrm{GeV}^2$, while instead in [24–26], the higher value of $t_c \approx 2.5\ \mathrm{GeV}^2$ has been favored. As a consequence, the value of $\mathcal{O}_8$ and other higher-dimension condensates obtained in these works are opposite in signs[4] with the ones from LSR and from MHA in large N_c. Taking the value of t_c in Eq. (10), we give the version of the FESR results of [24,26] in Table 1, where the slight difference is due to the different parametrizations of the τ-decay data (neural network in [26]) and to the different weights introduced for improving the original FESR.

4.5. Weighted finite energy sum rules

This FESR-like sum rule called “pinched-weight FESR” (hereafter denoted wFESR) by the authors [28] is an involved variant of the FESR in Eq. (17):

$$J_{\omega_n} \equiv \int_0^{t_c} dt\, \omega_n\left(\frac{t}{t_c}\right) \frac{1}{\pi} \operatorname{Im} \Pi_{LR}, \qquad (18)$$

[4] We have corrected the sign of $\mathcal{O}_8$ in the curve (a), Fig. 5 of [25]. Therefore curve (a) and (b) cross at $t_c \approx 1.3\ \mathrm{GeV}^2$ giving the value of $\mathcal{O}_8$ in Table 1. We have also rescaled the normalization by a factor 2 for consistency in our comparison.

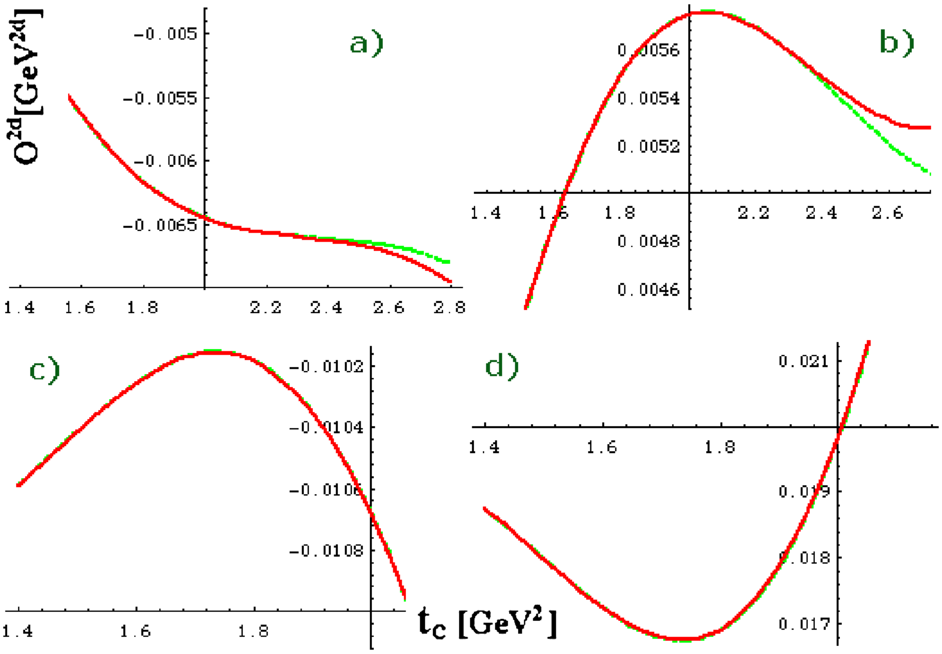

Fig. 3. t_c-behaviour in GeV^2 of different observables used in [28]: (a) $(t_c^2/7)J_{w_1}$, (b) $(t_c^2/2)J_{w_2}$, (c) $\mathcal{O}_6$, (d) $\mathcal{O}_8$ in units of GeV^{2d}, $2d$ being the dimensions of the condensates. The two curves delimit the region induced by the errors of the data. They coïncide in almost all regions except the ones above 2.4 GeV^2 where the data are inaccurate.

where the weight factor ω_n is:

$$\omega_n(x) = x\left[1 - \left(\frac{n}{n-1}\right)x + \left(\frac{1}{n-1}\right)x^n\right], \tag{19}$$

for $n = 2, 3, 4, 5, 6$, and corresponds to the so-called maximally safe analysis. The QCD expressions of these sum rules are given in Eq. (24) of Ref. [28] which we have checked the LO terms.

In order to test the results, we study the t_c-dependence of J_{ω_1} and J_{ω_2} as shown in Fig. 3(a) and (b). We include into the analysis, the known effect of $\mathcal{O}_4$, which we have also recovered in the previous section. We obtain a t_c-stability point (a compromise region between the convergence of the OPE (small t_c) and minimal dependence on the form of the QCD continuum (large t_c)) around 2.0 GeV^2, at which we can extract the optimal value of the condensates. However, one can notice from Fig. 3 that contrary to FESR, the estimates are not very sensitive (change in the last digit) to the values of t_c corresponding to the range in Eq. (9). Neglecting the small radiative corrections for illustration, one obtains in units of 10^{-3} GeV^6:

$$J_{\omega_1} \Rightarrow \mathcal{O}_{68}^{(1)} \equiv \mathcal{O}_6 + \frac{3}{7}\frac{\mathcal{O}_8}{t_c} \approx -6.6,$$
$$J_{\omega_2} \Rightarrow \mathcal{O}_{68}^{(2)} \equiv \mathcal{O}_6 + \frac{1}{2}\frac{\mathcal{O}_8}{t_c} \approx -5.8. \tag{20}$$

We insert into this expression the values in units of 10^{-3} of $\mathcal{O}_{6,8}$ fitted by [28] (mean value from ALEPH and OPAL fit):

$$\mathcal{O}_6 \simeq -4.9, \qquad \mathcal{O}_8 \simeq -3.8, \tag{21}$$

which gives:

$$\mathcal{O}_{68}^{(1)}(2.15) \simeq -5.7 \quad \text{and} \quad \mathcal{O}_{68}^{(2)}(2.15) \simeq -5.8. \tag{22}$$

This test shows the consistency between the results obtained using t_c stability in Eq. (20) and the least square fit in Eq. (21).

Alternatively, we can also solve the two equations J_{ω_1} and J_{ω_2} for extracting the two solutions $\mathcal{O}_6$ and $\mathcal{O}_8$. We study the t_c-dependence of the results in Fig. 3(c) and (d). Here, the stability is obtained at $t_c \simeq 1.7$ GeV^2 which differs from the one obtained previously. We may interpret this difference as due to the fact that we do not consider here the same observables as in Fig. 3(a) and (b). However, in order to have conservative results, we shall consider t_c in the range (1.7–2.0) GeV^2 where the two stabilities are obtained. One can inspect that, in this range, the estimate is only slightly affected by the t_c-values. The results are given in Table 1 with a good accuracy. We check again the consistency of the results by inserting these values into Eq. (20), which gives:

$$\mathcal{O}_{68}^{(1)}(2.15) \simeq -6.7 \quad \text{and} \quad \mathcal{O}_{68}^{(2)} \simeq -6.1. \tag{23}$$

Our test does not support the results given in [28] obtained from numerical fits. This may due to the fact that the terms in the series have alternate signs, and/or where the 2nd term is a small correction of the 1st one, and may be difficult to extract from the fitting procedure. Instead, we expect that the new results from this method which we give in Table 1 obtained using stability criteria, from solving the two equations J_{ω_1} and J_{ω_2} for extracting the two unknown $\mathcal{O}_6$ and $\mathcal{O}_8$ are more reliable.

Notice that in a large range of t_c, the two estimates of $\mathcal{O}_6$ and $\mathcal{O}_8$ do not flip sign, which, like in the case of LSR, can be due to the weight factor in the spectral integral. This is not the case of the basic FESR.

In principle, once one knows $\mathcal{O}_6$ and $\mathcal{O}_8$, one can extract the other high-dimension condensates from the set of equations given in Eq. (24) of [28]. $\mathcal{O}_{10}$ to $\mathcal{O}_{16}$ can be, e.g., extracted from J_{ω_3} to $J_{\omega_{10}}$. However, more we go to higher moments, less the accuracy on the estimate is reached as the high-dimension terms which one wishes to extract are tiny corrections to the leading order terms, while the method is not accurate enough to pick up these tiny corrections. For instance at $t_c \approx 2$ GeV^2, the QCD parts of the sum rules normalized to the leading $\mathcal{O}_6$ contributions read:

$$J_{\omega_5} \sim \# \left[1 + 0.005\left(\frac{2\ \text{GeV}^2}{t_c}\right)^4 \mathcal{O}_{14}\right],$$
$$J_{\omega_6} \sim \# \left[1 - 0.002\left(\frac{2\ \text{GeV}^2}{t_c}\right)^5 \mathcal{O}_{16}\right]. \tag{24}$$

The corrections are a factor 2 smaller for J_{ω_9} and $J_{\omega_{10}}$. This fact may explain why relatively large central values of the high-dimension condensates emerge from this method. A tentative extraction of $\mathcal{O}_{10}$ to $\mathcal{O}_{16}$ from J_{ω_3} to J_{ω_6} shows that the t_c-dependence present a flat stability around 1.25 GeV^2 and another extremum around 1.8 GeV^2. The values obtained at the second point are very sensitive to the input value of $\mathcal{O}_6$ and flip sign compared to the one at the flat plateau for $D \geqslant 14$, a feature similar to $\mathcal{O}_8$ from FESR analysis [24,26]. This may indicate that the weight factor is less efficient for high-dimension condensates. We have excluded the high-t_c solution similarly to the FESR case, and we deduce the values in Table 1.

4.6. Test of the factorization assumption

The $D = 6$ condensate contributions to Π^{LR} have been first derived in [7] using the leading order result of [8] for the vector and axial-vector correlators. The radiative corrections have been obtained in [33,34]. Using an anti-commuting γ_5 matrix and the choice of operator basis in [34], it reads by assuming a factorization of the four-quark condensates:

$$\mathcal{O}_6 = -\frac{64}{9}\pi\alpha_s\langle\bar{u}u\rangle^2\left[1 + \frac{\alpha_s}{\pi}\left(\frac{89}{48} - \frac{1}{4}\ln\frac{Q^2}{\nu^2}\right)\right]. \tag{25}$$

Using the NDLR or/and the HV regularization scheme, the same contribution reads, to leading order in N_c at $Q^2 = \nu^2$ [24]:

$$\mathcal{O}_6 = -8\pi\alpha_s\left[\langle\bar{u}u\rangle^2\left(1 + \frac{\alpha_s}{\pi}\frac{61}{12}\right) - \frac{1}{16\pi^2}A_{\text{LR}}\right], \tag{26}$$

where $A_{\text{LR}} \simeq (4.4 \pm 0.5) \times 10^{-3}$ is of order α_s^2.

The $D = 8$ four-quark condensate contributions have been obtained in [35] where a $1/N_c^2$ ambiguity has been noticed. The $D = 10$ condensates have been obtained in [22]. Assuming factorization, one can write:

$$\mathcal{O}_8 = \frac{64}{9}\pi\alpha_s\langle\bar{u}u\rangle^2 M_0^2,$$
$$\mathcal{O}_{10} = -\frac{8}{9}\pi\alpha_s\langle\bar{u}u\rangle^2\left[\frac{50}{9}M_0^2 + 32\pi\langle\alpha_s G^2\rangle\right]. \tag{27}$$

M_0^2 is the scale governing the mixed condensate and is equal to (0.8 ± 0.2) GeV^2 from the baryon sum rules [36,37], B–B^* mass-splitting [38] and string model [39]. We shall use the value of the gluon condensate $\langle\alpha_s G^2\rangle = (0.07 \pm 0.01)$ GeV^4 from e^+e^- data [12,40]. Within the factorization assumption, we shall include the log-dependence of the quark condensate and of α_s, which give:

$$\alpha_s \langle \bar{u}u \rangle^2|_{\rm fac} \simeq \frac{2}{9} \frac{\pi}{(\log Q/\Lambda)^{1/9}} \langle \widehat{\bar{u}u} \rangle^2, \tag{28}$$

which is 1.6×10^{-4} GeV^6 if one uses the invariant quark condensate $\langle \widehat{\bar{u}u} \rangle \simeq -(248\ \text{MeV})^3$ [41] and evaluate $\alpha_s(\tau)$ at $\tau = 1.5$ GeV^{-2} at which $\mathcal{O}_8$ has been extracted from the sum rule. Using these numerical inputs, we deduce in units of 10^{-3} GeV^D (D being the dimension of the condensate):

$$\mathcal{O}_6|_{\rm fac} \approx -3.6,$$
$$\mathcal{O}_8|_{\rm fac} = +2.9,$$
$$\mathcal{O}_{10}|_{\rm fac} = -4.7. \tag{29}$$

Comparing these values with the ones in Table 1 and Section 3, we conclude that the factorization assumption agrees in sign with these results but underestimate the absolute value of the condensates by a factor 2–5. This feature is similar to the case of the vector [12,14,42], axial-vector [11,43], baryon [37] sum rules and from the analysis of the V and $V + A$ τ-decay data [2,3,11]. From the theoretical point of view, the factorization assumption is only consistent with the renormalization of operators to leading order in $1/N_c$ due to mixing of different operators having the same dimensions [44].

5. Conclusions

We have used the V–A component of the hadronic tau decays data for exploring the vacuum structure of the $\Pi^{\rm LR}$ QCD correlator using a set of Laplace sum rules (LSR). We have also revisited different estimates based on FESR and its variant in Section 4. Our results are summarized in Table 1.

Contrary to most papers in the literature, we do not perform a least square fitting procedure for extracting simultaneously different condensates, but instead use the stability criteria (existence of minima or inflexion points) for our estimate of the condensates. Due to the alternate signs of the condensate contributions in the OPE and to the fact that in most methods, the high-dimension condensate contributions are corrections to the lowest-dimension condensates in the analysis, the approaches for extracting these high-dimension condensates can become inaccurate.

Instead, our strategy is to look for sum rules which disentangle, from the beginning, the relevant high-dimension condensates, and then makes the analysis cleaner and more transparent.

We have given a first estimate of the size of the $D = 18$ condensates, which will be interesting to check using alternative methods. The LSR estimate, which we expect to be more appropriate for extracting higher-dimension condensates than wFESR and FESR, shows that the size of the very high $D = 16$ and $D = 18$ condensates are relatively small which may indicate the good convergence of the OPE even at large τ-values.

During the analysis, as one can see in previous figures, the absolute values of the condensates are slightly affected by τ and t_c in the optimum region (minimum or inflexion point). However, it is important to notice that the results from LSR in large range of τ- and t_c-values do not flip sign, which is a great advantage compared to the ones from some finite energy like sum rules discussed in the literature.

The extension of the present analysis to some other channels are feasible though not straightforward. This is due to the relative importance of the continuum pQCD contribution for higher moments in some other channels, which is not the case of V–A, where this effect exactly cancels at higher energies. We plan to come back to these different channels in a future publication.

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