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Procedia Engineering 66 (2013) 354 – 368

**Procedia  
Engineering**[www.elsevier.com/locate/procedia](http://www.elsevier.com/locate/procedia)

5th Fatigue Design Conference, Fatigue Design 2013

## Experimental and numerical study for fatigue life prediction of bolted connection

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### Abstract

This paper presents results of an experimental study carried out for developing a model to predict the low-cycle fatigue life of bolted connection. The test specimens are cantilever beams with one end fixed by a bolted connection and the other end free. Each cantilever beam is subjected to a mass at the free end. Different levels of cyclic loading are applied to the beam system by means of a vibration shaker. Moreover, a numerical analysis is performed to study the uncertainty of the experimental model and to predict the lifetime scatter of the bolted connection. Monte Carlo simulations are adopted to include uncertainties by generating samples of the different random variables. The effects of these variables on the fatigue life of the bolted connection are identified.

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Selection and peer-review under responsibility of CETIM

*Keywords:* Bolted connection ; fatigue life ; shaker tests ; low cycle fatigue ; uncertainty ; Monte Carlo simulations

### 1. Introduction

Steel portal frames with bolted connections present numerous advantages. For both aesthetic and economic reasons, the demand of bolted connections in steel building frames has increased in recent years. However, the bolted connection between a beam and a column in a steel portal frame structure still raises design questions especially when it is subjected to dynamic loadings.

Beam-column connections in steel portal frames are proportioned and detailed to resist flexural, axial and shearing actions that result as a building sways through multiple inelastic displacement cycles during strong vibrations. However, such inelastic actions cause fatigue damage and if they continue long enough, the cumulative

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effects may exhaust the ductile capacity leading to fracture [1]. This indicates the importance of studying the fatigue damage of the beam-column connection in steel portal frame structures.

The bolted connection has been studied for many years to investigate and understand its behaviour under monotonic and cyclic loading. Experimental tests for predicting the resistance of the beam-column connection have been published with the aim of identifying the key parameters of resistance of the connection and verifying moment capacity prediction under monotonic static loading [2, 3].

However, a number of studies have been conducted on the behaviour of bolted connections subject to cyclic loading. Popov *et al.* [4] reported that the bolted connection has good hysteretic moment rotation behaviour. In addition, they concluded that the bolted connection exhibits excellent energy dissipation characteristics. Shi *et al.* [5] conducted a series of eight full-scale structural steel beam-to-column bolted connection specimens which was tested under cyclic loads. The test results indicated that bolted connections have adequate strength, rotational stiffness and ductility for use in steel portal frames.

While the research on the behaviour of bolted connection under cyclic loading has been conducted, very little information on the Low Cycle Fatigue (LCF) behaviour is available. The LCF causes a progressive and cumulative damage in the stiffness of the bolted connections [6, 7]. Bolts can loosen over time because of micro-macro slip in the bolt-nut and the assembled plates [8, 9]. Moreover, because of the stress concentration, cracks may appear in welds and bolts which were characterised by the different stages of micro and macro crack propagation and final fracture [10, 11]. Korol *et al.* [12] found that excessive yielding of the bolted connection made more prone to LCF and result in severe damage.

Krawinkler *et al.* [13] suggested developing mathematical models that permit the prediction of LCF life for random deformation histories. They modeled the Manson-Coffin equation using experimental results. LCF tests under constant displacement were performed and the linear damage accumulation theory (Miner's rule) was applied to predict the fatigue life.

Ballio *et al.* [14] conducted an experimental study of LCF behaviour of steel beam and column member. Two of the seven type specimens were bolted connections. The study showed that S-N (Stress - Number of cycles) curve fatigue model using Miner's rule, could be used to predict the LCF behaviour of bolted connections.

Xu *et al.* [15] verified the inelastic cyclic behaviour of bolted T-stub connections. They carried out a test program on 34 isolated bolted T-stubs focusing their attention mainly on the LCF models and on the influence of the loading history on the effectiveness of the application of the Miner's rule. In recent years, a variety of connection tests were performed by Lee *et al.* [16, 17] to evaluate the LCF behaviour. These research studies have utilized different types of connections but analysis procedures were similar.

Moreover, the fatigue process of structural elements under service loading is stochastic in nature. Life prediction and reliability evaluation is still a challenging problem for researchers in structural dynamics. Earlier developments of fatigue damage models reported in literature focus on deterministic nature of the process whereas in practice, damage accumulation according Yao *et al.* [18] is of stochastic nature. The stochasticity nature of the damage accumulation results from the randomness in fatigue resistance of material as well as that of the loading process. As a result of this, even under constant amplitude fatigue test at any given stress level, the fatigue life can show stochastic behavior [19].

Liu *et al.* [19] proposed a general methodology for stochastic fatigue life prediction under variable amplitude loading by combining a nonlinear fatigue damage accumulation rule and stochastic S-N curve representation technique. A deterministic damage accumulation rule proposed by Miner is used by Liu *et al.* [19]. This linear model defines damage as the ratio of the number of cycles of operation to the number of cycles to failure at any given stress level. Assuming no initial damage, the damage accumulation at single stress level is given as:  $n/N$ .

An appropriate uncertainty modeling technique is required to account for stochasticity in both material properties and external loading, which should accurately represent the randomness of the input variables and their covariance structure [19].

This paper focuses on the LCF behaviour of bolted connections. The objective of this experimental study is to examine the effects of LCF on bolted connection behaviour and to develop a LCF life prediction model of bolted connection. In order to study the behaviour of bolted connection under the effect of cyclic loading, quasi static tests or slow cyclic tests were used for many years. In this study, dynamic tests with a vibration shaker are performed for considering dynamic effects of the excitation on the bolted connection behaviour.

A bolted connection is tested under cyclic loading conditions. A total of twelve experimental tests with three different levels of constant peak displacement loading are conducted. The damaged bolts for each test specimen are demonstrated. The results of the experimental tests are used to develop a LCF life prediction model of the tested bolted connection.

The paper aim to study the uncertainty observed in the experimental results. To investigate the effect of the uncertainty, a simple stochastic approach which is called Monte Carlo Simulation method is adopted. The Monte Carlo simulation (MCS) is well adapted to include uncertainties in a deterministic finite element model by generating samples of the random parameter. A Monte Carlo simulation methodology is used to calculate the probabilistic life distribution of the beam. Finally, the probabilistic fatigue life prediction results of the Monte Carlo Simulations are compared with experimental data under variable loading.

## 2. Manson-Coffin fatigue life model

On the basis of the information from the S-N curve obtained experimentally, an analysis of the fatigue damage can be conducted. Generally, this curve can be divided into three domains [7]: oligocyclic domain, limited endurance domain and unlimited endurance domain, as shown in Fig. 1.

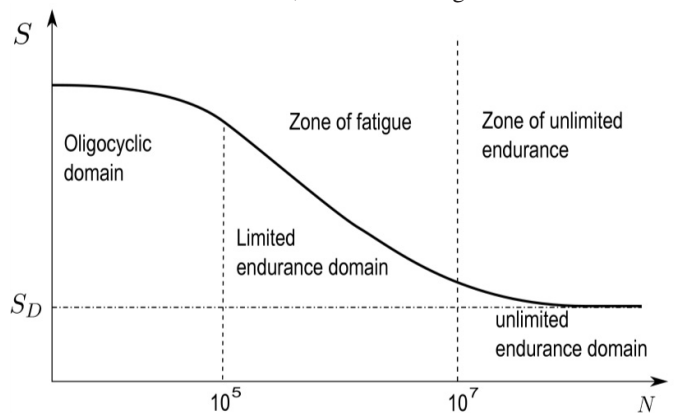


Fig. 1. Typical S-N curve.

The oligocyclic domain (LCF domain) corresponds to an applied constraint level (stress or strain) which is higher than the elastic limit of the constraint. The rupture occurs after a number of cycles less than  $10^5$  cycles, where each cycle produces significant damage.

Meanwhile in the unlimited endurance domain, the constraint level is close to the constraint limit of the fatigue ( $S_D$ ). This limit of the fatigue is defined as the constraint value which will not produce damage even for an infinite number of cycles more than  $10^7$ . The limited endurance domain is the domain intermediate between those two domains. In this domain, the rupture occurs in a number of cycles between  $10^5$  and  $10^7$  where the application of one cycle creates no harmful damage.

In experimental LCF studies, the fatigue lives are usually expressed as a function of total or plastic strain (Strain - Number of cycles). If the strain at critical locations in a structural element could be measured, fatigue lives caused by cyclic loading might be predicted accurately, by using a Manson-Coffin fatigue model.

However, for complex systems, it is almost impossible and impractical to make strain measurements at the critical locations of a connection. Instead, analogous models based on plastic connection rotation are used therein. A useful means of describing the LCF is expressed by Mander *et al.* [20] for a bolted connection.

Thus using the well-known Manson-Coffin relationship [21], the plastic rotation may be related to the number of cycles  $N_f$  by the following equation:

$$N_f = c(\Delta\phi_p)^{-b} \quad (1)$$

where  $\phi_p$  is the plastic rotation.  $c$  and  $b$  are the parameters of fatigue which depend on both the typology and the mechanical properties of the considered steel element.

The above-mentioned fatigue life relationship for this class of semi-rigid connection has been developed based on an analogy with the standard metal fatigue life relationships proposed by Manson-Coffin. This large fatigue based rotation capacity would generally well exceed the expected demands for most steel structural systems.

In order to incorporate LCF in a numerical model, a fatigue damage index must be implemented. Miner's rule is commonly employed to assess damage and failure in structural components under arbitrary loading histories [15]. It describes also the phenomena of the cumulative linearity of fatigue damage if another application stress or strain is employed. Some global parameter such as plastic rotation can be used rather than strains or stress for applying Miner's rule [16]. In the case of connection subjected to many cycles of rotation, Miner's rule is expressed by the following equation:

$$D_n = \sum_i \frac{n_i}{N_{fi}} \quad (2)$$

where  $n_i$  is the number of applied cycles for a given rotation level  $i$  and  $N_{fi}$  is the number of cycles to failure for the same rotation level  $i$ . Values of  $D_n$  greater than or equal to 1.0 indicate a low-cycle fatigue fracture of the member.

### 3. Experimental program

#### 3.1. Test specimens, properties and details

This investigation examines the behaviour of bolted connections. The system under consideration is a 0.75 m long straight beam with an I cross section. The beam is restrained at one end by a bolted connection and free at the other as shown in Fig. 2. The beam and column sections are IPE 80 and HEB 100, respectively.

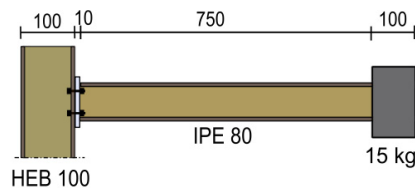


Fig. 2. Cantilever beam test set-up details (in mm)

The nominal values of yield strength ( $f_y$ ) and the ultimate values of tensile strength ( $f_u$ ) of the sections are respectively (260 MPa) and (450 MPa). The elastic modulus for the sections is ( $E = 210$  GPa). The sections were manufactured according to the requirements of NF EN 10025 with steel grade S275. The beam selected for the investigation in this study is a half-length of the IPE beam used in a two story frame. This frame was constructed at the Structural Dynamics Laboratory of *Ecole Centrale de Lyon* to examine the behavior of the frame with bolted connections. The portal frame model had a width of 1.6 m and a height of 2.42 m. The beams of the portal frame were a 1.4 m long straight beam with an I cross section. The beam to column connections were bolted connections as shown in Fig. 3. The two-story steel portal frame were conducted to verify the validity of a model developed by Saranik *et al.* [22] and to prove the efficacy of their nonlinear dynamic analysis techniques. The model developed by Saranik *et al.* depends on parameters of Manson-Coffin for the prediction of the lifetime and resistance of the two story frame under repeated loading.

A total of twelve connection tests are performed. Four standard bolts (M10-1.25 × 35 mm Grade 6.8 DIN 975 carbon steel) are used for each connection of the beam as illustrated in Fig. 3. The general layout of the bolted connection is shown in Fig. 4, with the thickness ( $t_p = 10$  mm) and dimensions ( $d_p = 120$  mm ×  $b_p = 76$  mm) of the end plate. The diameter of the hole for M10 bolts is 11 mm. The end plate material is S275 steel grade and the

elastic modulus for the plate material is ( $E = 210 \text{ GPa}$ ). Sketches of the test specimen are shown in Figs. 2 and 3.

To evaluate the initial stiffness, the equations of Eurocode 3 are used [23]. Based on the parameters of the connection (see Fig. 3), the initial stiffness is ( $k = 5.6 \times 10^5 \text{ N.m/rad}$ ).

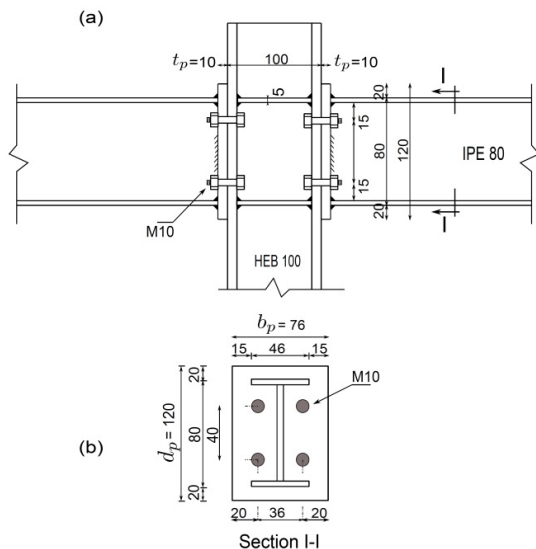


Fig. 3. (a) details of the bolted connection (in mm); (b) section I-I.

### 3.2. Experimental setup

A typical test set up consisted of a beam, which is connected to a stub-column by means of a bolted connection. The stub-column section is attached to a shaker. The basic configuration of a typical test set up, a test specimen, and the instrumentations are shown in Fig. 4. Two beams are used for the tests to maintain the balance of the shaker system when the harmonic excitation is applied.

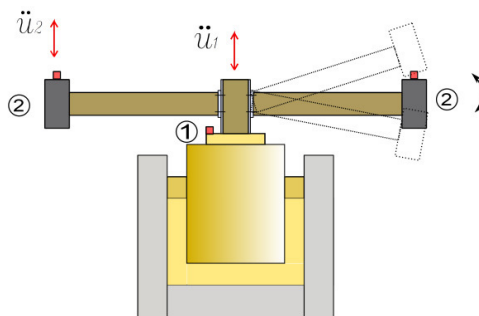


Fig. 4. Vibration Shaker System.

In order to reveal the effect of the moment on the connection, an additional mass of 15 kg is added at the extremity of one of the beams. Since this experiment represents the beam-column connection model on a small scale, the excitation parameter for the S-N curve is set as the rotation at the connection.

The cantilever beam system is excited with an electromagnetic shaker. The beam is supported by a vibration

shaker and excited with a sinusoidal excitation (see Fig. 4).

The experiment is performed by applying a vertical harmonic excitation and by measuring  $\pm \ddot{u}_1$  and  $\pm \ddot{u}_2$  as the responses in the base and at the extremity of the beam respectively. In order to vibrate the beam at the first vibration mode, the excitation frequency is equal to the first natural frequency of the beam system.

### 3.3. Equipment and instrumentations

The test assembly is instrumented with two accelerometers to measure the specimen's response. The accelerometers, with sensitivities around (100 mV/g), are mounted on the beam using a super glue and a duct tape. They collect the response at the free end of the beam and at the base.

The instrumentations are connected to an oscilloscope to observe the measured accelerations. They are also connected to a data acquisition system to record the measured data. Moreover, a hammer-shock is used to determine the natural frequency of the cantilever beam system. Fig. 5 illustrates the layout of instrumentation on the test specimens.

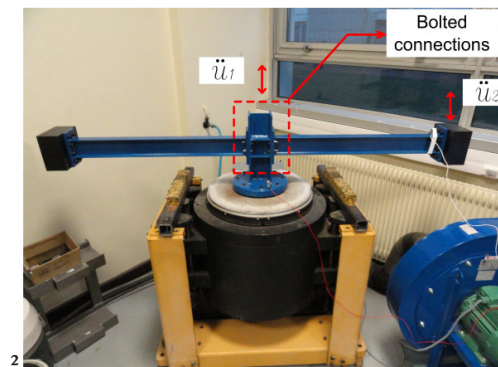


Fig. 5. The specimen with the vibration testing shaker.

A torque wrench is used to tighten the 4 bolts for each bolted connection to the same level for all tests, ensuring consistency in the boundary conditions. The bolts are tightened to the required tightening torque ( $T = 39 \text{ N.m}$ ). The tightening torque can be calculated, using the friction coefficients between threads and between bearing surfaces, by the following equation [24, 25]:

$$T = F_N (0.16P + 0.583d_2\mu_1 + \rho_m\mu_2) \quad (3)$$

where  $F_N$  is the clamping force.  $P$  is the thread pitch,  $d_2$  is the pitch diameter of bolt and  $\rho_m$  is the effective contact radius between the nut and joint surface.  $\mu_1$  and  $\mu_2$  are friction coefficients.

Properties related to the tightening characteristics such as the nut factors  $\rho_m$  and the friction coefficients  $\mu_1$  and  $\mu_2$  depend on the conditions of threads tolerance and manufacture, surface roughness and lubrication. The tightening characteristics can be obtained or estimated by the experiments representing those conditions in the actual bolted connection.

### 3.4. Test procedure

Each test commence with the assembly of the joint. The specimen is placed in position in the testing machine and the bolts are then assembled as shown in Fig. 5. Each bolt is then tightened to the required torque. During the test, manual control is exercised over: (1) the excitation frequency, in order to follow the natural frequency; (2) the

excitation amplitude, in order to keep the amplitude of the acceleration  $\ddot{u}_2$  always constant.

Due to repeated excitation, a loosening phenomenon reduces the clamping force of the bolts and subsequently the cyclic loading can cause sliding in the assembled elements, which can change the stress distribution and gradually produce fatigue damage. For the first result, the S-N curve can be obtained by relating the number of cycles to rupture and the level of the applied rotation. Meanwhile, the value of the applied rotation can be calculated through the information of  $\ddot{u}_1$  and  $\ddot{u}_2$ .

Each specimen is tested until the ultimate connection failure occurred. Then, the time at failure will be recorded and the level of rotation used for this test as well. These results will be used to determine the coordinates of a point in the S-N curve.

**4. Dynamic analysis of the system**

In this part, the equation of motion of the beam system is developed using the Virtual Work Method. The system has only one degree of freedom, and its motion can be represented by only one coordinate. It is the vertical displacement  $\eta$  of the mass  $M$  at the end, which is defined as positive in the upward direction. The velocity  $\dot{\eta}$  and the acceleration  $\ddot{\eta}$  will also be positive upwards at that point. The inertia, stiffness and the forces acting are shown in Fig. 6-a. The total work done by the forces (cf. Fig. 6-b) acting on the corresponding virtual displacements is, by the principle of virtual work, equal to zero:

$$M_l \frac{\delta(\eta)}{l} + \int_0^l m \ddot{\eta}(t) \frac{x^2}{l^2} \delta(\eta) dx + M \ddot{\eta}(t) \delta(\eta) \tag{4}$$

where  $m = \rho A$  is the mass of the beam in (kg/m),  $A$  is the area of the cross section and  $\rho$  is the density of the material of which the beam is made.  $l$  is the effective length of the beam.  $M_l$  is the moment at the base and it can be calculated by the following equation:

$$M_l = k\theta(t) = k \frac{\eta(t)}{l} \tag{5}$$

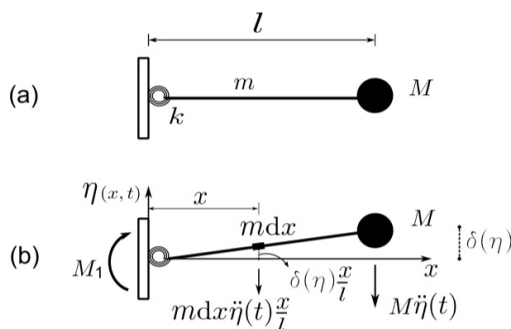


Fig. 6. (a) A cantilever beam model of the test system (b) Deflected shape: Application of the Virtual Work principle.

By dividing Eq. (4) by the virtual displacement, and simplifying, the equation of motion can be given by the following equation:

$$\frac{k}{l^2}\eta(t) + (\frac{ml}{3} + M)\ddot{\eta}(t) = 0 \tag{6}$$

It is seen that the above process has lumped the two mass ( $M$ ) and ( $ml$ ) into a single effective mass of ( $M_e = ml/3 + M$ ). Similarly, the stiffness has changed to ( $K_e = k/l^2$ ). The equation of motion can be written as follows:

$$K_e\eta(t) + M_e\ddot{\eta}(t) = 0 \tag{7}$$

Based on the method above, the natural frequency of the system is:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K_e}{M_e}} = \frac{1}{2\pi} \sqrt{\frac{k/l^2}{M + ml/3}} \tag{8}$$

4.1. Vibration of the system with a base excitation

The system discussed in Section 4, and illustrated in Fig. 6-a, was excited by applying a motion to the base, which was previously assumed semi-fixed. This important case is illustrated in Fig. 7. A translational system is shown, but a rotational system is also possible. Lumping may be required to reduce the system to this simplified form.

In Fig. 7, the absolute acceleration and the absolute displacement of the base are  $\ddot{u}_1$  and  $u_1$ , the displacement and the acceleration of the mass relative to the base are  $\eta$  and  $\ddot{\eta}$ .  $\ddot{u}_2$  and  $u_2$  are the absolute acceleration and the absolute displacement of the mass in space.

In the free body diagram, there are three forces acting on the mass as discussed previously, the equation of motion can be written as follows:

$$K_e\eta(t) + C_e\dot{\eta}(t) + M_e(\ddot{u}_1 + \ddot{\eta}(t)) = 0 \tag{9}$$

The same equation can be written as follows:

$$K_e\eta(t) + C_e\dot{\eta}(t) + M_e\ddot{\eta}(t) = -M_e\ddot{u}_1 \tag{10}$$

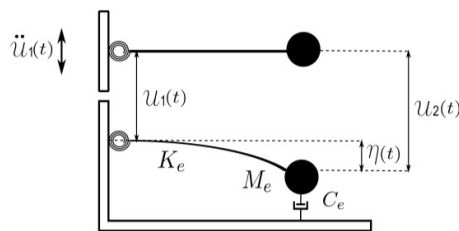


Fig. 7. Base-excited cantilever beam system with bolted connection.

It should be noted that  $\eta$  is a relative, not an absolute, coordinate, and that the inertia force acting on ( $m$ ) is ( $M_e\ddot{u}_2$ ), not ( $M_e\ddot{\eta}$ ). From Eq. (10), the force applied to the system is seen to be equal to ( $F = -M_e\ddot{u}_1$ ). This force is a sinusoidal one ( $F = -F_0 \sin(\omega t)$ ) where  $\omega$  is the frequency of the excitation signal generated by the shaker.

The damping coefficient can be calculated by the following equation:



$$C_e = 2M_e \omega_0 \xi \quad (11)$$

where  $\xi$  is the damping ratio and a value of 3% is adopted in the study.

The frequency response function of the system given in Eq. (10) can be written as:

$$H(\omega) = \frac{\eta_0}{F_0} = \frac{1}{K_e - \omega^2 M_e + i C_e \omega} \quad (12)$$

This complex-valued frequency response function can further be written as a magnitude as:

$$|H(\omega)| = \frac{|\eta_0|}{|F_0|} = \frac{1}{\sqrt{(K_e - \omega^2 M_e)^2 + (C_e \omega)^2}} \quad (13)$$

where  $|\eta_0|$  is the maximum relative displacement and  $|F_0|$  is the maximum force.

### 5. Fatigue damage analysis of the system

In this numerical simulation, harmonic excitation is applied in order to determine the lifetime of the system when fatigue damage reaches the critical value 1. The parameters of the system are given in Table 1.

A direct integration can be carried out, considering the stiffness degradation of the connection due to the fatigue damage. Since the excitation parameters which create the fatigue damage are the plastic rotation at the connection, the calculation of the plastic rotation for each cycle can be performed.

On the basis of the information of the S-N curve, the plastic rotation obtained for a cycle can be used to calculate the fatigue damage for one cycle. For the next cycles, the new value of the connection stiffness is applied as a function of the previous fatigue damage. Then the iteration loops are constructed as illustrated in Fig. 8.

Table 1. Parameters of cantilever beam system with bolted connection.

Parameter	Value
Effective length of the beam $l$	0.92 m
Section area of beam $A$	$7.64 \times 10^{-4}$ m <sup>2</sup>
Inertia of beam $I_b$	$80.1 \times 10^{-8}$ m <sup>4</sup>
Effective mass of the system $M_e$	17.4 kg
Young's modulus $E$	$2.1 \times 10^{11}$ N/m <sup>2</sup>
Density $\rho$	7800 kg/m <sup>3</sup>
Stiffness of bolted connection $k$	$5.6 \times 10^5$ Nm/rad
Damping ratio $\xi$	3%

The equation of motion with stiffness degradation is:

$$M_e \ddot{\eta} + C_e \dot{\eta} + K_e(D(t))\eta = -M_e \ddot{u}_1 \quad (14)$$

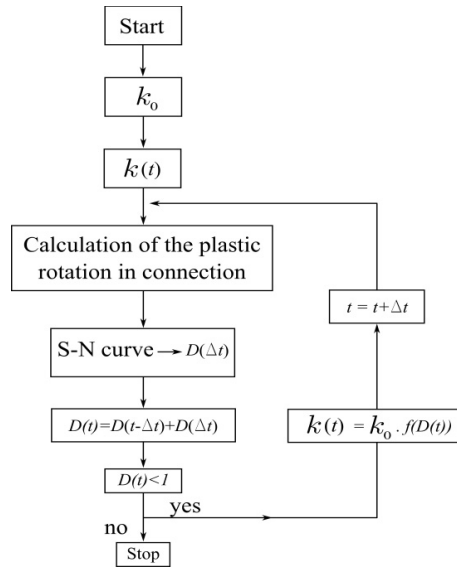


Fig. 8. Flowchart of the simulation of a structure’s lifetime

Accordingly, the magnitude of the frequency response function for the system can be determined by the following equation:

$$|H(\omega, D(t))| = \frac{1}{\sqrt{(K_e(D(t)) - \omega^2 M_e)^2 + (C_e \omega)^2}} \tag{15}$$

In this simulation, the method of calculating the lifetime takes into account the degradation of stiffness. The simulations are performed for three excitation forces at a frequency that is equal to the first natural frequency of the system  $f_n$ . Due to connection stiffness degradation, the first natural frequency can decrease from initial value  $f_n = 31$  Hz to a final value  $\bar{f}_n$ .

**6. Monte Carlo Simulation Methodology**

In literature several methods have been developed to take into account uncertainties in the response of complex mechanical systems such as Monte-Carlo simulation methodology. Then, a credible prediction of responses can be obtained.

In this study, a Monte-Carlo simulation methodology is used to calculate the probabilistic fatigue life distribution of the system. The flowchart presented in Fig. 8 is used to calculate the fatigue damage and the fatigue life.

Various uncertainties from material properties, structure geometry, applied loading and mechanical properties are included in the proposed calculation and used to study the uncertainty effects in fatigue life predictions. The basic deterministic parameters used in all cases are given in Table 1.

For a chosen random variable  $Y$ , the random input values for MCS are generated as follows [7]:

$$\tilde{Y} = \bar{Y}(1 + \delta_Y \zeta) \tag{16}$$

where  $\zeta$  is a Gaussian random variable,  $\bar{Y}$  and  $\delta_Y$  are respectively the mean value and the variation coefficient of the random variable  $Y$ .

Table 2 presents the values of variation coefficients of parameters considered in MCS calculations.

Table 2. Variation coefficients of parameters.

Parameter	Value
Maximum applied displacement $u_{0j}$	5%
Stiffness of bolted connection $k$	5%
Young's modulus $E$	3%
Effective mass of the system $M_e$	1%
Damping ratio $\xi$	5%

For Monte Carlo simulations, the number of samples is taken equal to 1000. Finally, a comparison of the predicted fatigue lives of the beam system can be obtained via experimental measures and via Monte-Carlo simulations.

## 7. Results and discussion

The experimental results demonstrated that two failure modes can be found in bolted connections subject to cyclic loading: self-loosening and fatigue failure which can be identified by a crack or a fracture of the bolt. Self-loosening happens at beginning and a backing off of the nut is observed. This phenomenon is a gradual loss of the clamping force in bolted connections under cyclic external loading. It can result in a decrease in the structural stiffness or the separation of clamped members. The self-loosening process of a bolted joint consists of two distinct stages. The first one occurs when there is no relative rotation between the nut and the bolt, and it is very difficult to observe with visual inspection. The second one is characterized by the backing off of the nut which is observed clearly in these tests.

For all tests, the maximum displacements  $\Delta_1$  and  $\Delta_2$  of the base and the tip mass are calculated from the recorded accelerations  $\ddot{u}_1$  and  $\ddot{u}_2$ . Follows, the total joint rotation is given by:

$$\phi_T = \frac{\Delta_1 - \Delta_2}{l} \quad (17)$$

Then, the elastic rotation due to beam deformation ( $\phi_{be}$ ) and the elastic rotation due to connection deformation ( $\phi_{ce}$ ) are calculated as follows:

$$\begin{cases} \phi_{be} = \frac{Fl^2}{3EI_b}; \\ \phi_{ce} = \frac{Fl}{k} \end{cases} \quad (18)$$

The plastic rotation of the connection ( $\phi_{cp}$ ) can be calculated as follows:

$$\phi_{cp} = \phi_T - \frac{Fl}{k} - \frac{Fl^2}{3EI_b} \quad (19)$$

Fig. 9 presents the fatigue life ( $2N_f$ ) versus the plastic rotation of the bolted connection using the proposed approach. The results of the twelve experiments, with three different levels of constant peak displacement loading, are also shown in Fig. 9. The range of the number of cycles to rupture of the S-N curve extends between  $4 \times 10^3$  and  $1 \times 10^5$  cycles where each point in Fig. 9 corresponds to the level of plastic rotation. After fitting, the proposed LCF life prediction model of bolted connection can be defined as:

$$N_f = 0.00022(\Delta\phi_p)^{-3} \tag{20}$$

where the parameters of fatigue are  $c = 0.00022$  and  $b = 3$ . This equation can be employed to determine the equivalent cycle numbers to failure for the bolted connection.

Fig. 10 presents the experimental test results for the bolted connection and the results of numerical analyses performed by the proposed approach. This figure demonstrates the uncertainty in fatigue parameters identified experimentally. The probabilistic fatigue life prediction results of the Monte Carlo Simulations are compared with experimental data. After fitting, the parameters of fatigue for the probabilistic LCF life prediction model of bolted connection are  $c = 0.00035$  and  $b = 2.91$ .

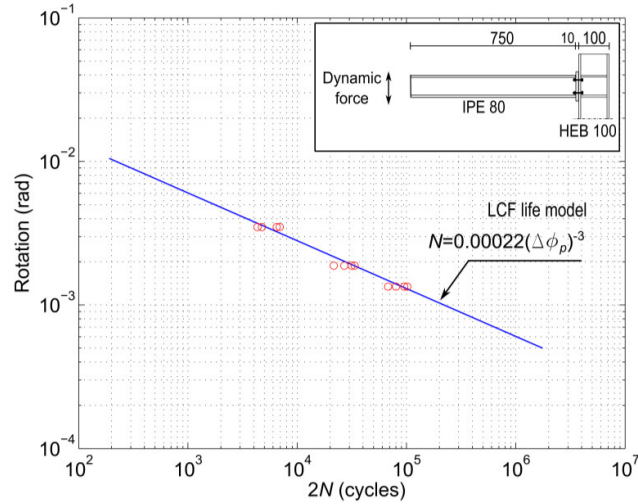


Fig. 9. Number of cycles to failure versus rotation of the connection according experimental test results : Experimental LCF life model.

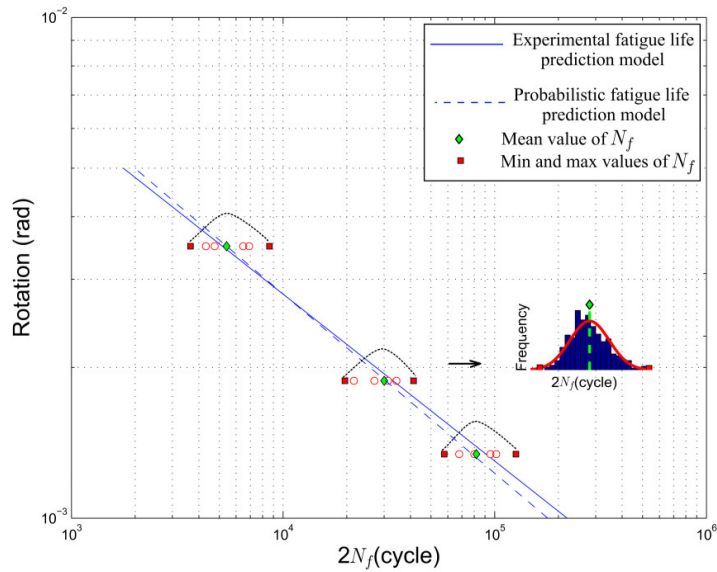


Fig. 10. Experimental and probabilistic LCF life prediction model.

From numerical results, the first natural frequency reduction depending on number of cycles can be estimated as shown in Fig. 11 and accordingly the fatigue life dispersion can be observed.

The MCS results demonstrate also the initial stiffness degradation of the bolted connection (see Fig. 12). This degradation is a direct result of the damage in the connection components. The uncertainty in the parameters of the bolted connection causes dispersion in mechanical and dynamic properties of the connection like the initial stiffness and the natural frequency. These uncertainties provide dispersion in the number of cycles to rupture for a bolted connection that can be estimated numerically by the proposed approach.

Finally, the results presented in this study demonstrate that the numerical analysis proposed in this study can be used to estimate the uncertainty of an experimental LCF life prediction model and to predict the lifetime scatter of a bolted connection. Monte Carlo simulations can be adopted to include uncertainties by generating samples of the different random variables. Then, the effects of these variables on the fatigue life of the bolted connection can be evaluated.

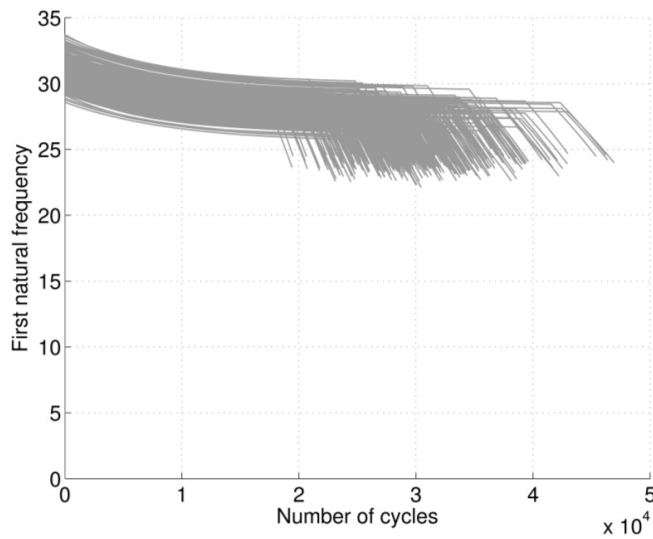


Fig. 11. Uncertainty in the degradation of the first natural frequency  $f_{n1}$  (Hz) versus the number of cycles under the second level of rotation

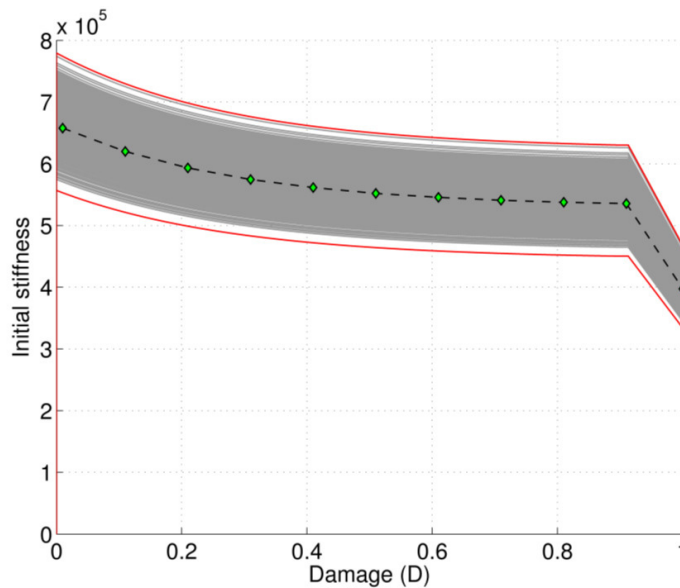


Fig. 12. Uncertainty in the degradation of the initial stiffness  $k$  (N.m/rad) versus the damage  $D$  under the second level of rotation.

## 8. Conclusion

In this paper, a LCF life prediction model of bolted connection was developed using 12 experimental tests. The experimental tests were performed on a cantilever beam with a tip mass. The beam system was excited by means of a vibration shaker. Two accelerometers were used to measure the vibration of the beam. These measurements were employed to compute the rotation of the beam system and the number of cycles to failure.

The experimental results showed that due to repeated excitation, a loosening phenomenon can reduce the clamping force of the bolts and then the cyclic loading causes sliding in the assembled elements which change the stress distribution and gradually produce fatigue damage in the bolted connection. The process of fatigue failure can be characterized also by the initiation of a small crack in the threaded part of the bolt, by the crack propagation and by the final failure which occurs very rapidly once the advancing crack has reached a critical size.

The information provided by the S-N curve and the proposed model can be used by engineers for the prediction of the lifetime of steel portal frames under repeated loadings. It is an effective tool for evaluating the number of cycles to failure for a considered rotation level. Additionally, the experimental data can facilitate the development and verification of design models for bolted connections. It can provide data on the fatigue damage characteristics of these connections. The effects of these variables on the fatigue life of the bolted connection were identified and the probabilistic life prediction model of the beam was identified by a Monte Carlo simulation methodology. The probabilistic fatigue life prediction results of the Monte Carlo Simulations are compared with experimental data under variable loading.

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