Theoretical Computer Science 86 (1991) 107-139
Elsevier

Unfold/fold transformation of stratified programs

Hirohisa Seki*

Central Research Laboratory, Mitsubishi Electric Corporation, 8-1-1 Tsukaguchi-Honmachi, Amagasaki, Hyogo 661, Japan

Abstract

Seki H., Unfold/fold transformation of stratified programs, Theoretical Computer Science 86 (1991) 107-139.

This paper describes some extensions of Tamaki-Sato's (1984) unfold/fold transformation of definite programs. We first propose unfold/fold rules also preserving the finite failure set (by SLD-resolution) of a definite program, which the original rules proposed by Tamaki and Sato do not. Then, we show that our unfold/fold rules can be extended to rules for stratified programs and prove that both the success set and the finite failure set (by SLDNF-resolution) of a stratified program are preserved. Preservation of equivalence of the perfect model semantics (Przymusinski (1988)) is also discussed.

1. Introduction

Program transformation provides a powerful methodology for program development, especially for derivation of an efficient program preserving the same meaning as that of an original and possibly inefficient program. Thus, one of the most important properties of program transformation is preservation of equivalence (Maher [10] investigated various formulations of equivalence for logic programs).

Tamaki and Sato proposed an elegant framework for unfold/fold transformation of logic programs [16]. Their transformation rules preserve the equivalence of a definite program in the sense of the least Herbrand model. Kawamura and Kanamori [7] recently proved that Tamaki-Sato's transformation also preserves the *success set* of a program, that is, a transformed program has the same computed answer substitution as that of the original program for any goal. Thus, the transformation rules by Tamaki and Sato seem to be sufficient, at least as far as positive information inferred from a program is concerned.

In general, however, their transformation does not always preserve the finite failure set (by SLD-resolution) of a definite program. The evaluation of a goal in

* This work was done while the author was at ICOT (Institute for New Generation Computer Technology).

a transformed program might not be terminating, even if the evaluation of that goal is finitely failed in the original program. Thus, when we are interested in negative information inferred from a program and Clark's negation as failure rule [3] is used, their transformation is not sufficient. Furthermore, when we consider an extension of their rules to a general logic program where the body of a clause may contain negative literals, the failure to preserve the finite failure of a program would lead to failure to preserve positive information inferred from the program.

In this paper, we propose unfold/fold rules which also preserve the finite failure set of a definite program. Then, we extend them to a stratified program and show that our transformation preserves both the success set and the finite failure set (by SLDNF-resolution) of a given stratified program. Preservation of equivalence of transformation in the perfect model semantics [12] is also discussed.

The organization of this paper is as follows. After summarizing preliminaries, Section 2 gives transformation rules which preserve the finite failure set of a definite program. Section 3 extends them to stratified programs. Section 4 discusses transformation rules which preserve the perfect model semantics. Finally, a summary of this work and a discussion of related work are given in Section 5.

Throughout this paper, we assume that the reader is familiar with the basic concept of logic programming, and the terminology follows that in [9]. As notation, variables are denoted by X, $Y, \ldots$, and atoms by A, $B, \ldots$. Multisets of atoms are denoted by L, K, $M, \ldots$, and θ, $\sigma, \ldots$ are used for substitutions.

2. Unfold/fold transformation

2.1. Preliminaries: Rules of transformation

This section describes Tamaki-Sato's unfold/fold transformation for definite programs [16]. The following descriptions of transformation rules are borrowed mainly from [15] and [7].

Definition 2.1.1 (*initial* (*definite*) *program*). An initial (definite) program P_0 is a definite program satisfying the following conditions:

(I1) P_0 is divided into two disjoint sets of clauses, P_{new} and P_{old}. The predicates defined in P_{new} are called *new predicates*, while those defined in P_{old} are called *old predicates*.

(I2) The new predicates appear neither in P_{old} nor in the bodies of the clauses in P_{new}.

Example 2.1.1. Let $P_0 = \{C_1, C_2, C_3\} \cup DB$, where

C_1: $reach(X, Y) \leftarrow arc(X, Y)$;

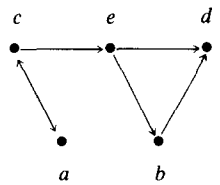
C_2: $reach(X, Y) \leftarrow arc(X, Z), reach(Z, Y)$;

C_3: $br(X, Y, N) \leftarrow reach(X, N), reach(Y, N)$;

and DB is a set of the following unit clauses defining predicate *arc*:

$arc(a, c)$. $arc(c, a)$. $arc(c, e)$.

$arc(e, b)$. $arc(e, d)$. $arc(b, d)$.



Predicate $reach(X, Y)$ is supposed to hold if there exists a path starting from node X and ending at node Y in a directed graph (shown above) whose relationship of arcs is given in DB. Predicate $br(X, Y, N)$ is supposed to hold if node N is reachable from both node X and node Y.

Let $P_{old} = \{C_1, C_2\} \cup DB$, $P_{new} = \{C_3\}$. Thus, "*br*" is a new predicate, while the other predicates are old predicates.

We call an atom A a *new atom* (an *old atom*) when the predicate of A is a new predicate (an old predicate), respectively.

Definition 2.1.2 (*unfolding*). Let P_i be a program and C a clause in P_i of the form: $H \leftarrow A, L$. Suppose that $C_1, \ldots, C_k$ are all the clauses in P_i such that C_j is of the form: $A_j \leftarrow K_j$ and A_j is unifiable with A, by an mgu, say θ_j, for each j $(1 \leqslant j \leqslant k)$.

Let C_j' $(1 \leqslant j \leqslant k)$ be the result of applying θ_j after replacing A in C with the body of C_j, namely, $C_j' = H\theta_j \leftarrow K_j\theta_j, L\theta_j$. Then, $P_{i+1} = (P_i - \{C\}) \cup \{C_1', \ldots, C_k'\}$. C is called the *unfolded clause* and $C_1, \ldots, C_k$ are called the *unfolding clauses.*

Example 2.1.2 (*continued from Example* 2.1.1). By unfolding C_3 at atom '$reach(X, N)$' in its body, program $P_1 = \{C_1, C_2, C_4, C_5\} \cup DB$ is obtained, where

C_4: $br(X, Y, N) \leftarrow arc(X, N), reach(Y, N)$.

C_5: $br(X, Y, N) \leftarrow arc(X, X_1), reach(X_1, N), reach(Y, N)$.

Definition 2.1.3 (*folding*). Let C be a clause in P_i of the form: $A \leftarrow K, L$ and D a clause in P_{new}[1] of the form: $B \leftarrow K'$. Suppose that there exists a substitution θ satisfying the following conditions:

(F1) $K'\theta = K$.

(F2) Let $X_1, \ldots, X_j, \ldots, X_m$ be internal variables of D, namely, appearing only in the body K' of D but not in B. Then, each $X_j\theta$ is a variable in C such that it appears in none of A, L and $B\theta$. Furthermore, $X_i\theta \neq X_j\theta$ if $i \neq j$.

(F3) D is the only clause in P_{new} whose head is unifiable with $B\theta$.

(F4) Either the predicate of A is an old predicate, or C is the result of applying unfolding at least once to a clause in P_0.

[1] Note that D is not necessarily in P_i.

Then, let C' be a clause of the form: $A \leftarrow B\theta$, L and let P_{i+1} be $(P_i - \{C\}) \cup \{C'\}$. C is called the *folded* clause and D is called the *folding* clause.

Example 2.1.3 (*continued from Example* 2.1.2) By folding the body of C_5 by C_3, program $P_2 = \{C_1, C_2, C_4, C_6\} \cup DB$ is obtained, where

$$C_6: \quad br(X, Y, N) \leftarrow arc(X, X_1), br(X_1, Y, N).$$

2.1.1. Previous results

Definition 2.1.4 (*transformation sequence*). Let P_0 be an initial program and P_{i+1} $(i \geqslant 0)$ a program obtained from P_i by applying either unfolding or folding. Then, the sequence of programs $P_0, P_1, \ldots, P_N$ is called a *transformation sequence starting from* P_0.

For the above unfold/fold transformation, Tamaki and Sato proved the following result [16].

Theorem 2.1.1 (Tamaki and Sato [16]). *The least Herbrand model M_{P_i} of any program P_i in a transformation sequence starting from initial program P_0, is identical to that of P_0.*

Recently, Kawamura and Kanamori [7] showed that Tanakai-Sato's transformation also preserves *answer substitutions* for any goal.

Definition 2.1.5 (*success set*). Let P be a (definite) program. The set of all the atom-substitution pairs (A, σ), such that there exists a successful SLD-derivation of $P \cup \{\leftarrow A\}$ with computed answer σ, is called the *success set* of P, and is denoted by $SS(P)$.

Theorem 2.1.2 (Kawamura and Kanamori [7]). *The success set $SS(P_i)$ of any program P_i in a transformation sequence starting from initial program P_0, is identical to that of P_0.*

Example 2.1.4 (*continued from Examples* 2.1.1 *and* 2.1.3). Since $br(a, c, e) \in M(P_0)$ holds, $br(a, c, e)$ is also in $M(P_2)$ from Theorem 2.1.1. More precisely, $(br(X, Y, N), \sigma = \{X/a, Y/c, N/e\})$ is in $SS(P_0)$, thus, that pair is also in $SS(P_2)$, from Theorem 2.1.2.

2.2. Modified folding rule and preservation of FF

2.2.1. Modified folding rule

This paper also considers the finite failure set (by SLD-resolution) of a program.

Definition 2.2.1 (*finite failure* (*FF*) *set*). Let P be a (definite) program. The set of all atoms A such that there exists a finitely failed SLD-tree for $P \cup \{\leftarrow A\}$, is called the (*SLD*) *finite failure set* of P, and is denoted by $FF(P)$.

The *partial correctness* of the transformation *w.r.t. FF* is easily shown.

Proposition 2.2.1 (partial correctness w.r.t. *FF*). *Let* $P_0, \ldots, P_N$ *be a transformation sequence. Then,* $FF(P_N) \subseteq FF(P_0)$ *for all* $N \geq 0$.

Proof. Let G be a definite goal, and suppose that $P_N \cup G$ has a finitely failed SLD-tree. From the soundness of SLD-resolution [3], $comp(P_N) \vdash G$. It is easy to see that $comp(P_0) \vdash comp(P_N)$ holds[2]. Thus, G is also a logical consequence of $comp(P_0)$. Then, from the completeness of SLD-resolution [5], $P_0 \cup G$ has a finitely failed SLD-tree. □

Tamaki-Sato's unfold/fold transformation, however, does *not* preserve the *total correctness* w.r.t. *FF*. That is, $FF(P_0) \subseteq FF(P_i)$ for all i $(N \geq i > 0)$ does not hold in general.

Example 2.2.1 (*continued from Example* 2.1.1, 2.1.3). The failure set of the original program P_0 is *not* preserved. For example, $br(a, b, e) \in FF(P_0)$, while $br(a, b, e)$ is not contained in $FF(P_2)$. In fact, any SLD-tree for $P_2 \cup \{\leftarrow br(a, b, e)\}$ has an infinite branch. Thus, $FF(P_0) \not\subseteq FF(P_2)$.

We now give a modified transformation rule which also preserves the total correctness w.r.t. *FF*. In order to specify such a rule, we need several definitions.

Definition 2.2.2 (*inherited atom*). Let $P_0, \ldots, P_N$ be a transformation sequence starting from P_0, and C a clause in P_i $(N \geq i \geq 0)$ whose head is a new atom. Then, an atom in the body of C is called an atom *inherited from* P_0 if one of the following conditions is satisfied:

(i) C is a clause in P_{new}. Then, each atom in the body of C is *inherited from* P_0.

(ii) Let C be the result of unfolding in P_i. Suppose that C_+ in P_{i-1} is the unfolded clause of the form: $A \leftarrow B, B_1, \ldots, B_n$, and that C_- in P_{i-1} is one of the unfolding clauses of the form: $B' \leftarrow K$. Thus, C is of the form: $A\theta \leftarrow K\theta, B_1\theta, \ldots, B_n\theta$, where θ is an mgu of B and B'. Then, each atom $B_j\theta$ $(1 \leq j \leq n)$ in C is *inherited from* P_0 if B_j in C_+ is inherited from P_0.

(iii) Let C be the result of folding in P_i. Suppose that C_+ in P_{i-1} is the folded clause of the form: $A \leftarrow K, B_1, \ldots, B_n$, and that D in P_{new} is the folding clause of the form: $B \leftarrow K'$. Thus, C is of the form: $A \leftarrow B\theta, B_1, \ldots, B_n$, where θ is an mgu such that $K'\theta = K$. Then, each atom B_j $(1 \leq j \leq n)$ in C is *inherited from* P_0 if B_j in C_+ is inherited from P_0.

[2] Note that the converse does not hold in general, that is, $comp(P_N) \not\vdash comp(P_0)$.

Intuitively, an inherited atom is (a possibly instantiated version of) an atom such that it was in the body of some clause in P_{new} and no unfolding has been applied to it.

Example 2.2.2. In Example 2.1.1, both "$reach(X, N)$" and "$reach(Y, N)$" in the body of C_3 are inherited atoms. In the body of clause C_5 (Example 2.1.2), atom "$reach(Y, N)$" is inherited from P_0, while neither "$arc(X, X_1)$" nor "$reach(X_1, N)$" is inherited from P_0.

Now, we can define a *modified* folding rule.

Definition 2.2.3 (*modified folding*). Let C and D be defined similarly in Definition 2.1.3, namely, C is a clause in P_i of the form: $A \leftarrow K, L$ and D is a clause in P_{new} of the form: $B \leftarrow K'$. Suppose that there exists a substitution θ satisfying the following conditions:

- (F1), (F2) and (F3) are the same as those defined in Definition 2.1.3.
- **(F4')** Either the predicate of A is an old predicate, or there is no atom in K which is inherited from P_0.

Example 2.2.3 (*continued from Example* 2.1.2). Consider clause C_5 in Example 2.1.2. As noted in Example 2.2.2, atom "$reach(Y, N)$" in its body is inherited from P_0, thus the modified folding does not allow it to be folded by C_3. Instead, by unfolding C_5 at atom "$reach(Y, N)$" in its body, program $P_2^m = \{C_1, C_2, C_4, C_7\} \cup DB$ is obtained, where

$$C_7: \quad br(X, Y, N) \leftarrow arc(X, X_1), arc(Y, Y_1), reach(X_1, N), reach(Y_1, N).$$

Now, atom "$reach(Y_1, N)$" in the body of C_7 is not inherited from P_0, so that the modified folding is now applicable to C_7. That is, by folding the body of C_7 by C_3, program $P_3^m = \{C_1, C_2, C_4, C_8\} \cup DB$ is obtained, where

$$C_8: \quad br(X, Y, N) \leftarrow arc(X, X_1), arc(Y, Y_1), br(X_1, Y_1, N).$$

Hereafter, except in Section 4, by *folding* we mean the *modified* folding defined in Definition 2.2.3, and by a *transformation sequence*, we mean the one obtained by applying either unfolding or modified folding.

2.2.2. Preservation of FF for definite clauses

In this subsection, we show that the unfold/fold transformation (using modified folding) guarantees the total correctness w.r.t. *FF* for definite programs. We need one more definition and a lemma.

Definition 2.2.4 (P_{new}-*expansion*). Let A be an atom. Suppose that $\bar{A}$ is defined as follows:

- When A is an old atom, $\bar{A}$ is A itself.
- When A is a new atom, $\bar{A}$ is either A, or a sequence of atoms "$B_1\theta, \ldots, B_n\theta$" such that there exists a (variant of a) clause in P_{new} of the form: $A_0 \leftarrow B_1, \ldots, B_n$ and θ is an mgu of A and A_0.

Then, $\bar{A}$ is called a P_{new}-expansion of A.

Similarly, let L be a sequence of atoms of the form: $A_1, \ldots, A_k$. Then a sequence of atoms $\overline{A_1}, \ldots, \overline{A_k}$ is called a *P_{new}-expansion of L*, and is denoted by $\bar{L}$.

Example 2.2.4 (*continued from Example* 2.1.1). Since "*reach*(X, Y)" is an old atom, a P_{new}-expansion of "*reach*(X, Y)" is itself. On the other hand, a P_{new}-expansion of $br(a, Y, N)$ is either itself, or a sequence of atoms "*reach*(a, N), *reach*(Y, N)".

Lemma 2.2.1 (P_0-simulation of SLD-derivation in P_N). *Let $P_0, \ldots, P_N$ be a transformation sequence. Let G be a goal, and suppose that there exists an SLD-derivation Dr of $P_N \cup \{G\}$, $G_0 = G, \ldots, G_k, \ldots$ using input clauses in P_N and substitutions $\theta_1, \ldots, \theta_k, \ldots$. Then, there exists an SLD-derivation Dr_0 of $P_0 \cup \{G\}$, $F_0 = G, \ldots, F_l, \ldots$ using input clauses in P_0 and substitutions $\sigma_1, \ldots, \sigma_l, \ldots$, satisfying the following conditions:*

(i) *For each k $(k \geq 0)$, there exists some $l(\geq 0)$ such that $F_l\sigma_1 \ldots \sigma_l$ is an P_{new}-expansion of $G_k\theta_1 \ldots \theta_k$, and*

(ii) *the restriction of $\sigma_1 \ldots \sigma_l$ to the variables in G is the same as that of $\theta_1 \ldots \theta_k$.*

(iii) (*fairness*) *Furthermore, if the SLD-derivation $G_0 = G, \ldots, G_k, \ldots$ is fair, then so is the SLD-derivation $F_0 = G, \ldots, F_l, \ldots$.*

Dr_0 is called a *P_0-simulation of Dr*.

The proof is shown in Appendix A.3, where a stronger version of the lemma is proved.

Example 2.2.5. Consider an SLD-derivation Dr_3 of $P_3^m \cup \{G_0 = \leftarrow br(a, b, e)\}$, where P_3^m was given in Example 2.2.3. See the right-hand side in Fig. 1. Dr_3 has a P_0-simulation $F_0 = G_0$, F_1, $F_2, \ldots, F_6$, which is shown in the left-hand side in the

F_0: $\leftarrow br(a, b, e)$
| C_3
F_1: $\leftarrow \underline{reach(a, e)}, reach(b, e)$
| C_2
F_2: $\leftarrow arc(a, X_1), reach(X_1, e), \underline{reach(b, e)}$
| C_2
F_3: $\leftarrow arc(a, X_1), reach(X_1, e), \underline{arc(b, Y_1)}, reach(Y_1, e)$
|
F_4: $\leftarrow arc(a, X_1), \underline{reach(X_1, e)}, reach(d, e)$
| C_2
F_5: $\leftarrow arc(a, X_1), arc(X_1, X_2), reach(X_2, e), \underline{reach(d, e)}$
| C_2
F_6: $\leftarrow arc(a, X_1), arc(X_1, X_2), reach(X_2, e), \underline{arc(d, Y_2)}, reach(Y_2, e)$
|
fail

G_0: $\leftarrow br(a, b, e)$
| C_8
G_1: $\leftarrow arc(a, X_1), \underline{arc(b, Y_1)}, br(X_1, Y_1, e)$
|
G_2: $\leftarrow arc(a, X_1), \underline{br(X_1, d, e)}$
| C_8
G_3: $\leftarrow arc(a, X_1), arc(X_1, X_2), \underline{arc(d, Y_2)}, br(X_2, Y_2, e)$
|
fail

Fig. 1. P_0-simulation (left) of an SLD-derivation of $P_3^m \cup \{\leftarrow br(a, b, e)\}$ (right).

figure (underlined atoms mean selected atoms). Note that F_3 (resp. F_4, F_6) is a P_{new}-expansion of G_1 (resp. G_2, G_3).

We can now show the total correctness w.r.t. *FF* for definite programs.

Proposition 2.2.2 (Total correctness w.r.t. FF). *Let $P_0, \ldots, P_N$ be a transformation sequence. Then, $FF(P_0) \subseteq FF(P_N)$ for all $N \geq 0$.*

Proof. For simiplicity of explanation, we assume here that G is a ground atom (a more general case is shown in Proposition 3.3.1). Suppose that an SLD-tree of $P_0 \cup \{\leftarrow G\}$ is finitely failed. Suppose further that $P_N \cup \{\leftarrow G\}$ has a *fair* SLD-tree which is not finitely failed. Obviously, no SLD-derivation $P_N \cup \{\leftarrow G\}$ ever succeeds; otherwise, a P_0-simulation of such a derivation would also succeed, which is a contradiction. Let *BR* be any nonfailed infinite branch in the fair SLD-tree for $P_N \cup \{\leftarrow G\}$. From Lemma 2.2.1, there exists a fair SLD-derivation Dr_0 of $P_0 \cup \{\leftarrow G\}$ which is a P_0-simulation of *BR*. Thus, Dr_0 is a nonfailed fair infinite derivation. From the result in [8], G is in the SLD finite failure set of P_0 iff every fair SLD-tree for $P_0 \cup \{\leftarrow G\}$ is finitely failed. Thus, Dr_0 should be finitely failed, which is a contradiction. □

3. Unfold/fold transformation of stratified programs

3.1. Preliminaries

We now consider an extension of the unfold/fold transformation from definite programs to stratified programs.

Definition 3.1.1 (*stratified program*). A general logic program P is *stratified* if its predicates can be partitioned into levels so that, in every program clause, $p \leftarrow L_1, \ldots, L_n$, the level of every predicate in a positive literal is less than or equal to the level of p and the level of every predicate in a negative literal is less than the level of p [1].

Throughout this paper, we assume that the levels of a stratified program are $1, \ldots, r$ for some integer r, where r is the *minimum* number satisfying the above definition. In this case, P is said to have the maximum level r and is denoted $P = \mathscr{P}^1 + \cdots + \mathscr{P}^r$, where $\mathscr{P}^i$ is a set of clauses whose head predicates have level i. Note that $\mathscr{P}^1$ is a set of definite clauses. When L is a literal whose predicate has level i, we denote it $level(L) = i$. Furthermore, the *stratum* [12] of a goal is defined as follows. For any positive atom A, let $stratum(A) = level(A)$ and $stratum(\neg A) = stratum(A) + 1$. Suppose that G is a goal of the form: $\leftarrow L_1, \ldots, L_n$, where $n \geq 0$ and L_i's are literals. Then, $stratum(G)$ is 0 if G is empty, and $\max\{stratum(L_i): 1 \leq i \leq n\}$, otherwise.

As in the previous section, we need to define an initial program, unfolding/folding and a transformation sequence for stratified programs. Although they are almost the same as the previous ones, we impose further restrictions on an initial stratified program.

Definition 3.1.2 (*initial (stratified) program*). An initial (stratified) program P_0 is a *stratified* program statisfying the following conditions:
- (I1) and (I2) are the same as those defined in Definition 2.1.1, and
- **(I3)** The definition of each new predicate consists of exactly one clause.

Condition (I3) above guarantees that a stratified program is also stratified after the unfold/fold transformation as shown below (Proposition 3.1.1), and most cases found in the literature seem to satisfy this condition.

Unfolding, (modified) folding and a transformation sequence are the same as those defined in Definition 2.1.2, Definition 2.2.3 and Definition 2.1.4, respectively. First, we have to confirm that our unfold/fold transformation preserves a stratification of an initial program.

Proposition 3.1.1 (preservation of stratification). *Let $P_0, \ldots, P_N$ be a transformation sequence. Then, if P_0 is a stratified program, so is P_i $(N \geq i \geq 0)$.*

Proof. Let p be a new predicate, and let $C \in P_{new}$ be its definition of the form: $p \leftarrow L$. Then, we define the level of p by $\mathrm{level}(p) = \max\{level(B_j) \mid B_j \in L\}$. Then, the proposition is obvious from the definitions of unfolding and folding. □

Example 3.1.1. Consider the following program:

C_9: $path(X, [X]) \leftarrow node(X)$;

C_{10}: $path(X, [X \mid L]) \leftarrow arc(X, Y), path(Y, L)$;

C_{11}: $good\_list([\,])$;

C_{12}: $good\_list([X \mid L]) \leftarrow \sim bad(X), good\_list(L)$;

C_{13}: $good\_path(X, L) \leftarrow path(X, L), good\_list(L)$;

where predicates *node* and *arc* are supposed to be defined by a set of unit clauses, and the definition of predicate *bad* is not material, but its level is assumed to be less than

$$\max\{level(path), level(good\_list)\}.$$

Suppose that a graph is given whose relationship of nodes and arcs is specified by the predicates *node* and *arc*, respectively. Then, predicate $good\_path(X, L)$ can be thought of as finding a path L such that it starts from node X and each node of L is a "good" (or not "bad") one.

Let P_{aux} be a set of definitions of predicates *node*, *arc* and *bad*, and let P_0^{str} be $\{C_9, C_{10}, C_{11}, C_{12}, C_{13}\} \cup P_{aux}$. Moreover, let $P_{old} = \{C_9, C_{10}, C_{11}\} \cup P_{aux}$, and $P_{new} = \{C_{13}\}$. Then, P_0^{str} satisfies the conditions of an initial stratified program.

By unfolding C_{13} at atom "$path(X, L)$" in its body, the following clauses $\{C_{14}, C_{15}\}$ are obtained, where

$$C_{14}: \quad good\_path(X, [X]) \leftarrow node(X), good\_list([X]).$$

$$C_{15}: \quad good\_path(X, [X \mid L]) \leftarrow arc(X, Y), path(Y, L), good\_list([X \mid L]).$$

Both C_{14} and C_{15} can be further unfolded, and we have:

$$C_{16}: \quad good\_path(X, [X]) \leftarrow node(X), \sim bad(X).$$
$$C_{17}: \quad good\_path(X, [X \mid L]) \leftarrow arc(X, Y), path(Y, L), \sim bad(X), good\_list(L).$$

By folding the body of C_{17} by C_{13}, program $P_3^{str} = P_0^{str} - \{C_{13}\} \cup \{C_{16}, C_{18}\}$ is obtained, where

$$C_{18}: \quad good\_path(X, [X \mid L]) \leftarrow arc(X, Y), \sim bad(X), good\_path(Y, L).$$

3.2. Partial correctness of transformation

The success set (*SS*) and the finite failure (*FF*) set of a stratified program are defined similarly to those of a definite program. That is, *SS* (*FF*) of a stratified program is defined by replacing "SLD-derivation (SLD-tree)" in Definition 2.1.5 (Definition 2.2.1) with "SLDNF-derivation (SLDNF-tree)" [9], respectively.

In this subsection, we show the partial correctness of our transformation w.r.t. both *SS* and *FF*.

Proposition 3.2.1 (partial correctness w.r.t. *SS* and *FF*). *Let $P_0, \ldots, P_N$ be a transformation sequence. Then, for $i = 0, \ldots, N-1$,*

(SS): *if $SS(P_i) = SS(P_0)$, then $SS(P_{i+1}) \subseteq SS(P_i)$.*

(FF): *if $FF(P_i) = FF(P_0)$, then $FF(P_{i+1}) \subseteq FF(P_i)$.*

The proof of the above proposition is shown in Appendix A.2.

3.3. Total correctness of transformation

3.3.1. Total correctness w.r.t. FF

We now show the total correctness of our unfold/fold transformation. We prove the total correctness w.r.t. *FF* first. As in the case for definite programs, we show Lemma 2.2.1 for stratified programs, replacing "SLD-derivation" in it with "SLDNF-derivation". That is,

Lemma 3.3.1 (P_0-simulation of SLDNF-derivation in P_N). *Let $P_0, \ldots, P_N$ be a transformation sequence. Let G be a goal, and suppose that there exists an SLDNF-derivation Dr of $P_N \cup \{G\}$, $G_0 = G, \ldots, G_k, \ldots$ using input clauses in P_N and substitutions $\theta_1, \ldots, \theta_k, \ldots$. Then, there exists an SLDNF-derivation Dr_0 of $P_0 \cup \{G\}$, $F_0 = G, \ldots, F_l, \ldots$ using input clauses in P_0 and substitutions $\sigma_1, \ldots, \sigma_l, \ldots$, satisfying the following conditions:*

(i) *For each k $(k \geq 0)$, there exists some $l(\geq 0)$ such that $F_l\sigma_1 \ldots \sigma_l$ is a P_{new}-expansion of $G_k\theta_1 \ldots \theta_k$, and*

(ii) *the restriction of $\sigma_1 \ldots \sigma_l$ to the variables in G is the same as that of $\theta_1 \ldots \theta_k$.*

(iii) *(fairness) Furthermore, if the SLDNF-derivation $G_0 = G, \ldots, G_k, \ldots$ is fair, then so is the SLDNF-derivation $F_0 = G, \ldots, F_l, \ldots$.*

Dr_0 is called a P_0-simulation of Dr.

The proof is given in Appendix A.3. Now we can show the total correctness w.r.t. *FF*.

Proposition 3.3.1 (total correctness w.r.t. *FF*). *Let $P_0, \ldots, P_N$ be a transformation sequence, where P_0 is an initial stratified program. Then, for all $N \geq 0$, $FF(P_0) \subseteq FF(P_N)$.*

Proof. Suppose that an SLDNF-tree of $P_0 \cup \{\leftarrow A\}$ is finitely failed. Obviously, no SLDNF-derivation $P_0 \cup \{\leftarrow A\}$ ever succeeds. Furthermore, it does not flounder, from the proposition shown by Shepherdson [14], which says that, if a query Q flounders under a computation rule, then it cannot fail under *any* computation rule.

Suppose that $P_N \cup \{\leftarrow A\}$ has a *fair* SLDNF-tree which is *not* finitely failed. Let BR_N be any nonfailed branch in that fair SLDNF-tree for $P_N \cup \{\leftarrow A\}$.

From Lemma 3.3.1, there exists a *fair* SLDNF-derivation BR_0 for $P_0 \cup \{\leftarrow A\}$ which is a P_0-simulation of BR_N. BR_0 neither succeeds nor flounders as noted above. Thus, BR_0 is a nonfailed fair infinite derivation. Then, we can show that $comp(P_0) \cup \{\exists A\}$ has a model, using similar methods in the proofs of completeness of negagtion as failure rule by [9, 2], which is a contradiction. □

3.3.2. Total correctness w.r.t. SS

Finally, we state the total correctness w.r.t. *SS*, whose proof is given in Appendix A.4.

Proposition 3.3.2 (total correctness w.r.t. *SS*). *Let $P_0, \ldots, P_N$ be a transformation sequence, where P_0 is an initial stratified program. Then, $SS(P_0) \subseteq SS(P_N)$ for all $N \geq 0$.*

4. On preservation of perfect model semantics

The semantics we have considered is somewhat operational, in that the success set and the finite failure set of a stratified program are given by specific procedures

such as SLD(NF)-resolution. In this section, we consider more declarative semantics, that is, the standard (minimal Herbrand) model M_P [1, 17], or, more generally, the *perfect model semantics* for stratified programs introduced by Przymusinski [12].

It seems to be a more direct extension from Tamaki-Sato's original unfold/fold rules to consider transformation rules preserving the equivalence of M_P or the perfect model semantics, since their framework preserves the least Herbrand model for a definite program. Recall that, Tamaki-Sato's unfold/fold transformation does not preserve the finite failure set. However, from the viewpoint of the perfect model semantics, it poses no problems, since a goal: "$\leftarrow G$" which has neither a successful SLD-derivation nor a finite failed SLD-tree is simply considered to be false. We assume familiarity with the perfect model semantics (see [12]).

Definitions of an initial program, unfolding rule and folding rule are the same as those in Definition 3.1.2, Definition 2.1.2 and Definition 2.1.3, respectively. Note that we do not have to consider the modified folding rule. A transformation sequence is also defined similarly to Definition 2.1.4. Then, we have the following proposition.

Proposition 4.1 (preservation of perfect model semantics). *The perfect model semantics of any program P_i in a transformation sequence starting from initial program P_0, is identical to that of P_0.*

The proof is given in Appendix A.5.

5. Conclusion

There have been several studies on equivalence-preserving transformation of logic programs. Tamaki and Sato's result [16] and its elaboration by Kawamura and Kanamori [7] are already described in Section 2.1.1. Maher extensively studied various formulations of equivalence for definite programs [10]. In that paper, he considered a transformation system similar to that of Tamaki and Sato, and stated that his unfold/fold rules preserve logical equivalence of completions, while, as stated in Section 2.2.1, those of Tamaki-Sato do not preserve it. Kanamori and Horiuchi [6] proposed a framework for transformation and synthesis based on generalized unfold/fold rules. Their system was shown to preserve the minimum Herbrand model semantics, but the finite failure set is not preserved in general. In a very recent paper, Gardner and Shepherdson [4] proposed a framework for unfold/fold transformation of normal programs, where negative literals are allowed in the bodies of clauses, and they showed that their transformation preserves procedural equivalence based on SLDNF-resolution. Their work, however, is not comparable with our version, nor with that of [16] and [7]; their folding rule [4] specifies that, when a program P_{i+1} is obtained from P_i by folding $C \in P_i$ by D, D should be in P_i, while, in our framework like [16] and [7], D is not necessarily in P_i.

Compared with previous work, the contributions of this paper will be summarized as follows:

(1) The modified folding rule for a definite program was proposed. The unfolding rule together with the modified folding rule was shown to preserve the finite failure set (by SLD-resolution) of a program as well as the success set. This guarantees a safer use of Tamaki-Sato's transformation when negation as failure rule is used.

(2) The unfold/fold rules for stratified programs were proposed. The modified folding rule has made it possible to extend the applicability of unfold/fold transformation rules to a stratified program, so that they preserve both the success set and the finite failure set of a stratified program by SLDNF-resolution.

(3) Preservation of equivalence of the perfect model semantics was discussed. We showed that unfold/fold rules by Tamaki and Sato can be extended to rules for a stratified program and preserve the equivalence of the perfect model semantics.

Appendix A.

A.1. Preliminaries

In the following proofs, for the ease of understanding and simplicity, we sometimes use such a representation that unifiers in SLD(NF)-resolution appear only implicitly and instead, we write the equations corresponding to the unifiers explicitly. For example, let $G_0 = \leftarrow B_1, \ldots, B_k, \ldots, B_n$ be a goal in an SLD(NF)-resolution, where B_k is the selected (positive) atom and C is an input clause $H \leftarrow \Gamma$. Then, the derived goal G_1 from G_0 and C is written:

$$\leftarrow B_1, \ldots, B_{k-1}, \Gamma, B_{k+1}, \ldots, B_n, B_k = H.$$

Namely, an mgu θ of B_k and H is not applied to G_1, but the equation $B_k = H$ corresponding to θ is added at the end of the goal. This formulation of SLD-resolution was proposed and studied by [18]. Since properties of this formulation play a crucial role in our proofs, we cite here the relationship between a usual SLD-derivation and the above formulation [18].

Consider an SLD-derivation Dr. Let $(A_0, A_1\theta_0, \ldots, A_n\theta_0 \circ \cdots \circ \theta_{n-1})$ be the list of selected atoms in the goals of Dr, written in the order in which they have been selected and let $(H_0, H_1, \ldots, H_n)$ be the list of corresponding heads of the input clauses used in the derivation and $(\theta_0, \theta_1, \ldots, \theta_n)$ the list of the mgu's such that

$$A_0\theta_0 \equiv H_0\theta_0,$$

$$A_n\theta_0 \circ \theta_1 \circ \cdots \circ \theta_{n-1} \circ \theta_n \equiv H_n\theta_n.$$

We assume the process of standardizing the variables in the input clauses apart as usual. Then, $\theta_0, \ldots, \theta_{n-1}$ do not affect H_n, so that $H_n\theta_n \equiv H_n\theta_0 \circ \theta_1 \circ \cdots \circ \theta_n$. The sequence of identities built by the SLD-derivation can therefore be rewritten as

$$A_0\theta_0 \equiv H_0\theta_0$$

$$A_n\theta_0 \circ \theta_1 \circ \cdots \circ \theta_n \equiv H_n\theta_0 \circ \theta_1 \circ \cdots \circ \theta_n.$$

It then shows that the SLD-derivation attempts to compute an mgu $\theta = \theta_0 \circ \theta_1 \circ \cdots \circ \theta_n$ (if it exists) which is a solution to the set of equations:

$$\mathcal{S} = \{A_0 = H_0, A_1 = H_1, \ldots, A_n = H_n\}.$$

On the other hand, when we consider a variant Dr' of Dr where each mgu is not applied to a goal but an equation corresponding to the mgu appears explicitly, we have exactly the same set of equations $\mathcal{S}$ in the last goal of Dr'.

Due to the unification theorem [13, 11], $\mathcal{S}$ gives the same mgu as θ modulo renaming if and only if it exists. Moreover, the order in which the substitutions are computed is immaterial. It is easy to see that this discussion can be extended to the case of SLDNF-derivation.

Based on this observation, we sometimes utilize the following notation. Let Γ be a sequence of literals, $\mathcal{S}$ a set of equations such that it gives an mgu θ. Then, an expression F of the form $\Gamma\theta$ is denoted also by "Γ, $\mathcal{S}$". We call Γ the *literal part* of F, while $\mathcal{S}$ is called the *equation part* of F. As an example of this formulation, we prove the following lemma.

Lemma A.1.1. *Let C_+ (resp. C_-) be a clause in a program P of the form: $H \leftarrow B_+$, L (resp. $B_- \leftarrow K$) such that B_+ is unifiable with B_- by an mgu θ, and C_+ shares no variables with C_-. Let G be a goal $\leftarrow A, \Delta$, where A is an atom unifiable with H, and Δ is a (possibly empty) sequence of literals, and variables in G appear neither in C_+ nor in C_-. Consider an SLDNF-derivation of $P \cup \{G\}$ consisting of goals $G_0 = G$, G_1, G_2, where G_1 (resp. G_2) is derived from G_0 (resp. G_1) and C_+ (resp. C_-), selecting A (resp. possibly an instantiated version of B_+).*

On the other hand, let C be the result of applying unfolding to C_+ at B_+ by C_-, i.e., C is the clause of the form: $H\theta \leftarrow K\theta$, $L\theta$. Consider a resolvent G_1' of G and C, selecting A. Then G_2 is equivalent to G_1' modulo variable renaming.

Proof. Using the above-mentioned notation, G_1 and G_2 can be written as follows:

$$G_1: \quad \leftarrow B_+, L, \Delta, H = A$$

$$G_2: \quad \leftarrow K, L, \Delta, B_+ = B_-, H = A.$$

On the other hand, G_1' is of the form

$$G_1': \quad \leftarrow K\theta, L\theta, \Delta, H\theta = A.$$

Since the equation $B_+ = B_-$ gives the substitution θ and variables among A and Δ are not affected by θ, G_2 can be rewritten as $\leftarrow K\theta, L\theta, \Delta, H\theta = A$, which is equivalent to G_1'. □

We prove one more technical lemma.

Lemma A.1.2. *Let C be a clause of the form: $H \leftarrow J$, K and D a clause of the form: $B \leftarrow J_0$ such that $J_0\theta = J$ for some substitution θ, and C, D and θ satisfy the conditions*

of folding (F1)-(F4) *in Definition* 2.1.3. *Let D' be a variant of D of the form: $B' \leftarrow J_0'$ such that variables in D' appear neither in C nor in D. Then, J is a variant of $J_0'\tau$, where τ is an mgu of B' and $B\theta$ such that $B'\tau = B\theta$. Moreover, J is different from $J_0'\tau$ only with respect to those variables in C which occur only in J but neither in H nor in K.*

Proof. Let x_i (resp. x_i') be those internal variables which occur only in J_0 (resp. J_0') but not in B (resp. B'), and let y_j (resp. y_j') be those variables which occur in B (resp. B') ($i, j \geq 0$). We thus denote J_0 (resp. J_0') by $J_0(x_i; y_j)$ (resp. $J_0(x_i'; y_j')$). From the conditions of folding, substitution θ can be written in the form: $\theta = \theta_{iv} \cup \theta_{hv}$, where $\theta_{iv} = \{x_i/z_i\}$ and $\theta_{hv} = \{y_j/t_j\}$ such that

(i) θ_{iv} is a renaming substitution and each variable z_j appears only in J but in none of H, K, and $y_j\theta$, and

(ii) t_j does not contain any z_i.

Therefore, τ is equivalent to $\theta_{hv}' = \{y_j'/t_j\}$. Thus, $J_0'\tau = J_0'\theta_{hv}' = J_0(x_i'; t_j)$. On the other hand, $J = J_0\theta = J_0(x_i; y_j)(\{x_i/z_i\} \cup \{y_j/t_j\}) = J_0(z_i; t_j)$. Comparing $J_0(x_i'; t_j)$ with $J_0(z_i; t_j)$, the lemma follows. □

A.2. Proofs of partial correctness

Instead of proving Proposition 3.2.1, we show a more general proposition. For this, we also generalize the definitions of the success set and the finite failure set of a given program as follows.

Definition A.2.1 (*success set*). Let P be a program and Γ a sequence of literals. The set of all pairs (Γ, σ) such that there exists a successful SLDNF-derivation of $P \cup \{\leftarrow \Gamma\}$ with computed answer substitution σ, is called the *success set* of P, and is denoted by $SS(P)$.

Definition A.2.2 (*finite failure* (*FF*) *set*). Let P be a program and Γ a sequence of literals. The set of all Γ such that there exists a finitely failed SLDNF-tree for $P \cup \{\leftarrow \Gamma\}$, is called the (*SLDNF*) *finite failure set* of P, and is denoted by $FF(P)$.

Moreover, we use the following notation convention. Let G be a goal of the form: $\leftarrow \Delta$, where Δ is a (possibly empty) sequence of literals. Then, when $(\Delta, \sigma) \in SS(P)$ (resp. $\Delta \in FF(P)$) holds, we denote it simply by $(G, \sigma) \in SS(P)$ (resp. $G \in FF(P)$).

Proposition A.2.1 (partial correctness w.r.t. *SS* and *FF*). *Let $P_0, \ldots, P_N$ be a transformation sequence. Then,*

(SS): *If $SS(P_i) = SS(P_0)$, then $SS(P_{i+1}) \subseteq SS(P_i)$ for $i = 0, \ldots, N-1$.*

(FF): *If $FF(P_i) = FF(P_0)$, then $FF(P_{i+1}) \subseteq FF(P_i)$ for $i = 0, \ldots, N-1$.*

Proof. The proof is by mutual induction on $s = stratum(G_0)$ of goal G_0. It is obvious when $s = 0$. Suppose that the proposition has been proved for all goals G_0' whose $stratum(G_0') \leq s$, where $s \geq 0$. We first prove **(SS)**.

Suppose there exists an SLDNF-refutation Dr_{i+1} of $P_{i+1} \cup \{G_0\}$ with the computed answer substitution σ, where *stratum*(G_0) is $s+1$. The proof is by induction on the length[3] of the SLDNF-refutation of $P_{i+1} \cup \{G_0\}$. Let $G_0 = \leftarrow A, \Delta$ where A is a literal and Δ is a (possibly empty) sequence of literals. Suppose further that A is the selected literal in G_0.

When $A = \sim A'$ is a negative atom, A should be ground and there exists a finitely failed SLDNF-tree for $P_{i+1} \cup \{\leftarrow A'\}$ and G_0 has the successor $G_1 = \leftarrow \Delta$. Since *stratum*$(\leftarrow A')$ is less than *stratum*(G_0), $P_i \cup \{\leftarrow A'\}$ has a finitely failed SLDNF-tree from the induction hypothesis on the partial correctness w.r.t. FF. Let Dr_i be an SLDNF-derivation of $P_i \cup \{G_0\}$. Then G_0 has the successor G_1 also in Dr_i. From the induction hypothesis of the length of an SLDNF-refutation, $(G_1, \sigma) \in SS(P_i)$, thus $(G_0, \sigma) \in SS(P_i)$.

Next, suppose that A is a positive atom. Let C be the input clause.

Case 1: C is inherited from P_i. Then, the proof is obvious from the induction hypothesis.

Case 2: C is the result of unfolding. Let $C_+ \in P_i$ be the unfolded clause of the form: $H \leftarrow B_+, L$ and $C_- \in P_i$ be the unfolding clause of the form: $B_- \leftarrow K$. Then, C can be written as $H\theta \leftarrow K\theta, L\theta$, where θ is an mgu of B_+ and B_-. Then, in the SLDNF-refutation Dr_{i+1} of $P_{i+1} \cup \{G_0\}$, G_0 has the successor G_1 of the form:

$$G_1: \quad \leftarrow K\theta, L\theta, \Delta, A = H\theta$$

and $(G_1, \sigma) \in SS(P_{i+1})$. On the other hand, consider an SLDNF-derivation Dr_i of $P_i \cup \{G_0\}$. Using C_+ as an input clause, G_0 has the successor G_1^i of the form:

$$G_1^i: \quad \leftarrow B_+, L, \Delta, A = H.$$

Again, using C_- as an input clause, G_1^i has the successor G_2^i of the form:

$$G_2^i: \quad \leftarrow K, L, \Delta, B_+ = B_-, A = H.$$

Since $(G_1, \sigma) \in SS(P_i)$ from the induction hypothesis and G_1 is equivalent (modulo renaming) to G_2^i from Lemma A.1.1, it is shown that $(G_0, \sigma) \in SS(P_i)$.

Case 3: C is the result of folding. Let $C_+ \in P_i$ be the folded clause of the form: $H \leftarrow J, K$ and $D \in P_{new}$ be the folding clause of the form: $B \leftarrow J_0$, where $J_0\theta = J$ for some substitution θ. Then C is $H \leftarrow B\theta, K$. In the SLDNF-refutation Dr_{i+1} of $P_{i+1} \cup \{G_0\}$, G_0 has the successor

$$G_1: \quad \leftarrow B\theta, K, \Delta, A = H.$$

Since $(G_1, \sigma) \in SS(P_i)$ from the induction hypothesis and $SS(P_i) = SS(P_0)$ from the assumption of the proposition, it follows that $(G_1, \sigma) \in SS(P_0)$. Let G_2 be the derived goal from G_1 and an input clause D' in P_0 with $B\theta$ as the selected atom, where D' is a variant of D, say, $D' = B' \leftarrow J_0'$ such that no variables in D' appear elsewhere. Then, G_2 is of the form:

$$G_2: \quad \leftarrow J_0', K, \Delta, A = H, B\theta = B'.$$

[3] The *length* of SLDNF-refutation is defined in a similar way to that of SLD-refutation (see [9]).

From the folding condition (F3), D' is the only clause in P_0 which is unifiable with $B\theta$, so (G_2, σ) is also in $SS(P_0)$. Again, from the assumption of the proposition, (G_2, σ). is also in $SS(P_i)$.

On the other hand, consider an SLDNF-derivation Dr_i of $P_i \cup \{G_0\}$. Using C_+ as an input clause, G_0 has the successor G_1^i of the form:

$$G_1^i: \quad \leftarrow J, K, \Delta, A = H.$$

From Lemma A.1.2, G_1^i is a variant of G_2. It follows that $(G_1, \sigma) \in SS(P_i)$, thus (G_0, σ) is also in $SS(P_i)$.

Proof of (*FF*): Let $G_0 = \leftarrow A, \Delta$ be a goal, and suppose that there exists a finitely failed SLDNF-tree Tr_{i+1} for $P_{i+1} \cup \{G_0\}$. We show that $P_i \cup \{G_0\}$ also has a finitely failed SLDNF-tree Tr_i. The proof is by induction on the size (the number of nodes) of Tr_{i+1}. Suppose that A is the selected atom in G_0 of Tr_{i+1}.

Induction Basis: Suppose that the size of Tr_{i+1} is 1. Then the following two cases are to be considered.

(i) A is a positive atom and there is no clause in P_{i+1} whose head is unifiable with A. When P_i has no clause whose head is unifiable with A, the proposition is obvious. Otherwise, since only unfolding might change the head of a clause during unfold/fold transformation, there exists only one clause C in P_i such that the head of C is unifiable with A and that, for the head H of each unfolded clause of C, H is not unifiable with A. In this case, it is straightforward to show that there exists a finitely failed SLDNF-tree for $P_i \cup \{\leftarrow A\}$.

(ii) A is a ground negative literal, say, $\sim A'$, and there exists a successful SLDNF-derivation of $P_{i+1} \cup \{\leftarrow A'\}$. From the partial correctness w.r.t. SS, $(A', \varepsilon) \in SS(P_i)$, where ε is an empty substitution. Thus $G_0 \in FF(P_i)$.

Induction Step: Suppose that the proposition has been proved for any goal whose finitely failed SLDNF-tree has the size less than t (≥ 1) and that the size of a finitely failed SLDNF-tree for $P_{i+1} \cup \{G_0\}$ is $t+1$.

(i) When A is a negative atom, say, $\sim A'$, A' should be ground and there exists a finitely failed SLDNF-tree for $P_{i+1} \cup \{\leftarrow A'\}$. In this case $P_{i+1} \cup \{\leftarrow \Delta\}$ also has a finitely failed SLDNF-tree. Then, the proposition holds from the induction hypothesis.

(ii) Suppose that A is a positive atom. In the SLDNF-tree Tr_i for $P_i \cup \{G_0\}$, let A be the selected atom in G_0. Let $G_{11}, \ldots, G_{1k}$ be the children of G_0 in Tr_i, and $C_1, \ldots, C_k$ the corresponding input clauses. We show that there exists a finitely failed SLDNF-tree for $P_i \cup \{G_{1j}\}$ for each j ($j = 1, \ldots, k$).

When C_j is inherited to P_{i+1}, the proposition is obvious. When folding is applied to some C_j, it is easy to see that the proposition holds from the similar discussion in the above proof of (SS) (case 3). Thus, we prove the proposition when unfolding is applied to some C_j.

Let C_j be of the form: $H \leftarrow B_+, L$ and suppose that unfolding is applied to B_+. Let $C_-^1, \ldots, C_-^n$ ($n > 0$) be all the clauses in P_i such that C_-^l ($1 \leq l \leq n$) is $B_-^l \leftarrow K_l$ and B_-^l is unifiable with B_+ by an mgu, say, θ_l. Then, the result of unfolding is

$P_{i+1} = (P_i - \{C_j\}) \cup \{C'_1, \ldots, C'_n\}$, where $C'_l = H\theta_l \leftarrow K_l\theta_l, L\theta_l$. Note that G_{1j} is denoted by

$$G_{1j}: \quad \leftarrow B_+, L, \Delta, H = A.$$

Consider an SLDNF-derivation of $P_i \cup \{G_{1j}\}$ with B_+ as the selected atom. Since $C^1_-, \ldots, C^n_-$ are all the clauses in P_i whose heads are unifiable with B_+, G_{1j} has the children $G_{21}, \ldots, G_{2n}$, where

$$G_{2l}: \quad \leftarrow K_l, L, \Delta, H = A, B_+ = B^l_-$$

assuming that any variable in C^l_- does not appear elsewhere. On the other hand, consider the finitely failed SLDNF-tree Tr_{i+1} for $P_{i+1} \cup \{G_0\}$. Recall that A is the selected atom. Each C'_l has two cases: either A is unifiable with the head $H\theta_l$ of C'_l or not. When it is not unifiable, the set of equations: $\{H = A, B_+ = B^l_-\}$ has no solution. Thus goal G_{2l} is finitely failed. Otherwise, G_0 in Tr_{i+1} has a child

$$G^{i+1}_{1l}: \quad \leftarrow K_l\theta_l, L\theta_l, \Delta, H\theta_l = A$$

which is finitely failed in P_{i+1}. From the induction hypothesis on the size of a finitely failed tree, G^{i+1}_{1l} has a finitely failed SLDNF-tree also in P_i. Since $B_+ = B^l_-$ gives the substitution θ_l, G^{i+1}_{1l} is equivalent to G_{2l}. Thus, G_{2l} has also a finitely failed SLDNF-tree in P_i. This completes the proof. □

A.3. Proof of Lemma 3.3.1.

In this subsection, we give a proof of Lemma 3.3.1. For this, we first prove the following lemma, which says that, for a one-step SLDNF-derivation of $P_N \cup \{\leftarrow A\}$, there exists a "corresponding" (possibly several steps) SLDNF-derivation of $P_0 \cup \{\leftarrow A\}$, where $P_0, \ldots, P_N$ is a transformation sequence and A is an atom. In the following, for a clause C of the form: $H \leftarrow B_1, \ldots, B_m$, we denote its head H by $head(C)$ and its body $B_1, \ldots, B_m$ by $body(C)$.

Lemma A.3.1 (P_0-simulation of one-step SLDNF-derivation in P_N). *Let $P_0, \ldots, P_N$ ($N \geq 0$) be a transformation sequence and $G^N_0 = \leftarrow A$ be a goal, where A is an atom. Let C be a clause in P_N of the form: $H \leftarrow B_1, \ldots, B_m$ ($m \geq 0$) and let G^N_1 be a resolvent of G^N_0 and C, written in the form: $\leftarrow B_1, \ldots, B_m, A = H$.*

(i) *Then, there exists an SLDNF-derivation Dr_0 of $P_0 \cup \{\leftarrow A\}$ consisting of $G_0 = \leftarrow A, \ldots, G_k$ ($k \geq 0$) using input clauses in P_0 and substitutions $\sigma_1, \ldots, \sigma_k$ such that the literal part of G_k is a P_{new}-expansion of that of G^N_1. Namely, the following condition is satisfied:*

(D1) *G_k is denoted by $\leftarrow \bar{B}_1, \ldots, \bar{B}_m, A = H$, where there exists a bijection φ from the multiset $\{B_1, \ldots, B_m\}$ to the multiset $\{\bar{B}_1, \ldots, \bar{B}_m\}$ such that*

$$\varphi(B_i) = \bar{B}_i = \begin{cases} B_i & \text{if } B_i \text{ is either an old atom} \\ & \text{or a negative literal} \\ body(D_i), head(D_i) = B_i & \text{otherwise} \end{cases}$$

where $D_i \in P_{new}$ is a clause whose head is unifiable with B_i ($i = 1, \ldots, m$) (see Fig. 2).

(ii) *Moreover, when A is a new atom, there exists a variant C_0 of some clause in P_{new} such that C_0 is $A_0 \leftarrow L_0$ and we can construct an SLDNF-derivation Dr'_0 of $P_0 \cup \{\leftarrow L_0, A_0 = A\}$, consisting of $G'_0 = \text{“}\rightarrow L_0, A_0 = A\text{”}, \ldots, G'_{k'}$ $(k' \geq 0)$ using input clauses in P_0 and substitutions $\sigma'_1, \ldots, \sigma'_{k'}$, satisfying the condition* (D1) *replacing k with k'. Furthermore,*

(D2) *For each U in L_0 such that U is left unresolved in the SLDNF-derivation Dr'_0, let $\bar{B}_j$ (for some j, $m \geq j \geq 0$) in $G'_{k'}$ be the possibly instantiated version of U. Then, $B_j = \varphi^{-1}(\bar{B}_j)$ is an inherited atom in C.*

Proof. The proof is done by induction on the length of a transformation sequence N.

Induction Basis: The base case ($N = 0$) trivially holds, since it suffices to consider the SLDNF-derivation of $P_0 \cup \{\leftarrow A\}$ using the same clause C as its input clause. Moreover, when A is a new atom, let C_0 be C itself. Then, it is easy to see that the above (ii) is satisfied.

Induction Step: Suppose that the above proposition holds until $N-1$. We consider the following three cases.

Case 1: C is inherited from P_{N-1}. Then it is obvious by the induction hypothesis.

Case 2: C is the result of unfolding. Let $C_+ \in P_{N-1}$ be the unfolded clause of the form: $H \leftarrow B_+, J$ and $C_- \in P_{N-1}$ the unfolding clause: $B_- \leftarrow K$. Then C is $H\theta \leftarrow K\theta, J\theta$, where θ is an mgu of B_+ and B_-. The resolvent G_1^N of C and $G_0^N = \leftarrow A$ can be denoted by

$$G_1^N: \quad \leftarrow K\theta, J\theta, H\theta = A.$$

(i) First, we show that there exists an SLDNF-derivation Dr_0 of $P_0 \cup \{\leftarrow A\}$ satisfying the condition (D1). Consider an SLDNF-derivation of $P_{N-1} \cup \{\leftarrow A\}$. Using C_+ as an input clause, $G_0^{N-1} = \leftarrow A$ has the successor G_1^{N-1} of the form: $\leftarrow B_+, J, H = A$. Since the above lemma holds for P_{N-1} from the induction hypothesis, $P_0 \cup \{\leftarrow A\}$ has an SLDNF-derivation which satisfies the condition (D1), consisting of $G_0 = \leftarrow A, \ldots, G_{k_1}$ for some k_1 (≥ 0) such that $G_{k_1} = \leftarrow \overline{B_+}, \bar{J}, H = A$. Let φ_{k_1} be a bijection from the multiset $\{B_+, J\}$ to the multiset $\{\overline{B_+}, \bar{J}\}$ such that φ_{k_1} satisfies the condition in (D1). We consider the following two cases depending on whether $\overline{B_+} = B_+$ (i.e., B_+ is an old atom) or not.

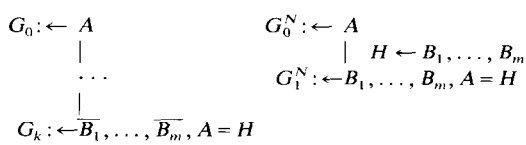


Fig. 2. P_0-simulation (left) of an SLDNF-derivation of $P_N \cup \{\leftarrow A\}$ (right).

(i-1) Suppose that $\overline{B_+} = B_+$. Consider a resolvent of $\leftarrow B_+$ and $C_- \in P_{N-1}$, which is denoted by $\leftarrow K$, $B_- = B_+$. Since the above lemma holds for P_{N-1} from the induction hypothesis, there exists an SLDNF-derivation $Dr_0(B_+)$ of $P_0 \cup \{\leftarrow B_+\}$ which satisfies (D1), consisting of $F_0 = \leftarrow B_+, \ldots, F_{k_2} = \leftarrow \bar{K}$, $B_- = B_+$ for some k_2 ($\geqslant 0$). Let φ_{k_2} be a bijection from the multiset $\{K\}$ to the multiset $\{\bar{K}\}$ such that φ_{k_2} satisfies the condition in (D1). Thus, by concatenating the SLDNF-derivation $Dr_0(B_+)$ to B_+ in G_{k_1}, the SLDNF-derivation of $P_0 \cup \{\leftarrow A\}$ consisting of $G_0 = \leftarrow A, \ldots, G_{k_1}$ can be extended to the one consisting of $G_0 = \leftarrow A, \ldots, G_{k_1}, \ldots, G_{k_1+k_2}$, where $G_{k_1+k_2}$ is of the form: $\leftarrow \bar{K}, \bar{J}, B_- = B_+, H = A$. Recall that θ is an mgu of B_- and B_+ and, we can assume that θ does not affect those variables in A, thus $A\theta = A$. Moreover, it is easy to see that, for a (possibly empty) sequence of literals Γ and a substitution τ, $\bar{\Gamma}\tau$ is equivalent to $\overline{\Gamma\tau}$. Consequently, $G_{k_1+k_2}$ can be written as:

$$G_{k_1+k_2}: \quad \leftarrow \overline{K\theta}, \overline{J\theta}, H\theta = A,$$

whose literal part is exactly a P_{new}-expansion of that of G_1^N. Moreover, a bijection φ from the multiset of the literal part of G_1^N to that of $G_{k_1+k_2}$ is defined in an obvious way from φ_{k_1} and φ_{k_2}.

(i-2) Suppose that $\overline{B_+}$ is of the form: "$L_1, A_1 = B_+$" for a variant $A_1 \leftarrow L_1$ of some clause in P_{new}. From the induction hypothesis for P_{N-1}, there exists an SLDNF-derivation $Dr_0(\overline{B_+})$ of $P_0 \cup \{\leftarrow L_1, A_1 = B_+\}$ which satisfies (D1), consisting of $F_0 = \leftarrow L_1, A_1 = B_+, \ldots, F_{k_2} = \leftarrow \bar{K}$, $B_- = B_+$ for some k_2 ($\geqslant 0$). Then the proposition follows from a discussion similar to (i-1).

(ii) Next, suppose that A is a new atom. We show that there exists a clause C_0: $A_0 \leftarrow L_0 \in P_{new}$ and an SLDNF-derivation Dr_0' of $P_0 \cup \{\leftarrow L_0, A_0 = A\}$ satisfying both conditions (D1) and (D2). From the induction hypothesis for P_{N-1}, there exists an SLDNF-derivation of $P_0 \cup \{\leftarrow L_0, A_0 = A\}$ which satisfies the conditions (D1) and (D2), consisting of $G_0 = "\leftarrow L_0, A_0 = A", \ldots, G_{k_1}$ for some k_1 ($\geqslant 0$) such that $G_{k_1} = \leftarrow \overline{B_+}, \bar{J}, H = A$. Again, we consider the following two cases depending on whether $\overline{B_+} = B_+$ (i.e., B_+ is an old atom) or not.

(ii-1) Suppose that $\overline{B_+} = B_+$. Consider a resolvent of $\leftarrow B_+$ and $C_- \in P_{N-1}$, which is denoted by $\leftarrow K$, $B_- = B_+$. Again, from the induction hypothesis, there exists an SLDNF-derivation $Dr_0(B_+)$ of $P_0 \cup \{\leftarrow B_+\}$ which satisfies (D1), consisting of $F_0 = \leftarrow B_+, \ldots, F_{k_2} = \leftarrow \bar{K}$, $B_- = B_+$ for some k_2 ($\geqslant 0$). From the similar discussion to that in (i), there exists an SLDNF-derivation Dr_0' of $P_0 \cup \{\leftarrow L_0, A_0 = A\}$ consisting of $G_0 = "\leftarrow L_0, A_0 = A", \ldots, G_{k_1}, \ldots, G_{k_1+k_2}$, where $G_{k_1+k_2}$ is of the form: $\leftarrow \overline{K\theta}, \overline{J\theta}$, $H\theta = A$. The proof that Dr_0' satisfies (D1) is quite similar to that of the case (i). We thus only show that Dr_0' satisfies (D2). Let φ_{k_1}, φ be the bijections defined as in (i). If there exists an atom, say U, in L_0 such that it is left unresolved in Dr_0', its possibly instantiated version, say, $\overline{B_j}$ should be contained in $\bar{J}$. Then such an unresolved atom U exists in Dr_0' if and only if $B_j = \varphi_{k_1}^{-1}(\overline{B_j})$ in J is an inherited atom in C_+ from the induction hypothesis, which holds if and only if $B_j\theta$ is an inherited atom in C from the definition of an inherited atom. Thus the condition (D2) holds for Dr_0'.

(ii-2) Next, suppose that B_+ is a new atom. As for the condition (D1), the proof is quite similar to that of (i-1). Moreover the condition (D2) is also shown from a discussion similar to (ii-1).

Case 3: C is the result of folding. Let $C_+ \in P_{N-1}$ be the folded clause of the form: $H \leftarrow J$, K and $D \in P_{new}$ be the folding clause: $B \leftarrow J_0$, where $J_0\theta = J$ for some substitution θ. Let θ_{IV} be the restriction of θ to those variables occurring only in J_0. From the condition of folding (F2), θ_{IV} is a renaming substitution of the form: $\{X_i/Z_i\}$ $(i \geq 0)$, where X_i is an internal variable in D and Z_i is a variable occurring only in J. The result of folding C is $H \leftarrow B\theta$, K. The resolvent G_1^N of C and $G_0^N = \leftarrow A$ can be denoted by

$$G_1^N: \quad \leftarrow B\theta,\ K,\ H = A.$$

(i) First, we show that there exists an SLDNF-derivation Dr_0 of $P_0 \cup \{\leftarrow A\}$ satisfying the condition (D1). Since $B\theta$ is a new atom, what we should prove is that $P_0 \cup \{\leftarrow A\}$ has an SLDNF-derivation Dr_0, consisting of $G_0 = \leftarrow A, \ldots, G_{k_1}$ for some k_1 (≥ 0) such that

$$G_{k_1}: \quad \leftarrow \tilde{J}_0,\ \bar{K},\ \tilde{B} = B\theta,\ H = A,$$

where $\tilde{D}$ is an arbitrarily chosen and fixed variant of D of the form: $\tilde{B} \leftarrow \tilde{J}_0$ such that none of the variables in $\tilde{D}$ appear elsewhere. Moreover, let ι be a renaming substitution from variables in D to those in $\tilde{D}$. Consider an SLDNF-derivation of $P_{N-1} \cup \{\leftarrow A\}$. Let C'_+ be a variant of C_+ of the form: $H \leftarrow \tilde{J}^*$, K, where $\tilde{J}^*$ is $J\theta_{IV}^{-1} \circ \iota$, that is, replacing variable Z_i $(= \theta_{IV}(X_i))$ in J by $\iota(X_i)$. Using C'_+ as an input clause, $G_0^{N-1} = \leftarrow A$ has the successor G_1^{N-1}: $\leftarrow \tilde{J}^*$, K, $H = A$. From the induction hypothesis and the fact that $\tilde{J}^*$ consists only of old atoms, there exists an SLDNF-derivation Dr_0 of $P_0 \cup \{\leftarrow A\}$ which satisfies the condition (D1), consisting of $G_0 = \leftarrow A, \ldots, G_{k'_1}$ for some k'_1 (≥ 0) such that

$$G_{k'_1}: \quad \leftarrow \tilde{J}^*,\ \bar{K},\ H = A.$$

From the similar discussion in Lemma A.1.2 and the definition of $\tilde{J}^*$, it is easy to see that "$\tilde{J}_0$, $\tilde{B} = B\theta$" is equivalent to $\tilde{J}^*$, which means that $G_{k'_1}$ is actually a P_{new}-expansion of G_1^N and that Dr_0 satisfies the condition (D1).

(ii) Next, suppose that A is a new atom. We show that there exists a clause C_0: $A_0 \leftarrow L_0$ in P_{new} and an SLDNF-derivation Dr'_0 of $P_0 \cup \{\leftarrow L_0, A_0 = A\}$ satisfying the both conditions (D1) and (D2). From the induction hypothesis for P_{N-1} and the similar discussion in (i), there exists an SLDNF-derivation Dr'_0 of $P_0 \cup \{\leftarrow L_0, A_0 = A\}$ which satisfies the conditions (D1) and (D2), consisting of $G_0 = \leftarrow L_0$, $A_0 = A, \ldots, G_{k_1}$ for some k_1 (≥ 0) such that $G_{k_1} = \leftarrow \tilde{J}^*$, $\bar{K}$, $H = A$, where $\tilde{J}^*$ is defined in the above (i). The condition (D1) is shown similarly to the case (i). As for the condition (D2), note that an unresolved atom in Dr'_0 (if any), say U, is not contained in $\tilde{J}^*$; otherwise, let $B'_j \in \tilde{J}^*$ be the possibly instantiated version of U. Then, B'_j is an inherited atom in C'_+, thus folding can not be applied to C'_+, nor to C_+, which contradicts the assumption. Consequently, the unresolved atom U, if

it exists, would be contained in $\bar{K}$. Again, from the induction hypothesis, it is an inherited atom in C'_+, and so is in C_+. From the definition of the inherited atom, it is also an inherited atom in $C \in P_N$, which proves the condition (D2). □

Using the above lemma, we can now show Lemma 3.3.1.

Lemma A.3.2 (P_0-simulation of SLDNF-derivation in P_N). *Let $P_0, \ldots, P_N$ be a transformation sequence and let G be a goal. Suppose that there exists an SLDNF-derivation Dr^N of $P_N \cup \{G\}$, $G_0 = G, \ldots, G_k, \ldots$, using input clauses in P_N and substitutions $\theta_1, \ldots, \theta_k, \ldots$ Then, there exists an SLDNF-derivation Dr^0 of $P_0 \cup \{G\}$, $F_0 = G, \ldots, F_{l_k}, \ldots$, using input clauses in P_0 and substitutions $\sigma_1, \ldots, \sigma_{l_k}, \ldots$, satisfying the following conditions:*

(i) *For each k ($k \geq 0$), there exists some l_k (≥ 0) such that $F_{l_k}\sigma_1 \circ \cdots \circ \sigma_{l_k}$ is a P_{new}-expansion of $G_k\theta_1 \circ \cdots \circ \theta_k$, and*

(ii) *the restriction of $\sigma_1 \circ \cdots \circ \sigma_{l_k}$ to the variables in G is the same as that of $\theta_1 \circ \cdots \circ \theta_k$.*

(iii) (fairness) *Furthermore, if the SLDNF-derivation $G_0 = G, \ldots, G_k, \ldots$ is fair, then so is the SLDNF-derivation $F_0 = G, \ldots, F_{l_k}, \ldots$*

Dr^0 is called a P_0-simulation of Dr^N.

Proof. The proof is done by induction on the length k of SLDNF-derivation Dr^N. The induction basis (i.e., $k = 0$) is obvious. In the following, we define a bijection φ_k from the multiset of literals in G_k to that of literals in F_{l_k}. For $k = 0$, $G_0 = F_0$ and let φ_0 be an identity. Suppose that the proposition holds until $k-1$ ($k > 0$) and φ_{k-1} is already defined. Let Dr^N_{k-1} be the segment of Dr^N from G_0 to G_{k-1}, and let G_{k-1} be of the form: $\leftarrow A, \Delta$, where Δ is a possibly empty sequence of literals and A is the selected literal in G_{k-1}. We first show conditions (i) and (ii).

- When $A = \sim A'$ is a negative literal, A should be ground and there exists a finitely failed SLDNF-tree for $P_N \cup \{\leftarrow A'\}$. In this case, G_k is of the form: $\leftarrow \Delta$ and θ_k is an identity substitution. From the partial correctness w.r.t. FF, $P_0 \cup \{\leftarrow A\}$ also has a finitely failed SLDNF-tree. Thus, it is easy to see that the above (i) and (ii) hold. φ_k is φ_{k-1} except that the selected atom A is deleted from its domain.
- Otherwise (namely, A is a positive atom), suppose that A is an old atom. Let $C \in P_N$ be an input clause of the form: $H \leftarrow L$, where L is a possibly empty sequence of literals. Then, the resolvent G_k of G_{k-1} and C, is of the form: $\leftarrow L, \Delta, A = H$, where θ_k is a substitution given by the equation $A = H$. On the other hand, from the induction hypothesis, there exists an SLDNF-derivation $Dr^0_{l_{k-1}}$ corresponding to Dr^N_{k-1}, which satisfies the conditions in the lemma. Suppose that $Dr^0_{l_{k-1}}$ consists of $F_0 = G, \ldots, F_{l_{k-1}}$. Note that $F_{l_{k-1}}$ can be denoted by $\leftarrow \bar{A}, \bar{\Delta}$. Since A is assumed to be an old atom, $\bar{A}$ is equivalent to A. Moreover, due to Lemma A.3.1, there exists an SLDNF-derivation Dr^0_A of $P_0 \cup \{\leftarrow A\}$ consisting of a sequence of goals $\leftarrow A, \ldots, \leftarrow \bar{L}, A = H$, which satisfies the conditions given

therein. Thus, using Dr'^{0}_{A}, it is easy to see that we can extend $Dr'^{0}_{l_{k-1}}$ into an SLDNF-derivation $Dr'^{0}_{l_k}$ so that it satisfies the conditions (i) and (ii) in the lemma. Let φ_L be a bijection from the multiset $\{L\}$ to $\{\bar{L}\}$. Then, using φ_L, φ_k is obtained by extending φ_{k-1} in an obvious way.

- The proof of the conditions (i) and (ii) for the case where A is a new atom is done similarly to the above-mentioned case.

  Finally, we show the fairness condition (iii). Suppose that A is selected in some goal G_i of Dr'^{N} and let G_{i+1} be the resolvent of G_i and an input clause C. Then, in Dr'^{0}, the corresponding subgoal $\leftarrow \varphi_i(A)$ in F_{l_i} is tried, obtaining some descendent node $F_{l_{i+1}}$ of F_{l_i} which corresponds to G_{i+1}. Thus, in order to show the fairness condition, it suffices to consider the case where A is a new atom. Let $\varphi_i(A)$ be of the form: $B_1, \ldots, B_n$, $H = A$, where $H \leftarrow B_1, \ldots, B_n$ is a variant of a clause in P_{new} whose head is unifiable with A. If there exists any literal, say B_j, in F_{l_i} such that it is left unresolved from F_{l_i} to $F_{l_{i+1}}$, it follows from Lemma A.3.1 that it should be an inherited atom in C. Thus $\varphi^{-1}_{i+1}(B_j)$ is an old atom in G_{i+1}. From the fairness condition of Dr'^{N}, its possibly instantiated version will be eventually selected, which means that B_j in Dr'^{0} will be also eventually selected. □

A.4. Proof of total correctness w.r.t. SS

In order to prove the total correctness w.r.t. *SS*, we need to prepare several definitions and notations. Most of the following definitions are originally given in [15] and [7] for definite programs. We extend them for general programs in a suitable manner. Let θ be a substitution and A an atom. Then, the restriction of θ to the variables in A is denoted by $\theta|_A$.

Definition A.4.1 (*weight of derivation*). Let P_0 be the initial program of a transformation sequence and Γ a sequence of literals. Let Dr be a finite SLDNF-derivation of $P_0 \cup \{\leftarrow \Gamma\}$ consisting of goals $G_0 = \leftarrow \Gamma, G_1, \ldots, G_n$. The *weight* of Dr is defined as follows:

(1) When Dr ends in an empty clause (i.e., Dr is a successful derivation), the *weight* of Dr is the number of those goals in Dr whose selected literals are either old atoms or negative literals.

(2) When Dr ends in a goal $G_n = \leftarrow \sim B_1, \ldots, \sim B_k$ $(k \geq 1)$ such that each B_i $(k \geq i \geq 1)$ is a nonground atom, the *weight* of Dr is the number of those goals in Dr whose selected literals are either old literals or negative literals, plus k (i.e., the number of negative literals in G_n).

Definition A.4.2 (*weight of atom-substitution pair*). Let P_0 be the initial program of a transformation sequence. Let A be an atom and σ a substitution. Then, the *weight* of a pair (A, σ), denoted by $w(A, \sigma)$, is defined to be the minimum of the weight of the SLDNF-derivation of $P_0 \cup \{\leftarrow A\}$, consisting of goals $G_0 = \leftarrow A, G_1, \ldots, G_n$ and substitutions $\theta_1, \ldots, \theta_n$ such that

(i) G_n is either an empty goal or a goal consisting only of negative nonground literals, and

(ii) σ is the restriction of $\theta_1 \circ \cdots \circ \theta_n$ to the variables in A.

Similarly, let Γ be a sequence of literals and τ a substitution. Then, the above definition is extended to the definition of the weight of a pair (Γ, τ), denoted by $w(\Gamma, \tau)$, in an obvious way.

The following notion of a descent clause plays an important role in the proof of the total correctness w.r.t. *SS*.

Definition A.4.3 (*descent clause*). Let P_i be a program in a transformation sequence starting from an initial program P_0. Let C be a clause in P_i of the form: $H \leftarrow L$, where L is a possibly empty sequence of literals. Suppose that A is an atom such that $(A, \sigma) \in SS(P_0)$ for some substitution σ. Then C is called a *descent clause* for (A, σ), if there exists a substitution τ such that

(W1) $((L, H = A), \tau) \in SS(P_0)$ and the restriction of τ to the variables in A is σ,

(W2) $w(A, \sigma) \geqslant w((L, H = A), \tau)$, and

(W3) if C satisfies the folding condition (F4), then $w(A, \sigma) > w((L, H = A), \tau)$.

Definition A.4.4 (*weight completeness*). Let P_i be a program in a transformation sequence starting from the initial program P_0. Then, P_i is *weight complete* if and only if, for any atom-substitution pair $(A, \sigma) \in SS(P_0)$, there exists a descent clause in P_i for that pair.

After showing the following lemma, we proceed by proving the total correctness w.r.t. *SS*.

Lemma A.4.1. *Let $P_0, \ldots, P_N$ be a sequence of program transformation and C be a clause in P_i $(0 \leqslant i \leqslant N)$. If C does not satisfy the folding condition* (F4) *in Definition* 2.1.3, *then all the atoms in the body of C are old atoms.*

Proof. Since C does not satisfy the condition (F4), the head of C is a new atom and unfolding has not been applied to C during the transformation. Thus, C should be inherited as it is from P_0. Then, the lemma obviously holds from the definition of an initial program P_0. □

Following [15], the outline of the proof of the total correctness is as follows:

(1) We first show that the weight completeness is a sufficient condition for the total correctness w.r.t. *SS* (Lemma A.4.2).

(2) Next, the initial program P_0 of a transformation sequence is shown to be weight complete (Lemma A.4.3).

(3) Finally, the weight completeness is preserved during program transformation (Lemma A.4.4).

Lemma A.4.2 (weight completeness is sufficient for total correctness w.r.t. *SS*). Let $P_0, \ldots, P_N$ *be a sequence of program transformation. If* P_i *is weight complete, then* $SS(P_i) \supseteq SS(P_0)$ $(N \geq i \geq 0)$.

Proof. We prove a more general proposition that, under the same condition, if $(\Gamma, \sigma) \in SS(P_0)$, then $(\Gamma, \sigma) \in SS(P_i)$, where Γ is a (possibly empty) sequence of literals.

First, we introduce the following well-founded ordering $>$ into the set of pairs (Γ, σ) in $SS(P_0)$, i.e., $(\Gamma_1, \sigma_1) > (\Gamma_2, \sigma_2)$ if and only if

(1) $w(\Gamma_1, \sigma_1) > w(\Gamma_2, \sigma_2)$, or

(2) $w(\Gamma_1, \sigma_1) = w(\Gamma_2, \sigma_2)$ and the number of new atoms in Γ_1 is greater than that of new atoms in Γ_2.

We show the lemma by induction on the above defined well-founded ordering.

As for the induction basis, i.e., when Γ is empty, the lemma is obvious. Next, suppose that Γ is of the form: A, Δ, where Δ is a possibly empty sequence of literals and A is the selected atom in the initial goal G_0 of an SLDNF-refutation of $P_0 \cup \{G_0 = \leftarrow A, \Delta\}$ with the computed answer substitution σ. When A is a negative literal, say, $\sim A'$, A' should be ground and there exists a finitely failed SLDNF-tree for $P_0 \cup \{\leftarrow A'\}$. Thus, G_0 has the child node $G_1 = \leftarrow \Delta$ such that $(\Delta, \sigma) \in SS(P_0)$. On the other hand, from the total correctness w.r.t. *FF* (Proposition 3.3.1), $P_i \cup \{\leftarrow A'\}$ also has a finitely failed SLDNF-tree. Thus, also in an SLDNF-derivation of $P_i \cup \{\leftarrow A, \Delta\}$, when A is selected in the initial node G_0, G_0 has the child node G_1. From the induction hypothesis on the well-founded ordering $>$, (Δ, σ) is in $SS(P_i)$. Thus, so is (Γ, σ). Otherwise, suppose that A is a positive literal. From the definition that $(\Gamma, \sigma) = ((A, \Delta), \sigma)$ is in $SS(P_0)$, there exists an SLDNF-derivation of $P_0 \cup \{\leftarrow A, \Delta\}$ with a computed answer substitution σ. Thus, it follows that its subgoal $P_0 \cup \{\leftarrow \Delta\}$ has an SLDNF-derivation which satisfies the following conditions:

(D1) It ends in a goal $G_n = \leftarrow \mathcal{N}$, where $\mathcal{N}$ is a possibly empty sequence of negative non-ground literals.

(D2) Let σ_Δ be a substitution for variables in $\leftarrow \Delta$, computed during this derivation. Then, there exists a substitution σ_1 such that $(A\sigma_\Delta, \sigma_1) \in SS(P_0)$ and $\sigma_\Delta \circ \sigma_1 = \sigma$.

(D3) Moreover, $\mathcal{N}\sigma_1$ is ground and $\leftarrow \mathcal{N}\sigma_1$ has a successful SLDNF-derivation.

In the following, we consider such an SLDNF-derivation of $P_0 \cup \{\leftarrow \Delta\}$ that it satisfies (D1)-(D3) and the weight $w(\Delta, \sigma_\Delta)$ is the minimum.

As P_i is weight complete and $(A\sigma_\Delta, \sigma_1) \in SS(P_0)$, there exists a descent clause C for $(A\sigma_\Delta, \sigma_1)$ in P_i. Let C be of the form: $H \leftarrow L$, where no variables in C appear elsewhere. Then, from the definition of the descent clause, the following conditions hold:

(W1) $((L, H = A\sigma_\Delta), \tau) \in SS(P_0)$ and the restriction of τ to the variables in $A\sigma_\Delta$ is σ_1,

(W2) $w(A\sigma_\Delta, \sigma_1) \geq w((L, H = A\sigma_\Delta), \tau)$, and

(W3) if C satisfies the folding condition (F4), then $w(A\sigma_\Delta, \sigma_1) > w((L, H = A\sigma_\Delta), \tau)$.

Now, consider an SLDNF-derivation Dr of $P_i \cup \{\leftarrow A, \Delta\}$. Then, using C as an input clause, the initial goal $G_0 = \leftarrow A, \Delta$ has the successor G_1: $\leftarrow L, \Delta, A = H$. We first show that there exists an SLDNF-refutation Dr_0 of $P_0 \cup \{G_1\}$ with the computed answer substitution whose restriction to the variables in G_1 is σ. Resolving the subgoal Δ in G_1 first, from (D1) in the above, G_1 has a descendent G_{l_1} of the form: $\leftarrow L, \mathcal{N}, H = A\sigma_\Delta$ with the substitution σ_Δ computed from G_1 to G_{l_1}.

From the above condition (W1), $P_0 \cup \{\leftarrow L, H = A\sigma_\Delta\}$ has a successful SLDNF-derivation with the computed answer substitution τ. Thus, G_{l_1} has a descendent $G_{l_2} = \leftarrow \mathcal{N}\tau$. Since the restriction of τ to the variables in $A\sigma_\Delta$ is σ_1 and, as noted in (D3), $\mathcal{N}\sigma_1$ is ground and $\leftarrow \mathcal{N}\sigma_1$ has an SLDNF-refutation, it follows that $P_0 \cup \{G_1\}$ has an SLDNF-refutation with the computed answer substitution $\sigma_\Delta \circ \tau$ whose restriction to the variables in G_0 is σ (see Fig. 3). Thus, the weight $w((L, \Delta, H = A), \sigma_\Delta \circ \tau)$ is defined.

$$\begin{aligned} w((A, \Delta), \sigma) &= w(A\sigma_\Delta, \sigma_1) + w(\Delta, \sigma_\Delta) \\ &\geq w((L, H = A\sigma_\Delta), \tau) + w(\Delta, \sigma_\Delta) \quad \text{(from (W2))} \\ &= w((L, \Delta, H = A), \sigma_\Delta \circ \tau). \end{aligned}$$

When $w((A, \Delta), \sigma) > w((L, \Delta, H = A), \sigma_\Delta \circ \tau)$ holds, from the induction hypothesis on the well-founded ordering $>$, $P_i \cup \{G_1\}$ has a successful SLDNF-derivation with a computed answer $\sigma_\Delta \circ \tau$. Thus $P_i \cup \{G_0\}$ also has a successful SLDNF-derivation with a computed $\sigma_\Delta \circ \tau$ whose restriction to the variables in G_0 is σ. So, we have that $((A, \Delta), \sigma) = (\Gamma, \sigma) \in SS(P_i)$.

On the other hand, when $w((A, \Delta), \sigma) = w((L, \Delta, H = A), \sigma_\Delta \circ \tau)$ holds, from the condition (W3), C does not satisfy the folding condition (F4). From Lemma A.4.1,

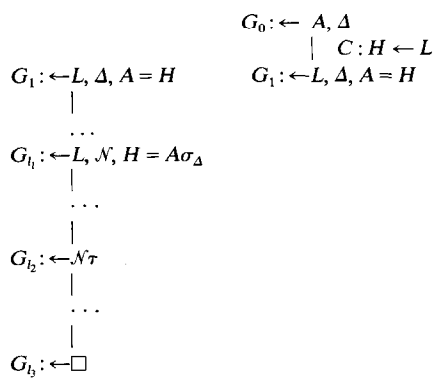


Fig. 3. An SLDNF-derivation of $P_0 \cup \{G_1\}$ (left) and an SLDNF-derivation of $P_i \cup \{G_0\}$ (right).

the predicate of H is a new predicate and so is that of A, while all atoms in L are old atoms. Thus, it follows that $((A, \Delta), \sigma) > ((L, H = A, \Delta), \sigma_\Delta \circ \tau)$. Consequently, from the induction hypothesis, $P_i \cup \{\leftarrow L, \Delta, H = A\}$ has a successful SLDNF-derivation with a computed answer $\sigma_\Delta \circ \tau$. From the same discussion as above, it is shown that $((A, \Delta), \sigma) = (\Gamma, \sigma) \in SS(P_i)$. □

Lemma A.4.3. *The initial program P_0 is weight complete.*

Proof. Let (A, σ) be an atom-substitution pair in $SS(P_0)$ and let Dr be an SLDNF-refutation of $P_0 \cup \{G_0 = \leftarrow A\}$ with answer substitution σ such that the weight of Dr is $w(A, \sigma)$. Furthermore, let $C = H \leftarrow L$ be the input clause used in G_0. Then, G_0 has the child node $G_1 = \leftarrow L, H = A$ and there exists an SLDNF-refutation of $P_0 \cup \{G_1\}$ with answer substitution τ such that the restriction of τ to the variables in A is σ. It is easy to see that C satisfies conditions (W1) and (W2) of the definition of a descent clause. Moreover, C satisfies the folding condition (F4) if and only if H is an old atom. Thus, it follows that $w(A, \sigma) > w((L, H = A), \tau)$, so C satisfies also the condition (W3). □

Lemma A.4.4 (preservation of weight completeness). *Let $P_0, \ldots, P_N$ be a sequence of program transformation. If P_i is weight complete, then P_{i+1} is also weight complete $(N - 1 \geqslant i \geqslant 0)$.*

Proof. Let (A, σ) be an atom-answer substitution pair in $SS(P_0)$. Since P_i is weight complete, there exists a descent clause C_0 for (A, σ) in P_i. We will show that there exists a descent clause for (A, σ) also in P_{i+1}, by considering the following three cases.

Case 1: C_0 is in P_{i+1}. Then C_0 itself is a descent clause for (A, σ) also in P_{i+1}.

Case 2: C_0 is unfolded. Let C_0 be $H \leftarrow B_+, J$, where J is a possibly empty sequence of literals and suppose that atom B_+ is unfolded. Since C_0 is a descent clause for (A, σ), there exists an SLDNF-refutation of $P_0 \cup \{\leftarrow B_+, J, H = A\}$ with a computed answer substitution τ such that the restriction of τ to the variables in A is σ. Thus, it follows that its subgoal $P_0 \cup \{\leftarrow J, H = A\}$ has an SLDNF-derivation which satisfies the following conditions:

(D1) It ends in a goal of the form: $\leftarrow \mathcal{N}$, where $\mathcal{N}$ is a (possibly empty) sequence consisting only of negative literals. Let τ_1 be a substitution computed during this SLDNF-derivation.

Then, it is easy to see that $P_0 \cup \{\leftarrow B_+\tau_1\}$ has an SLDNF-refutation with a computed answer substitution τ_2 such that

(D2) $\mathcal{N}\tau_2$ is ground and $P_0 \cup \{\leftarrow \mathcal{N}\tau_2\}$ has an SLDNF-refutation, and

(D3) $\tau_1 \circ \tau_2 = \tau$.

Again, from the weight-completeness of P_i, there exists a descent clause C_- in P_i for $(B_+\tau_1, \tau_2)$. Let C_- be $B_- \leftarrow K$. Then,

(D4) $P_0 \cup \{\leftarrow K, B_- = B_+\tau_1\}$ has an SLDNF-refutation with a computed answer substitution η such that the restriction of η to the variables in $B_+\tau_1$ is τ_2.

Since B_- and $B_+\tau_1$ are unifiable, so are B_+ and B_-. Thus, C_0 is unfolded by C_-, obtaining an unfolded clause C in P_{i+1} of the form: $H\beta \leftarrow K\beta, J\beta$, where β is an mgu of B_+ and B_-. Now, we show that C is a descent clause for (A, σ).

Condition (W1): Consider an SLDNF-derivation Dr of $P_0 \cup \{G_0 = \leftarrow K\beta, J\beta, H\beta = A\}$. Due to the unification theorem, an SLDNF-derivation Dr' of $P_0 \cup \{G_0' = \leftarrow K, B_- = B_+, J, H = A\}$ is equivalent to Dr, as far as a computed answer substitution restricted to the variables in A is concerned. From (D1), G_0' has a descendent node G_1': $\leftarrow K, B_- = B_+\tau_1, \mathcal{N}$. From (D4), G_1' has a descendent node G_2': $\leftarrow \mathcal{N}\eta$. Finally, from (D2) and from the fact that the restriction of η to the variables in $B_+\tau_1$ is τ_2, Dr' has a successful SLDNF-derivation with a computed answer substitution $\tau_1 \circ \eta$. Moreover,

$$(\tau_1 \circ \eta)|_A = (\tau_1 \circ \eta|_{B_+\tau_1})|_A = (\tau_1 \circ \tau_2)|_A = \tau|_A = \sigma.$$

Condition (W2): Since C_0 (resp. C_-) is a descent clauses for (A, σ) (resp. $(B_+\tau_1, \tau_2)$), we have

$$w(A, \sigma) \geqslant w((B_+, J, H = A), \tau),$$

$$w(B_+\tau_1, \tau_2) \geqslant w((K, B_- = B_+\tau_1), \eta).$$

Thus,

$$\begin{aligned} w(A, \sigma) &\geqslant w((B_+, J, H = A), \tau) && (1)\\ &= w((B_+\tau_1, \tau_2) + w((J, H = A), \tau_1) \\ &\geqslant w((K, B_- = B_+\tau_1), \eta) + w((J, H = A), \tau_1) && (2)\\ &= w((K, B_- = B_+, J, H = A), \tau_1 \circ \eta) \\ &= w((K\beta, J\beta, H\beta = A), \tau_1 \circ \eta). \end{aligned}$$

Condition (W3): Note that C satisfies the condition (F4). Thus, we have to show that

$$w(A, \sigma) > w((K\beta, J\beta, H\beta = A), \tau_1 \circ \eta). \qquad (3)$$

When B_+ is a new atom, C satisfies (F4). Thus, the strict inequality in (1) holds. Otherwise (i.e., when B_+ is an old atom), C_- satisfies (F4), thus the strict inequality in (2) holds. Consequently, in either case, it is shown that the strict inequality holds in (3). Thus, it is shown that C is a descent clause for (A, σ).

Case 3: C_0 is folded. Let C_0 be $H \leftarrow J_+, K$ and $D \in P_{new}$ be the folding clause of the form $B \leftarrow J_-$, where $J_-\theta = J_+$ for some substitution θ. Then, the result of folding $C \in P_{i+1}$ is $H \leftarrow B\theta, K$. Since C_0 is a descent clause for (A, σ), there exists an SLDNF-refutation $P_0 \cup \{G_0 = \leftarrow J_+, K, H = A\}$. Let τ be its computed answer substitution such that the restriction of τ to the variables in A is σ. We show that this C is a descent clause for (A, σ).

Condition (W1): Consider an SLDNF-derivation Dr of $P_0 \cup \{G_0' = \leftarrow B\theta, K, H = A\}$. Let D' be a variant of D, say, $D' = B' \leftarrow J_-'$ such that all variables in D' are newly introduced ones. Then G_0' has a successor G_1' of the form: $\leftarrow J_-'\beta, K,$

$H = A$, where β is an mgu of B' and $B\theta$ such that $B'\beta = B\theta$. From Lemma A.1.2, G_1' is a variant of G_0 such that they are different only with respect to the internal variables in J'_-. Thus $P_0 \cup \{G_0'\}$ has an SLDNF-refutation with a computed answer substitution τ' such that the restriction of τ' to the variables in A is σ.

Condition (W2) *and* (W3): Note that C_0 is a descent clause for (A, σ) and it satisfies the condition (F4). Thus, we have

$$w(A, \sigma) > w((J_+, K, H = A), \tau) = w((J'_-\beta, K, H = A), \tau').$$

Since B is a new atom, it is shown from the definition of the weight that

$$w(A, \sigma) > w((B\theta, K, H = A), \tau'). \qquad \square$$

A.5. Proof of preservation of perfect model semantics

In this section, we give the proof of Proposition 4.1. The important property of the perfect model semantics we will use in the following proof is that every perfect model is *supported* [12]. That is, we first fix a pre-interpretation (e.g., [9]) J of a program P. Let M be the perfect model of P based on J. Then, for every J-ground atom A in M, there exists a J-ground instance of a clause in P such that its head is equal to A, all positive premises belong to M and none of the negative premises belong to M.

From this property, we can consider a "ground proof derivation" analogous to an SLDNF-derivation. Namely, let Γ be a (possibly empty) sequence of ground literals such that $M \models \Gamma$. Then, a *ground proof derivation of* $P \cup \{G_0 = \leftarrow \Gamma\}$ consists of a sequence of goals: $G_0, G_1, \ldots, G_n = \square$ (an empty goal), a sequence $C_1, \ldots, C_{n-1}$ of J-ground instances of clauses (called input clauses) in P or negative ground literals, satisfying the following conditions:

(i) Let G_i be of the form: $\leftarrow A_1, \ldots, A_m, \ldots, A_n$, where A_m is the selected positive literal in G_i. Let $C_{i+1} \in P$ be an input clause of the form: $A_m \leftarrow L$, where L is a possibly empty sequence of ground literals and $M \models L$. Then G_{i+1} is $\leftarrow A_1, \ldots, A_{m-1}, L, A_{m+1}, \ldots, A_n$.

(ii) G_i is $\leftarrow A_1, \ldots, A_m, \ldots, A_n$, where $A_m = \sim A'$ is the selected negative literal in G_i such that $M \models \sim A'$. Then G_{i+1} is $\leftarrow A_1, \ldots, A_{m-1}, A_{m+1}, \ldots, A_n$.

The purpose of defining the above ground proof derivation is to prove Proposition 4.1 by following exactly the same lines as in the proofs of Proposition 3.2.1 and Proposition 3.3.2.

In the following, for a fixed pre-interpretation J, we denote the perfect model of P based on J by $PERF(P)$. Moreover, as we did in Section A.2, we consider $PERF(P)$ as a set of all (possibly empty) sequences of ground literals Γ such that $PERF(P) \models \Gamma$, or equivalently, $P \cup \{\leftarrow \Gamma\}$ has a ground proof derivation. As we did in Section A.4, we also need the definitions of weights and descent clauses modified suitably for the current purpose.

Definition A.5.1 (*weight of a ground proof derivation*). Let P_0 be the initial program of a transformation sequence and Γ *a sequence of ground literals. Let* Dr be a ground

proof derivation of $P_0 \cup \{\leftarrow \Gamma\}$ consisting of goals $G_0 = \leftarrow \Gamma, G_1, \ldots, G_n$. The *weight* of Dr is defined to be the number of those goals in Dr whose selected literals are either old atoms or negative literals.

Definition A.5.2 (*weight of atom*). Let P_0 be the initial program of a transformation sequence and A a ground atom such that $PERF(P_0) \models A$. Then the *weight* of A, denoted by $w(A)$, is defined to be the minimum of the weight of the ground proof derivation of $P_0 \cup \{\leftarrow A\}$. Similarly, let Γ be a sequence of ground literals such that $PERF(P_0) \models \Gamma$. Then, the above definition is extended to the definition of the weight of Γ, denoted by $w(\Gamma)$, in an obvious way.

Definition A.5.3 (*descent clause*). Let P_i be a program in a transformation sequence starting from an initial program P_0. Let C be a ground instance of a clause in P_i of the form: $A \leftarrow L$, where L is a possibly empty sequence of ground literals. Suppose that A is an atom such that $PERF(P_0) \models A$ holds. Then C is called a *descent clause* for A if the following conditions are satisfied:

(W1) $PERF(P_0) \models L$ holds,

(W2) $w(A) \geqslant w(L)$, and

(W3) if C satisfies the folding condition (F4), then $w(A) > w(L)$.

Definition A.5.4 (*weight completeness*). Let P_i be a program in a transformation sequence starting from the initial program P_0. Then, P_i is *weight complete* if and only if, for any ground atom $A \in PERF(P_0)$, there exists a descent clause in P_i for A.

Proposition A.5.1 (preservation of perfect model semantics). *Let $P_0, \ldots, P_N$ be a transformation sequence. Then,*

(PC): *If* $PERF(P_i) = PERF(P_0)$, *then* $PERF(P_{i+1}) \subseteq PERF(P_i)$ *for* $i = 0, \ldots, N-1$.

(TC): *If* $PERF(P_i) = PERF(P_0)$, *then* $PERF(P_i) \subseteq PERF(P_{i+1})$ *for* $i = 0, \ldots, N-1$.

Proof. The proof is by mutual induction on $s = stratum(G_0)$ of goal $G_0 = \leftarrow \Gamma$. It is obvious when $s = 0$. Suppose that the proposition has been proved for all goals G_0' whose $stratum(G_0') \leqslant s$, where $s \geqslant 0$. We first prove (PC).

Suppose there exists a ground proof derivation Dr_{i+1} of $P_{i+1} \cup \{G_0\}$. The proof is by induction on the length of the ground proof derivation of $P_{i+1} \cup \{G_0 = \leftarrow \Gamma\}$. Let Γ be of the form: A, Δ, where A is a ground literal and Δ is a (possibly empty) sequence of ground literals. Suppose that $stratum(G_0)$ is $s+1$. Suppose further that A is the selected atom in G_0. When $A = \sim A'$ is a negative literal, $PERF(P_{i+1}) \models \sim A'$

holds and G_0 has the successor $G_1 = \leftarrow \Delta$. Since $stratum(\leftarrow A')$ is less than $stratum(G_0)$, $PERF(P_i) \models \sim A'$ from the induction hypothesis on the total correctness (TC). Let Dr_i be a ground proof derivation of $P_i \cup \{G_0\}$. Then G_0 has the successor G_1 also in Dr_i. From the induction hypothesis of the length of a ground proof derivation, $PERF(P_i) \models \Delta$, thus $PERF(P_i) \models \Gamma$.

Next, suppose that A is a positive atom. Then, the proof is quite similar to that of Proposition 3.2.1 (see Section A.2), except that we should consider a ground proof derivation instead of an SLDNF-refutation. So we omit the proof.

Proof of (TC): The proof of the total correctness (TC) is done quite similarly to that of Proposition 3.3.2. First, note that Lemma A.4.1 and Lemma A.4.3 hold also in this case. Thus, what we should prove are those lemmas corresponding to Lemma A.4.2 and Lemma A.4.4. Since the proofs of both lemmas are shown as in their counterparts in the previous section, we only show in the following the proof of the counterpart of Lemma A.4.2.

Lemma A.5.1 (weight completeness is sufficient for total correctness w.r.t. the perfect model semantics). *Let $P_0, \ldots, P_N$ be a sequence of program transformation. If P_{i+1} is weight complete, then $PERF(P_{i+1}) \supseteq PERF(P_0)$ $(N-1 \geq i \geq 0)$.*

Proof. First, we introduce the following well-founded ordering $>$ into the set of a (possibly empty) sequence of ground literals Γ in $PERF(P_0)$, i.e., $\Gamma_1 > \Gamma_2$ if and only if

(1) $w(\Gamma_1) > w(\Gamma_2)$, or

(2) $w(\Gamma_1) = w(\Gamma_2)$ and the number of new atoms in Γ_1 is greater than that of new atoms in Γ_2.

We show the lemma by induction on the above defined well-founded ordering.

As for the induction basis, i.e., when Γ is empty, the lemma is obvious. Next, suppose that Γ is of the form: A, Δ, where Δ is a possibly empty sequence of ground literals and A is the selected literal in G_0 of a ground proof derivation of $P_0 \cup \{G_0 = \leftarrow A, \Delta\}$. When A is a negative literal, say, $\sim A'$, $PERF(P_0) \models \sim A'$ holds. Thus G_0 has the child node $G_1 = \leftarrow \Delta$ such that $\Delta \in PERF(P_0)$. On the other hand, from the partial correctness (PC), $PERF(P_{i+1}) \models \sim A'$ holds. Thus, also in a ground proof derivation of $P_{i+1} \cup \{G_0 = \leftarrow A, \Delta\}$, when A is selected in G_0, G_0 has the child node G_1. From the induction hypothesis on the well-founded ordering $>$, Δ is in $PERF(P_{i+1})$. Thus so is Γ. Otherwise, suppose that A is a positive ground literal. From the definition that $\Gamma = A, \Delta$ is in $PERF(P_0)$, there exists a ground proof derivation of $P_0 \cup \{\leftarrow A, \Delta\}$.

As P_{i+1} is weight complete and $A \in PERF(P_0)$, there exists a descent clause C for A in P_{i+1}. Let C be of the form: $A \leftarrow L$. Then, from the definition of the descent clause, the following conditions hold:

(W1) $PERF(P_0) \models L$,

(W2) $w(A) \geq w(L)$, and

(W3) if C satisfies the folding condition (F4), then $w(A) > w(L)$.

Now, consider a ground proof derivation Dr of $P_{i+1} \cup \{\leftarrow A, \Delta\}$. Then, using C as an input clause, the initial goal $G_0 = \leftarrow A, \Delta$ has the successor G_1: $\leftarrow L, \Delta$.

If $w(A) > w(L)$ holds, then $(A, \Delta) > (L, \Delta)$. Thus, from the induction hypothesis on the well-founded ordering $\succ$, we have that $PERF(P_{i+1}) \models A, \Delta$.

On the other hand, when $w(A) = w(L)$ holds, from the condition (W3), C does not satisfy the folding condition (F4). From Lemma A.4.1, A is a new atom, while all atoms in L are old atoms. Thus it follows that $(A, \Delta) > (L, \Delta)$. Consequently, from the induction hypothesis, it is shown that $PERF(P_{i+1}) \models A, \Delta$. □

Acknowledgment

This work is based on the result by Tamaki and Sato, and the succeeding work by Kawamura and Kanamori. The author would like to express deep gratitude to them for their stimulating work. The idea of modified folding arose from discussions with Kazunori Ueda and Tadashi Kanamori. The author would like to thank anonymous referees for their useful comments.

References

[1] K.R. Apt, H. Blair, and A. Walker, Towards a theory of declarative knowledge, in: J. Minker, ed., *Foundations of Deductive Databases and Logic Programming* (Morgan Kaufmann, 1987. Los Altos, CA) 89-148.

[2] L. Cavedon and J.W. Lloyd, A completeness theorem for SLDNF-resolution, Technical Report CS-87-06, Computer Science Department, University Walk, Bristol, 1987.

[3] K.L. Clark, Negation as failure, in: H. Gallaire and J. Minker, eds. *Logic and Database* (Plenum, New York, 1978) 293-322.

[4] P.A. Gardner and J.C. Shepherdson, Unfold/fold transformations of logic programs, submitted for publication.

[5] J. Jaffar, J.-L. Lassez and J.W. Lloyd, Completeness of the negation as failure rule, in: *Proc. IJCAI-83* (1983) 500-506.

[6] T. Kanamori and K. Horiuchi, Construction of logic programs based on generalized unfold/fold rules, in: *Proc. 4th Internat. Conf. on Logic Programming* (1987) 744-768.

[7] T. Kawamura and T. Kanamori, Preservation of stronger equivalence in unfold/fold logic program transformation, ICOT Technical Report, ICOT, 1988; also in FGCS'88.

[8] J.-L. Lassez and M.J. Maher, Closures and fairness in the semantics of programming logic, *Theoret. Comput. Sci.* **29** (1984) 167-184.

[9] J.W. Lloyd, *Foundations of Logic Programming* (Springer, Berlin, 2nd ed., 1987).

[10] M.J. Maher, Equivalences of Logic Programs, in: *Proc. 3rd Internat. Conf. on Logic Programming*, (1986) 410-424; also in: J. Minker ed., *Foundations of Deductive Databases and Logic Programming* (Morgan Kaufmann, Los Altos, CA, 1987) 627-658.

[11] A. Martelli and U. Montanari, An efficient unification algorithm, *ACM TOPLAS* 4(2) (1982) 258-282.

[12] T.C. Przymusinski, On the declarative and procedural semantics of logic programs, submitted for publication. Its extended abstract appears in *Proc. 5th Internat. Conf. Symp. on Logic Programming*, Seattle (1988).

[13] J.A. Robinson, A machine-oriented logic based on the resolution principle, *J. ACM* **12**(1) (1965) 23-41.

[14] J.C. Shepherdson, Negation as failure: a comparison of Clark's completed data base and Reiter's closed world assumption. *J. Logic Programming* **1** (1984) 51-79.

[15] H. Tamaki, *Program Transformation in Logic Programming* (*Kyoritsu Pub. Co.*, 1987) 39-62, (*in Japanese*).

[16] H. Tamaki and T. Sato, Unfold/fold transformation of logic programs, in: *Proc. 2nd Internat. Logic Programming Conf.* (1984) 127-138.

[17] A. Van Gelder, Negation as failure using tight derivations for general logic programs, in: *Proc. 1986 Symp. on Logic Programming*, (1986) 127-138.

[18] D.A. Wolfram, M.J. Maher and J.-L. Lassez, A unified treatment of resolution strategies for logic programs, in: *Proc. 2nd Internat. Logic Programming Conf.* (1984) 263-276.