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HISTORIOGRAPHIC VICES
I. LOGICAL ATTRIBUTION

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SUMMARIES

In the first of this series is considered the fallacious attribution of results that are logical antecedents or consequents of established knowledge.

Ce premier essai d'une série discutte l'attribution illusoire des résultats qui sont conséquents ou antécédents des connaissances bien établies.

Editors are well situated to observe offenses against good scholarship, since much of their work consists in crime prevention. This is particularly true in a hybrid discipline such as history of mathematics in which most practitioners are, or have recently been, professionals in at most one of the fields. The collection here begun is miscellaneous -- to be continued as I observe, recall, or retrieve examples. Since the purpose is not to correct published errors and still less to pillory individuals, examples are given without attribution or in fictitious form similar to the original. Correspondence is invited. I thank Gregory Moore for comments on a draft.

A canonical deduction schema for logical attribution is:

(1) X knew A.
B is a logical consequence of A.
Therefore, X knew B.

Since mathematicians know how hard it is to find nontrivial consequences, such a schema is rarely explicit. But it is surprising how often a writer discovers A in the works of X, practices his mathematical skills to derive B, and then delightedly credits X with B.

For example, a medievalist discovers in a fifteenth century work a verbal formula for approximating the sine by a product that can be calculated for an arbitrarily large number of factors. When expressed in modern notation, this formula does not immediately suggest a series, but well chosen manipulations reduce it to the first n terms of Taylor's series for the sine. The author does not argue explicitly that the fifteenth century mathematician must have known Taylor's series or that he even had any concept of infinite series, nor does he present any evidence that he could have done the manipulations, but he blandly asserts that Taylor's series was known 200 years before

Newton. The conclusion is doubly unfortunate, not only untenable but distracting attention for more interesting questions, e.g. how was such a good result obtained *without* the methods of calculus, and what does it tell us about mathematics in terms meaningful at that time and place?

A more plausible version of (1) is obtained by replacing the middle term by:

(2) B is an *obvious* logical consequence of A.

But this is only slightly more likely to lead to a correct conclusion, because "obviousness" is as time-tagged as every other aspect of mathematics. This is true even within the development of an individual. What mathematician has not had the experience of suddenly finding obvious what for a long time had been unclear, doubtful, or even unsuspected! We have the familiar anecdote attributed to S. Lefschetz and others. The hero is lecturing to a seminar. "Now the following result is obvious. Hmmm. Or is it? Hmmm. Excuse me, I'll be back in a moment." He leaves. Twenty minutes pass. He returns. "Yes, it's obvious!"

One sees instances of (1) with the middle term replaced by:

(3) A is a logical consequence of B.

Again, since it is commonplace that unconscious assumptions have always abounded in mathematics, that the antecedents of propositions are potentially infinite, and that knowledge of a result always *precedes* its placement in a logical structure, this schema is not explicitly used or asserted. But some writers cannot resist the temptation of imposing on the past the logical structures with which they are familiar.

A famous logician once wrote: "We are told that Thales knew that the angle subtended at the circumference by the diameter of a circle is a right angle. It is difficult to see how he could know this, unless he had given some sort of deductive proof of it." Why difficult? At that time a great deal of geometric knowledge had accumulated without the construction of any deductive synthesis. Indeed the deductive organization of mathematics by Euclid and his predecessors *presupposed* such knowledge. The theorem in question could easily be discovered and verified with a craftsman's square, in the course of other practical activities, or through experience with geometrical drawings.

A well known controversy based on fallacy (3) is the extended discussion of how the Egyptians could have found the correct formula for the volume of a frustrum of a square pyramid [May 1973, 487]. Some have argued that it is a great mystery, since Max Dehn's famous result of 1900 showed that they could not have proved the rule without resort to methods developed only much later. But one can take this seriously only if he imagines that mathematicians find results by deduction, or is ignorant of the empirical basis of Egyptian mathematics. They could have found the rule in many ways, e.g. from their experience in building pyramids and/or from considering the special case of the six congruent pyramids with bases the faces of a cube and common apex at the center, in each case supplemented by some generalization and algebraic manipulation of which their capability is established.

These vices arise, as do most of those I shall discuss, from looking at mathematics as a timeless, static, logical structure gradually revealed to man, instead of as a historically evolving human phenomenon. Of course, the historian must for some purposes make inferences from directly documented knowledge to what is plausibly inferred, but valid argement for such inferences cannot rest on logical connections alone. It requires historical analysis of the knowledge, thinking, and environment of the persons involved. Moreover, it is such analysis that is the distinctive task of the historian. The mathematicians' craft is to trace the formal logical connections.

REFERENCE

May, Kenneth O. 1973 *Bibliography and Research Manual of the History of Mathematics* Toronto (University of Toronto Press)