Investigation on Mechanism of Altitude Characteristic for Air-breathing Pulsed Laser Thruster

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Abstract

Altitude characteristic is of great importance for studying when an air-breathing pulsed laser thruster works in the dense atmosphere condition of 0-30 km altitude. The experimental findings all over the world show that the similar relationship between impulse coupling coefficient and altitude. According to strong explosion theory and an ideal gas model, a dimensionless factor indicating energy law of similitude is introduced, and formula of impulse coupling coefficient is deducted. Then theoretical study of altitude characteristic is carried out and mechanism of altitude characteristic is further explained. The results indicate that there is a maximum value of impulse coupling coefficient if the dimensionless factor equals to 0.41 in theory, and whether the phenomena of maximum appear or not depends on the range of the dimensionless factor related to altitude. As to a conical nozzle with the fixed length of 120 mm, the relationship between the sonic velocity and the dimensionless factor causes the maximum phenomenon at the altitude of about 12.5 km, and maximum theoretical impulse coupling coefficient is also found in the experimental investigations. The mechanism of altitude characteristic for air-breathing pulsed laser thruster is discovered in this article, which will provide reference for further research on altitude characteristic.

Keywords: laser propulsion; air-breathing mode; altitude characteristic; dimensionless factor; maximum phenomenon

1. Introduction

Altitude characteristic suggests the effects of altitude on a certain kind of performance parameters. For different kinds of engines, it refers to the relationship between those important performance parameters and flight height. On the one hand, there is a design height for an engine; On the other hand, engines with a fixed design height will take on different index parameters at different flight heights. Pointing out the working performances and ranges of engines, altitude characteristic becomes a fundamental basis for trajectory design and attitude control on vehicles. In terms of aeroengine, altitude characteristic refers to the change relation of parameters of thrust, fuel consumption rate, etc. to flight height with other conditions unchanged[1]. For turbine engine, altitude characteristic refers to that both engine thrust and fuel consumption rate decrease with the enhancement of flight height when flight Mach number remains constant. While for rocket engine, it refers to the change relation of thrust to flight height. The thrust increases with enhancement of height and gets to its maximum in vacuum condition[2-3]. Since the atmosphere is available as working matter in the air-breathing pulsed laser thruster, no material onboard can be consumed. In reality, it has an infinite specific impulse and a considerable impulse coupling coefficient, which is a most important performance parameter for laser propulsion distinguishing from other propulsion types[4]. The concept of altitude characteristic for air-breathing laser thruster refers to the change relation of impulse coupling coefficient to flight height when flight Mach number remains constant. Single-pulsed altitude characteristic refers to altitude characteristic under the eradiation of a single laser pulse. Since the effects of atmosphere density decreasing with altitude rising on air-breathing laser propulsion are great, air-breathing laser propulsion can only work in a range of relatively dense atmosphere, and impulse coupling coefficient will significantly decrease with further increasing of altitude. The necessity of investigation on altitude characteris-
tic is increasingly apparent.

Investigation on altitude characteristic for air-breathing pulsed laser propulsion is of great practical significance and the change relation of impulse coupling coefficient to flight height must be revealed from altitude characteristic to determine the scope of application for air-breathing mode in space launching. On the other hand, to carry out height design for laser thruster according to altitude characteristic, it can provide a basis for vehicle trajectory design. At present, investigators all over the world have paid much attention to this field. A number of experimental studies and numerical simulations have been made and some phenomenal conclusions have been obtained. There are certain differences among these conclusions, which cannot be explained by experimental errors. Based on analyzing experimental data of altitude characteristic and starting with altitude characteristic theory for conical nozzles, the mechanism of altitude characteristic for air-breathing pulsed laser thruster is disclosed elaborately in this article, which lays a foundation for further study.

2. Analysis on Experimental Data of Altitude Characteristic at Home and Abroad

At present, there are mainly three experimental results of single-pulsed altitude characteristic for parabolic nozzles at home and abroad, given by German W. O. Schall team\(^{[5-8]}\), Chinese Y. J. Hong team\(^{[9-10]}\), Chinese Z. P. Tang team\(^{[11]}\) and R. Q. Tan team\(^{[12-13]}\) in cooperation respectively. The three results of impulse coupling coefficient \(C_m\) for different atmosphere altitude \(H\) are shown in Fig. 1. There is a platform in the range of 0-10 km in W. O. Schall team’s \(C_m\) results, and then \(C_m\) decreases rapidly as the altitude rises. Y. J. Hong team’s results are similar to those of W. O. Schall team, except for a difference in detail that \(C_m\) decreases slowly in the range of 0-10 km. Z. P. Tang team think that \(C_m\) has maximums and maximum positions depend on the nozzle.

Though experimental conditions of the three teams such as laser parameters, nozzle sizes, measurement methods and so on are not the same (see Table 1), differences in the experimental results cannot all be explained by different experimental conditions and measurement errors. Because there is not a reasonable theoretical model of impulse coupling characteristics for air-breathing laser propulsion yet, the three teams only interpret the \(C_m\) maximum experimental phenomenon in altitude characteristic qualitatively and consider the phenomenon influenced by coupling of detonation velocity and atmosphere density without convincing arguments. However, altitude characteristic is of very great importance to working characteristics of air-breathing laser thrusters, which is worth careful researching into theoretically.

3. Theoretical Study on Altitude Characteristic

There are many complex physical and chemical phenomena in working process of pulsed laser propulsion, which induce that theoretical study on altitude characteristic for complicated nozzle geometries is very hard. Among different possible geometrical performances of nozzles, conical nozzles which correspond to linearity characteristics will be discussed. When the energy of a laser beam is deposited at the apex of a conical nozzle transiently and a strong shock wave is formed, Sedov model of one-dimensional spherically-symmetric point-focusing explosion can be used here to depict the evolution process...
the evolution process of shock waves and flow field, which has a self-similar solution (see Fig. 2). Ref. [14] has done some painstaking research on this problem and the analytical solution of $C_m$ is

$$C_m = \frac{10\pi \eta_{de} (1 + \cos \theta)}{9c_0 \sqrt{I_1}} \left[ \frac{48\gamma I_1 L_2}{25(\gamma + 1)} \right] \bar{R}^{3/2}$$  \hspace{1cm} (1)

where $c_0$ is ambient gas sonic velocity, $\eta_{de}$ energy conversion rate from laser energy to plasma internal energy, i.e., laser energy deposition rate, $p_0$ ambient gas pressure, $E_{in}$ incident laser energy, $\theta$ half cone angle; $\gamma$ means specific heat ratio, and $I_1$ and $I_2$ are constants related to $\gamma$. Attention should be paid to the fact that $\bar{R}$ is a dimensionless factor introduced here and $R$ the fixed length.

$$\bar{R} = \frac{R}{R'} = \left[ \frac{2E_{in} \eta_{de}}{p_0 (1 - \cos \theta)} \right]^{1/3}$$ \hspace{1cm} (2)

where $c_0$ is ambient gas sonic velocity, $\eta_{de}$ energy conversion rate from laser energy to plasma internal energy, i.e., laser energy deposition rate, $p_0$ ambient gas pressure, $E_{in}$ incident laser energy, $\theta$ half cone angle; $\gamma$ means specific heat ratio, and $I_1$ and $I_2$ are constants related to $\gamma$. Attention should be paid to the fact that $\bar{R}$ is a dimensionless factor introduced here and $R$ the fixed length.

Table 1  Experimental conditions and parameters.

<table>
<thead>
<tr>
<th>Research team</th>
<th>Incident Laser energy / J</th>
<th>Parabolic equation</th>
<th>Aperture diameter / mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>W. O. Schall</td>
<td>128.288</td>
<td>$y^2=40x$</td>
<td>4100</td>
</tr>
<tr>
<td>Y. J. Hong</td>
<td>64.7-64.8</td>
<td>$y^2=20x$</td>
<td>1900</td>
</tr>
<tr>
<td>Z. P. Tang and R. Q. Tan</td>
<td>10.19-8.6</td>
<td>A:$y^2=20x$</td>
<td>480</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B:$y^2=40x$</td>
<td>650</td>
</tr>
</tbody>
</table>

From Eqs. (1)-(2), it can be seen that with the eradiation of a fixed-energy laser pulse, the change of $C_m$ versus $R$ mainly depends on these two para-meters $c_0$ and $p_0$. While $c_0$ is a function of $p_0$ and initial density $\rho_0$, $C_m$ can be calculated according to standard atmosphere parameters at different altitudes on condition that other parameters have been given.

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Fig.2 Conical nozzle and spherical shock wave.

3.1. Change of impulse coupling coefficient versus dimensionless factor

For air, the value of $\gamma$ equals 1.4, and we suppose that $\eta_{de}$ equals about 50%. The change curves of $C_m$ versus $\bar{R}$ for a conical nozzle with $\theta=30^\circ$ at four different altitudes are shown in Fig. 3.

$$C_m = \frac{20\pi \eta_{de} (1 + \cos \theta)}{9c_0 \sqrt{I_1}} \left[ \frac{16\gamma I_1 L_2}{25(\gamma + 1)} \right]^{3/2}$$ \hspace{1cm} (4)

It can be seen that $C_m$ has maximums at different altitudes, while maximum position $\bar{R}_{\text{max}}$ and maximum value $C_{m, \text{max}}$ are expressed as follows respectively:

$$\bar{R}_{\text{max}} = \left[ \frac{16\gamma I_1 L_2}{25(\gamma + 1)} \right]^{1/3}$$ \hspace{1cm} (3)

It is interesting that $\bar{R}_{\text{max}}$ is only related to gas specific heat ratio, independent of incident laser energy, nozzle cone angle, energy deposition rate and gas initial parameters. For a gas with $\gamma=1.4$, $\bar{R}_{\text{max}}=0.41$, the $C_{m, \text{max}}$ is not only related to gas specific heat ratio, but also dependent on nozzle cone angle, energy deposition rate, and gas initial parameters. However, $C_{m, \text{max}}$ is unrelated to incident laser energy. The characteristics called energy law of similitude are very important to nozzle design.

There is an interesting phenomenon which can be found in Fig.3. The relationship between $C_m$ and atmosphere altitude is rather complex: Neither is linear, nor is monotonic. Fig.3 shows that $C_m$ has maximums when $\bar{R}_{\text{max}}=0.41$ without designating the change of $C_m$ with $\bar{R}$ when the nozzle length is constant.

3.2. Change of impulse coupling coefficient versus atmosphere altitude

In order to explain inconsistency of experimental results obtained by the three teams given in Fig.1 and reveal the mechanism of atmosphere altitude characteristic, conical nozzles in length range of 40-90 mm covering all the nozzle lengths used in the experiments are chosen to be discussed here. Under the conditions of $E_{in}=60$ J, $\eta_{de}=50\%$, $\theta=30^\circ$ and $\gamma=1.4$, ...
the theoretical change curves of $\bar{R}$ versus $H$ for three conical nozzles with $R=40$ mm, 120 mm and 360 mm respectively are shown in Fig.4(a), and that of $C_m$ versus $H$ under the same condition in Fig.4(b).

It can be seen from Fig. 4(a) that: 1) $\bar{R}$ increases monotonously with nozzle length increasing for fixed altitude; 2) The bigger the nozzle length is, the more rapidly $\bar{R}$ decreases with $H$ increasing for fixed nozzle length. This is worthy of being considered seriously. $\bar{R}$ is always decreasing monotonously with $H$ increasing despite of different nozzle lengths, i.e., maximum dimensionless factor $\bar{R}_{\text{max}}$ is gained at sea level for every nozzle. It has been discussed in Section 3.1 that $C_m$ has maximums when $\bar{R}$ falls to 0.41, i.e., $C_m$ first increases to its maximum value along curves on the right of the maximum point in Fig.3 and then decreases along the left curves in Fig.3, which corresponds to the $R=120$ mm and $R=360$ mm curves in Fig.4(b). $C_m$ maximum positions of altitude are different for different nozzle lengths, as shown in Fig. 4(b).

4. Mechanism of Altitude Characteristic

To explain the mechanism of altitude characteristic for air-breathing laser thrusters, the basic relationship among $C_m$, $\rho_0$ and $p_0$ is to be analyzed firstly. When $p_0$ remains constant ($p_0=30$ kPa), the change curve of $C_m$ versus $\rho_0$ is shown in Fig.5(a). Since $C_m$ has a monotonic increasing trend with $\rho_0$ increasing, it is the largest at sea level and then decreases gradually when $p_0$ remains constant. When $\rho_0$ remains constant ($\rho_0=1.225$ kg/m$^3$), the change curve of $C_m$ versus $p_0$ is shown in Fig. 5(b). The other conditions are $E_{\text{in}}=60$ J, $\eta_{\text{le}}=50\%$, $\theta=30^\circ$, $R=120$ mm and $\gamma=1.4$. $C_m$ has a monotonic decreasing trend with $p_0$ increasing. Both of the natural atmosphere parameters $\rho_0$ and $p_0$ decrease with the enhancement of altitude height, which illustrates that the $C_m$ maximum phenomenon
is the cooperation result of the two parameters. The parameters related to $\rho_0$ and $p_0$ in Eq. (1) are $c_0$ and $R$. Considering $\rho_0$ and $p_0$ at the same time, the change curve of $c_0$ versus $H$ is shown in Fig. 6. $c_0$ almost decreases linearly in the range of 0-11 km and increases monotonously above 20 km. The value of $c_0$ changes little in the range of 11-20 km. Moreover, the change curve of $c_0$ versus $H$ (see Fig. 7) displays that $C_m$ decreases monotonously with $c_0$ increasing. Therefore, the change curve of $C_m$ versus $H$ is reciprocal to that in Fig. 6.

It can be seen from Fig. 3 and Fig. 4(a) that for fixed length nozzles, $\bar{R}$ decreases monotonously with $H$ increasing and the change trend of $C_m$ depends on the descending cover range. For the fixed 120 mm length nozzle in Fig. 4(a), $\bar{R}_0$ at sea level is about 0.72, and then leaps over $\bar{R}_{\text{max}}=0.41$, inducing $C_m$ is moving along a direction designated by the thin dotted line in Fig. 3 and a maximum value appears.

Comprehensive comparison among Figs. 3-4(a), and Figs. 6-7 reveals that:

1. In the range of 0-11 km, $c_0$ decreases 85% monotonously relative to the value at sea level, which results in the fact that $C_m$ increases 18.2% monotonously. At the same time, $\bar{R}$ decreases to 0.48 from 0.72, causing $C_m$ to increase a few times (see Fig. 3), too. The fast ascending section of 0-11 km in Fig. 4 (b) is the cooperation result of these two parameters.

2. In the range of 11.0-12.5 km, $c_0$ is nearly 290 m/s which changes gently and has little impact on $C_m$. However, $\bar{R}$ keeps on decreasing from 0.48 to $\bar{R}_{\text{max}}=0.41$, inducing $C_m$ goes on increasing. It is displayed in Fig. 3 that $C_m$ increases relatively slowly, which corresponds to the slowly varying section near maximum positions in Fig. 4 (b).

3. In the range of 12.5-20.0 km, $c_0$ is still nearly unchanged and has little impact on $C_m$. However, after $\bar{R}$ strides to the maximum point, $C_m$ begins to decrease and thus the maximum value appears at $\bar{R}_{\text{max}}=0.41$.

4. Above 20 km, $c_0$ begins to increase inversely and results in a reduced $C_m$ directly. At the same time, $\bar{R}$ continues to decrease with altitude increasing and leads to a negative growth of $C_m$, too. The cooperation causes $C_m$ to go on decreasing above 20 km in Fig. 4 (b). This descending trend of $C_m$ will be gentler and gentler influenced by the mitigation of $c_0$ and $R$.

Therefore, different nozzle lengths chosen in experiments will determine whether impulse coupling coefficient maximums appear or not. When choosing nozzles, it is expected that the dimensionless factor covers the section near maximum as much as possible. Since the parabolic nozzles used by the three teams are different, the laser energy deposition rate is difficult to be determined experimentally and theoretically.

5. Conclusions

Based on different experimental results of the three research teams, the corresponding theoretical study on altitude characteristic for air-breathing laser propulsion in this article shows that:

1. At different flight heights, impulse coupling coefficient first increases and then decreases with enhancement of the dimensionless factor and there is always a maximum, whose position is only related to gas specific heat ratio.

2. For nozzles with fixed length, the dimensionless factor corresponding to a given altitude decreases monotonously with flight height increasing, and whether impulse coupling coefficient has maximum values is related to covering ranges of the dimensionless factor.

3. There are two different covering cases of the dimensionless factor for nozzles with fixed length at different altitudes. The maximum phenomenon in the experiments is judged to belong to the second covering case.

The mechanism of altitude characteristic is dis-
closed theoretically in this article. The qualitative regularity is consistent with the experimental results. Further study is awaited.

References


Biography:

Hong Yanji  Born in 1963, she received M. S. and Ph.D. degrees from National University of Defense Technology in 1994 and 1997 respectively, and then became a professor in Academy of Equipment Command & Technology. Her main research interest is advanced propulsion technology. E-mail: hongyanji@vip.sina.com