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Non-linear models for the prediction of specified design strengths of concretes development profile

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KEYWORDS

Compressive strength;
Strength prediction;
Model correlation;
Model validation

Abstract Different concrete structures are designed according to their concrete strength requirements. Consequently, concrete strength is one of the prime properties of concrete structures. In this study, compressive strength behavioral pattern of seven design strength concretes 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa at curing ages of 3, 7, 14, 21, 28, 56, 90 and 180 days was examined. In order to evaluate the long term effects on compressive strength of target design concretes, 360 cylindrical samples were cast. On the basis of the existing experimental tested strength data, a polynomial equation based model having 2 degrees with fractional power of 0.5 degree interval of each term was found to have acceptable correlation for describing the compressive strength gaining profile with the tested concrete ages. Correlation of proposed model was justified against the statistical point of view for examining the best fit profile with the observations. Apart from the correlation approach, the accuracy of the proposed model was validated with corresponding experimental observations of target design concretes followed by the model parameters estimation with 95% confidence interval. From the predicted results, the study revealed that proposed polynomial equation based model possessed strong potential for predicting 3, 7, 14, 21, 28, 56, 90 and 180 days compressive strength of design concretes with high accuracy and trivial error rates.

Introduction

Compressive strength is the design property of concrete. An overall view of concrete quality is reflected by the concrete strength [1]. In addition, compressive strength is a structural engineering performance measure, employed for designing concrete structures [2]. In practice, design engineers use different specified concrete strengths to design the structural components. For instance, minimum compressive strengths (severe exposure) of concrete for interior slabs, foundations walls

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Nomenclature

(f'_c)	compressive strength of concrete (MPa)	t	curing age of concrete (day)
E_c, ME	modulus of elasticity of concrete (MPa)	R^2 (R-squared)	coefficient of determination
f_r, MR	modulus of rupture of concrete (MPa)	RMSE	root mean square error
λ	modification factor for type of concrete	NSE	Nash-Sutcliffe efficiency
f_{sp}, STS	splitting tensile strength of concrete (MPa)	CMO	correlated model option
P	maximum applied load on concrete (N)	OPC	ordinary Portland cement
l	length of the concrete sample (mm)	SD	standard deviation
d	diameter of the concrete sample (mm)	ASTM	American Society for Testing and Materials
M_S	weight of specimen at fully saturated condition	M_D	weight of oven dried specimen
a & b	model constant parameter	ACI	American Concrete Institute

and garage floor slabs are 17 MPa, 21 MPa and 24 MPa respectively [3].

Rashid and Mansur [4] provided data that concrete of compressive strength of 30 MPa was regarded as high strength in the 1950s. Gradually, concretes of compressive strength of 40–50 MPa in the 1960s, 60 MPa in the 1970s, and 100 MPa and beyond in the 1980s have evolved and were used in structures. Moreover, PCA [5] classified compressive strength of concrete as normal, high, very high and ultra-high strength of ranges from < 50 MPa, 50–100 MPa, 100–150 MPa and > 150 MPa respectively.

Dead loads and size of structural members can be reduced using the specified design high strength concrete than the normal strength concrete. Also, not only stronger but also lightweight durable structure can be designed with increasing the high strength concrete application that also minimizes the cost of the structures [5].

A numerous empirical equations have been used to predict the physical properties and compressive strength of concrete to design structural members. Therefore, prediction of concrete strength has been considered as an active area of research and a considerable number of studies have been carried out. Many attempts have been made to obtain a suitable mathematical Model which is capable of predicting concrete strength at various ages with acceptable high accuracy [6,7]. Additionally, early age strength prediction is very useful in reducing construction cost and ensuring safety of construction works. Furthermore, early age strength prediction has several practical applications [8]. Besides, a rapid and reliable concrete strength prediction would be of great significance [7] for the overall construction processes.

In this research, the findings were divided into three sections, physical properties of concrete constituent materials, different behavioral pattern of specified strength concrete, and the third approach was to develop a suitable high accuracy mathematical Model for predicting the compressive strength development profile of specified design concrete strength for 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa on the basis of curing age. ACI mix design procedure was applied for all design strength concrete. All tests were conducted in the laboratory of the Department of Civil Engineering, Leading University, Sylhet, Bangladesh.

In this study, some non-linear models such as power equation, exponential equation, logarithmic equation and polynomial equation based model were applied to observe the best correlated model profile for 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa strength development with curing age. The correlated model equations for each tested design strength concrete may be potential of use for observing the strength gaining pattern with the age of structures. The accuracy of the concrete strength predicted models was justified through statistic evaluation followed by validation and checking the 95% confidence level of the estimated model parameters.

Materials and experimental program

Physical properties of concrete materials

Coarse aggregate (CA)

Conventional CA was collected from the local stone crusher areas. The higher and lower sizes of CA were 19 mm and 12 mm. The unit weight (UW), specific gravity (SG), water absorption percentage (WA %) and water content of coarse aggregates were determined according to the ASTM standard methods [9–11] respectively. A summary of test results is presented in Table 1.

Fine aggregate (FA)

In the study, FA was collected from locally available natural valley sand collecting areas. Physical parameters of FA such as unit weight, specific gravity, water absorption percentage and fineness modulus (FM) were determined using the ASTM standard methods [9,12–14] respectively. These parameters were analyzed to compare the effect of sand on concrete prop-

Table 1 Physical properties of aggregates.

Properties	UW (g/cm^3)	SG	WA (%)	FM
CA	1.48 ± 0.12	2.75 ± 0.08	1.5 ± 0.05	–
FM	1.54 ± 0.1	2.6 ± 0.15	1.21 ± 0.06	2.45 ± 0.12

erties. Each test was performed 3 times for confirming the reproducibility of experimental data [15]. Test results of FA are presented in Table 1.

Binding materials

In this research, only OPC was used as a binding material. OPC was purchased commercially from locally available cement factory. Some crucial physical properties of OPC such as normal fineness, consistency, initial setting time, final setting time and specific gravity were determined using the ASTM standard methods [16–19] respectively. To check the compressive strength of cement mortar, test was conducted using the method adopted by ASTM [18] at 3 and 7 days curing ages. Cement used for concrete samples, was type-I and 52.5 grade. A summary of the test results is shown in Table 2.

Design of concrete mixes

The concrete mixes were designed based on the nominal 28-day compressive strength of 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa, following the ACI mix design procedure. Accordingly, the mixed proportions of tested design strengths are shown in Table 3.

Preparation of concrete samples

The concrete specimens were compacted, vibrated and molded in a cylindrical mold of dimensions 6 in. (150 mm) dia. × 12 in. (300 mm) height [20]. After 24 h of casting, cylindrical samples were demolded and cured in distilled water under the average humidity of 94% ± 1% at 25^0 ± 2 °C in the laboratory room. pH of curing water was 7.0 using distilled water. ASTM standards were applied regarding sampling, curing and testing [21].

Concrete properties

A total number of 360 concrete specimens were tested to determine concrete properties including compressive strength, modulus of elasticity, modulus of rupture and splitting tensile strength of specified compressive strengths ranging from 21 MPa to 42 MPa (3000–6000 psi). The variable investigated in this study was only curing age.

Workability test

Concrete workability was measured in terms of slump values following the method adopted by ASTM standard method [22]. Slump test was performed three times on each mix for confirming the accuracy of the results. A summary of slump test results is shown in Table 3.

Table 3 Concrete mixes of seven different specified compressive strengths.

Design strength of concrete (MPa)	Mixing proportion (OPC:FA:CA)	W/C ratio	Slump (cm)
21	1:2.52:3.40	0.59	7.3152 ± 0.45
24	1:2.20:3.10	0.535	6.35 ± 0.35
28	1:1.89:2.77	0.48	5.6642 ± 0.25
31	1:1.55:2.53	0.44	5.3848 ± 0.32
35	1:1.33:2.30	0.4	4.9276 ± 0.42
38	1:1.11:2.07	0.36	4.191 ± 0.28
42	1:0.90:1.84	0.32	3.429 ± 0.22

Water absorption test

The water absorption was measured in order to evaluate whether there was an increase in the concrete structure's pore space as a result of constituents mixing and compactness. Water absorption of all samples was determined at ages 3, 7, 14, 21, 28, 56, 90 and 180. Concrete samples were kept in fully immersion condition until testing age. Water absorption after immersion was calculated using the following equation [23]:

$$\% \text{ Water absorption } (W) = [(M_S - M_D)/M_D] * 100 \quad (1)$$

Water absorption is expressed in percentage and the water uptake relative to the dry mass. Test results are represented in Table 4.

Compressive strength

Compressive strength (f'_c) of studied specified design concrete samples (21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa) samples was tested at concrete ages of 3, 7, 14, 21, 28, 56, 90 and 180 days and was moist cured until the time of testing. The samples were taken out of water approximately 24 h before testing and were kept in the air dry condition in the laboratory [24]. Concrete samples were tested according to ASTM [25].

Modulus of elasticity

The modulus of elasticity is a measure of stiffness which can be determined from the compressive strength test of concrete cylindrical samples. In this study, modulus of elasticity was

Table 2 Characteristics of OPC.

SL No.	Characteristics	Experimented values (Average ± SD)	ASTM standard
01	Fineness (%)	96.2 ± 1.2	Not less than 90%
02	Consistency (%)	26.2 ± 1.5	22–30% by cement weight
03	Initial setting time (min.)	126 ± 6	Not less than 45 min.
04	Final setting time (min.)	251 ± 13	Not more than 375 min.
05	Specific gravity	3.06 ± 0.03	IS-2720 (3.15)
06	Compressive strength (MPa)		
	3 days	1.43 ± 1.2	Minimum 12.0 MPa (ASTM standard)
	7 days	21 ± 1.65	Minimum 19.0 MPa (ASTM standard)
	28 days	37.2 ± 2.03	Minimum 28.0 MPa (ASTM standard)

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Table 4 Water absorption (%) results of all target design concretes strengths samples.

Design strength of concrete (MPa)	21	24	28	31	35	38	42
3 day	5.7 ± 1.9	4.5 ± 1.5	4.1 ± 1.36	3.65 ± 1.21	2.95 ± 0.98	1.98 ± 0.66	1.34 ± 0.44
7 day	6.6 ± 0.94	5.43 ± 0.77	5.15 ± 0.73	4.12 ± 0.58	3.35 ± 0.47	2.45 ± 0.35	1.76 ± 0.25
14 day	7.12 ± 0.50	5.85 ± 0.41	5.56 ± 0.39	4.42 ± 0.31	3.67 ± 0.26	2.63 ± 0.18	1.93 ± 0.13
21 day	7.39 ± 0.35	6.08 ± 0.28	5.71 ± 0.27	4.63 ± 0.22	3.95 ± 0.18	2.78 ± 0.13	2.03 ± 0.09
28 day	7.51 ± 0.26	6.21 ± 0.22	5.86 ± 0.20	4.8 ± 0.17	4.1 ± 0.14	2.89 ± 0.10	2.19 ± 0.07
56 day	7.68 ± 0.13	6.35 ± 0.11	6.02 ± 0.10	4.97 ± 0.08	4.21 ± 0.07	2.97 ± 0.05	2.28 ± 0.04
90 day	7.79 ± 0.08	6.45 ± 0.07	6.13 ± 0.06	5.12 ± 0.05	4.34 ± 0.04	3.08 ± 0.03	2.34 ± 0.02
180 day	7.86 ± 0.04	6.49 ± 0.03	6.23 ± 0.03	5.19 ± 0.02	4.43 ± 0.02	3.13 ± 0.01	2.4 ± 0.02

calculated using the ACI provided formula considering the secant modulus [26]:

$$E_c = 57,000\sqrt{f'_c}\ (\text{psi}) = 4700\sqrt{f'_c}\ (\text{MPa}) \tag{2}$$

The Eq. (2) can be used for concretes with strength up to 6000 psi (42 MPa).

Modulus of rupture

Modulus of rupture is a measure of concrete strength before rupture. It is also referred to as bending strength. In this study, modulus of rupture was calculated according to ACI Code equation [26]:

$$f_r = 7.5\lambda\sqrt{f'_c}\ (\text{psi}) = 0.62\lambda\sqrt{f'_c}\ (\text{MPa}) \tag{3}$$

where λ is a modification factor that bears the values of 1.0, 0.85 and 0.75 for normal-weight, sand-lightweight and all-lightweight concrete respectively.

Splitting tensile strength

Generally, splitting tensile strength is used in the design of structural concrete members to assess the shear resistance provided by concrete materials. Splitting tensile strength of specified concrete strength was calculated using the standard method given by ASTM [27]:

$$f_{sp} = \frac{2P}{\pi l d} \tag{4}$$

Concrete strength prediction models

Concrete compressive strength is influenced by many factors including water/cement ratio, cement content and properties, aggregate type and its properties, etc. [8]. This study introduces simple mathematical model that can help to predict the rate of compressive strength gain for different design concrete strengths at different ages. For better understanding the development pattern of some specified design concrete compressive strengths such as 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa with curing ages, the four non-linear well-known empirical equation based models were used. Many researchers also used these models as only one independent variable (curing age) type model equation. The mathematical features of the models were described below.

Polynomial equation based model

Plecas [28] adopted the polynomial equation based model up to its fourth terms with fraction power values to observe the better leaching behavior of 137Cs from radioactive waste formation. In this study, this model equation was adopted to predict the compressive strength of specified design concrete strength:

$$f'_c = a_1 + a_2t^{1/2} + a_3t^1 + a_4t^{3/2} \quad [\text{model 01}] \tag{5}$$

Logarithmic equation based model

Abd elaty [8], Ukpata et al. Yeh [29] and Yeh [30] applied the logarithmic equation based model to predict the compressive strength of concrete in different conditions. The model equation is expressed below:

$$f'_c = a + b\ln(t) \quad [\text{model 02}] \tag{6}$$

Power equation based model

Resheidat and Madanat [21] and Yeh [30] adopted the power equation based model to predict the compressive strength where the dependent variable was compressive strength (f'_c) and curing age (t) is an independent variable. The model equation is expressed below:

$$f'_c = at^b \quad [\text{model 03}] \tag{7}$$

Exponential equation based model

As the concrete strength development pattern is non-linear, exponential equation based model can be used to predict the strength of different curing ages where the curing time (t, day) is only an independent variable parameter. The model equation is expressed below:

$$f'_c = ae^{bt} \quad [\text{model 04}] \tag{8}$$

Model evaluation

The adopted four non-linear models were compared to experimental generated data from laboratory tests followed by the justification of the accuracy using statistical parameters such as R^2, RMSE and NSE. In general, the values of R^2 range from 0 to 1, with higher values indicating less error variance. However typically the values of R^2 greater than 0.5 are considered acceptable [31–34]. RMSE values approaching toward 0 and NSE values being 1 indicate the best fit of the model profile with the observations [31].

Model extension

In the current study, polynomial equation based model was extended in different longer terms with 0.5 and 0.25 degree

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intervals as trial basis using the least squares procedure for demonstrating the best fit model of compressive strength development profile in a better way with high accuracy. The current research adopted the following model options as trial basis to well correlate the proposed model for demonstrating the specified design concrete strength development with concrete ages:

$$f'_c = a_1 + a_2t + a_3t^2 \quad [29] \quad [\text{CMO1}] \tag{9}$$

$$f'_c = a_1 + a_2t^{1/2} + a_3t + a_4t^{3/2} \quad [28] \quad [\text{CMO2}] \tag{10}$$

$$f'_c = a_1 + a_2t^{1/2} + a_3t + a_4t^{3/2} + a_5t^2 \quad [35] \quad [\text{CMO3}] \tag{11}$$

$$f'_c = a_1 + a_2t^{1/4} + a_3t^{1/2} + a_4t^{3/4} + a_5t + a_6t^{5/4} + a_7t^{3/2} \quad [\text{CMO4}] \tag{12}$$

Results and discussion

Characteristics of design strengths

Cement based materials develop strength with continued hydration. The rate of gain of strength is faster at start and the rate is reduced with age. In spite of considering the 28-day compressive strength for design purposes, actually concrete develops strength beyond 28 days as well [8,36]. Fig. 1 represents the average compressive strength development pattern with standard deviation values at different concrete ages of specified design concrete such as 21 MPa (3000 psi), 24 MPa (3500 psi), 28 MPa (4000 psi), 31 MPa (4500 psi), 35 MPa (5000 psi), 38 MPa (5500 psi) and 42 MPa (6000 psi). Compressive strength of all design concretes increased appreciably with concrete ages due to prolonged cement hydration. It can be observed that the rapid strength at curing age ranges from 3 to 28 days. But increasing with lower rate of strength gaining profile was from 28 to 180 days. Moreover, the analysis revealed the higher increasing trend of compressive strength with the higher design strength of concrete. Among the specified design strength, the highest compressive strength development trend at different concrete ages was achieved for 42 MPa because of the cement content. Accordingly, the decreased pattern of concrete strength was observed with the reduction of cement content. Comparatively the lowest strength gaining profile at each curing ages was observed for design strength 21 MPa than the other design strengths. As all the specified concrete strengths were designed based on the nominal 28-day compressive strength, the crucial matter is that samples of all concrete types exceeded the design strength of 28 days.

The increase of concrete strength with time relative to the 28-day strength is shown in Fig. 2. As the concrete mixes were designed based on the 28-day strength, the strength increasing ratio was taken 1 at 28-day concrete age. At 3 days curing age, strength increasing ratios were found to be 0.518, 0.497, 0.450, 0.469, 0.479, 0.463 and 0.470 for 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa respectively. This means almost 45–50% strength gained within 3 days curing age. In addition, the study observed the strength development ranges from 65% to 71% of the design strengths at 7 curing period of concrete ages (Fig. 2). The cylinder test results showed that concrete strength at 28 day was 1.45–1.5 times of the 7 day age strength. According to Aziz [37], strength increasing ratios of the 28–7 days lie generally between 1.3 and 1.7 but the majority of the results were above 1.5. Similar data were also reported by Hassoun and Al-Manasser [26], and concrete strength at 28 days is 1.5 times that of 7 days. The analysis revealed the lower rate of concrete strength attainment after 28 day of concrete age. Slight increase with lower rate of strength development pattern was also observed from ages of 90 to 180 days for each tested specified design concrete.

Common trend in concrete technology is to use compressive strength as a quantitative measure for other properties of concrete [8,36]. In this study, the secant modulus of elasticity (E_c), modulus of rupture (f_r) and splitting tensile strength (f_{sp}) of design strength concretes are presented in Figs. 3–9 as the function of tested compressive strength results. The analysis shows that secant modulus of elasticity, modulus of rupture and splitting tensile strength of design concretes increased with the increase of compressive strength. In addition, results showed that the higher increasing trend of E_c, f_r and f_{sp} was observed regarding the higher design strength of concrete (Figs. 3–9). Moreover, linearly increasing trends were relative to the compressive strength increases for all tested design strength concrete. Other researchers [38–40,24] also reported similar findings. A least-square linear regression analysis of the E_c, f_r and f_{sp} trend values of tested design strengths of concretes showed best-fitness with the square-root of cylinder compressive strength for different concrete ages (Figs. 3–9). Best-fitness was evaluated using the statistical parameter such

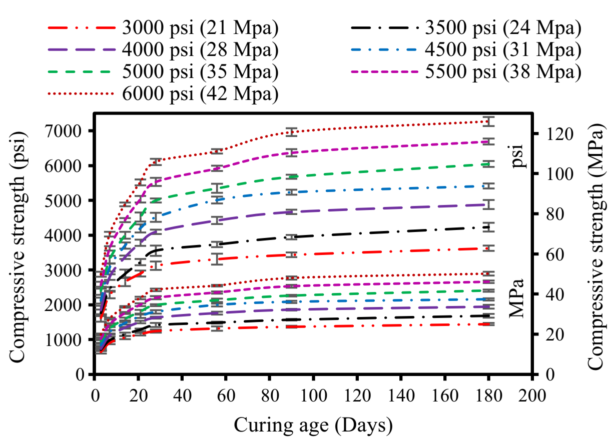

Fig. 1 Compressive strength development profile of specified design strength of concrete.

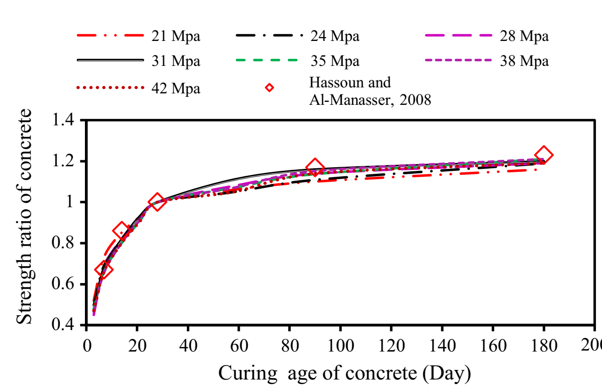

Fig. 2 Compressive strength increasing ratio with concrete age.

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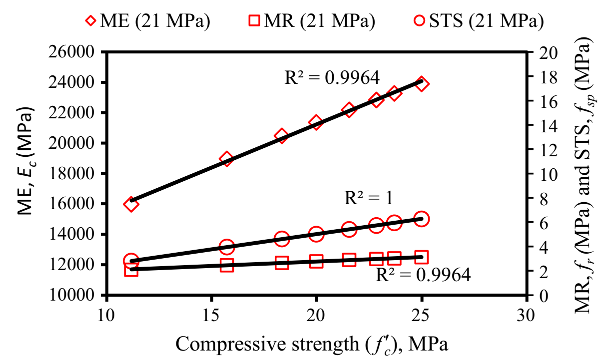

Fig. 3 Variation of ME, MR and STS for design strength of 21 MPa.

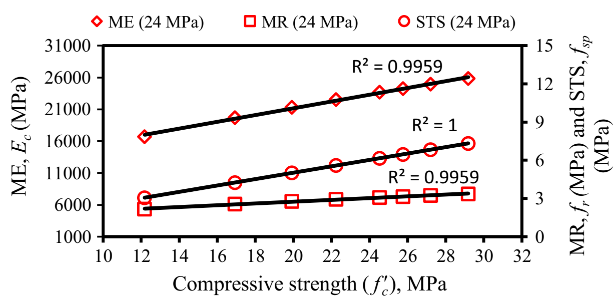

Fig. 4 Variation of ME, MR and STS for design strength of 24 MPa.

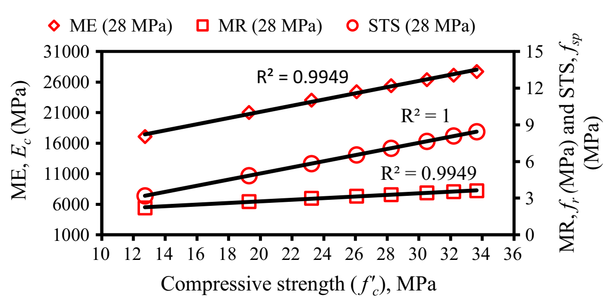

Fig. 5 Variation of ME, MR and STS for design strength of 28 MPa.

as coefficient of determination (R^2). The proposed relationships of E_c, f_r and f_{sp} with the cylinder compressive strength and ACI Code suggested relationships which are shown in Table 5 and may be used to estimate the tested properties of concrete for design strengths of concretes of 21–42 MPa.

Correlation of non-linear models

Figs. 10–16 represents the correlation of non-linear models for predicting the compressive strength (f'_c) of specified design concrete strengths of like 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 28 MPa and 42 MPa respectively with the curing ages. Four different non-linear equation based models were compared with experimental observed data to get a better understanding of strength development pattern on long term

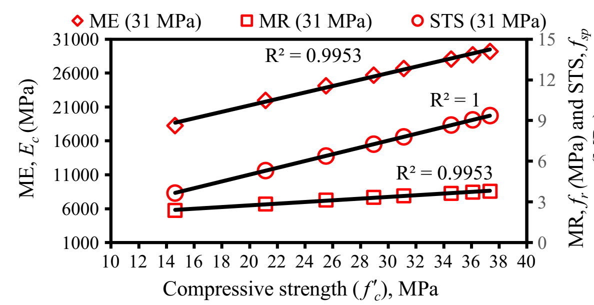

Fig. 6 Variation of ME, MR and STS for design strength of 31 MPa.

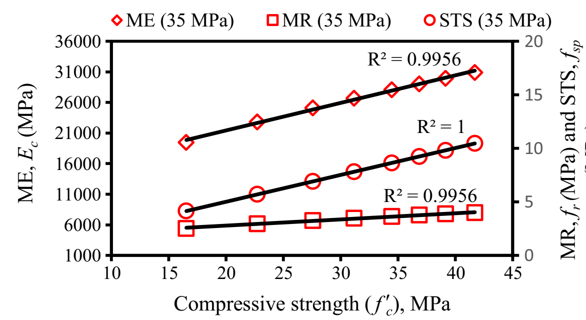

Fig. 7 Variation of ME, MR and STS for design strength of 35 MPa.

curing ages. Model parameter estimation was conducted following non-linear techniques using software. Parameters were estimated with 95% confidence level. In order to evaluate the models correlation statistically, R^2, RMSE and NSE values were measured (Figs. 10–16). Based on comparison between the proposed equations, it was observed that R^2 and NSE values of the four non-linear models for all target design strengths of concretes were greater in model 01 (polynomial equation based model) than the other three models. In addition, model 01 also showed lowest value of RMSE (Figs. 10–16). Therefore, the model 01 showed the best correlation to the experimental observation for evaluating the compressive strength gaining characteristics during the curing ages, whereas models 2, 3 and 4 failed to describe the strength development pattern properly. Model 04 demonstrated the unrealistic strength gaining profile compared with the experimental results for all

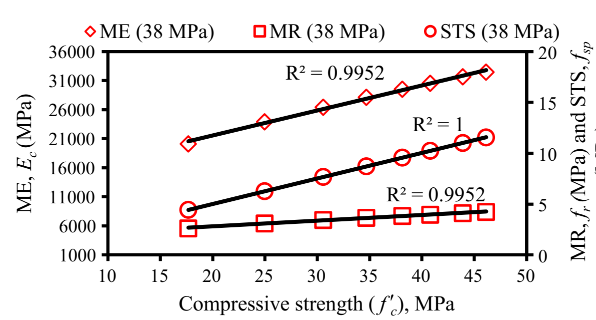

Fig. 8 Variation of ME, MR and STS for design strength of 38 MPa.

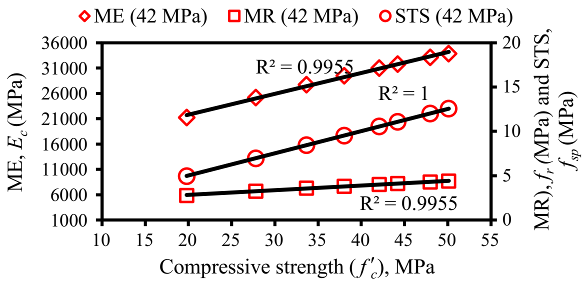

Fig. 9 Variation of ME, MR and STS for design strength of 42 MPa.

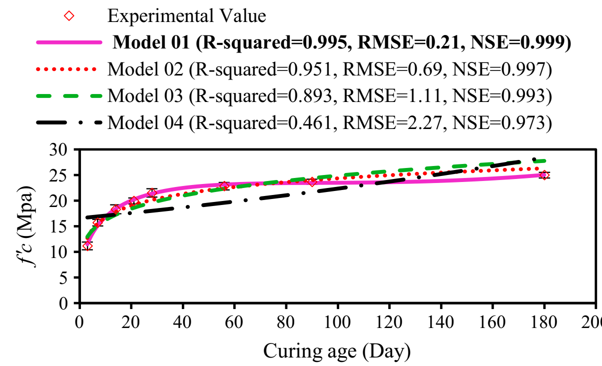

Fig. 10 Correlation of non-linear models for strength development prediction of 21 MPa.

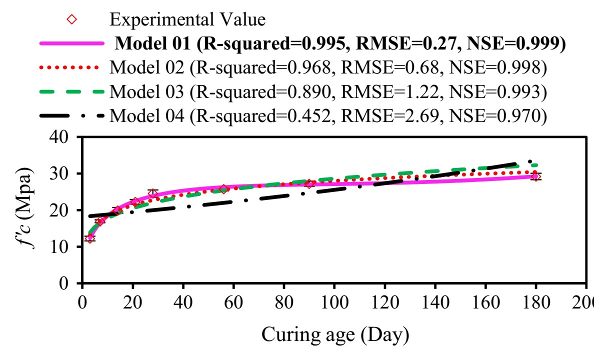

Fig. 11 Correlation of non-linear models for strength development prediction of 24 MPa.

design concrete (Figs. 10–16). Accordingly, model 01 was the well correlated model with high accuracy among the studied four non-linear models to represent the increasing rate of studied design concrete strength with the concrete ages in full immersion curing condition.

Correlation of extended models

In this study, correlation of polynomial equation based on four extended models denoted as COM1, CMO2, CMO3 and CMO4 was tested respectively. Figs. 17–23 presents the comparison of models profiles with the experimental observation data for specified design strengths of concretes 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa respectively. The study revealed preferable correlation trend with the breakdown of independent variable's (curing age) full power to fractional power. CMO1 (Quadratic least square regression equation) showed the most deviated strength gaining profile with the experimental observations than the other model options for all target design strengths of concretes (Figs. 17–23). In CMO1, independent variable parameter contains the power 1 and 2 whereas in CMO2 and CMO3 each term of independent variable's power was extended with fractional power like 0.5 degree. The study showed that CMO2 and CMO3 described the better correlation of strength development than CMO1. This means, fractional power based model equation shows well fitness to the observations with high accuracy and low errors than whole (non-fractional) power based model equation (Figs. 17–23). Moreover, CMO4 was also extended. But model profile of CMO4 showed higher deviation form with greater RMSE values than CMO2 and CMO3. The study found that if the power of polynomial based equation, breaks down into the fractional smaller values

Table 5 Proposed relationship of specified design concretes properties with compressive strength.

Design strength (MPa)	Proposed equation		
	Modulus of elasticity (MPa)	Modulus of rupture (MPa)	Splitting tensile strength (MPa)
21	$E_c = 569.52\sqrt{f'_c} + 9852$	$f_r = 0.0739\sqrt{f'_c} + 1.28$	$f_{sp} = 0.2509f'_c - 2 \times 10^{-14}$
24	$E_c = 533.88\sqrt{f'_c} + 10{,}489$	$f_r = 0.0691\sqrt{f'_c} + 1.36$	$f_{sp} = 0.2509\sqrt{f'_c}$
28	$E_c = 504.77\sqrt{f'_c} + 11{,}026$	$f_r = 0.0655\sqrt{f'_c} + 1.43$	$f_{sp} = 0.2509\sqrt{f'_c}$
31	$E_c = 474.73\sqrt{f'_c} + 11{,}742$	$f_r = 0.0616\sqrt{f'_c} + 1.52$	$f_{sp} = 0.2509\sqrt{f'_c} - 2 \times 10^{-14}$
35	$E_c = 450.34\sqrt{f'_c} + 12{,}401$	$f_r = 0.0584\sqrt{f'_c} + 1.61$	$f_{sp} = 0.2509\sqrt{f'_c}$
38	$E_c = 430.21\sqrt{f'_c} + 12{,}954$	$f_r = 0.0558\sqrt{f'_c} + 1.68$	$f_{sp} = 0.2509\sqrt{f'_c} - 7 \times 10^{-14}$
42	$E_c = 410.33\sqrt{f'_c} + 13{,}601$	$f_r = 0.0532\sqrt{f'_c} + 1.76$	$f_{sp} = 0.2509\sqrt{f'_c} - 2 \times 10^{-14}$
ACI suggested equation	$E_c = 4700\sqrt{f'_c}$ (For concrete strength up to 42 MPa)	$f_r = 0.62\sqrt{f'_c}$	$f_{sp} = \frac{2P}{\pi ld}$ [ASTM recommended equation]

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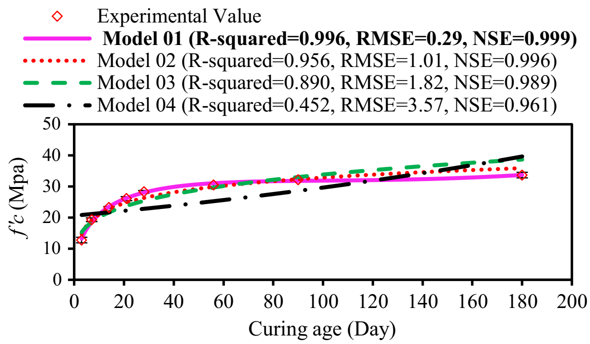

Fig. 12 Correlation of non-linear models for strength development prediction of 28 MPa.

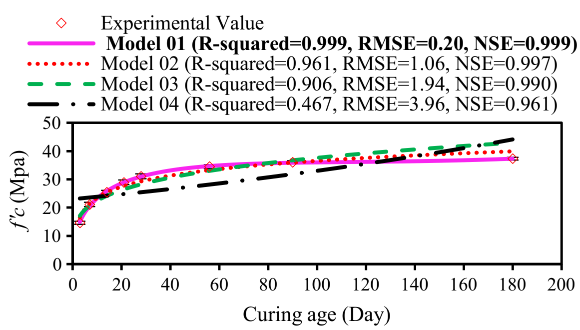

Fig. 13 Correlation of non-linear models for strength development prediction of 31 MPa.

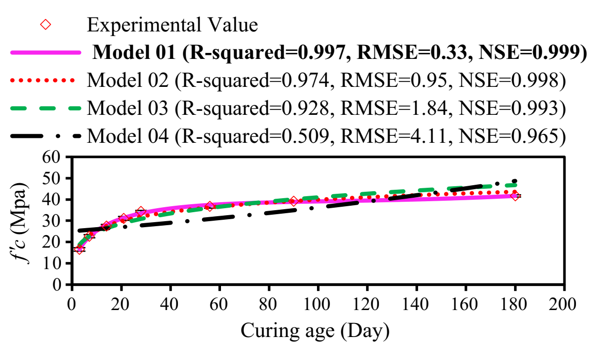

Fig. 14 Correlation of non-linear models for strength development prediction of 35 MPa.

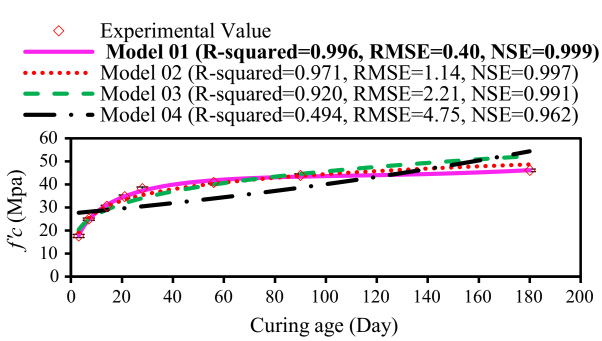

Fig. 15 Correlation of non-linear models for strength development prediction of 38 MPa.

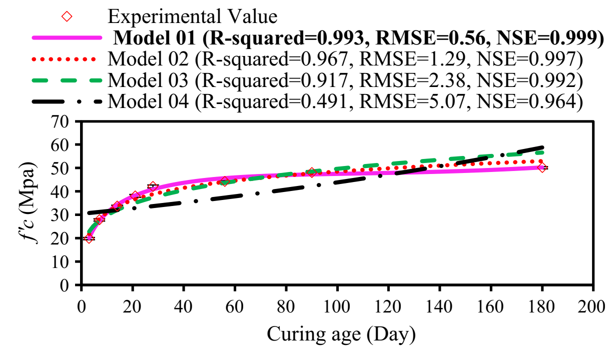

Fig. 16 Correlation of non-linear models for strength development prediction of 42 MPa.

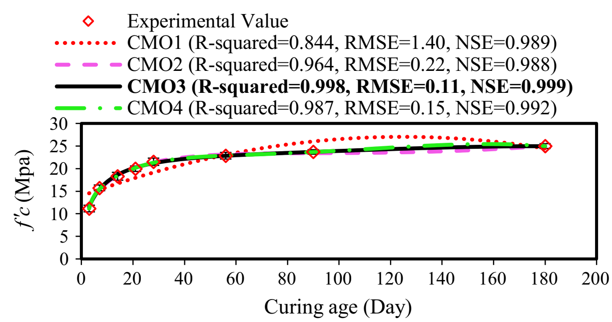

Fig. 17 Correlation of extended models for demonstrating the development of 21 MPa.

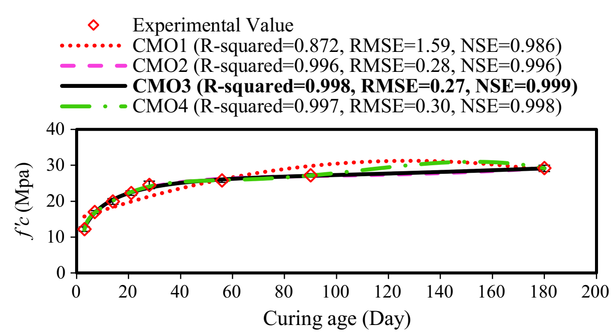

Fig. 18 Correlation of extended models for demonstrating the development of 24 MPa.

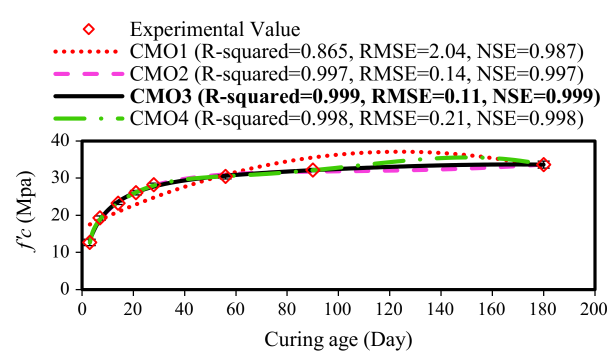

Fig. 19 Correlation of extended models for demonstrating the development of 28 MPa.

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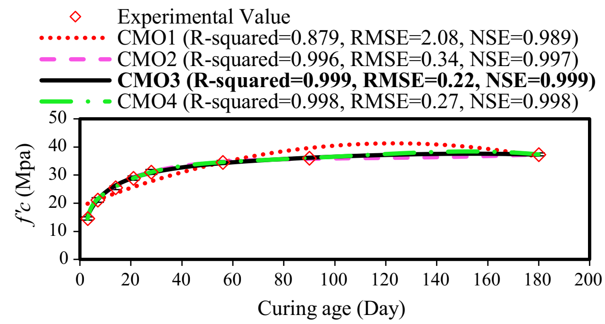

Fig. 20 Correlation of extended models for demonstrating the development of 31 MPa.

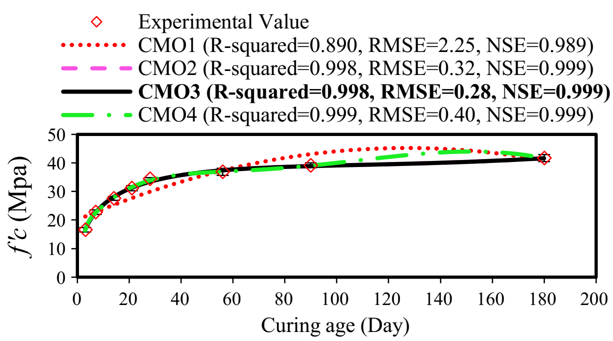

Fig. 21 Correlation of extended models for demonstrating the development of 35 MPa.

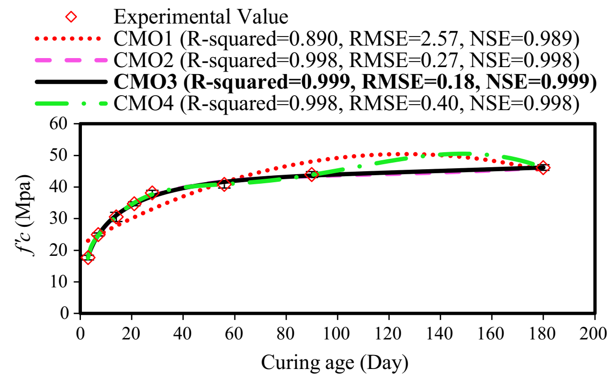

Fig. 22 Correlation of extended models for demonstrating the development of 38 MPa.

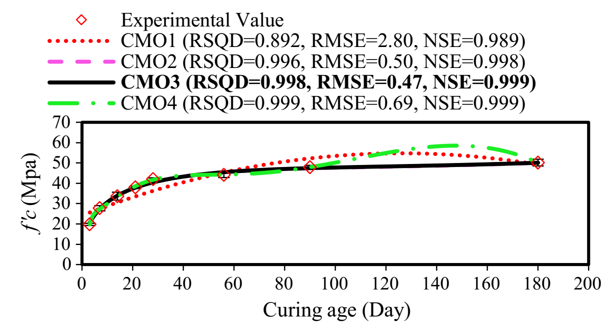

Fig. 23 Correlation of extended models for demonstrating the development of 42 MPa.

arbitrarily, models may not be well correlated with the observations. Although, the extended terms of polynomial equation based regression model's power increase, with the smaller fractional values, it was found to be well correlated with the observation points but deviated from the profile path with high error values. Table 4 shows the comparison of extended models options with respect to the statistical parameters such as R^2, RMSE and NSE to check the best fitness to their experimental observation data.

Based on the statistical analysis results of three parameters (Table 6), CMO3 showed the best correlation to the experimental observation data than CMO1, CMO2 and CMO4 for all target design strengths of concretes (Figs. 17–23), and this indicates that the most appropriate model option is CMO3 for experimental data interpretation of compressive strength development profile with curing ages.

The parameter estimation outcome of the proposed correlated well correlated model equation (CMO3) for all target design concrete strengths is represented in Table 7. Parameters were estimated with their 95% confidence level.

Validation of the proposed strength prediction model

In the current study, the proposed model was validated for the prediction of target design concretes strengths development profile with concrete ages using the 2nd observation strengths data of same seven mixes and samples which were again obtained from the laboratory following the procedure of initial compressive strength tests. Figs. 24–30 show the validation graph of the proposed model (CMO3) with the experimental observations (2nd observation) corresponding to the design concretes strengths: 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 28 MPa and 42 MPa respectively. The accuracy of the CMO3 was justified through statistical analyses (i. e. R^2, RMSE and NSE). The R^2, RMSE and NSE statistical values which were calculated for comparing experimental data with CMO3 results are shown in Figs. 24–30. The R^2 and NSE values of CMO3 with respect to the experimental compressive strength data for target design concretes strengths were found to be close to 1, indicating the validity of the proposed model. Furthermore, the calculated RMSE values of CMO3 fitness to the observation patterns were found to be quite reasonable. The study revealed that the predicted strengths results using CMO3 are quite similar to the experimental results. Consequently, compressive strength values of the tested design concretes' strengths can be predicted using CMO3 without conducting experiments in a quite short period of time with trivial error rates. Also, the experiments cost can be saved by using CMO3 predicted results for the tested design concretes' strengths at different concrete ages without any further testing.

All results of target design concretes obtained from experimental observations and CMO3 predicted values for 3, 7, 14, 21, 28, 56, 90 and 180 curing ages are plotted in Figs. 31–37 respectively. On these figures 1:1 line indicates the visual comparison results between the test results and predicted values. Comparing the experimental compressive strength results with those obtained from CMO3 model it can be seen that they are obviously similar. This shows that the experimental results are in harmony with the CMO3 Model results and very close

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Table 6 Evaluation of statistical parameters among the extended models for checking the best correlation.

Design strength of concrete (MPa)	R^2 (*R*-squared)	RMSE	NSE
21	CMO3 > CMO4 > CMO2 > CMO1	CMO1 > CMO4 > CMO2 > CMO3	CMO3 > CMO4 > CMO2 > CMO1
24	CMO3 > CMO4 > CMO2 > CMO1	CMO1 > CMO2 > CMO4 > CMO3	CMO3 > CMO4 > CMO2 > CMO1
28	CMO3 > CMO4 > CMO2 > CMO1	CMO1 > CMO4 > CMO2 > CMO3	CMO3 > CMO4 > CMO2 > CMO1
31	CMO3 > CMO4 > CMO2 > CMO1	CMO1 > CMO2 > CMO4 > CMO3	CMO3 > CMO4 > CMO2 > CMO1
35	CMO4 > CMO3 > CMO2 > CMO1	CMO1 > CMO4 > CMO2 > CMO3	CMO4 = CMO3 = CMO2 > CMO1
38	CMO3 > CMO4 = CMO2 > CMO1	CMO1 > CMO4 > CMO2 > CMO3	CMO3 > CMO4 = CMO2 > CMO1
42	CMO4 > CMO3 > CMO2 > CMO1	CMO1 > CMO4 > CMO2 > CMO3	CMO4 = CMO3 > CMO2 > CMO1

Table 7 Estimated parameters of proposed well correlated model equations (CMO3).

Model parameter	Design strengths of concretes						
	21 MPa	24 MPa	28 MPa	31 MPa	35 MPa	38 MPa	42 MPa
*a*1	−0.416	0.903	−3.428	−1.846	1.886	−0.188	2.866
*a*2	8.867	8.296	12.259	12.646	10.120	12.664	12.044
*a*3	−1.322	−1.020	−1.743	−1.783	−0.948	−1.401	−1.151
*a*4	0.091	0.057	0.118	0.124	0.031	0.070	0.043
*a*5	-2.36×10^{-3}	-1.11×10^{-3}	-3.05×10^{-3}	-3.4×10^{-3}	1.34×10^{-5}	-1.3×10^{-3}	-3.37×10^{-4}

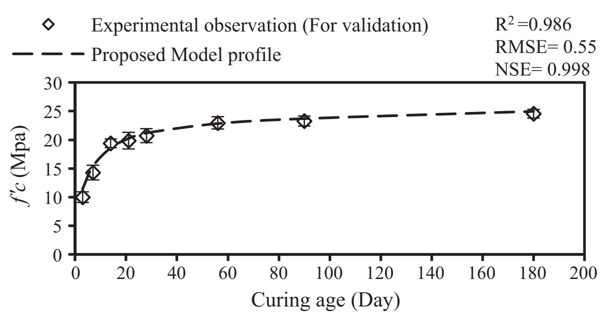

Fig. 24 Validation of CMO3 for development of 21 MPa with concrete ages.

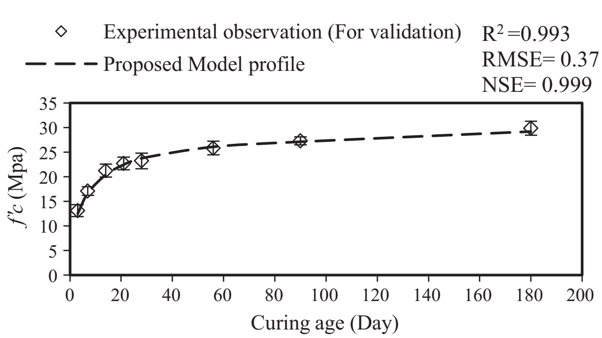

Fig. 25 Validation of CMO3 for development of 24 MPa with concrete ages.

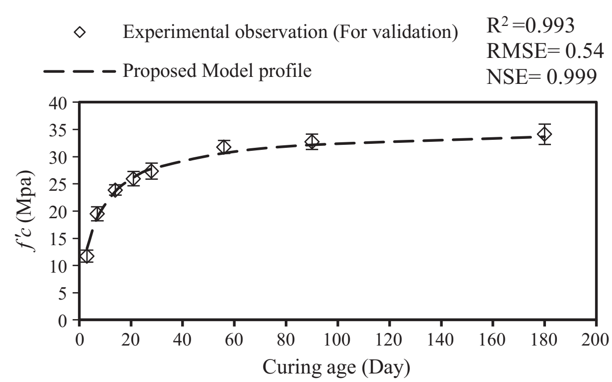

Fig. 26 Validation of CMO3 for development of 28 MPa with concrete ages.

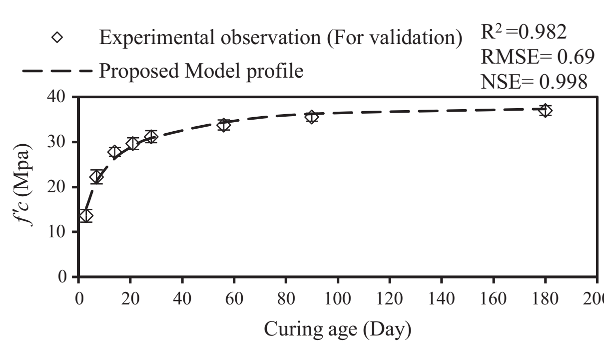

Fig. 27 Validation of CMO3 for development of 31 MPa with concrete ages.

values to each other with insignificant difference. The equation of linear least square fit line and the R^2 values is given in Figs. 31–37 for the observed and CMO3 model predicted values.

The error percentage (%) of the proposed model (CMO3) and validation data (2nd observation) when compared with experimental data (1st observation) are represented in Table 8. As it can be seen in the calculated error results, the experimental

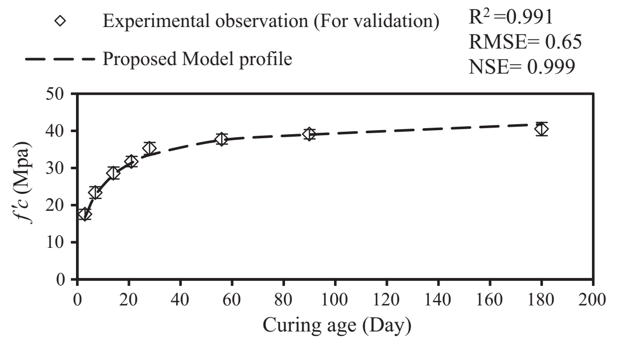

Fig. 28 Validation of CMO3 for development of 35 MPa with concrete ages.

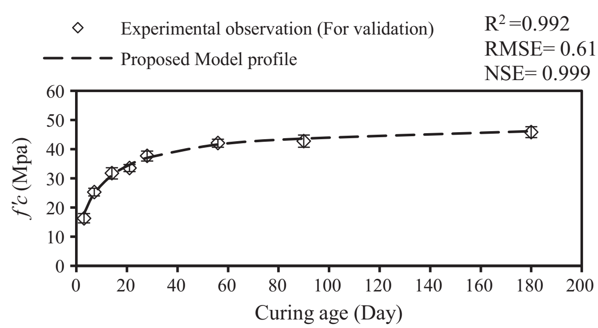

Fig. 29 Validation of CMO3 for development of 38 MPa with concrete ages.

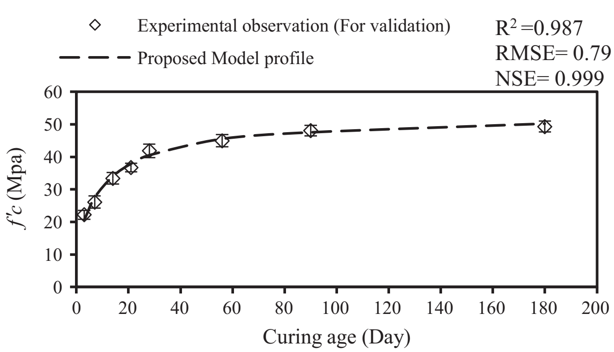

Fig. 30 Validation of CMO3 for development of 42 MPa with concrete ages.

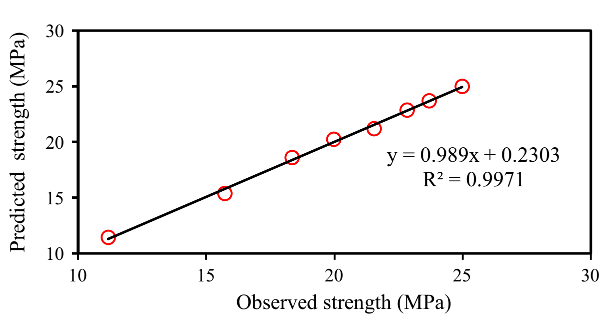

Fig. 31 Correlation between the observed and model predicted CS for design concrete 21 MPa.

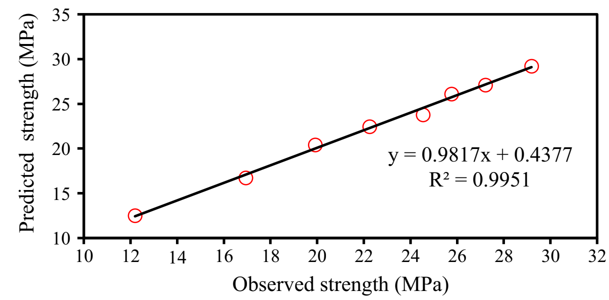

Fig. 32 Correlation between the observed and model predicted CS for design concrete 24 MPa.

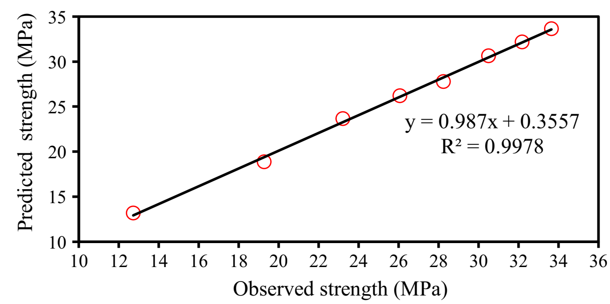

Fig. 33 Correlation between the observed and model predicted CS for design concrete 28 MPa.

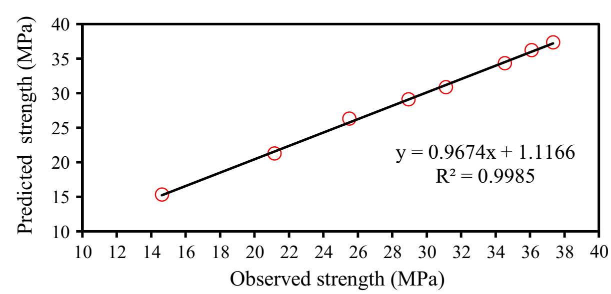

Fig. 34 Correlation between the observed and model predicted CS for design concrete 31 MPa.

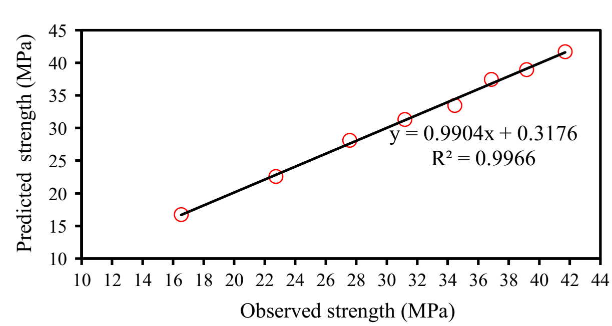

Fig. 35 Correlation between the observed and model predicted CS for design concrete 35 MPa.

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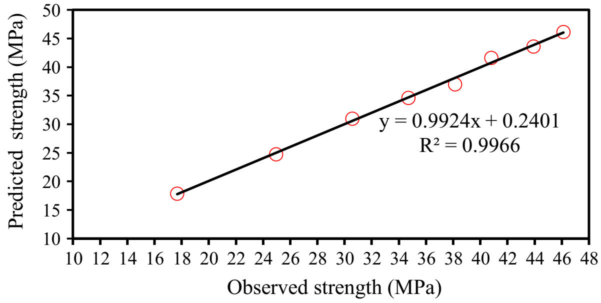

Fig. 36 Correlation between the observed and model predicted CS for design concrete 38 MPa.

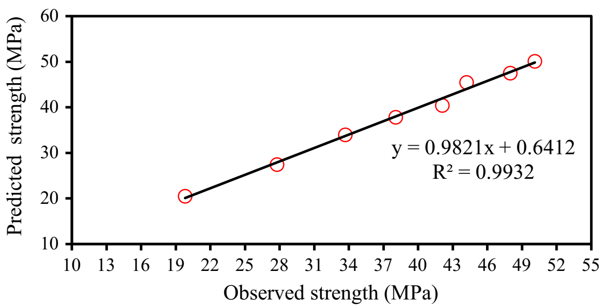

Fig. 37 Correlation between the observed and model predicted CS for design concrete 42 MPa.

compressive strength results do not vary significantly as compared to the obtained predicted data using CMO3 model. There is little difference between the experimental and predicted results. Using the CMO3 Model, average error was found to be 0.156%, 0.299%, 0.26%, 1.01%, 0.14%, 0.016% and 0.106% for 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa respectively in the tested concrete ages. In the experimental validation observations average error was found to be −1.65%, 1.91%, 0.01%, 0.169%, 1.919%, −0.825% and −0.17% for the all target design strengths concretes respectively. Table 6 shows that the error in difference between the model predicted values and the experimental results is less than 10%.

Conclusions

The main goal of this study was to characterize the compressive strength behavioral pattern of different specified design concretes strengths: 21 MPa, 24 MPa, 28 MPa, 31 MPa, 35 MPa, 38 MPa and 42 MPa with the increase of curing ages to predict the compressive strength development rate without performing experimental studies. Properties such as modulus of elasticity, modulus of rupture and splitting tensile strength of target design concretes were measured with respect to the compressive strengths to estimate mathematical expressions. In order to predict the long term effects on compressive strength of design concretes, four non-linear models were used along with their parameter estimation on the basis of compressive strength. The results of the analyses showed that polynomial equation describes strength development profile more accurately than the other three non-linear models. The

Table 8 Error calculation (%) of proposed model predicted and validation data (2nd observation) with respect to the experimental results (1st observation).

Curing days	21 MPa		24 MPa		28 MPa		31 MPa		35 MPa		38 MPa		42 MPa	
	Proposed model	Observation for validation	Proposed model	Observation for validation	Proposed model	Observation for validation	Proposed model	Observation for validation	Proposed model	Observation for validation	Proposed model	Observation for validation	Proposed model	Observation for validation
3	2.30	−5.67	2.51	7.68	3.42	−7.66	4.78	−6.78	1.16	5.81	1.26	−7.50	3.44	9.08
7	−2.25	−6.18	−1.34	1.02	−2.24	1.28	0.51	5.19	−0.63	2.66	−0.83	1.24	−1.20	−5.96
14	1.37	5.68	2.47	6.69	1.80	2.83	3.28	5.01	1.93	3.78	1.46	3.85	0.96	−0.82
21	1.29	−0.41	0.98	2.15	0.39	−0.44	0.53	2.43	0.45	1.67	−0.19	−3.21	−0.46	−3.41
28	−1.56	−3.75	−3.14	−5.30	−1.70	−3.16	−0.82	0.20	-2.88	2.30	−2.91	−1.10	−3.86	−0.59
56	0.07	0.55	1.28	0.35	0.48	4.15	−0.63	−2.19	1.64	2.35	1.98	3.18	2.91	1.71
90	0.05	−1.71	−0.38	0.32	−0.08	1.65	0.38	−1.55	−0.55	−0.25	−0.68	−2.52	−1.01	0.21
180	−0.02	−1.70	0.01	2.31	0.01	1.40	−0.02	−0.96	0.02	−2.97	0.04	−0.54	0.07	−1.58

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polynomial equation was also extended further to, four different longer terms to achieve better correlation results using the least squares procedure. It is found that strength gaining rate of concretes follows the polynomial equation having 2-degrees with fractional power like 0.5 degree interval for each term regarding the scope of works. The values were very closer to the experimental data obtained from proposed polynomial equation based model. The model was justified using statistical parameters such as R^2, RMSE and NSE along with the validation of the model profile with experimental observation and found to be satisfactory with trivial error difference for demonstrating the compressive strength development phenomena. As a result, compressive strength values of target design concretes strengths can be predicted using the proposed model profile.

Conflict of interest

The authors state that there is no conflict of interest.

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