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Physics Letters B

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Source of the Kerr–Newman solution as a supersymmetric domain-wall bubble: 50 years of the problem



Alexander Burinskii

Theor. Phys. Lab. NSI, Russian Academy of Sciences, B. Tulkaya 52, Moscow 115191, Russia

ARTICLE INFO

Article history:

Received 31 December 2015

Accepted 13 January 2016

Available online 15 January 2016

Editor: M. Cvetič

ABSTRACT

We consider the chiral field model of the source of the Kerr–Newman (KN) solution and obtain that it represents a supersymmetric spinning soliton, bounded by the chiral domain wall (DW) of the ellipsoidal form. The known method for transformation of the planar DW to Bogomolnyi form we generalize to the curved DW-bubble adapted to the Kerr coordinate system and obtain the supersymmetric BPS-saturated source of the KN solution, having some remarkable features, in particular, the quantum angular momentum. The main new result is that the source forms a breather, i.e. the DW–antiDW combination. Taking into account that the KN solution describes the spinning particles with gyromagnetic ratio $g = 2$, as that of the Dirac electron, we touch the problem of the compatibility of the spinning particles with gravity.

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1. Introduction

Last year we marked 50 years of the problem of source of the Kerr–Newman (KN) solution. Starting in 1965 in the paper by Newman and Janis [1], where it was mentioned that the KN space is two-sheeted, the diverse attempts and suggestions to solve this problem are debating up to now [2–4]. In this paper we develop the line started by W. Israel [5], who suggested to truncate the second sheet of the Kerr geometry along the *disk* spanned by the Kerr singular ring. After analysis of the Israel source by Hamity [6], a modified *disk-like source* was suggested by C. López [7] as an ellipsoidal vacuum bubble – a thin shell covering the Kerr singular ring and matching with the external KN solution. In the papers [4, 8] we considered a generalization of the López model, in which the thin shell is replaced by a *domain-wall* (DW) forming a spinning bubble-soliton [9], based on the $N = 1$ supersymmetric chiral field model.¹ The corresponding Hamiltonian was reduced to Bogomolnyi form, and it was shown that this soliton forms a supersymmetric, BPS saturated state. In this letter we would like pay attention to the used intricate method reduction to Bogomolnyi form, which was apparently first suggested in [11] for a two-dimensional kink solution, and then successfully used for the planar DW in [12–16]. We generalize this method to the much more complex case of the

Kerr geometry, in which the source is spinning and bounded by the DW of ellipsoidal form. Besides, it is formed by the system of chiral fields, when one of them depends on the Kerr angular coordinate and time. With respect to previous treatment [4, 8], we obtain new very important feature of the DW source – formation of the DW–antiDW (breather) structure.

In addition to the purely academic interest, the problem of the source of the KN solution is also important for resolution of the conflict between quantum theory and gravity. As is known, the KN solution has the gyromagnetic ratio of the Dirac electron [17], and therefore, it corresponds to the external gravitational and em field of the electron [18]. Structure of the source of the KN solution could shed the light on the question, which peculiarities of the spinning particles provide their consistency with gravity, and we come to the conclusion that the supersymmetric DW phase transition is one of the conditions for this consistency.

2. Spinning soliton

Metric of the KN solution in the Kerr–Schild (KS) form [18] is

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_{\mu}k_{\nu}, \quad (1)$$

where $\eta_{\mu\nu}$ is metric of the auxiliary Minkowski space² \mathbb{M}^4 ,

¹ E-mail address: bur@ibrae.ac.ru.

¹ Earlier story of the problem of the KN source was described by W. Israel [5], and later by A. Krasinski [10].

² We use signature $(-+++)$ and the dimensionless units $G = c = \hbar = 1$.

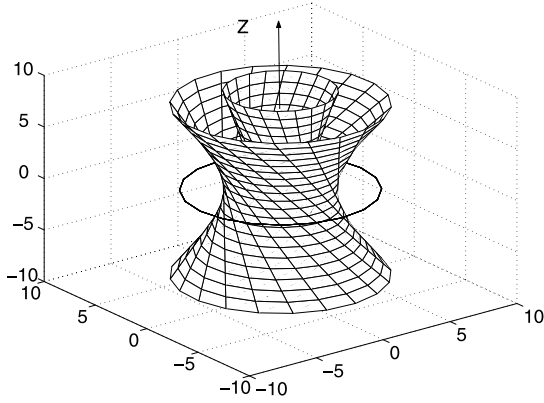


Fig. 1. The Kerr congruence, generated by null vector field k^μ , is extended analytically through the Kerr singular ring to ‘negative’ sheet of the Kerr space, $r < 0$, creating two-sheeted space-time.

$$H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta} \quad (2)$$

is a scalar function, expressed in the Kerr oblate ellipsoidal coordinates r, θ and ϕ_K , related with the Cartesian coordinates $(t, x, y, z) \in \mathbb{M}^4$ as follows [18]

$$\begin{aligned} x + iy &= (r + ia)e^{i\phi_K} \sin \theta, \\ z &= r \cos \theta, \\ t &= \rho - r. \end{aligned} \quad (3)$$

The null vector field

$$k_\mu dx^\mu = dr - dt - a \sin^2 \theta d\phi_K, \quad (4)$$

($k_\mu k^\mu = 0$) is tangent to Principal Null Congruence (PNC) \mathcal{K} of the Kerr geometry, forming a vortex-like polarization of the Kerr space-time. Similarly to the metric, the vector potential

$$A_\mu dx^\mu = -\text{Re} \left[\left(\frac{e}{r + ia \cos \theta} \right) \right] k_\mu \quad (5)$$

is also aligned to the PNC direction k_μ .

The surface $r = 0$ represents a disk-like “door” from negative sheet $r < 0$ to positive one $r > 0$, see Fig. 1. The null vector fields $k^{\mu\pm}(x)$ turns out to be different on these sheets, and two different null congruences \mathcal{K}^\pm , create two different metrics $g_{\mu\nu}^\pm = \eta_{\mu\nu} + 2Hk_\mu^\pm k_\nu^\pm$ and two different EM fields on the same Minkowski background.

The mysterious two-sheeted structure of the Kerr geometry created the problem of a “realistic” source of the KN solution. Relevant “regularization” of the KN solution was suggested by López [7], who excised singular region together with negative sheet and replace it by a regular core with a flat internal metric $\eta_{\mu\nu}$. The external KN metric (1) matches with flat metric inside the bubble at the condition $H(r) = 0$, which determines the bubble boundary $r = R$ as the surface of “zero-gravity potential”,

$$H|_{r=R}(r) = 0. \quad (6)$$

In accordance with (2) it yields

$$R = r_e = \frac{e^2}{2m}. \quad (7)$$

Since r is the Kerr oblate radial coordinate, the bubble surface forms the disk with radius $r_c \sim a$ and thickness $r_e = e^2/2m$. The matter sources and currents are distributed over the surface of this disk, and the EM vector potential takes maximal value in the equatorial plane $\cos \theta = 0$, at the disk boundary $r = r_e$, where the vector

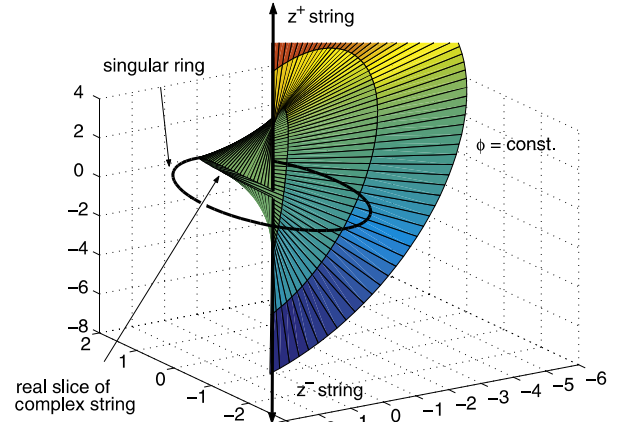


Fig. 2. The Kerr coordinate surface $\phi_K = \text{const.}$ Generators of the Kerr congruence k_μ are tangent to the Kerr singular ring at $\theta = \pi/2$, and vector potential forms a closed loop along the border of the KN source.

potential is dragged by the Kerr singular ring, see Fig. 2, forming a closed loop along the sharp border of the source.

In [9] this model was extended to a spinning soliton-bubble model representing the domain-wall (DW) interpolating between internal flat space-time and the exact KN solution outside the bubble. It was assumed [9,19] that the regulated smooth KN solution is described by a “deformed” KS metric suggested by Gürses and Gürsey [20], which retains the KS form (1), but has the “deformed” function

$$H = H_{GG} = \frac{f(r)}{r^2 + a^2 \cos^2 \theta}, \quad (8)$$

where $f(r)$ interpolates between zero value inside: $f = 0$ by $r < r_e$, and the exact KN form outside the bubble:

$$f \equiv f_{KN} = mr - e^2/2. \quad (9)$$

For cosmic black holes (BH) of large mass, this source may be considered as interior of the BH hidden by the BH horizon. For the KN solutions with parameters of the elementary particles, the spin/mass ratio is extremely large, $a \gg m$, the horizons disappear, and the rotating source is alternative to the naked singularity of the ultra-extreme KN solution [4,9,19].

3. Supersymmetric phase transition

As in many other soliton models, the considered in [9] soliton was formed by the Higgs mechanism of symmetry breaking. However, it was shown that the typical quartic potential term $V = g(\bar{\sigma}\sigma - \eta^2)^2$, where σ is v.e.v. of the Higgs field $\Phi(x)$, does not suit for the source of the Kerr–Newman solution, since it leads to the location of the Higgs condensate outside the source that breaks the gauge symmetry of the external EM field. The correct scheme of the phase transition leading to location of the Higgs field inside the source was used in [9]. It is N = 1 D = 4 supersymmetric field model [21] with three chiral fields $\Phi^{(i)}$, $i = 1, 2, 3$, forming a domain wall (DW) bubble.

The Lagrangian corresponding to bosonic part of the N = 1 supersymmetric field model has the form [21]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \sum_i (\mathcal{D}_\mu^{(i)} \Phi^{(i)}) (\mathcal{D}^{(i)\mu} \Phi^{(i)})^* - V. \quad (10)$$

Due to condition (6) the DW boundary of the source may be considered on the flat background, $g_{\mu\nu} = \eta_{\mu\nu}$, and the covariant derivatives $\mathcal{D}_\mu^{(i)} \equiv \partial_\mu + ieA_\mu^i$ are taken to be flat.

Among the three chiral fields $\Phi^{(i)}$, $i = 1, 2, 3$, we select the field $\Phi^{(1)}$ as the Higgs field $\Phi \equiv \Phi^{(1)}$, and consider only one gauge field $A_\mu^1 \equiv A_\mu$ as the vector potential of the KN EM field, setting $A_\mu^2 = A_\mu^3 = 0$.

The potential V of the supersymmetric chiral model is determined by the superpotential

$$V(r) = \sum_i |\partial_i W|^2, \quad \partial_i W \equiv \partial W / \partial \Phi^i, \quad (11)$$

which must provide the necessary concentration of the Higgs field inside the bag [9]. The corresponding superpotential was suggested by Morris in [22] in the form $W(\Phi^i, \bar{\Phi}^i) = Z(\Sigma \bar{\Sigma} - \eta^2) + (Z + \mu)\Phi \bar{\Phi}$, where μ and η are real constants, and $(\Phi, Z, \Sigma) \equiv (\Phi^1, \Phi^2, \Phi^3)$. The vacuum states are determined by the conditions $\partial_i W = 0$, which yield:

- (I) internal vacuum: $r < R - \delta$, $V(r) = 0$, $|\Phi| = \eta = \text{const.}$, $Z = -\mu$, $\Sigma = 0$, $W_{in} = \mu\eta^2$, and
- (II) external vacuum: $r > R$, $V(r) = 0$, $\Phi = 0$, $Z = 0$, $\Sigma = \eta$, $W_{ext} = 0$.

We select also the phase transition zone $R - \delta < r < R$,

- (III) where the vacua (I) and (II) are separated by a positive spike of the potential V , and the surface currents are concentrated.

The part of the Lagrangian which is related with the nonzero Higgs field $\Phi \equiv \Phi^{(1)}$ is just the same as the Lagrangian used by Nielsen–Olesen in the model of the vortex string in superconducting media [23]. Similarly, the KN source may be considered as a disk formed of a superconducting matter. This part of the Lagrangian leads to the equations, [4,9],

$$\partial_\nu \partial^\nu A_\mu = J_\mu = e|\Phi|^2(\chi_{,\mu} + eA_\mu) \quad (12)$$

which describe interaction between the complex Higgs field $\Phi(x) = |\Phi(x)|e^{i\chi(x)}$ and the KN vector potential A_μ penetrating inside the source. Regularization of the KN electromagnetic field inside the source occurs via compensation of the vector potential A_μ by gradient of the phase of the Higgs field $\chi_{,\mu}$, leading to the vanishing of the current J_μ inside the source and its localization on the border of DW, which is typical for superconductors. In agreement with (12), the surface currents compensate the difference between the internal and external values of the vector-potential. As was shown in [4,9], this mechanism determines phase of the Higgs field $\chi = n\phi + \omega t$, leading to two important features of the source:

- (A): the Higgs field oscillates with frequency $\omega = 2m$,
- (B): angular momentum is quantum, $J = n/2$, $n = 1, 2, 3$.

4. Bogomolnyi equations

Stress-energy tensor of the considered system consists of the pure em part $T_{\mu\nu}^{(em)}$ and contributions from the chiral fields $T_{\mu\nu}^{(ch)}$,

$$T_{\mu\nu}^{(tot)} = T_{\mu\nu}^{(em)} + \sum_i (\mathcal{D}_\mu^{(i)} \Phi^i) \overline{(\mathcal{D}_\nu^{(i)} \Phi^i)} - \frac{1}{2} g_{\mu\nu} [\sum_i (\mathcal{D}_\lambda^{(i)} \Phi^i) \overline{(\mathcal{D}^{(i)\lambda} \Phi^i)} + V]. \quad (13)$$

Flatness of the metric inside the bubble and in the vicinity of the domain wall boundary leads to vanishing of the cross-terms in the metric tensor, and using (11) we can simplify the chiral part of the Hamiltonian to the form

$$H^{(ch)} \equiv \mu = \frac{1}{2} \sum_{i=1}^3 \left[\sum_{\mu=0}^3 |\mathcal{D}_\mu^{(i)} \Phi^i|^2 + |\partial_i W|^2 \right], \quad (14)$$

which represents the mass density of the domain wall μ .

Absence of the cross-terms indicates that the flat interior of the bubble and its DW boundary are not rotating. However, the strong influence of gravity is saved in the ellipsoidal shape of the DW and in the strong drag effect acting of the KN electromagnetic field, which is to be aligned with twisted null direction k_μ of the Kerr congruence even in the limit of the flat space-time, see Fig. 2. Taking it into account, we use the Kerr coordinate system (3) which is adapted to the Kerr congruence (4) and to ellipsoidal shape of the bubble-source.

In previous treatments [4,9] we obtained that the Higgs field confined inside the source depends on angular coordinate and time

$$\Phi \equiv \Phi^1(x) = |\Phi^1(r)|e^{i\chi(t,\phi)}, \quad (15)$$

while the other chiral fields depend only on r . We separate in (14) the terms depending on ϕ and t ,

$$2H^{(ch)} = |\mathcal{D}_t^{(1)} \Phi^1|^2 + |\mathcal{D}_\phi^{(1)} \Phi^1|^2 + \sum_{i=1}^3 [|\mathcal{D}_r^{(i)} \Phi^i|^2 + |\partial_i W|^2], \quad (16)$$

and note that they are positive definite, and we obtain two Bogomolnyi equations which are valid for interior of the source,

$$\mathcal{D}_t^{(1)} \Phi^1 = 0, \quad \mathcal{D}_\phi^{(1)} \Phi^1 = 0. \quad (17)$$

One sees that for interior of the source, where $|\Phi^1|$ is constant and $J_\mu = 0$, (17) is analog (12),

$$\mathcal{D}_\mu^{(1)} \Phi^1 = i\Phi(\chi_{,\mu} + eA_\mu) = (i/e\Phi)J_\mu. \quad (18)$$

Reduction of the last term is cumbersome and was considered earlier for the planar chiral DW models in [11–16]. There are introduced some constant phase factors $e^{i\alpha_i}$, which allowed to rewrite last term in the form

$$H^{(ch-r)} = \sum_{i=1}^3 \frac{1}{2} |\mathcal{D}_r^{(i)} \Phi^i - e^{i\alpha_i} \partial \bar{W} / \partial \bar{\Phi}^i|^2 + \text{Re } e^{-i\alpha_i} (\partial \bar{W} / \partial \bar{\Phi}^i) \mathcal{D}_r^{(i)} \Phi^i. \quad (19)$$

These phases regarded as constants, which looked somewhat artificial. In the case of Kerr DW the phase α_1 turns out to be related with phase of the Higgs field and takes the physical meaning of the compensating field for A_ϕ and A_t . Therefore, we allow this phase to be functions of coordinates, except the coordinate r that parametrizes profile DW. From (19) we obtain Bogomolnyi equation

$$\mathcal{D}_r^{(i)} \Phi^i - e^{i\alpha_i} \partial \bar{W} / \partial \bar{\Phi}^i = 0, \quad (20)$$

in which the coordinate r parametrizes the oblate surface of the bag similar to the parallel surfaces of the planar domain wall, see Fig. 3.

The function W and Z are real, and without loss of generality, we can also set a real Φ^3 , which allows us to take $\alpha_2 = \alpha_3 = 0$. From (20) and (15) we obtain $\alpha_1 = 2\chi(t, \phi)$, leading to $\mathcal{D}_r^{(1)} \Phi = e^{2i\chi(t,\phi)} \mathcal{D}_r^{(1)} \bar{\Phi}$ and

$$H^{(ch-r)} = \sum_{i=1}^3 \frac{1}{2} |\partial_r \Phi^i - \partial W / \partial \Phi^i|^2 + \text{Re } (\partial W / \partial \Phi^i) \partial_r \Phi^i, \quad (21)$$

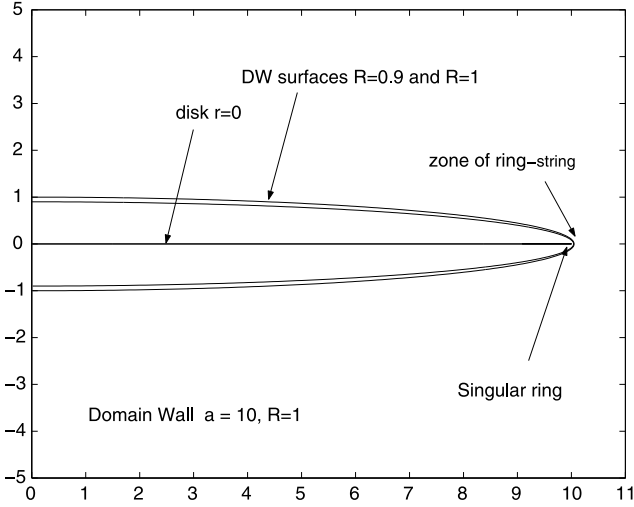


Fig. 3. Profile of the spheroidal domain wall phase transition.

where the replacement of the covariant derivatives $\mathcal{D}_r^{(1)}$ to partial ∂_r is valid due to concrete form of the used superpotential.

Minimum of the energy density $H^{(ch-r)}$ is achieved for

$$\mathcal{D}_r^{(i)} \Phi^i = \partial W / \partial \Phi^i \quad \mathcal{D}_r^{(i)} \bar{\Phi}^i = \partial \bar{W} / \partial \bar{\Phi}^i, \quad (22)$$

which corresponds to saturated Bogomolnyi bound. The expression (21) turns into full differential

$$H^{(ch-r)} = Re (\partial W / \partial \Phi^i) \partial_r \Phi^i = \partial W / \partial r. \quad (23)$$

The mass-energy of the supersymmetric interior of the bubble together with DW boundary is

$$M_{ch} = \int dx^3 \sqrt{-g} T_0^{0(ch)}, \quad (24)$$

where $\sqrt{-g} = (r^2 + a^2 \cos^2 \theta) \sin \theta$. Axial symmetry allows us to integrate over ϕ , leading to

$$M_{ch} = 2\pi \int dr d\theta (r^2 + a^2 \cos^2 \theta) \sin \theta T_0^{0(ch)}. \quad (25)$$

Using (23) we obtain

$$M_{ch} = 2\pi \int dr d\theta (r^2 + a^2 \cos^2 \theta) \sin \theta \partial_r W. \quad (26)$$

Taking into account that superpotential $W(r)$ is constant inside and outside the source,

$$W_{int} = \mu \eta^2, \quad W_{ext} = 0, \quad (27)$$

we have $\partial_r W = 0$ inside and outside the bag and, by crossing the bag boundary, we get the incursion $\Delta W = W(R + \delta) - W(R - \delta) = -\mu \eta^2$. After integration over $r \in [0, R]$, and then over $X = \cos \theta$ we obtain

$$M_{ch} = 2\pi \Delta W \int_{-1}^1 dX (R^2 + a^2 X^2) = 4\pi (R^2 + \frac{1}{3} a^2) \Delta W. \quad (28)$$

However, working in the very specific Kerr coordinate system, we note that coordinates extend analytically to negative r , enabling the continuation for $r < 0$ the vector potential and the chiral fields. Applying that to superpotential, we obtain on the negative sheet an anti-DW which gives just the same contribution to total mass, but with opposite sign. It means that on the negative sheet we have

an anti-DW, and the DW source of the KN solution may be considered as a DW–antiDW system similar to the known kink–antikink solutions, called also as ‘breathers’, see for example [24].

Integration (26) over $r \in [-R, R]$, shows that the mass-energy of the DW breather is zero, and the total mass of the KN source will only be determined by the surface currents and by contribution from the external KN EM field.

The KN breather regularizes gravitational field of the KN solution, and we have to specify the regularization of the EM field now. Taking into account the relation $d\phi_K = d\phi + a(r^2 + a^2)^{-1} dr$ we note that r - component of the vector-potential $er(r^2 + a^2)^{-1} dr$ is regular, and moreover, it is perfect differential and can be dropped. Singularity of the vector-potential is places in the equatorial plane $\cos \theta = 0$, and outside the source, it takes maximal value at the border of the disk, $r = r_e = e^2/2m$, $\cos \theta = 0$, where from (5) and (4) we obtain

$$A_\mu^{(max)} dx^\mu = \frac{e}{r_e} (dt + a \sin^2 \theta d\phi). \quad (29)$$

This border locates very close to the former Kerr singular ring, and the boundary $r_e = e^2/2m$ appears as the corresponding cut off parameter. This maximal value extends inside the source under the control of the Higgs field according to (12) which is equivalent to Bogomolnyi equations (17). As a result, the vector-potential takes inside the source gradient form, and the EM strength $F^{\mu\nu}$ vanishes together with its contribution to the total mass in (13). However, matching of the internal potential with external EM field turns out to be broken on the border of source outside the equatorial plane, and there appear the surface currents $J_\mu(r)$, penetrating inside the source according to (12) or (18). The corresponding contribution to the mass density is

$$\mu_J(r) = |\mathcal{D}_t^{(1)} \Phi^1|^2 + |\mathcal{D}_\phi^{(1)} \Phi^1|^2 = \frac{|J_t|^2 + |J_\phi|^2}{e^2 \Phi \bar{\Phi}}. \quad (30)$$

We note that although we deleted the negative sheet of metric and interior of the source is flat, and also all components of the energy density of the source are regularized, the vector-potential A_μ inside the source remains singular and two-valued because the Kerr coordinate system allows analytic extension to negative values of the coordinate r .

5. Conclusion

We considered supersymmetric source of the KN solution formed by the $N = 1$ DW-bubble model. We obtain that this source represents indeed the bubble–antibubble system, analog of the kink–antikink solution. The known for planar DW transformation to Bogomolnyi form is generalized to the curved DW-bubble of the Kerr geometry, where it acquires the space-time dependence demonstrating its efficiency at full capacity.

We noted that supersymmetry determines uniquely the shape of the source of the KN solution, and therefore, its stability. Any deviation of the domain wall boundary from the surface of “zero gravity potential”, determined by the equation (6), must break supersymmetry, adding gravitating terms to the supersymmetric vacuum states and breaking the Bogomolnyi bound for the domain wall phase transition. This mechanism works for any value of the gravitational constant, and therefore, the Einstein–Maxwell gravity controls the shape and stability of the source, despite the known view on the weakness of the gravitational interaction.

As is known, the Kerr solution describes gravitational field of a spinning particle, and in particular, since the gyromagnetic ratio of the KN solution is $g = 2$, as that of the Dirac electron, [17], it must correspond to the gravitational and em field of the electron, and the problem of the source of the KN solution may not be purely

academic, but also it can be related to the problem of the role gravity beyond Standard Model, shedding the light on the features of the spinning particles which are required for consistency with gravity. In particular, it was shown recently, [4,8], that the source of the KN solution reflects some features of the famous MIT and SLAC bag models, but the typically used there quartic potential of the Higgs field conflicts with the external KN gravity, and the consistency with gravity requires the supersymmetric scheme of phase transition. This aspect of the problem of source of KN solution is beyond the scope of this letter.

This research is supported by the RFBR grant No. 13-01-00602.

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