Terminal-zone wake vortex safety assessment in the context of UAS integration in the NAS

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Abstract

The paper presents results of the vortex safety analysis conducted for selected unmanned aircraft systems (UAS) operating alongside commercial aircraft in the terminal zone. The approach is developed on the basis of a dynamic model of an aircraft entering an upstream wake vortex that allows evaluating changes in the important aerodynamic parameters following the wake vortex interaction and assessing the operational safety risks in application to variable-size UAS. The benchmark parametric study estimates UAS responses in terms of the roll control ratios (RCR) developed by the follower aircraft depending on their position and orientation relative to the wake shed by the leader B737 aircraft, as well as the duration of the wake-UAS interaction. The UAS dynamic reaction is examined in the context of several possible flight control scenarios that include a time-delayed maximum aileron deviation and application of a robust feedback-loop flight controller.

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1. Introduction

The integration of unmanned aircraft systems (UAS) in the National Aerospace System (NAS) presents a number of challenges currently addressed by Federal Aviation Administration (FAA) in the context of the operational safety implications. In particular, further development and improvements to the Integrated Safety Assessment Model (ISAM) should fully incorporate UAS operations both for current and future risk analyses through careful update of the event-sequence diagrams (ESDs) and the corresponding fault-tree (FT) structures. In this context, much needed probabilistic assessment of the wake vortex interference events in the terminal zone relies on the accuracy of the predictive models incorporating various classes of UAS in the comprehensive wake vortex interference studies.
In recent years, a number of parametric wake vortex prediction models have been developed such as the AVOSS (Robins et al, 2002) and TDAWP (Proctor et al, 2006) models developed by NASA, and D2P/P2P/S2P models developed by DLR (Holzäpfel et al, 2007). These models output a wake vortex’s circulation strength and position considering its generation, decay and advection processes. The initial vortex strength depends on aircraft parameters such as lift (weight) and airspeed. For instance, S2P model is capable of providing a probabilistic distribution of wake vortex circulation strength and vertical/lateral position in real-time. Considering wake decay and advection processes, the S2P model predicts the uncertainty bounds and the upper and lower limits of wake vortex parameters, and then applies prescribed probability density distributions (PDDs) normalized by the calculated uncertainty bounds to express wake vortex random behaviors (Sugiura et al, 2011).

The first part of this paper presents the developed dynamic model of an aircraft entering a wake vortex that allows evaluating changes in the important flight parameters following the wake vortex interaction and assessing the operational safety risks. This model is created as part of SITAR WVSS simulation module. Several scenarios involving UAS operation in the terminal zone are further evaluated for risk assessments in a benchmark study examining the wake interaction response of selected UAS systems. The developed methodology and the conducted studies pave the way to further optimization, testing and validation of the wake interference models including appropriate estimation of uncertainties in aircraft and weather inputs, developing inputs to fault tree analyses within ISAM structure, and eventually employing the developed models for adjusting procedures for UAS operations in the terminal zone.

2. Dynamic wake interaction model

The task of integrating UAS systems (particularly small-size) with the routine air-traffic operations in the terminal zones reveals a number of challenges. One likely scenario examined in the current work is the interaction of a follower UAS (considered in a steady-level flight) with the wake vortex induced by a larger leader aircraft. As the UAS approaches the wake, it is affected by a rolling moment induced by the upstream vortex that may lead to a loss of the aircraft control. The subsequent sections first discuss the models implemented within Safety Investigation Toolkit for Analysis and Reporting Wake Vortex Safety System (SITAR WVSS, or SITAR) module employed in the current study to predict the effects of the UAS wake interaction process.

2.1. Dynamic model of aircraft entering a wake vortex

The developed approach incorporates a simplified aircraft dynamic model to evaluate the effects of the wake vortex interaction. In this model, only the banking moment perturbation is considered while the side forces resulting from the wake interaction are not taken into consideration (note that the magnitude of the induced rolling moment may thus be slightly overestimated since accounting for the lateral motion would reduce the wake interaction period). In addition, pitching and yaw moments are also neglected. Additional parameters include sideslip, which is assumed to be zero, thrust force, which counteracts the drag, and G-force equal to 1 for a steady horizontal flight. Based on the described assumptions, the dynamics of the aircraft can be described by (e.g., McCormick, 1995),

\[
\frac{dV}{dt} = -\sin \Theta \tag{1}
\]

\[
\frac{d\Theta}{dt} = \frac{g}{V} \left( \cos \gamma (1 + \Delta p) - \cos \Theta \right) \tag{2}
\]

\[
\frac{d\Psi}{dt} = -\frac{g}{V \cos \Theta} \sin \gamma (1 + \Delta p) \tag{3}
\]
Correspondingly, the dynamics of the aircraft position in spatial coordinates is governed by,

\[
\frac{dh}{dt} = V \sin \Theta \\
\frac{dx}{dt} = V \cos \Theta \cos \Psi \\
\frac{dz}{dt} = -V \cos \Theta \sin \Psi
\]  
(4)  
(5)  
(6)

Furthermore, the induced rolling moment, $\Delta M_x$, and the control input, $M_{ux}$, are introduced in the following linearized equations governing the roll angular velocity $\omega_x$ and the bank angle $\gamma$,

\[
\frac{d\omega_x}{dt} = \frac{\rho V S l^2}{2 m_x} \rho m_x \omega_x + \Delta M_x - M_{ux} \\
\frac{d\gamma}{dt} = \omega_x
\]  
(7)  
(8)

Note that the aerodynamic forces, moments, and the magnitude of $\Delta M_x$ depend on such factors as the wake intensity and the aircraft geometry and can be obtained using the strip model described, e.g., in (MacCormick, 1995). According to such approach, the induced rolling moment is the resultant of the sum of moments created by particular “strips” of wing and fins.

2.2. Wake vortex modeling

Modeling of the wake vortex is achieved using the Kutta-Joukowski theorem that allows the initial vortex circulation being written as,

\[
\Gamma_0 = \frac{W_L}{\rho V_b b_L \frac{\pi}{4}}
\]  
(9)

where $W_L$ is the maximum takeoff mass of the leader aircraft, $\rho$ is standard atmospheric density (a function of the altitude), and $b_L$ is the wing span of the leader aircraft, with the result corresponding to the elliptical distribution of the wing loading. In accordance with measurements from flight tests, the initial core radius of the wake is taken equal to 3.5% of the generator wing span (e.g., Schwarz et al, 2010),

\[
r_{\epsilon_0} = 0.035b_L
\]  
(10)
Velocities induced by the wake are calculated using Burnham-Hallock model with a constant core radius (Schwarz et al., 2010)

\[ V = \frac{\Gamma_L}{2\pi r_c^2 + r^2} \]  

(11)

The two-phase transport and decay model is used to predict the evolution of the vortex strength incorporating the diffusion phase followed by the rapid decay phase (Baranov et al., 2013). The equations describing the employed model are,

\[
\begin{align*}
\Gamma &= \Gamma_0 e^{-\frac{C_1}{T^*}}T^*, \quad t < t^* \\
\Gamma &= \Gamma_0 e^{-\frac{(C_1 - C_2 (t-t^*)V_0)}{T^*b}}, \quad t > t^*
\end{align*}
\]  

(12)

where \( T^* \) is a nonlinear function of \( \varepsilon \), and \( T = \frac{tV_0}{b_0} \), \( C_1, C_2 \) are constants.

Modeling the wake vortices near the ground (as the aircraft takes off or descends for landing) is obtained though the classical inviscid theory (e.g., Baranov et al., 2013) As the vortices approach the “in-ground” vicinity (where the distance to the ground is less than or equal to 2\( b \)), a thin vortex sheet (a boundary layer) is generated due to the zero velocity at the ground (impermeability condition). Whereas ground is assumed to be flat, the velocity field at any position above the ground is easily obtained by using the image (mirror) vortices below the ground plane instead of the vortex sheet. Cross-wind influence and vortex decay due to the ambient turbulence are also taken into account in the model.

2.3. Safety assessment model implementation

The SITAR WVSS system enables modeling of various scenarios to assess the operational safety risks of UAS by utilizing a list of aircrafts with their characteristics and track files containing information regarding the aircraft’s spatial movement as inputs. The input track files contain a set of coordinates in space (altitude, latitude, longitude) and time corresponding to each point. If the points contained in the track files are not sufficient, interpolation can be used to fill every points along the time axis every N seconds, where N is the interpolation time period. Different interpolation time periods can be set up and the flight divided into discrete frames corresponding to the interpolated points. The N-seconds flight through or in the wake can be simulated with a specified response model, which are described in the subsequent sections. This capability allows SITAR’s simulations to account for up to ten aircrafts and their wake vortex interaction simultaneously in one scenario.

As illustrated in Fig. 1, the simulations show the aircraft model with the accompanying trailing wake vortices (in the form of rings) on the interface. Each vortex ring is color coded in green, yellow, or red signifying the level of intensities for three categories of aircraft: Light, Medium, and Heavy according to ICAO wake turbulence category. When the follower aircraft encounters the wake vortex, the impact is calculated using the previously mentioned dynamic model. This also enables the prediction of the follower aircraft’s flight trajectory while interacting with the wake vortex. Simulation time can be prescribed and parameters such as maximum bank angle, altitude change, and the angular velocity are calculated.

There is an opportunity to create an encounter artificially by supplying the horizontal (\( dZ \)) and vertical (\( dH \)) offsets and the closing angle (\( d\text{Angle} \)) (Fig. 2).
3. Simulation results

3.1. Benchmark case study scenario

The parametric study considers several variable-size UAS systems interacting with the wake generated by a leader Boeing 737 aircraft for two different cases.

The first case corresponds to an encounter at an altitude of 860 m and VL= 86 m/s. In this scenario, the leader aircraft is at steady level flight with a follower 5500 m behind cruising at a speed of 30 m/s. Vertical and horizontal offsets of the follower UAS vary between -20 m to 20 m and -30 m to 30 m, respectively, with respect to the leader position. The wake vortex encounter is predicted for each position with a step of 1 m and flight-time through the wake of 1 s.

In the second case, the encounter occurs after the take-off of a Boeing 737 at a height of 300 m. The distance between the two aircraft is 3900 m and their respective velocities are 98 m/s for the leader and 8 m/s for the follower aircraft with a flight-time through the wake of 5 s. In this case,

The UAS selected as follower aircraft in the case studies are the Osprey small UAV (sUAV), MQ-1 Predator UAV, and Northrop Grumman’s RQ-4 Global Hawk UAV. Their key characteristics are presented in Table 1.

Table 1. UAV characteristics parameters used in simulations.

<table>
<thead>
<tr>
<th>UAV</th>
<th>Weight (kg)</th>
<th>J_{xx} (kg*m^2)</th>
<th>Wing span (m)</th>
<th>Wing area (m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Osprey</td>
<td>29.48</td>
<td>5.17</td>
<td>3.35</td>
<td>1.87</td>
</tr>
<tr>
<td>MQ-1 Predator</td>
<td>1020</td>
<td>3547</td>
<td>14.8</td>
<td>11.5</td>
</tr>
<tr>
<td>Northrop Grumman RQ-4 Global Hawk</td>
<td>12133</td>
<td>241660</td>
<td>35.42</td>
<td>50.1</td>
</tr>
</tbody>
</table>
Rolling moment of inertia $J_{xx}$ was estimated using the technique described in (Roskam, 1985). The calculation is based on the assumption that within each airplane category (based on the weight, number and position of engines, etc), the dimensionless radius of gyration, $R_{x,y,z}$ is the same and can be used for calculation of $J_{xx}$.

3.2. Wake induced UAS reaction

To assess the wake induced UAS reaction, a roll control ratio is used as the impact severity criteria (Schwarz et al, 2010),

$$RCR = \left| \frac{C_{l,\text{WV}}}{C_l(\delta_{\alpha,\text{max}})} \right|$$

(13)

For the maximum roll control power of the three aircrafts considered in the paper, a value of $C_l(\delta_{\alpha,\text{max}}) = -0.14$ was used.

The values of RCR behind the B-737 for the three UAS are illustrated in Fig. 3. Zones with RCR>1 are considered hazardous due to the induced rolling disturbance exceeding the aircraft’s maximum roll control power, thus resulting in a loss of control. Results for the Osprey sUAV reveals the impact with a wake from B-737 to be hazardous at the horizontal offset of 15 m from the leaders’ flightpath. These regions are spaced approximately 30 meters from one another and show an area in between inducing zero rolling disturbance on the sUAV. Therefore, a stable flight is possible for an Osprey operating behind a B-737 due to its small size and short wingspan, however it would have to operate in the areas with RCR=0.

Results for the Global Hawk and Predator UAVs (Fig. 3) reveal the zones of the wake vortex impact to be much greater. Such differences are primarily a result of the larger aircraft dimension and wingspan. Although the zone of influence is much bigger than Osprey’s, the maximum value of RCR experienced by the Global Hawk does not exceed 0.51 due to the wake vortex having less impact on the larger airframe.

Fig. 3. Wake vortex induced Roll Control Ratio (RCR) contours. (Left picture – Osprey UAV, middle – Predator, right – Global Hawk).

3.3. UAS time response scenarios

The aircraft response model incorporated in the SITAR code assumes an immediate reaction of the aircraft to the wake-induced disturbance in the form of the maximum aileron deflection to compensate for the exerted rolling moment. The control input $M_{ux}$ described in the previous sections is used to simulate the autopilot response using one of three models:

- The first model realizes the maximum aileron deflection at time $T_1 = 0.5$ s after the entry. The moment of entry is $T = 0$.
- In the second model, the ailerons are fully deflected (30 deg) after the delay of $T_2 = 0.1$ s if the bank angle exceeds the maximum of 10 deg or if the bank angular velocity exceeds the limit of 15 deg/s.
In the third model, there is no response input.

Fig. 4 compares the bank deviations with the corresponding response models when entering a wake vortex for different UAS. Fig. 4 (upper left) shows the Osprey sUAV subjected to a hazardous impact at the distance of 15 m from the leader’s flight-path in both directions. The results show the most significant bank deviation of 120 deg to occur when the response input is zero (model 3). However, in the presence of response (models 1 and 2), the wake vortex impact is significantly reduced. As previously mentioned, the RCR at these positions exceeds 1, and the maximum bank deviation of 70 deg can lead to the loss of control. Fig. 4 (bottom) reveals the Global Hawk to be more resistant to the wake impact due to its greater moment of inertia \( J_{xx} \), thus only reaching a bank deviation of 3 deg. Also, for the Predator, the no-reaction model and the model with full deflection of flaps after \( T_1 \) produce similar results.

![Fig. 4. Maximum bank deviations for Osprey (upper left), Predator (upper right) and Global Hawk (bottom). Legend: Blue – no reaction, Green – full deflection after \( T_1 \), Red – full deflection after \( T_2 \).](image)

The altitude change of the follower UAV for both benchmark cases was also analyzed. In Fig. 5, the altitude deviations of Osprey UAV, Predator and Global hawk are presented for the first case. The Osprey UAV underwent the maximum deviation of 11 m at \( dZ = \pm 1 \) m. Predator’s and Global Hawk’s deviations were less than 4 m and 2 m, correspondingly.

![Fig. 5. Maximum altitude deviations for Osprey (left), Global Hawk (middle) and Predator (right).](image)

For the Osprey sUAV (Fig. 6), the zone with the maximum deviation is in the range of \( dZ = \pm 10 \) m and the peak altitude deviation is approximately 71 m. Predator and Global hawk exhibit 13.7 m and 3 m altitude change, correspondingly. Taking into consideration the low velocity of the follower aircraft and the proximity of the ground in this case, such deviations for UAV can lead to an accident.

The above-described aircraft response model incorporated in the SITAR code assumes an immediate reaction of the aircraft to the wake-induced disturbance in the form of the maximum aileron deflection to compensate for the exerted rolling moment. A recent study (Golubev et al, 2015) has developed another scenario of the generic gust-induced UAS response involving a robust flight controller that employs a smart array of synthetic-jet actuators.
(SJAs) in lieu of the mechanical control surfaces (i.e., ailerons). The aircraft dynamic model assumes to contain parametric uncertainties due to linearization errors and unmodeled nonlinearities. Specifically, the aircraft system can be modeled via a quasi-linear state space system as,

\[ \dot{x} = Ax + Bu + f(x, t) \]  

(14)

where \( A \in \mathbb{R}^{n \times n} \) denotes the uncertain state matrix, \( B \in \mathbb{R}^{n \times m} \) represents the uncertain input matrix, and \( f(x, t) \in \mathbb{R}^{n} \) is a state- and time-dependent unknown, nonlinear disturbance. For example, \( f(x, t) \) could include exogenous disturbances such as gusts or nonlinearities not captured in the linearized dynamic model. In (Golubev et al., 2015), a vertical discrete wind gust model was employed to model the impact of an upstream non-uniformity that could be induced by a wake vortex. The following model defined by Federal Aviation Regulations (FAR) as a bounded nonlinearity in the longitudinal axis was used to investigate the impact of different magnitudes of a wind gust on the trajectory tracking capability of a UAS subject to the sudden upstream disturbance:

\[ f(x, t) = \begin{bmatrix} -11.1 \\ 7.2 \\ 37.4 \\ 0 \end{bmatrix} \frac{1}{V_0} \left\{ \frac{U_{ds}}{2} \left[ 1 - \cos \left( \frac{\pi s}{H} \right) \right] \right\}, \]  

(15)

where \( H \) denotes the distance (between 35 feet and 350 feet) along the airplane’s flight path for the gust to reach its peak velocity, \( V_0 \) is the forward velocity of the aircraft when it enters the gust, \( s \in [0, 2H] \) represents the distance penetrated into the gust (e.g., \( s = \int_{t_0}^{t_1} V(t) \, dt \) where \( V(t) \) is the forward velocity element of the state vector \( x \)), and \( U_{ds} \) is the design gust velocity as specified in FAR.

Test simulations of the performance of SJAs-based robust flight controller were based on the linearized model for the Osprey aircraft for the level flight at an airspeed of 25 m/s at an altitude of 60 m. Note that the examined longitudinal controller tracks the pitch rate and the forward velocity commands.

Fig. 7 (left) shows the gust velocity, followed by the closed-loop trajectory tracking results in the presence of a 10 m/s wind gust. The objective is for the aircraft to maintain straight and level flight at a trim condition of 25 m/s forward velocity and 0 deg/s pitch rate, while minimizing the altitude deviation. Fig. 7 further illustrates the aircraft response in terms of the resulting time history of the pitch angle and the altitude deviation under the robust closed-loop control. The results in this test case demonstrate that the sUAS can compensate for the wind gust disturbance and reliably track the reference trajectory within safety constraints. Specifically, the maximum altitude deviation remained within a 0.95 m magnitude, which is well within the recently reduced vertical separation minimum of 500 feet (152 m), as specified by FAR. Note that the stability analysis utilized to develop the current robust control...
law cannot be used to prove that the altitude is regulated to a desired set point with zero steady-state error, with the challenge to be addressed in future work.

4. Conclusions

The current paper described application of SITAR WVSS simulation module to assess possible wake safety risks associated with integration of UAS in the NAS. The assessment was based on examining the roll control ratios (RCR) of 3 variable-size UAVs (Osprey, Global Hawk and Predator) induced by the wake of the leader B-737 aircraft. The impact areas were shown in terms of the vertical and horizontal offsets of the follower UAS with respect to the leader position. The larger Global Hawk and Predator aircraft revealed expanded zones of the wake vortex impact compared to the sUAV (Osprey). However, for the considered separation distances from the leader, the maximum RCR far exceeded the critical value for the sUAV, in contrast to the larger UAVs. Three different autopilot response scenarios were further investigated including the time-delayed responses. In all cases, the sUAV revealed significant acquired bank and altitude deviations with much greater sensitivity to the considered response scenarios, while the larger UAVs appeared much more resistant to the wake impact due to their greater moments of inertia. Finally, a separate study examined application of a robust feedback-loop flight controller for sUAV based on a smart array of synthetic-jet actuators (SJAs) employed in lieu of the mechanical control surfaces. Results for the test case of sUAV subject to a wind gust demonstrated that the controller was able to successfully compensate for the gust disturbance and reliably track the reference trajectory within safety constraints.

References


