ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 660 (2008) 293–298
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Reconstructing $f(R)$ theory according to holographic dark energy

Xing Wu, Zong-Hong Zhu *

Department of Astronomy, Beijing Normal University, Beijing 100875, China

Received 17 September 2007; accepted 23 December 2007

Available online 4 January 2008

Editor: T. Yanagida

Abstract

In this Letter a connection between the holographic dark energy model and the $f(R)$ theory is established. We treat the $f(R)$ theory as an effective description for the holographic dark energy and reconstruct the function $f(R)$ with the parameter $c>1$, $c=1$ and $c<1$, respectively. We show the distinctive behavior of each cases realized in $f(R)$ theory, especially for the future evolution.

PACS: 98.80.-k; 95.36.+x; 04.50.+h; 98.80.Es

Keywords: Holographic dark energy; $f(R)$ theory; Reconstruction

1. Introduction

Since 1998, the Type Ia Supernovae observations [1] have indicated that the expansion of the universe is currently accelerating. This result has then been further confirmed by independent observations of Cosmic Microwave Background (CMB) [2] and Large Scale Structure (LSS) [3]. One explanation for the cosmic acceleration is ascribed to adding an exotic energy component with negative pressure, dubbed the dark energy, of which the origin and nature is still a mystery. Various dark energy models have been proposed in the literature (see [4] for a detailed review). Among others, the simplest candidate for dark energy is the cosmological constant or the vacuum energy. Fitted quit well with observational data though it is, this model suffers from the famous cosmological constant problem [5]. This problem arises primarily due to the fact that the vacuum energy is considered within the framework of quantum field theory in Minkowski background. As we known, however, at cosmological scales where the effect of gravity has to be taken into account, the above description of the vacuum energy would break down, and it is believed that the correct theoretical value of the vacuum energy will be predicted by a complete theory of quantum gravity. Although we are far from reaching such a fundamental theory, we do know some features of it. The holographic principle [6] is an important feature which can shed some light on the cosmological constant problem and the dark energy problem. According to this principle, considering gravity, the number of the degree of freedom of a local quantum field theory system is related to the area of its boundary, rather than the volume of the system as expected when gravity is absent. Along this line, Cohen et al. [7] suggested an entanglement relation between the IR and UV cut-offs due to the limitation set by the formation of a black hole, which in effect sets an upper bound for the vacuum energy

$$L^3\rho_\Lambda \leqslant L M_p^2, \tag{1}$$

where ρ_Λ is related to the UV cut-off, L is the IR cut-off and M_p is the reduced Planck mass. The form of dark energy is proposed by saturating the bound as

$$\rho_\Lambda = \frac{3c^2 M_p^2}{L^2}, \tag{2}$$

where c is a numerical factor. It is easy to check that inserting $L = H_0^{-1}$ may give rise to the energy density compatible with current observation in orders of magnitude. However, as Hsu [8] pointed out, this cannot lead to a desired equation of state. Li [9]

* Corresponding author.
E-mail addresses: wxxwwxxw@mail.bnu.edu.cn (X. Wu), zhuzh@bnu.edu.cn (Z.-H. Zhu).

0370-2693
doi:10.1016/j.physletb.2007.12.031

proposed the holographic dark energy model, where L is chosen to be the future event horizon

$$R_{eh} = a\int_t^\infty \frac{dt'}{a(t')} = a\int_a^\infty \frac{da'}{Ha'^2}. \tag{3}$$

This model has been tested to be well consistent with current observations [10], and it is a compelling candidate for solving the dark energy problem.

Adding new component of dark energy to the whole energy budget is one way, there are also other promising ways without resorting to new forms of energy. Since general relativity is only tested within solar system up to now, we may well consider the modification to the Einstein–Hilbert action in Einstein gravity at larger scales with higher order curvature invariant terms such as R^2, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, or $R\,\Box^k\,R$ as well as nonminimally coupled scalar fields with terms like $\phi^2 R$. Furthermore, these terms naturally emerge as quantum corrections in the low energy effective action of quantum gravity or string theory [11,12]. Here we focus on the $f(R)$ theories where the modification is a function of the Ricci scalar only. In [13], the author first introduced an additional R^2 to the Einstein–Hilbert action leading to an inflationary solution for the early universe. As for applications to the dark energy problem, it is shown that adding an term $1/R^n$ [14] or more generalized $c_1/R^n + c_2R^m$ [15] can lead to late time acceleration originated from pure geometrical effect, equivalent to introducing an effective dark energy in the Einstein frame (see, for example, [16], Section XVI in [4] and the references therein for more works dedicated to solving the dark energy problem with $f(R)$ theories).

It can be shown [17] that through conformal transformations, extended theories with higher order terms and nonminimally coupled scalar fields correspond to Einstein gravity with some minimally coupled scalar fields (quintessence) suggesting, to some extent, an equivalence between dynamic dark energy models and $f(R)$ theories. An approach was proposed in [18] to reconstruct the form of $f(R)$ from a given expansion history $H(z)$. From observational data such as SNe distance modulus vs redshift, we can obtain the luminosity distance $D_L(z)$ through which $H(z)$ is given by

$$H(z) = \left\{\frac{d}{dz}\left[\frac{D_L(z)}{1+z}\right]\right\}^{-1}. \tag{4}$$

In fact, however, large errors in current observational data prevent using this procedure to determine the exact form of $f(R)$. But the important thing is that we can use a given $H(z)$ predicted from a dark energy model to reconstruct its equivalent $f(R)$ theory. For example, the $f(R)$ theories reconstructed from the $H(z)$ given by quiessence and the Chaplygin gas respectively were obtained in [18]. In this Letter, we treat the holographic dark energy model as one inspired by the holographic principle, an important feature of a more fundamental theory of quantum gravity, and to reconstruct the corresponding $f(R)$ theory as an equivalent description. Compared with previous works, where the holographic dark energy is reconstructed within scalar field models like ghost condensate [19], quintessence [20], tachyon [21] and hessence [22], here we perform the reconstruction in $f(R)$ theory without resorting to any additional dark energy component, that is, the holographic dark energy is effectively described by the modification of gravity.

In addition we note that there are in fact two strategies in $f(R)$ theories: the metric formalism, where the action is varied with respect to the metric only; and the Palatini formalism [23], where the metric and the connection are treated as two independent variables with respect to which the action is varied. It is only in Einstein gravity $f(R) = R$ that both approaches reach the same result. In general $f(R)$ theories, the problem of which approach should be used is still an open question and the final solution may be determined by further observations and theoretical development. At present we assume the metric formalism in this Letter.

2. Reconstruction of $f(R)$ theory

Now let us consider a homogeneous and isotropic universe with flat spatial geometry consisting of matter and the vacuum energy given by (2). The Friedmann equation reads

$$3M_p^2 H^2 = \rho_m + \rho_\Lambda, \tag{5}$$

where $\rho_m = \rho_{m0}(1+z)^3$ by the equation of energy conservation, and a subscript 0 denotes the value at present. By introducing $\Omega_\Lambda = \frac{\rho_\Lambda}{3M_p^2H^2}$ and $\Omega_m = \frac{\rho_m}{3M_p^2H_0^2} = \Omega_{m0}(1+z)^3$, we obtain the Hubble parameter

$$H(z) = H_0\sqrt{\frac{\Omega_m}{1-\Omega_\Lambda}}. \tag{6}$$

Clearly, once we determine the evolution of $\Omega_\Lambda(z)$, the whole expansion history $H(z)$ is determined. Combining the definitions of holographic dark energy and the event horizon (2) and (3) we get

$$\int_a^\infty \frac{da'}{Ha'^2} = \frac{c}{Ha\sqrt{\Omega_\Lambda}}, \tag{7}$$

with the initial condition given by setting $z = 0$ in (6)

$$\Omega_{m0} + \Omega_{\Lambda 0} = 1. \tag{8}$$

Inserting (6) into the above equation and taking derivative with respect to z on both sides (using $1+z = 1/a$), we obtain the differential equation of Ω_Λ

$$\Omega'_\Lambda = -\frac{1}{(1+z)}\Omega_\Lambda(1-\Omega_\Lambda)\left(1+\frac{2}{c}\sqrt{\Omega_\Lambda}\right), \tag{9}$$

where the prime denotes derivative with respect to z. We can see that c is the only parameter determining the dynamics of the holographic dark energy. In fact, we can use the equation of energy conservation of the dark energy to get the equation of state (EoS) of the dark energy

$$w_\Lambda = -\frac{1}{3} - \frac{2}{3c}\sqrt{\Omega_\Lambda}. \tag{10}$$

It is easy to see that when $\Omega_\Lambda \to 1$ in the future, for $c > 1$ the EoS will always be greater than -1 showing quintessence-like behavior; for $c = 1$ the universe will end up with a de Sitter phase; and for $c < 1$ the universe will end up with a phantom phase and the EoS crossing -1 occurs during the evolution exhibiting quintom-like behavior. Therefore the parameter c plays a very important role in determining the evolutionary nature of the holographic dark energy. Many works [10] have been devoted to constraining this parameter by observations such as SNe, CMB and galaxy clusters etc. Almost all the best fits indicate $c < 1$, although $c > 1$ is also compatible with the data within 1σ.

Once we fix c and Ω_{m0}, the evolution of $\Omega_\Lambda(z)$ can be determined by solving (9) with the initial condition (8). Then by (6) we can obtain $H(z)$. Now we assume the holographic dark energy as an underlying theory of dark energy, and we want to find the corresponding $f(R)$ theory as an effective description. We follow the method proposed in [18]. Using the Robertson–Walker metric, the Ricci scalar can be expressed in terms of the Hubble parameter:

$$R = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right), \tag{11}$$

where the dot represents derivative with respect to time t. In this Letter we set $k = 0$. Note that here we assume the signature as $\{+, -, -, -\}$, the same as in [18], and therefore R is always negative (so it is with f). Once we choose $\{-, +, +, +\}$, the minus sign in front of (11) would disappear. This is just a matter of convention, which means no physical difference. Let us start from the action

$$S = \int d^4x\,\sqrt{-g}\left[f(R) + \mathcal{L}_m\right], \tag{12}$$

where $\mathcal{L}_m$ is the matter Lagrangian. We use the units $M_p = c = \hbar = 1$. Variation with respect to the metric leads to the modified field equation [24]

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T^{(\mathrm{curv})}_{\mu\nu} + T^{(m)}_{\mu\nu}, \tag{13}$$

where $G_{\mu\nu}$ is the Einstein tensor, and an effective stress-energy tensor containing the higher order contributions is defined by

$$T^{(\mathrm{curv})}_{\mu\nu} = \frac{1}{f'(R)}\Big\{g_{\mu\nu}\left[f(R) - Rf'(R)\right]/2 \\ + f'(R)^{;\alpha\beta}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\nu}g_{\alpha\beta})\Big\} \tag{14}$$

and the matter's contribution is in $T^{(m)}_{\mu\nu} = \tilde{T}^{(m)}_{\mu\nu}/f'(R)$ with $\tilde{T}^{(m)}_{\mu\nu}$ the standard minimally coupled matter stress-energy tensor. With the RW metric, we obtain the modified Friedmann equations

$$H^2 + \frac{k}{a^2} = \frac{1}{3}\left[\rho_{\mathrm{curv}} + \frac{\rho_m}{f'(R)}\right], \tag{15}$$

$$2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} = -(p_{\mathrm{curv}} + p_m), \tag{16}$$

and the continuity equation

$$\dot{\rho}_{\mathrm{tot}} + 3H(\rho_{\mathrm{tot}} + p_{\mathrm{tot}}) = 0. \tag{17}$$

The three equations are not independent and we combine them to get one equation

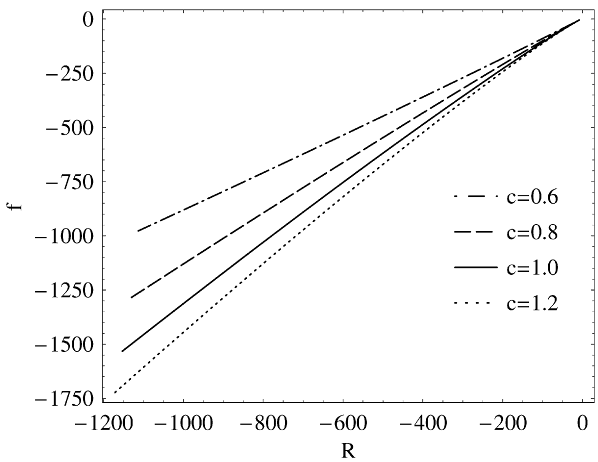

Fig. 1. Reconstructed $f(R)$ with $0 \leqslant z \leqslant 10$ and $c = 0.6$ (dash-dotted), $c = 0.8$ (dashed), $c = 1.0$ (solid) and $c = 1.2$ (dotted).

$$\dot{H} = -\frac{1}{2f'(R)}\Big\{3H_0^2\Omega_{m0}(1+z)^3 + \ddot{R}f''(R) \\ + \dot{R}\left[\dot{R}f'''(R) - Hf''(R)\right]\Big\}, \tag{18}$$

where the prime denotes derivative with respect to R. Using the relation $d/dt = -(1+z)Hd/dz$ to replace the variable t by z, (18) can be transformed into a third order differential equation of $f(z)$

$$\mathcal{C}_3(z)\frac{d^3f}{dz^3} + \mathcal{C}_2(z)\frac{d^2f}{dz^2} + \mathcal{C}_1(z)\frac{df}{dz} = -3H_0^2\Omega_{m0}(1+z)^3, \tag{19}$$

where $\mathcal{C}_n(z)$ consists of $H(z)$ and its derivatives. Once we know the function $H(z)$, the coefficients $\mathcal{C}_n(z)$'s can be calculated. However, in our case of the holographic dark energy model, $H(z)$ cannot be derived analytically. By (6) and (9), $H(z)$ and its derivatives can be expressed by $\Omega_\Lambda(z)$, therefore, after painful derivation, the coefficients $\mathcal{C}_n(z)$'s can be ultimately expressed by the combinations of $\Omega_\Lambda(z)$. Although Ω_Λ itself can be solved by (9) alone, it has to be considered as a part of the whole set of differential equations in order to solve $f(z)$. So we consider (9) and (19) together as a differential equation set, of which one initial condition is (8). According to [18], the other initial conditions are

$$\left(\frac{df}{dz}\right)_{z=0} = \left(\frac{dR}{dz}\right)_{z=0}, \tag{20}$$

$$\left(\frac{d^2f}{dz^2}\right)_{z=0} = \left(\frac{d^2R}{dz^2}\right)_{z=0}, \tag{21}$$

$$f(z=0) = f(R_0) = 6H_0^2(1 - \Omega_{m0}) + R_0. \tag{22}$$

Given Ω_{m0} and c, with these four initial conditions (8) and (20)–(22), the differential equation set (9) and (19) can be solved numerically. The reconstructed function $f(R)$ is presented in Fig. 1, where we set $\Omega_{m0} = 0.29$ and $c = 0.6$, 0.8, 1.0, 1.2 respectively. For the sake of comparison, we also show the same result on a lf–lR plane in Fig. 2, where $lf \equiv \ln(-f)$ and $lR \equiv \ln(-R)$ as used in [18]. Note that we set the values of c within the range $0.6 \leqslant c \leqslant 1.2$, which is consistent with fitting results according to the works in [10]. Compared with the

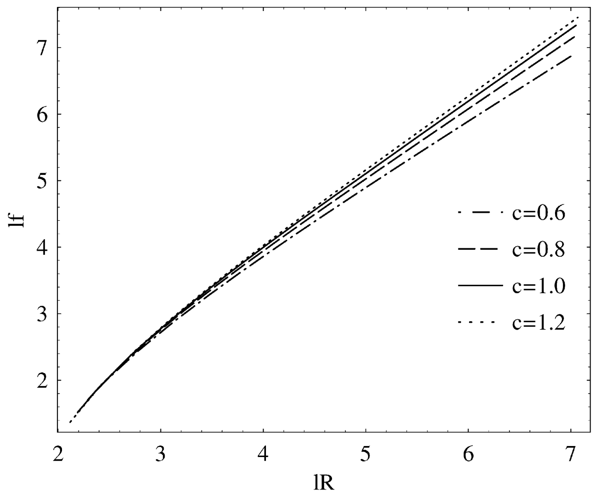

Fig. 2. Reconstructed $f(R)$ in lf–lR plane with $0 \leqslant z \leqslant 10$. $lf \equiv \ln(-f)$ and $lR \equiv \ln(-R)$.

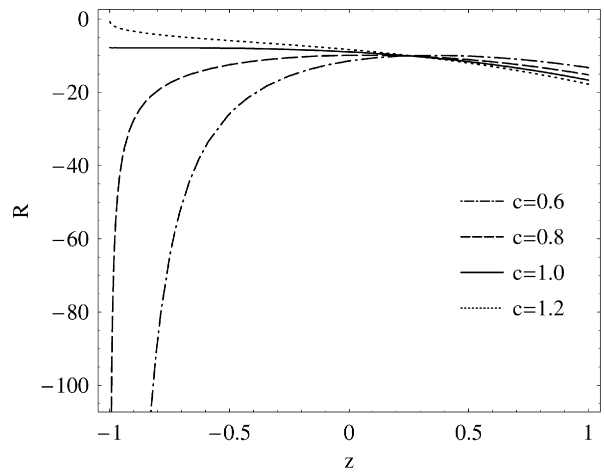

Fig. 3. The future evolution of R.

reconstructed function $f(R)$ for quiessence and that for Chaplygin gas (Fig. 2 and Fig. 4 in [18]), we can see that the three figures are similar. This is because the values of the parameters are set to be around their best fits. This is in effect approximately equivalent to reconstruct $f(R)$ with $H(z)$ directly from observational data. We expect future observations with more accurate data will discriminate between these models.

3. Discussion and conclusion

The reconstructed $f(R)$ theory is naturally consistent with the solar system experiment by the reconstruction method. In fact, this requirement is just the physical motivation for the initial conditions (20) and (21). For one thing, if we rewrite (15) explicitly with $8\pi G$

$$H^2 = \frac{8\pi G}{3}\left[\rho_{\rm curv} + \frac{\rho_m}{f'(R)}\right], \tag{23}$$

we can see that $f'(R)$ effectively modifies the Newton gravitational constant G as $G/f'(R)$, that is, a variable gravitational coupling. In order to be compatible with solar system experiments, at $z = 0$, we must require $G/f'(R_0) = G$ or $f'(R_0) = 1$. By

$$f'(R_0) = \left[\left(\frac{dR}{dz}\right)^{-1}\frac{df}{dz}\right]_{z=0} = 1 \tag{24}$$

this leads to (20). For the other, it is shown [25] that the consistency with solar system test also requires $f''(R_0) = 0$, which directly gives rise to (21). As for the remote past, there is no reason for us to impose this requirement since the experiments are done today and the validity of the result holds only at $z \sim 0$.

Fig. 1 shows that for small $|R|$ (small z also), the functions $f(R)$ are indistinguishable for different parameter c. As we mentioned before, c is a crucial parameter characterizing the nature of the holographic dark energy. Differences between the corresponding $f(R)$ functions become significant as $|R|$ (or z) increases. To further illustrate that the reconstructed theory does reflect the distinctive effect of c, we consider the future evolution scenario. Fig. 3 shows the future evolution of R. As is expected, for $c < 1$, the curves indicate the typical phantom behavior: $|R| \to \infty$. This is because the dark energy with EoS < -1 dominates over matter and the phantom energy density increases with time, tears apart structures and a Big Rip is unavoidable. For $c = 1$, $|R|$ varies little and the dark energy becomes more and more like a cosmological constant. For $c > 1$, R vanishes in the future. In Fig. 4, we can see that the difference is more distinctively reflected by the function $f(R)$ reconstructed based upon the future evolution of the holographic dark energy model. For $c = 1.2$, as R approaches zero, f increases from negative to positive, which may indicate an inverse power law dependence of f on R. This is consistent with the models proposed in [14] and [15]. For $c = 1$, the straight line manifests a linear dependence on R up to a constant, which is consistent with the de Sitter phase where $f = R + 2\Lambda$. For $c = 0.8$, the curve first meets a turnaround point, at which the decreasing $|R|$ begins to increase due to the domination of the phantom-like dark energy. It can be checked that in this case, the turnaround redshift is in the near future for $c = 0.8$ while it is in the near past for $c = 0.6$, namely, the domination of phantom-like dark energy begins earlier for smaller c. As the universe evolves, $|R|$ keeps growing, and f decreases first and then increases to become positive. Both f and R become divergent in the final Big Rip. Further analysis shows that for $c = 0.6$, $f \to -\infty$; for $c = 0.8$, $f \to +\infty$, which clearly reveals that the form of the function $f(R)$ may be significantly different for different c. We note that the existence of the turnaround point is a universal feature for all the phantom-dark energy models realized in $f(R)$ theories, due to the competition between dark energy and matter.

In conclusion, we have reconstructed the function $f(R)$ in the extended theory of gravity according to the holographic dark energy. The basic reconstruction procedure can be simply summarized as: first, express R as a function of z; then f can also be considered as a function of z by $f(z) = f[R(z)]$; thirdly obtain a third order differential equation for $f(z)$ and solve it with some initial conditions; and finally reconstruct $f(R)$ from $R(z)$ and $f(z)$. Note that in this procedure, the Hub-

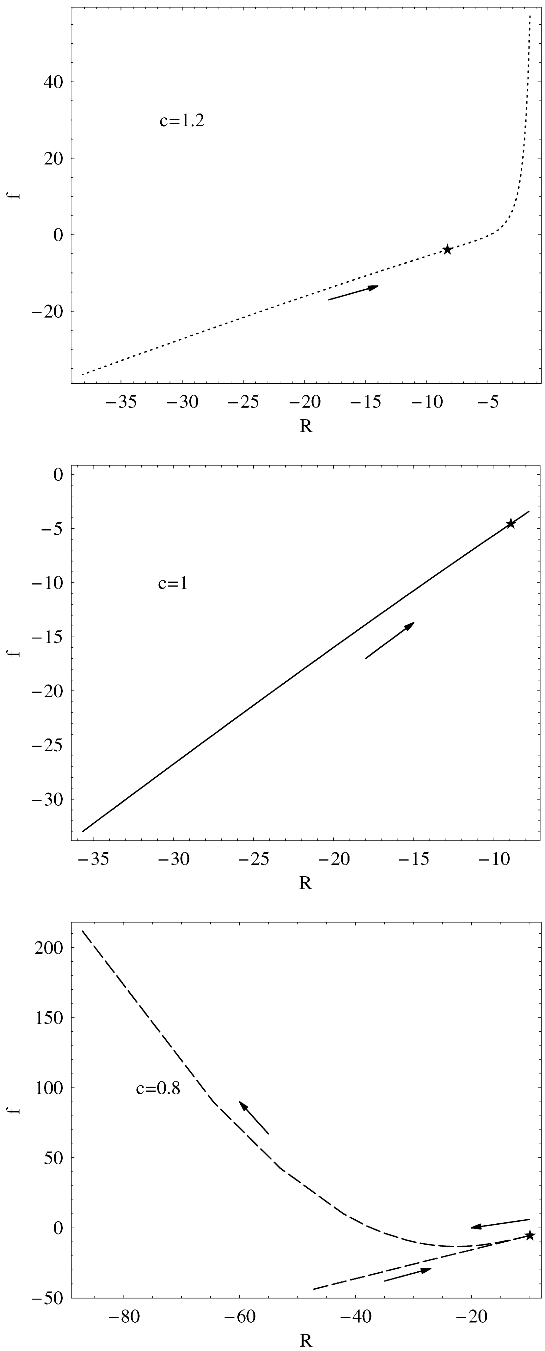

Fig. 4. Reconstructed $f(R)$. The curves are plotted with z from around 2 down to -1. The arrow denotes the direction of z increasing. The star denotes current value at $z=0$. The turnaround point is the point where the total energy density starts to increase after phantom energy dominates over matter.

ble parameter $H(z)$ and its derivatives with respect to z enter into R and the coefficients of the differential equation for $f(z)$. Since $H(z)$ and its derivatives can be expressed by Ω_Λ, what we are dealing with can be treated essentially as a differential equation set with the unknown functions $f(z)$ and $\Omega_\Lambda(z)$ to be solved. With some initial conditions imposed by physical consideration, we solved the differential equation set and found the function $f(R)$ numerically.

In addition, it should be emphasized that the holographic dark energy is the result obtained within the framework of general relativity, rather than any other extended theory such as $f(R)$ theory. What we have done is to reconstruct the $f(R)$ theory which effectively describes the holographic dark energy in Einstein gravity. Whether the holographic vacuum energy can be generalized to $f(R)$ theories and what it looks like are questions worth further investigation.

Acknowledgements

We thank Rong-Gen Cai for helpful discussion. This work was supported by the National Natural Science Foundation of China, under Grant No. 10533010, 973 Program No. 2007CB815401 and Program for New Century Excellent Talents in University (NCET) of China.

Appendix A

According to [18], here we list some expressions essential for practical calculation. The coefficients in (19) are

$$\mathcal{C}_1 = \dot{R}^2\left(\frac{dR}{dz}\right)^{-4}\left[3\left(\frac{dR}{dz}\right)^{-1}\left(\frac{d^2R}{dz^2}\right)^2 - \frac{d^3R}{dz^3}\right] - (\ddot{R} - \dot{R}H)\left(\frac{dR}{dz}\right)^{-3}\frac{d^2R}{dz^2} - 2(1+z)H\frac{dH}{dz}\left(\frac{dR}{dz}\right)^{-1}, \tag{A.1}$$

$$\mathcal{C}_2 = (\ddot{R} - \dot{R}H)\left(\frac{dR}{dz}\right)^{-2} - 3\dot{R}^2\left(\frac{dR}{dz}\right)^{-4}\frac{d^2R}{dz^2}, \tag{A.2}$$

$$\mathcal{C}_3 = \dot{R}^2\left(\frac{dR}{dz}\right)^{-3}. \tag{A.3}$$

$R(z)$ and its derivatives are

$$R = -6\left[2H^2 - (1+z)H\frac{dH}{dz}\right], \tag{A.4}$$

$$\frac{dR}{dz} = -6\left\{-(1+z)\left(\frac{dH}{dz}\right)^2 + H\left[3\frac{dH}{dz} - (1+z)\frac{d^2H}{dz^2}\right]\right\}, \tag{A.5}$$

$$\dot{R} = -(1+z)H\frac{dR}{dz}, \tag{A.6}$$

$$\ddot{R} - \dot{R}H = 6(1+z)H^2\left\{3(1+z)^2\frac{dH}{dz}\frac{d^2H}{dz^2} + H\left[(1+z)^2\frac{d^3H}{dz^3} - 6\frac{dH}{dz}\right]\right\}. \tag{A.7}$$

Higher order derivatives of $H(z)$ are too complicated to be listed here. In practical calculation, we use computer program for derivation. In addition, the code for *Mathematica* 5.0 we used is available on request.

References

[1] A.G. Riess, et al., Astron. J. 116 (1998) 1009, astro-ph/9805201;
S. Perlmutter, et al., Astrophys. J. 517 (1999) 565, astro-ph/9812133.

[2] D.N. Spergel, et al., astro-ph/0603449;
L. Page, et al., astro-ph/0603450;
G. Hinshaw, et al., astro-ph/0603451;
N. Jarosik, et al., astro-ph/0603452.

[3] M. Tegmark, et al., Phys. Rev. D 69 (2004) 103501, astro-ph/0310723;
M. Tegmark, et al., Astrophys. J. 606 (2004) 702, astro-ph/0310725;
U. Seljak, et al., Phys. Rev. D 71 (2005) 103515, astro-ph/0407372;
J.K. Adelman-McCarthy, et al., SDSS Collaboration, Astrophys. J. Suppl. 162 (2006) 38, astro-ph/0507711;
K. Abazajian, et al., astro-ph/0410239;
K. Abazajian, et al., astro-ph/0403325;
K. Abazajian, et al., astro-ph/0305492;
M. Tegmark, et al., Phys. Rev. D 74 (2006) 123507, astro-ph/0608632.

[4] E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) 1753, hep-th/0603057.

[5] P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 559, astro-ph/0207347;
S.M. Carroll, Living Rev. Rel. 4 (2001) 1, astro-ph/0004075;
S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.

[6] G. 't Hooft, gr-qc/9310026;
L. Susskind, J. Math. Phys. 36 (1995) 6377, hep-th/9409089.

[7] A. Cohen, D. Kaplan, A. Nelson, Phys. Rev. Lett. 82 (1999) 4971, hep-th/9803132.

[8] S.D.H. Hsu, hep-th/0403052.

[9] M. Li, Phys. Lett. B 603 (2004) 1, hep-th/0403127.

[10] Q.G. Huang, Y.G. Gong, JCAP 0408 (2004) 006, astro-ph/0403590;
X. Zhang, F.Q. Wu, Phys. Rev. D 72 (2005) 043524, astro-ph/0506310;
Z. Chang, F.Q. Wu, X. Zhang, Phys. Lett. B 633 (2006) 14, astro-ph/0509531;
Z.L. Yi, T.J. Zhang, Mod. Phys. Lett. A 22 (2007) 41, astro-ph/0605596;
X. Zhang, F.-Q. Wu, Phys. Rev. D 76 (2007) 023502, astro-ph/0701405.

[11] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, Effective Action in Quantum Gravity, IOP Publishing, Bristol, 1992;
N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, Cambridge Univ. Press, Cambridge, 1982;
R. Utiyama, B.S. De Witt, J. Math. Phys. 3 (1962) 608.

[12] M. Green, J. Schwarz, E. Witten, Superstring Theory, Cambridge Univ. Press, Cambridge, 1987;
A.A. Tseytlin, C. Vafa, Nucl. Phys. B 372 (1992) 443;
G. Veneziano, Phys. Lett. B 265 (1991) 287;
M. Gasperini, J. Maharana, G. Veneziano, Phys. Lett. B 272 (1991) 277;
K.A. Meissner, G. Veneziano, Phys. Lett. B 267 (1991) 33.

[13] A.A. Starobinsky, Phys. Lett. B 91 (1980) 99.

[14] S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner, Phys. Rev. D 70 (2003) 043528.

[15] S. Nojiri, S.D. Odintsov, Phys. Rev. D 68 (2003) 123512.

[16] S. Nojiri, S.D. Odintsov, Phys. Rev. D 74 (2006) 086005, hep-th/0608008;
S. Nojiri, S.D. Odintsov, J. Phys. Conf. Ser. 66 (2007) 012005, hep-th/0611071.

[17] P. Teyssandier, J. Math. Phys. 24 (1983) 2793;
K. Maeda, Phys. Rev. D 39 (1989) 3159;
D. Wands, Class. Quantum Grav. 11 (1994) 269;
S. Capozziello, R. de Ritis, A.A. Marino, Gen. Relativ. Gravit. 30 (1998) 1247;
S. Capozziello, G. Lambiase, Gen. Relativ. Gravit. 32 (2000) 295;
S. Gottlöber, H.-J. Schmidt, A.A. Starobinsky, Class. Quantum Grav. 7 (1990) 893.

[18] S. Capozziello, V.F. Cardone, A. Troisi, Phys. Rev. D 71 (2005) 0403503.

[19] X. Zhang, Phys. Rev. D 74 (2006) 103505, astro-ph/0609699.

[20] X. Zhang, Phys. Lett. B 648 (2007) 1, astro-ph/0604484.

[21] J. Zhang, X. Zhang, H. Liu, Phys. Lett. B 651 (2007) 84, arXiv: 0706.1185 [astro-ph].

[22] W. Zhao, arXiv: 0706.2211 [astro-ph].

[23] M. Ferraris, M. Francaviglia, I. Volovich, Class. Quantum Grav. 11 (1994) 1505;
G. Allemandi, A. Borowiec, M. Francaviglia, Phys. Rev. D 70 (2004) 043524;
G. Allemandi, A. Borowiec, M. Francaviglia, Phys. Rev. D 70 (2004) 103503.

[24] S. Capozziello, Int. J. Mod. Phys. D 11 (2002) 483;
S. Capozziello, S. Carloni, A. Troisi, astro-ph/0303041.

[25] R. Dick, Gen. Relativ. Gravit. 36 (2004) 217;
A.E. Dominguez, D.E. Barraco, Phys. Rev. D 70 (2004) 043505.