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Semi-analytical force model for grinding operations

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Abstract

Grinding process is generally used to improve the tolerance integrity and surface quality of a workpiece. However, in case of some hard-to-machine materials grinding can also be a cost effective alternative even for roughing operations. It is crucial to know process forces since they are necessary to identify the conditions for surface burn which is one of the most important issues in grinding applications. In this paper, a new semi-analytical force model for grinding process is developed by modeling abrasive grits and their interaction with the workpiece individually. Grits are examined to determine their geometrical properties and distribution on the grinding wheel. Semi-analytical equations for total normal and tangential force components as well as average force per grit are established by using the micro milling analogy. Fundamental parameters such as shear stress and friction coefficient between the grits and the work material are identified. The model can then be used in prediction of the forces for different cases involving the same material and the abrasive grain however with different conditions. The model predictions are verified by several experiments and also using Johnson-Cook material model.

1. Introduction

Grinding is generally considered as a finishing operation, nevertheless process quality and process parameter selection depends to a large extent on the experience of the operator. Since abrasive wheels have a stochastic nature, even if an operator achieve optimum parameters by experience or practical knowledge; it is hard to get a repeatable process. In order to overcome these issues and predict the outcomes of the operation beforehand, modeling of the process is required. There are several parameters which govern the cutting process; however, force values can be considered as the most essential ones since they lead us to the temperature and chatter vibration issues. To be able to calculate process forces, solid understanding of the process geometry, mechanics and abrasive wheel topography are required [1]. As the optic and other types of measurement systems develop, getting an insight or performing actual topography measurements of abrasive wheel surface become possible and this advancement led researchers to realize that each grain performs cutting action individually similar to the milling process. This property of the abrasive wheel makes the grinding process similar to a milling operation since both tools use its teeth to remove chips from workpiece in a microscopic scale [14].

Grinding forces are composed of chip formation, rubbing and ploughing forces. At the very first stage of the interaction between abrasive grit and the workpiece, plastic deformation occurs, temperature of the workpiece increases and let normal stress to exceed yield stress of the material. After a certain point, abrasive grit starts to penetrate into the material and starts to displace it, which is responsible for the ploughing forces. Finally, grit starts to shearing action and removes the chip from workpiece [4]. For the sake of achieving optimum conditions, process modeling and understanding of the cutting action per grain are necessary.

Nomenclature

a	Axial depth of cut (mm)
b	Radial depth of cut (mm)
feed	Workpiece velocity (mm/s)
$feed_r$	Workpiece velocity per rev (mm/rev)
V_c	Cutting velocity (m/s)
θ	Grain position angle (degrees)
K_{tc}	Tangential cutting force coefficient (Mpa)
K_{nc}	Normal cutting force coefficient (Mpa)
K_{tp}	Tangential ploughing force coefficient (Mpa)
K_{np}	Normal ploughing force coefficient (Mpa)
l_c	Geometric length of cutting zone (mm)
$l_{c\text{-}area}$	Area of cutting zone (mm^2)
D	Diameter of grinding wheel (mm)
R	Radius of grinding wheel (mm)
W	Width of grinding wheel (mm)
C	Grain number per mm^2
W_{area}	Area of grinding wheel surface (mm^2)
T_{grains}	Total number of grains on wheel
A_g	Active grain number
α	Grit rake angle (degrees)
r	Grit edge radius
F_{tc}	Force in tangential direction (N)
F_{nc}	Force in normal direction (N)
F_{tp}	Ploughing force in tangential direction (N)
F_{np}	Ploughing force in normal direction (N)
$F_{tc\text{-}g}$	Force per grain in tangential direction (N)
$F_{nc\text{-}g}$	Force per grain in normal direction (N)
$F_{tp\text{-}g}$	Ploughing force per grain in tangential direction (N)
$F_{np\text{-}g}$	Ploughing force per grain in normal direction (N)
MRR	Material removal rate (mm^3/s)
$Ø_s$	Shear Angle (degrees)
τ	Shear Stress (MPa)

2. Literature review

As mentioned before, grinding process has a stochastic nature, therefore process models and approaches vary in a large scale. The distribution and shape of the grain cutting edges strongly influence the force and surface finish [13]. Tönshoff et al. stated that the kinematics of the process is characterized by a series of statistically irregular and separate engagements. Both chip thickness and force models are presented and different approaches are compared [13]. As Brinksmeier et al. claimed the grinding process is the sum of the interactions among the wheel topology, process kinematics and the workpiece properties. Physical, empirical and heuristic process models are presented and most of them are related to the topography of wheel, workpiece properties and chip thickness analysis [16].

To begin with the experimental or mechanistic models, Fan and Miller [6] conducted grinding experiments and calibrated constants which depend on the workpiece material, grinding wheel and several other process parameters, in the formulation. Experiments should be performed to identify these constants for different arrangements of workpiece-wheel pair and process parameters. Johnson et al. [7] determined

force equations for face grinding operation by regression analysis from experimental data and identified the constants for various grinding wheel-workpiece pairs. The model is claimed to be implemented in industry quickly which is the main advantage of the experimental models. However, lack of accuracy and need for excessive experimental effort are drawbacks of these models.

There are semi-analytical force models in the literature as well [1,4,5,8,9]. They need experimental calibration of few parameters in semi-analytic formulations. Once these parameters are determined correctly, it is claimed that process forces can be calculated by presented semi-analytic force equations. Durgumahanti et al. [4] used this approach by assuming variable friction coefficient focusing mainly on the ploughing force. They established force equations for ploughing and cutting phases and need experimental calibration for certain parameters. Single grit tests were performed in order to understand the ploughing mechanism and the measured values are used to calculate the total process forces. Single grit analysis is beneficial since we can get more deterministic data about that particular grit without considering stochastic nature of them on the wheel. Chang and Wang focus more on stochastic nature of the abrasive wheel and tried to establish a force model as a function of the grit distribution on the wheel [5]. It is tricky to identify grit density function and require correct assumptions on grit locations and adequate generalizations. Hecker et al. [1] followed a more deterministic way by analyzing the wheel topography and then generalized the measured data through the entire wheel surface. Afterwards they examined the force per grit and identified the experimental constants. Kinematic analysis of grit trajectories during cutting were performed and chip thickness per grain assumed as a probabilistic random variable which is defined by Rayleigh probability density function [8]. Rausch et al. [9] focused on diamond grits by modeling their geometric and distributive nature individually rather than examining them on the abrasive wheel. Regular hexahedron or octahedron shapes of the grits are investigated and the model is capable of calculating engagement status for each grain on the tool and thus the total process forces. Koshy et al. developed a methodology to place abrasive grains on a wheel with a specific spatial pattern and examined these engineered wheels' performance [3]. Similar methods can be used to obtain the optimum abrasive wheel for a specific operation in the future.

It is hard to construct complete analytic models for abrasive processes considering their stochastic nature and the geometric properties of abrasive grits which highly depend on the dressing conditions. In this study, micro milling analogy for grinding operations is used. Once average geometrical properties of grains are identified, force analyses are performed by assuming each of the abrasive grain similar to the micro milling tool teeth. Then, fundamental parameters for chip formation are identified. In addition, as a double check, by measuring grinding temperature during the process, forces calculated based on the Johnson Cook material model are verified. Micro milling analogy and modeling of abrasive grits

will also be useful in expanding this force model to thermal and stability analyses.

3. Abrasive Wheel Topography

It is essential to identify the wheel topography in order to model mechanics and dynamics of the grinding process. There are several methodologies for measuring abrasive grits, their distribution and geometrical properties. In this research, optical measurement systems are used and necessary parameters are obtained through evaluations. In order to determine grits which are in the cutting zone and interact with the workpiece, a camera system with a 50x lens is utilized.
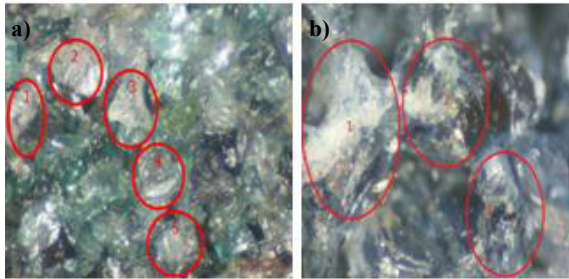

Figure 1 - (a) Alumina and (b) SiC wheel grain per mm^2 measurement

Both alumina and silicon carbide grinding wheels are measured and C parameter is identified [4,5]. Field of views of 1x1 mm for Alumina and 0.5x0.5 mm for SiC wheel were used due to focusing properties of the lens. Once the active number of abrasive grains in cutting area is determined, the actual feed per grain and thus the uncut chip thickness and process forces can be calculated. As it can be seen in Figure 1, grain number per millimeter square is 5 and 6 for Alumina and SiC grinding wheels, respectively. Height of the grains should also be checked to determine whether they are active or not as explained by Jiang et al. [11].
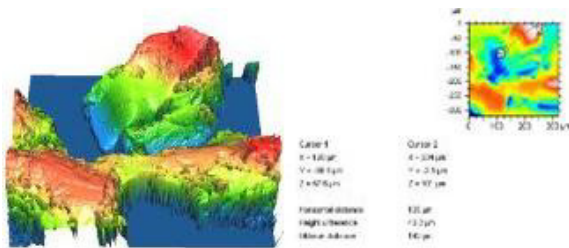

Figure 2 - Grit profile (rake angle and edge radius identification)

Once the C value is determined, the next step is to measure abrasive grains individually in order to determine average values of geometric parameters such as rake angle and edge radius. Measurements are done by a special areal confocal 3D measurement system and sufficient number of grains is scanned. After obtaining 3D scan of an abrasive grit, it is straightforward to determine rake angle and edge radius. One should carefully investigate the cutting direction while measuring rake angle since grits gave random shapes and

locations. Measurements are done on both type of wheels (Alumina and SiC), but presented results are for SiC as illustrated in Figure 3. Rake angle and edge radius values have a normal distribution as agreed in the literature [1]. Mean values for the rake angle is -17° and the edge radius is 0.01 mm with standard deviations of 4.58 and 0.002 mm, respectively.
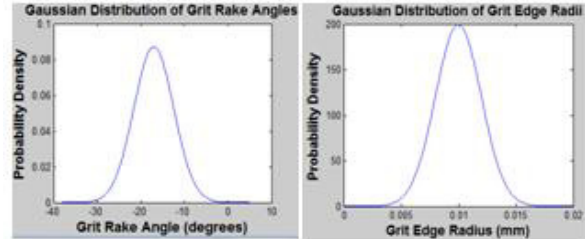

Figure 3 - Grit rake angle and edge radius distribution

As mentioned before, these properties highly depend on the wheel type and dressing conditions. Therefore, one may obtain different geometric properties with different dressing arrangements. In this work, it is assumed that abrasive grits will always have the same properties with same dressing procedure.

4. Force Model

As mentioned before, milling force equations are used with some modifications for prediction of grinding forces.

$$F_{t-grain} = K_{tc} \times f \times \sin(\theta) \times b + K_{tp} \times b$$
$$F_{n-grain} = K_{nc} \times f \times \sin(\theta) \times b + K_{np} \times b$$

(1)

In these equations K_{tc}, K_{tp}, K_{nc} and K_{np} are to be identified empirically but can be used for other cases with the same abrasive wheel and workpiece pair. K_{tc} and K_{nc} coefficients are for chip formation; K_{tp} and K_{np} are for ploughing forces. Ploughing forces are obtained by linear regression analysis and subtracted from the total forces in order to identify the average chip formation force per grain. After that step, K coefficients for both chip formation and ploughing components are identified. Expressions for other geometrical parameters, i.e. contact length, wheel surface area, total number of abrasive grits, number of active cutting grains and contact area are given below in order.

$$l_c = \sqrt{D \times a}$$ (2)

$$W_{area} = 2 \times \pi \times (D/2) \times w$$ (3)

$$T_{grains} = W_{area} \times C$$ (4)

$$A_g = l_{real-area} \times C$$ (5)

$$l_{c-area} = l_c \times b$$ (6)

Contact length can be derived geometrically from wheel-workpiece interaction as illustrated in Figure 4-a. It was shown in the literature that the real contact length is

substantially larger than the geometric contact length [10,13,2]. The increased area of contact is mainly due to deflection of the wheel and grains under the action of the normal force [2]. Contact length estimation using thermocouple measurement method is used for better accuracy [2]. As it can be seen in Section 6, temperature values are measured during each operation and by using equation 7, real contact length can be estimated. P is power, q_w is the total heat transferred into the workpiece material through contact length and finally l_{real} is the estimated contact length.

$$q_w = \frac{P}{l_{real} \times b} \qquad (7)$$

Wheel surface area is straightforward to calculate which is perimeter times width of the wheel. Since we identify C value in Section 3, grain number per millimeter square times wheel area is equal to the total number of abrasive grit on the cutting tool. Active grain number is obtained by multiplying C with contact area; which is contact length times radial depth of cut.
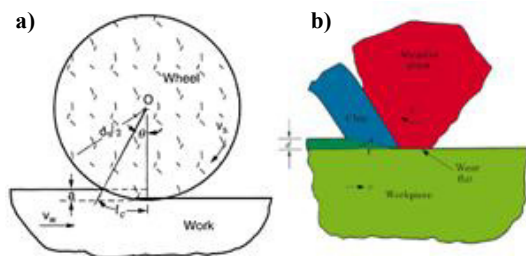
Figure 4- (a) wheel-workpiece (b) grit-workpiece interaction [13].

Using the position of a grain in the cutting zone the corresponding chip thickness at each time increment can be determined and used in force per grain calculation. Total process forces can be obtained by integrating them over the active number of grits.

$$F_{t-total} = \int_{i=1}^{A_g} F_{t-activegrain(i)}$$

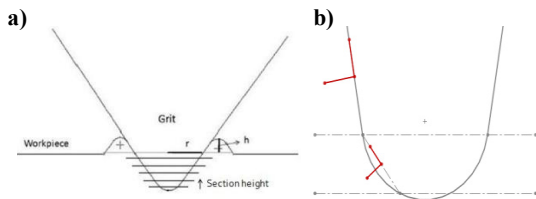$$F_{n-total} = \int_{i=1}^{A_g} F_{n-activegrain(i)} \qquad (8)$$

Figure 5 - (a) Grit engagement section and division into small regions (b) Force direction change due to edge radius.

One single grain's workpiece engagement section is divided into small portions and force analysis is done accordingly. As it is illustrated in Figure 5 [4], grit-workpiece engagement section is divided into sections in order to investigate the local angles such as side edge cutting and effective rake angle; afterwards, they are used to calculate forces at that particular section and projected into normal and tangential directions in order to get total process forces for that grain. Figure 5 (b) is an exaggeration in order to illustrate the methodology properly; section heights should be small enough to be precise in force calculations.

5. Experimental Procedure and Model Validation

In order to validate calculated forces by the presented semi-analytical model, force and temperature experiments have been conducted on TOS - FNK 2 NC milling machine tool. AISI 1050 steel and 125x25x20 SiC 80 M vitrified grinding wheel are used as workpiece and cutting tool respectively. Single point diamond dresser with 2 carat grade is used for dressing the wheel. Four different axial depth of cuts at 0.01, 0.02, 0.03 and 0.06 mm and at four feeds; 0.56, 0.69, 1.04 and 1.67 mm per revolution have been used as it can be seen in Table 1. Shear stress A and B are values for Johnson Cook material and Merchant model used cases respectively.

Table 1. Process Parameters and Shear Angle-Stress Results.

#	a (mm)	feed_r (mm/rev)	$\varnothing_s$ (degrees)	μ	Shear Stress A (MPa)	Shear Stress B (MPa)
1	0.01	0.56	29.52	0.70	475.55	452.90
2	0.02	0.56	35.35	0.56	1297.01	1284.18
3	0.03	0.56	36.24	0.54	1299.05	1286.13
4	0.06	0.56	30.13	0.68	1188.63	1061.21
5	0.01	0.69	32.78	0.63	750.73	544
6	0.02	0.69	32.33	0.64	1284.22	1088.32
7	0.03	0.69	28.15	0.72	1167.59	949.21
8	0.06	0.69	26.81	0.75	1083.72	1003.45
9	0.01	1.04	30.47	0.68	473.42	361.33
10	0.02	1.04	31.72	0.65	1058.72	745.58
11	0.03	1.04	29.15	0.70	1204.73	933.91
12	0.06	1.04	26.57	0.75	1036.39	835.87
13	0.01	1.67	30.82	0.67	330.73	318.12
14	0.02	1.67	31.12	0.66	1055.16	864.81
15	0.03	1.67	31.22	0.66	1124.40	858.39
16	0.06	1.67	29.93	0.69	940.04	776.84

Dressing conditions are determined from catalogue for single point diamond dresser as 0.2 mm depth to the wheel and 152 mm/sec feed rate. No coolant is used in these experiments in order to avoid miscalculations due to lack of convection heat transfer model when measuring process temperatures in Section 6. K_{tc} and K_{nc} are identified by the first four experiments conducted at 0.01 mm axial depth of cut and at four feeds. Experimental setup can be seen in Figure 6 [3,4,7].

Table 2. K coefficient identification.

	0.56(mm/rev)	0.69(mm/rev)	1.04(mm/rev)	1.67(mm/rev)
K_{tc}	2301 MPa	2521 MPa	2800 MPa	2401 MPa
K_{fc}	4501 MPa	6202 MPa	5701 MPa	4201 MPa

After the force coefficients are determined, the forces for the remaining 12 experiments can be calculated with the presented semi-analytical model. The experiments were repeated twice for both SiC and Alumina wheels for reliability. It is shown that if C and the force coefficients are identified correctly, process forces can be predicted accurately. C is 5 for the SiC wheel used in the tests. Ploughing force components are subtracted from total in order to get chip formation forces presented in Figure 7.
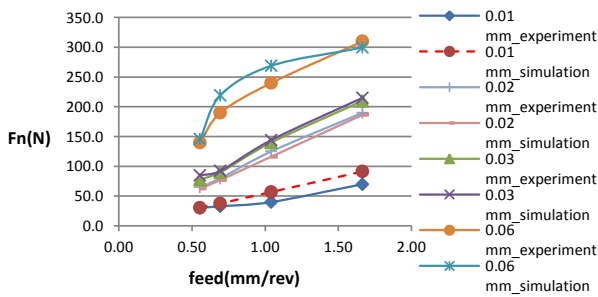

Figure 6- Experimental Setup

Figure 7- Experimental & Model Results (Total Forces)

As it can be seen in Figure 7, predicted chip formation forces are in a good agreement with measured forces in experiments. Grinding wheel is dressed after each axial depth of cut which corresponds to 4 times after 0.01, 0.02, 0.03 and 0.06 mm. Force per grit values can be calculated by using the K coefficients and geometric properties of grits identified in Section 3. Uncut chip thickness per grain, its rake angle, edge radius and average distance to its neighbor grain is known; therefore average force per grain can be calculated by the
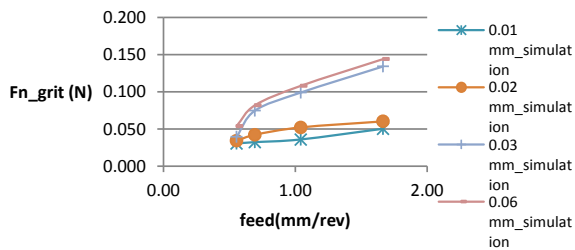

Figure 8- Calculated average force per grit

presented model. Linear regression analysis is used to identify ploughing force components and find the chip formation force as mentioned before.
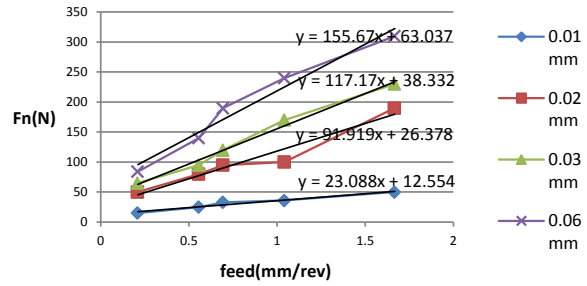

Figure 9 - Ploughing force identification

Table 3. Ploughing Force Estimation and Coefficients.

	0.01 mm	0.02 mm	0.03 mm	0.06 mm
F_{np}(N) experimental	12.55	26.37	38.33	63.03
F_{np}(N) calculated	12.55	22.10	33.65	75.30
K_{np}(N/mm)	1255			

Due to the plastic deformation, as abrasive grain meets with workpiece and exceeding yield stress of the metarial, rubbing forces occur. Next step is; grit starts to penetrate into the workpiece and be able to displace it and ploughing forces come into the scene. These two phases are taken as one and identified by linear regression analysis. They are considered as rubbing and ploughing forces at the third zone. By excluding the 3rd zone effect, it is assumed that the chip formation mechanism is similar to the milling operation [10].

By using Merchant's Circle and his theory for chip formation mechanism, shear and friction angles can be calculated. Geometric properties are known by topography investigations and friction coefficient can be calculated from experimental data. Shear angle and friction coefficient (μ) values are given in the Table 1. Shear angle is identified by using Merchant's Circle and shear stress can be predicted. Feed value per grain is known and width of grain is assumed as equal to its height [11]. The simulated forces are in a good agreement with experiments considering the %10.6 average error value for SiC and %13.7 for Alumina wheel.

6. Johnson Cook Material Model

Another verification approach is used as an indirect one. The identified shear stresses can be plugged into Johnson-Cook material model and the corresponding temperatures can be obtained. Furthermore, in-process temperatures in the cutting zone can be measured for comparison, also can be used and calculate shear stress values from Johnson-Cook material model presented in equation 9 and validate the presented semi-analytical force model.

$$\tau = \frac{1}{\sqrt{3}} \times \left[A + B \times (\frac{\gamma}{\sqrt{3}})^n \right] \times \left[1 + \ln(\frac{\gamma'}{\gamma_0})^m \right] \times \left[1 - (T^-)^v \right] \qquad (9)$$

Table 4. Johnson-Cook Parameters for AISI 1050 Steel [12].

A(MPa)	B(MPa)	n	m	v
880	500	0.234	0.0134	1

Strain and strain rate along shear plane formulations are crucial and can be derived by understanding the process geometry and chip formation mechanism [10].

$$\gamma = \frac{1}{2\sqrt{3}} \times \frac{\cos(\alpha_n)}{\cos(\phi_n - \alpha_n) \times \sin(\phi_n)} \qquad (10)$$

$$\gamma' = \frac{C_{ox}}{\sqrt{3}} \times \frac{V_{sh}}{l_s} \qquad (11)$$

C_{ox} is constant and V_{sh} is the shear velocity and l_s is the length of a shear plane. Shear velocity formulation is given in eq.

$$V_{sh} = V_s \times \frac{\cos(\alpha_n)}{\cos(\phi_n + \alpha_n)} \qquad (12)$$

Temperature measurement experiments are conducted in order to identify process temperatures and get shear stresses by using the Johnson-Cook formula. K type thermocouple is used for temperature measurements. It is embedded into the workpiece with epoxy in a 0.15 mm diameter blind hole opened by wire EDM. The hole is blind because when grinding wheel reaches to the thermocouple, thermocouple smears with the workpiece which ensures full contact between them [15]. Temperature measurements are done via taking continuous data from the contact zone between abrasive wheel and workpiece as illustrated in Figure 10. A sample measurement graph can be seen in Figure 11. 0.06 mm axial depth of cut and 1.04 mm/rev feed rate are used for the presented data in Figure 11. Peak temperature is measured as 875.7°C. After measuring the temperatures, shear stresses are calculated by the presented formulations in section 6. Average difference between shear stresses for Merchant and Johnson-Cook are %19.38 [Table 1].
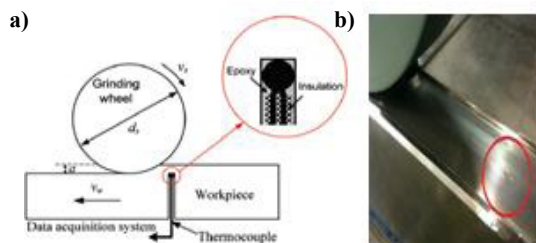

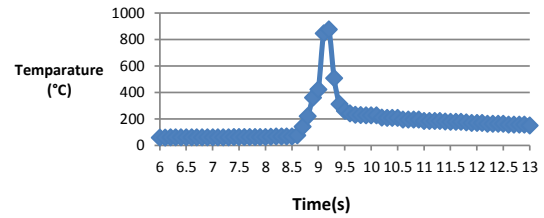Figure 10- (a) Temperature measurement setup (b) Exposed thermocouple after an operation

Figure 11- Temperature measurement data

7. Conclusion

A grinding process model is proposed based on surface topography of the abrasive wheel and milling process geometry analogy. Semi-analytical equations for forces per grain and total forces are presented. Temperature measurements are done for each operation in order to obtain shear stresses from J-C formula. Predicted values are in a good agreement with experimental results.

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