Invariant-mass spectroscopy for condensed single- and double- $\bar{K}$ nuclear clusters to be formed as residues in relativistic heavy-ion collisions

Toshimitsu Yamazaki $^a$, Akinobu Doté $^b$, Yoshinori Akaishi $^b$

$^a$ RI Beam Science Laboratory, RIKEN, Wako, Saitama 351-0198, Japan
$^b$ Institute of Particle and Nuclear Studies, KEK, Tsukuba, Ibaraki 305-0801, Japan

Received 26 September 2003; received in revised form 9 January 2004; accepted 9 January 2004

Editor: J.P. Schiffer

Abstract

Using a phenomenological $\bar{K}N$ interaction, we predict that few-body double-$\bar{K}$ nuclei, such as $pp\bar{K}^-$ and $ppn\bar{K}^-$, as well as single-$\bar{K}$ nuclei, are tightly bound compact systems with large binding energies and ultra-high nucleon densities. We point out that these $\bar{K}$ nuclear clusters can be produced as residual fragments in relativistic heavy-ion collisions, and that their invariant masses can be reconstructed from their decay particles.

© 2004 Elsevier B.V. Open access under CC BY license.

1. Introduction

Recently, exotic nuclear systems involving $\bar{K}$ ($K^-$ or $\bar{K}^0$) as a constituent have been investigated theoretically [1–5] based on phenomenologically constructed $\bar{K}N$ interactions (hereafter referred to as $AY$), which reproduce low-energy $KN$ scattering data [6], kaonic hydrogen atom data [7] and the binding energy and decay width of $\Lambda(1405)$. They are characterized by a strongly attractive $I = 0$ part, which essentially arises from the deep bound state of $\Lambda(1405)$, but is fully reconciled with the “repulsive-like” low-energy scattering lengths and kaonic hydrogen level shift. These empirically based bare $\bar{K}N$ interactions are consistent with theoretical predictions based on a chiral SU(3) effective Lagrangian [8–11]. Whereas these bare interactions lead to a shallow $K^-$ optical potential for continuum states in infinite nuclear matter [9–11], they persist to be strong in isolated bound states of finite nuclei. The effective interactions derived in [1] were examined and tested by comparing with exact few-body calculations using the bare interactions.

The strongly attractive $K^-p$ interaction is expected to cause not only enormous binding of $K^-$ in proton-rich nuclei, but also shrinkage of $\bar{K}$-bound nuclei. The calculated bound states in $pnn\bar{K}^-$ and $^3\text{Be}K^-$ lie below the $\Sigma\pi$ emission threshold, which is the main decay channel of $K^-N$ and, thus, are predicted to have narrow decay widths. These few-body treatments have been further extended to more
complex systems by the method of Antisymmetrized Molecular Dynamics (AMD) [3–5], which is now capable of calculating the structure with density distributions of individual constituent particles in an ab initio way without a priori assumption about the structure. The predicted K̅ bound states have central nucleon densities \( \rho_0 \), 4–9 times as much as the normal nuclear density \( \rho_0 = 0.17 \text{ fm}^{-3} \), with large binding energies \( E_K \approx 100 \text{ MeV} \). Such strongly bound compact systems can be called “K̅ nuclear clusters”. Since the predicted nucleon densities very much exceed the nucleon compaction limit, \( \rho_c \approx 2 / v_N \approx 2.3 \rho_0 \), with \( v_N \approx 2.5 \text{ fm}^3 \) being the nucleon volume, it may be questionable to apply the hadronic \( \bar{K}N \) and \( NN \) interactions to such dense systems. Although the K̅ clusters are expected to be in deconfined quark–gluon states [12–15], there is no theoretical treatment available on “dense and cold” microscopic systems. Thus, it is vitally important to pursue an experimental strategy to search for K̅ clusters and to examine their properties, in which we take the predicted binding energies and widths as a guiding reference, assuming that the hadronic picture persists. Interesting questions naturally arise: how about the structure of double-K̅ nuclei and how can they be produced and identified. In the present Letter we report on the results of our calculations on the structure of the simplest systems, ppK̅− and ppK̅−K̅−, and then propose to identify K̅ clusters as residues (“K̅ fragments”) after relativistic heavy-ion reactions. This decay-channel spectroscopy can be done by reconstructing invariant-mass spectra of decay particles of K̅ clusters, in contrast to formation-channel spectroscopy using direct reactions, such as (K̅−, n) [1,16] and (K̅−, π−) [2].

2. Double-K̅ clusters

2.1. ppK− K̅−

We applied the same theoretical treatments as were given in [1,4] for the double-K̅ systems. We performed four-body variational calculation using the Tamagaki potential (OPEG) [17] as a bare NN interaction and the AY KN interaction as a bare KN interaction, whereas we neglected the K−K̅ interaction simply because of a lack of information. We show the result in Fig. 1. The hitherto untouched ppK− system was predicted in a previous paper to be bound with a certain binding energy \( E_K = 48 \text{ MeV} \) and width \( \Gamma_K = 61 \text{ MeV} \) [2]. The p–p rms distance is 1.90 fm, close to the normal inter-nucleon distance. In the ppK−K̅− system, on the other hand, the binding energy and width were calculated to be \( E_K = 117 \text{ MeV} \) and \( \Gamma_K = 35 \text{ MeV} \), with a p–p rms distance very much reduced to 1.3 fm. Thus, the addition of a K̅ increases the binding energy and the nucleon density. Since these bound states lie above the \( \Sigma \pi \) emission threshold, their widths are dominated by the main decay channel (K−p → \( \Sigma \pi \)).

2.2. ppnK− K̅− and ppnK−K̅−

It has already been predicted that the ppnK− system has much stronger binding and a much higher density than ppK−, indicating that the addition of a neutron further strengthens the binding of the system. Thus, it is interesting to investigate the ppnK−K̅− system. We constructed an effective NN-central force and KN
force using the $g$-matrix method, and carried out an AMD calculation of $\bar{p}\bar{p}K^-K^-$. We found that the double-$\bar{K}$ cluster ($ppn\bar{K}^-K^-$) is indeed more tightly bound than the single-$\bar{K}$ cluster ($ppn\bar{K}^-$), as shown in Fig. 2, where we present the density contours of $^3$He, $ppn\bar{K}^-$ and $ppn\bar{K}^-K^-$. The central nucleon density reaches $\rho(0) \sim 3\text{ fm}^{-3}$. The $ppn\bar{K}^-K^-$ system is shown to be bound even deeper. We summarize these results in Table 1 together with the results on single-$\bar{K}$ clusters [4,5]. $\Gamma_K$, the width for decaying to $\Lambda\pi$ and $\Sigma\pi$, was evaluated by calculating the expectation value of the imaginary potential contained in the effective $AY\bar{K}N$ interaction with the wave function obtained by the AMD calculation. No additional widths of other origins were taken into account at this stage.

### Table 1: Summary of predicted $\bar{K}$ clusters

<table>
<thead>
<tr>
<th>$\bar{K}$ cluster</th>
<th>$M^2$ [MeV$^2$]</th>
<th>$E_K$ [MeV]</th>
<th>$\Gamma_K$ [MeV]</th>
<th>$\rho(0)$ [fm$^{-3}$]</th>
<th>$R_{rms}$ [fm]</th>
<th>$k_p$ [fm$^{-1}$]</th>
<th>$k_K$ [fm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p\bar{K}^-$</td>
<td>1407</td>
<td>27</td>
<td>40</td>
<td>0.59</td>
<td>0.45</td>
<td>1.37</td>
<td>1.37</td>
</tr>
<tr>
<td>$pp\bar{K}^-$</td>
<td>2322</td>
<td>48</td>
<td>61</td>
<td>0.52</td>
<td>0.99</td>
<td>1.49</td>
<td>1.18</td>
</tr>
<tr>
<td>$ppp\bar{K}^-$</td>
<td>3211</td>
<td>97</td>
<td>13</td>
<td>1.56</td>
<td>0.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ppn\bar{K}^-$</td>
<td>3192</td>
<td>118</td>
<td>21</td>
<td>1.30</td>
<td>0.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pppp\bar{K}^-$</td>
<td>4171</td>
<td>113</td>
<td>26</td>
<td>1.29</td>
<td>0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pppn\bar{K}^-$</td>
<td>4135</td>
<td>114</td>
<td>34</td>
<td>1.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ppK^-K^-$</td>
<td>2747</td>
<td>117</td>
<td>35</td>
<td>2.97</td>
<td>0.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pppK^-K^-$</td>
<td>3582</td>
<td>221</td>
<td>37</td>
<td>2.33</td>
<td>0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pppnK^-K^-$</td>
<td>4511</td>
<td>230</td>
<td>61</td>
<td>2.33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3. Possible suppression of the direct formation and decay of $\bar{K}$ clusters

The compact $\bar{K}$ clusters predicted here are very different from ordinary nuclei in many respects. Their structure can most likely be described in terms of de-confined quarks, rather than of “nucleons $+ K^-$”, and we expect that their decays to hadrons may be suppressed because of the need to rearrange the quarks and gluons into hadrons. On one hand, this would be a welcome feature, because the possible suppression of decays of $\bar{K}$ clusters would favour the discreteness of these bound states for better spectroscopic observation. On the other hand, we anticipate that the same mechanism would also reduce the formation probability of these clusters from normal target nuclei via direct reactions, such as $(K^-,\pi^-)$ and $(\pi^+,K^0)$ reactions for single-$\bar{K}$ nuclei [2]. Although the few-body double-$\bar{K}$ clusters can in principle be produced by $\Delta S = -2$ direct reactions: $d(K^-,K^0)p\bar{K}^-K^-$ and $^3\text{He}(K^-,K^+)ppn\bar{K}^-K^-$, the formation process may also be suppressed, and no visible peak may be revealed above a quasi-free back-
ground of $p(K^-, K^+)\Xi^-$ in an inclusive spectrum. This would be a serious problem.

3. $\bar{K}$ clusters as residues in heavy-ion collisions

Now, we point out that such $\bar{K}$ clusters should be found as residues of relativistic heavy-ion reactions, where $K^-$ mesons and $\Lambda$ hyperons are produced abundantly [21–23]. Usually, these strange particles are used as probes to study the size and temperature of fireballs produced in heavy-ion collisions. Here, we present a totally different view, namely, we propose to search for single-$\bar{K}$ and double-$\bar{K}$ clusters as residues of hot and dense fireballs, since the probability of forming strongly bound $\bar{K}$ clusters is expected to be rather high. Once a $\bar{K}$ cluster having a binding energy of $\sim 100$ MeV is produced in a chaotic nuclear medium, its tight binding will make its dissociation difficult, even at a high temperature of $50 \sim 100$ MeV. Thus, $\bar{K}$ clusters, once created, tend to survive through collisions, and escape in the freeze-out phase. They ultimately decay via their own decay modes, from which the invariant masses of parent $\bar{K}$ clusters may be reconstructed.

In the following we speculate about possible processes, (A) and (B), toward the formation and decay of $\bar{K}$ clusters.

(A) Evolution of $\bar{K}$ clusters as deep trapping centers

$K^-$ mesons are abundantly produced even in sub-threshold nuclear reactions [21,22]. This phenomenon is interpreted as being due to the decreased $K^-$ mass in the nuclear medium, which is caused by a strong attraction between $K^-$ and $p$. They are embedded in an attractive nuclear potential (the mass of $K^-$ is effectively reduced), and continue to undergo collisions with nucleons. Some of the $K^-$'s may escape from the nuclear medium, when they acquire sufficient energies from further collisions so as to be emitted as free $K^-$ mesons. This is a “heating-up + escaping” process. It is to be noted that the same attractive interaction is the origin of $\bar{K}$ clusters. In this sense, the “subthreshold” $K^-$ mesons are brothers of $\bar{K}$ clusters; both are born from the same parents, \textit{in-medium} $K^-$'s.

The $K^-$ mesons born in a fireball produce extra-deep and localized self-trapping potentials, which are intermittently accommodated by a few correlated nucleons (notably, $p^2$, $p^2n$ ($^3$He) and $p^2n^2$ ($^4$He)). Under such circumstances, the $K^-$'s become self-trapped together with an ensemble of [ppn], for example. Since $\bar{K}$ clusters once produced are hardly destroyed by further collisions because of their extremely large binding energies compared to the temperature, we expect a cascade evolution of $\bar{K}$ clusters, as shown below.

Single-$\bar{K}$ cluster formation:

\begin{align*}
p + K^- & \rightarrow \Lambda(1405), \quad \Lambda(1405) + p \rightarrow ppK^-, \quad (1) \\
pK^- + p & \rightarrow ppK^-, \quad (2) \\
ppK^- + p & \rightarrow pppK^-, \quad (3) \\
ppK^- + n & \rightarrow ppnK^-, \quad (4) \\
^3He + K^- & \rightarrow ppnK^-, \quad (5) \\
^4He + K^- & \rightarrow ppnnK^- \quad (6)
\end{align*}

Double-$\bar{K}$ cluster formation:

\begin{align*}
ppK^- + K^- & \rightarrow ppK^-K^-, \quad (7) \\
ppK^-K^- + n & \rightarrow ppnK^-K^-, \quad (8) \\
ppnK^- + K^- & \rightarrow ppnK^-K^-, \quad (9) \\
pppK^- + K^- & \rightarrow ppK^-K^- \quad (10)
\end{align*}

These processes occur as collisional capture processes, when aided by surrounding nucleons, which transfer energies and momenta to form $K$ clusters efficiently.

Productions of $\Lambda(1405)$ and $\Lambda(1520)$ in heavy-ion reactions can also be sources of $\bar{K}$ clusters, since $\Lambda(1405)$ is a bound state of $K^- + p$ and $\Lambda(1520)$ is a resonance state of $\bar{K} + N$. When they are produced in a nuclear medium, they proceed to kaonic bound states, forming $\bar{K}$ clusters. The role of these excited hyperons as doorways to kaonic systems was studied in the case of $(K^-, \pi^-)$ reactions [2]. Likewise, excited hyperons with $S = -2$ can be a doorway to double-$\bar{K}$ clusters.

The energy diagram for this cascade evolution was calculated, as shown in Fig. 3. The deepest trapping center among the single-$\bar{K}$ clusters is $ppnK^-$. The double-$\bar{K}$ clusters, $ppnK^-K^-$ and $pppnK^-K^-$, are the deepest among the double-$\bar{K}$ clusters. The probability of forming such deep traps can be estimated by a coalescence model [24,25]. Realistic simulations for heavy-ion reaction residues, such as RQMD [27] and
HSD [28], can be extended so as to include the $\bar{K}$ cluster productions.

(B) Direct formation of $\bar{K}$ clusters from QGP

In central collisions of relativistic heavy ions, a dense and hot fireball is produced. When the temperature of a primordial fireball exceeds a QCD transition temperature ($T > 150$ MeV) it is expected to be in quark–gluon plasma (QGP). Since the $\bar{K}$ clusters are by themselves dense, and are likely to be in a deconfined quark–gluon phase, as in QGP, they will be spontaneously formed in a self-organized way, like clusterized islands, remaining in a cooling and expanding hadron-gas medium throughout the freeze-out phase, as schematically shown in Fig. 4. Here, the $s$-quarks in a primordial QGP act as seeds for $\bar{K}$ clusters. This process is different from the cascade evolution process considered above, and the probability of each $s$-quark to proceed to a $\bar{K}$ cluster (even to a double-$\bar{K}$ cluster) is expected to be high. The time for their formation as well as the time for their decay are close to the freeze-out time.

Recently, it was shown that particle emission data including strange particles are well accounted for by a hadro-chemical equilibrium model in terms of the freeze-out temperature, the baryon chemical potential and the fireball volume as parameters [18–20]. In this model all particles (or states) are treated on equal footing, and the yields of various $\bar{K}$ clusters have been calculated by Andronic et al. [26], as discussed in the next section.

4. $\bar{K}$-cluster invariant-mass spectroscopy

Eventually, the $\bar{K}$ clusters formed in heavy-ion collisions decay via strong interactions by their own intrinsic decay modes. Whether these decays occur inside or outside the nuclear collision volume, is a key problem. The condition to observe the free decay of a $\bar{K}$ cluster with a decay width $\Gamma_K$ is

$$\tau_K = \frac{\hbar}{\Gamma_K} > \tau_f,$$

where $\tau_f$ is the freeze-out time. For the predicted decay width of $\Gamma_K \approx 20$ MeV, $\tau_K \approx 10 \text{ fm/c}$, which is marginally longer than the calculated freeze-out time, $\tau_f \sim 5 \text{ fm/c}$ [27–30]. Thus, most $\bar{K}$ clusters formed in (and before) the freeze-out phase are likely to survive and undergo free decays.

The unique signature for $\bar{K}$ cluster formation is a clear peak to be revealed in the invariant-mass spectra of its decay particles, if all of the decay particles with their energies and momenta are correctly identified. This method applies to limited cases, where $\bar{K}$ clusters can decay to trackable particles, such as

(i) $pp\bar{K}^{-} \rightarrow \Lambda + p,$

(ii) $ppn\bar{K}^{-} \rightarrow \Lambda + d,$
(iii) \( \text{pppK}^– \rightarrow \Lambda + p + p \), \( (14) \)
(iv) \( \text{ppnnK}^– \rightarrow \Lambda + t \), \( (15) \)
(v) \( \text{pppnK}^– \rightarrow \Lambda + ^3\text{He} \), \( (16) \)
(vi) \( \text{ppK}^– \rightarrow \Lambda + \Lambda \), \( (17) \)
(vii) \( \text{pppK}^– \rightarrow \Lambda + \Lambda + p \), \( (18) \)
(viii) \( \text{pppnK}^– \rightarrow \Lambda + \Lambda + d \). \( (19) \)

These decay processes are energetically the most favoured, though their branching ratios are not known.

Experimentally, however, a genuine peak in an invariant mass spectrum may be masked by a large continuous background of accidental combinatorial origin. As an example, the ratio of the peak intensity to the total accidental background in a \( \Lambda + d \) invariant mass spectrum is expressed by

\[
R = \frac{Y(\text{pppK}^–)\text{BR}(\text{pppK}^– \rightarrow \Lambda + d)}{Y(\Lambda)Y(d)}, \quad (20)
\]

where \( Y(\text{pppK}^–) \), \( Y(\Lambda) \) and \( Y(d) \) are the average yields of the \( \text{K}^- \) cluster, \( \Lambda \) and \( d \) per collision (namely, multiplicities), respectively, and \( \text{BR} \) is the decay branching ratio. Usually, the combinatorial background can be estimated and subtracted by the event mixing method. Thus, a signal with a ratio of \( R \approx 0.01 \) can be identified with confidence. In the following, we show that the proposed search is indeed feasible.

Recently, \( \Lambda \) hyperons have been identified in high-energy heavy-ion reactions at GSI-SIS from the energies and momenta of their decay vertices, \( p + \pi^- \), by a large 4π detector (FOPI) [23]. The average multiplicity of \( \Lambda \) at a h.i. energy of 2 GeV/u is \( Y(\Lambda) \sim 0.15 \), whereas \( Y(p) \sim 40 \) and \( Y(d) \sim 2 \) [23]. We expect that the invariant-mass spectra for the above processes can be composed from charged-particle tracks (\( p, d, t \) and \( ^3\text{He} \)) in connection with a \( \Lambda \). The first goal may be to identify the simplest single-\( \text{K}^- \) clusters, ppK\(^-\) and pppK\(^-\). It is to be noted that an experimental indication for the pppK\(^-\) bound state has recently been obtained from a \(^4\text{He}\)(stopped \( K^- \), n) experiment at KEK [31]. This formation-channel data will be examined in comparison with decay-channel data, as proposed here.

Recent calculations of Andronic et al. [26] based on a hadron-gas model [18] give \( Y(\text{pppK}^–) \sim 3 \times 10^{-3} \) per total charged pion, or \( \sim 0.06 \) per collision, when the incident energy is 2 GeV/u. It is interesting to note that this yield is larger than \( Y(K^-) \). This means that, even if the decay branching to \( \Lambda + d \) is 0.1, the pppK\(^-\) \( \rightarrow \Lambda + d \) signal can be identified with a ratio of \( R \sim 0.02 \) over a large combinatorial background.

Once single-\( \text{K}^- \) clusters are found, the next step will be to pursue double-\( \text{K}^- \) clusters. Here, we need a fireball with a large multiplicity of strangeness. Recently, abundant productions of \( \Lambda \) are observed at the RHIC energy by PHENIX [32] and STAR [33]. The future accelerator at GSI will provide 30–40 GeV/u heavy ions [34], which is suitable for double-\( \text{K}^- \) cluster invariant-mass spectroscopy in view of the large baryon density to be achieved in collisions, and also of abundant strangeness production. The calculated yield of the double-\( \text{K}^- \) cluster, ppK\(^-\)K\(^-\), has a maximum at the c.m. collision energy of \( \sqrt{s} = 5–10 \text{ GeV/u} \) (the incident energy around 30 GeV/u), and amounts to \( Y(\text{ppK}^-\text{K}^-) \sim 2 \times 10^{-4} \) per total charged pion, or \( \sim 0.01 \) per collision [26]. In view of such a large yield the invariant-mass spectroscopy for \( \Lambda + \Lambda \) may also be feasible.

5. Concluding remarks

The present proposal is, in a sense, a revisit of an old proposal of density isomers by Lee and Wick [35]. The time scale in the present case is around 10 fm/c, not ns–\( \mu \)s or longer, but the object is still metastable compared with the orbiting time of \( K^- \) in nuclei (\( \Gamma_K \approx E_K \)). Another remark is that the invariant-mass method proposed here is a valid procedure because the decay that concerns us takes place in free space after the freeze-out phase. On the other hand, invariant-mass spectroscopy for in-medium hadrons suffers from various in-medium disturbances, as remarked in [36]. When \( \text{K}^- \) clusters decay in a dense fireball region, their “invariant masses” will be shifted to lower values (collision-induced red shifts) by \( \Delta M \approx -q^2/(2M_V) \), as calculated in [36]. For \( \rho \approx 3\rho_0 \) this “red shift” amounts to \( \sim 50 \text{ MeV} \).

We can conceive of a further extension of the double-\( \text{K}^- \) systems to multi-\( \text{K}^- \) nuclear matter. Whereas the nucleons and hyperons are hard to compress, presumably because of the Pauli repulsion in the quark sector, multi-\( \text{K}^- \) systems, such as \( (p\text{K}^-)^n\text{n}^a \), become self-compressed dense matter without the aid of gravity. The characteristic feature of \( \text{K}^- \) in producing
dense nuclear systems may be intuitively understood as a result of the non-existence of Pauli blocking in the $(u,d)$ quark sector by implanting $K^-$, since $K^-$ is composed of $\bar{s}u$. Here, kaon condensation may also play an essential role [37,38]. Fig. 5 shows schematically the speculated dependences of multi-$\bar{K}$ bound states as compared with non-$\bar{K}$ nuclei. The $\bar{K}$ matter with a large $K$ fraction ($K^-/N \sim 1$) may be more stable than the corresponding non-strange matter.

So far, the present treatment does not contain the effect of chiral symmetry restoration at high density. If the KN interaction is increased along with a restoration of the chiral symmetry in accordance with the Tomozawa–Weinberg relation [39,40], similar to an effect recently observed in the $\pi N$ interaction in a nuclear medium [41], the $K^-$ energy line is bent downward with an increase of $\rho$; the $\bar{K}$ clusters may be more bound and denser, and the $\bar{K}$ matter may become more stable. The $\bar{K}$ (or $s$-quark) clusters, which we propose to study experimentally, will provide not only a unique playground to study possible quark–gluon phases of dense and bound nuclear systems, but also an important access toward the understanding of strange matter and stars.

Acknowledgements

We would like to thank Professor Paul Kienle for his stimulating and encouraging comments. We also thank Professors H. Horiuchi, N. Herrmann and R.S. Hayano, and Dr. K. Iida for the helpful discussion, and Professor P. Braun-Munzinger and Dr. A. Andronic for informing us of their results of calculations prior to publication. The present work is supported by Grant-in-Aid of Monbukagakusho of Japan. One of the authors (A.D.) acknowledges the receipt of a JSPS Postdoctoral Fellowship.

References