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Tensor form factors of $B \to K_1$ transition from QCD light cone sum rules

T.M. Aliev [a,1], M. Savcı [a,*], Kwei-Chou Yang [b]

[a] Physics Department, Middle East Technical University, 06531 Ankara, Turkey
[b] Department of Physics, Chung Yuan Christian University, Chung-Li 320, Taiwan

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ABSTRACT

The tensor form factors of B into p-wave axial vector meson transition are calculated within light cone QCD sum rules method. The parametrizations of the tensor form factors based on the series expansion are presented.

1. Introduction

Rare decays due to the flavor-changing neutral current $b \to s$ ($b \to d$) transitions constitute one of the most important classes of decays in carefully checking the predictions of the Standard Model (SM) at tree level, since they are forbidden in SM at loop level. In SM the flavor changing neutral current (FCNC) processes $b \to s\ell^+\ell^-$ proceed through the electroweak penguin and box diagrams. These decays are also very suitable in looking for new physics (NP) beyond the SM, via contributions of the new particles to the penguin and box diagrams, that are absent in the SM.

The $B \to K^*(892)\ell^+\ell^-$ decay has been observed in [1,2]. Moreover, the forward–backward asymmetry has been measured in [3,4]. The longitudinal polarization and forward–backward asymmetry of $B \to K^*(892)\ell^+\ell^-$ and the isospin asymmetry of $B^0 \to K^{*0}(892)\ell^+\ell^-$ and $B^\pm \to K^{*\pm}(892)\ell^+\ell^-$ are also measured by BaBar Collaboration in [5] and [6], respectively. The experimental results are more or less in agreement with the predictions of SM. However, the precision of experiments is currently too low to make the final conclusion. The situation should considerably be improved at LHCb.

The radiative decays of B meson, involving $K_1(1270)$, where K_1 is the orbitally excited state, is observed by BELLE. The other radiative and semileptonic decay modes involving $K_1(1270)$ and $K_1(1400)$ are hopefully expected to be measured soon.

Similar to the $B \to K^*(892)\ell^+\ell^-$ decay the $B \to K_1\ell^+\ell^-$ decay is also a very good object for probing the new physics effects beyond the SM. Here the problem becomes more sophisticated due to the mixing of $K_{1A}(1^1P_3)$ and $K_{1B}(1^1P_1)$ state. The physical states $K_1(1270)$ and $K_1(1400)$ are determined by

$$\begin{pmatrix} |K_1(1400)\rangle \\ |K_1(1270)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}. \qquad (1)$$

In the present work we calculate the tensor form factors for the $B \to K_{1A(B)}$ transition in the framework of the light cone QCD sum rules method (LCSR) (for more about LCSR see [7,8]).

* Corresponding author.
E-mail addresses: taliev@metu.edu.tr (T.M. Aliev), savci@metu.edu.tr (M. Savcı), kcyang@cycu.edu.tw (K.-C. Yang).
[1] Permanent address: Institute of Physics, Baku, Azerbaijan.

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The Letter is organized in the following way. In Section 2 we derive the LCSR for the tensor form factors describing the $B \to K_{1A(B)}$ transition. Section 3 is devoted to the numerical analysis of the sum rules for the form factors. We also summarize our results in this section.

2. Light cone QCD sum rules for the tensor form factors of the $B \to K_{1A(B)}$ transition

The $B \to K_{1A(B)}\ell^+\ell^-$ decay is described by $b \to s\ell^+\ell^-$ transition at quark level. The effective Hamiltonian responsible for the $b \to s\ell^+\ell^-$ transition is given by,

$$\mathcal{H} = -4\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}(\mu), \tag{2}$$

where the form of the local Wilson operators $\mathcal{O}_i$ $(i = 1, \ldots, 10)$ is given in [9]. This effective Hamiltonian leads to the following result for the $b \to s\ell^+\ell^-$ decay amplitude

$$\mathcal{M} = \frac{G_F}{2\sqrt{2}}\frac{\alpha_{\rm em}}{\pi} V_{tb}V_{ts}^* \left\{ C_9^{\rm eff}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu\ell + C_{10}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu\gamma_5\ell - 2\frac{m_b}{q^2}C_7\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)\bar{\ell}\gamma_\mu\ell \right\}, \tag{3}$$

where the Wilson coefficient $C_9^{\rm eff} = C_9 + Y$, with $Y = Y_{pert} + Y_{LD}$, contains both the perturbative and the long distance contribution parts. The explicit expression of C_7, C_9, Y_{pert} and C_{10} are given in [9]. The long distance effects generated by the four-quark operators with the c-quark have recently been calculated for the $B \to K^*\ell^+\ell^-$ and $B \to K\ell^+\ell^-$ decays in [10] and it is obtained that below the charmonium region of q^2 this effect can change the value of C_9 around 5% and 20% for $B \to K$ and $B \to K^*$ transitions, respectively. Similar calculations for $B \to K_1$ transition has not yet been calculated. For simplicity, in the following discussions we denote K_{1A} and K_{1B} as K_1.

It follows from Eq. (3) that for the calculation of the $B \to K_1$ transition, the matrix elements $\langle K_1(p,\lambda)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle$ and $\langle K_1(p,\lambda)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle$ are needed. For the $B \to K_1$ transition, these matrix elements are defined in terms of the form factors as follows:

$$\begin{aligned}
&\big\langle K_1(p,\lambda)\big|\bar{s}\gamma_\mu(1-\gamma_5)b\big|B(p_B)\big\rangle \\
&\quad = -i\frac{2}{m_B+m_{K_1}}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{(\lambda)*\nu}p_B^\alpha p^\beta A^{K_1}(q^2) - \left[(m_B+m_{K_1})\varepsilon_\mu^{(\lambda)*}V_1^{K_1}(q^2) - (p_B+p)_\mu\big(\varepsilon^{(\lambda)*}p_B\big)\frac{V_2^{K_1}(q^2)}{m_B+m_{K_1}}\right] \\
&\qquad + 2m_{K_1}\frac{(\varepsilon^{(\lambda)*}p_B)}{q^2}q_\mu\big[V_3^{K_1}(q^2) - V_0^{K_1}(q^2)\big],
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\big\langle K_1(p,\lambda)\big|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\big|B(p_B)\big\rangle \\
&\quad = 2T_1^{K_1}(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{(\lambda)*\nu}p_B^\alpha p^\beta - iT_2^{K_1}(q^2)\big[(m_B^2-m_{K_1}^2)\varepsilon_\mu^{(\lambda)*} - \big(\varepsilon^{(\lambda)*}q\big)(p_B+p)_\mu\big] \\
&\qquad - iT_3^{K_1}(q^2)\big(\varepsilon^{(\lambda)*}q\big)\left[q_\mu - \frac{q^2}{m_B^2-m_{K_1}^2}(p_B+p)_\mu\right],
\end{aligned} \tag{5}$$

where $q = p_B - p$. There are the following relations between the form factors:

$$V_3^{K_1}(q^2) = \frac{m_B+m_{K_1}}{2m_{K_1}}V_1^{K_1}(q^2) - \frac{m_B-m_{K_1}}{2m_{K_1}}V_2^{K_1}(q^2), \qquad V_3^{K_1}(0) = V_0^{K_1}(0), \quad \text{and} \quad T_1^{K_1}(0) = T_2^{K_1}(0). \tag{6}$$

To be able to calculate the form factors responsible for the $B \to K_1$ transition we consider the following two correlation functions:

$$\Pi_\mu = i\int d^4x\, e^{iqx}\big\langle K_1(p,\lambda)\big|T\big\{\bar{s}(x)\gamma_\mu(1-\gamma_5)b(x)\bar{b}(0)i\gamma_5 d(0)\big\}|0\rangle, \tag{7}$$

$$\Pi_{\mu\nu} = i\int d^4x\, e^{iqx}\big\langle K_1(p,\lambda)\big|T\big\{\bar{s}(x)\sigma_{\mu\nu}\bar{b}(x)b(0)i\gamma_5 d(0)\big\}|0\rangle. \tag{8}$$

In order to construct the sum rules for the form factors responsible for the $B \to K_1$ transition these correlation functions should be calculated in two different languages, in terms of hadrons and quark and gluon degrees of freedom. The calculation of the correlation function in terms of quark and gluon degrees of freedom is carried out at virtualities $m_b^2 - p_B^2 \geqslant \Lambda_{QCD}m_b$ and $m_b^2 - q^2 \geqslant \Lambda_{QCD}m_b$. Using the operator product expansion, the sum rules are obtained by equating these two representations through the dispersion relations.

Phenomenological parts of the correlation functions (7) and (8) can be obtained by inserting complete set of hadrons with the same quantum numbers as the interpolating current, and separating the ground state one can easily obtain

$$\Pi_\mu = -\frac{\langle K_1(p,\lambda)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle\langle B(p_B)|\bar{b}i\gamma_5 d|0\rangle}{p_B^2 - m_B^2} + \cdots, \tag{9}$$

$$\Pi_{\mu\nu} = -\frac{\langle K_1(p,\lambda)|\bar{s}\sigma_{\mu\nu}b|B(p_B)\rangle\langle B(p_B)|\bar{b}i\gamma_5 d|0\rangle}{p_B^2 - m_B^2} + \cdots, \tag{10}$$

where "$\cdots$" describes the contributions coming from higher states and continuum, and the matrix element $\langle K_1(p,\lambda)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle$ is given in Eq. (4). The second matrix element in Eq. (9) is expressed in the standard way

$$\langle B(p_B)\big|\bar{b}i\gamma_5 d|0\rangle = \frac{f_B m_B^2}{m_b}, \tag{11}$$

where f_B is the B-decay constant and m_b is the b-quark mass. The matrix element $\langle K_1(p,\lambda)|\bar{s}\sigma_{\mu\nu}b|B(p_B)\rangle$ is defined as

$$\begin{aligned}&\langle K_1(p,\lambda)\big|\bar{s}\sigma_{\mu\nu}b\big|B(p_B)\rangle\\&\quad = -iA(q^2)\big[\varepsilon_\mu^{(\lambda)*}(p+p_B)_\nu - \varepsilon_\nu^{(\lambda)*}(p+p_B)_\mu\big] + iB(q^2)\big(\varepsilon_\mu^{(\lambda)*}q_\nu - \varepsilon_\nu^{(\lambda)*}q_\mu\big) + i\frac{2C(q^2)}{m_B^2 - m_{K_1}}(p_\mu q_\nu - p_\nu q_\mu).\end{aligned} \tag{12}$$

Contracting Eq. (12) with the momentum q^ν and using the relation

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta},$$

the following relations among A, B and C can easily be obtained:

$$T_1^{K_1}(q^2) = A(q^2), \qquad T_2^{K_1}(q^2) = A(q^2) - \frac{q^2}{m_B^2 - m_{K_1}^2}B(q^2), \qquad T_3^{K_1}(q^2) = B(q^2) + C(q^2). \tag{13}$$

Using Eqs. (11) and (12), for the phenomenological parts of the correlation functions we get

$$\begin{aligned}\Pi_\mu = -\frac{f_B m_B^2}{m_b}\frac{1}{p_B^2 - m_B^2}\bigg\{&-\frac{2i}{m_B^2 - m_{K_1}^2}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{(\lambda)*\nu}p_B^\alpha p^\beta A^{K_1}(q^2)\\&-\bigg[(m_B + m_{K_1})\varepsilon_\mu^{(\lambda)*}V_1^{K_1}(q^2) - P_\mu\big(\varepsilon^{(\lambda)*}q\big)\frac{V_2^{K_1}(q^2)}{m_B + m_{K_1}}\bigg] + 2m_{K_1}\frac{\varepsilon^{(\lambda)*}q}{q^2}q_\mu\big[V_3^{K_1}(q^2) - V_0(q^2)\big]\bigg\},\end{aligned} \tag{14}$$

$$\Pi_{\mu\nu} = -\frac{f_B m_B^2}{m_b}\frac{1}{p_B^2 - m_B^2}\bigg\{-iA(q^2)\big(\varepsilon_\mu^{(\lambda)*}P_\nu - \varepsilon_\nu^{(\lambda)*}P_\mu\big) + iB(q^2)\big(\varepsilon_\mu^{(\lambda)*}q_\nu - \varepsilon_\nu^{(\lambda)*}q_\mu\big) + 2iC(q^2)\frac{\varepsilon^{(\lambda)*}q}{m_B^2 - m_{K_1}^2}(p_\mu q_\nu - p_\nu q_\mu)\bigg\}, \tag{15}$$

where $P = p_B + p$.

We now proceed to calculate the theoretical part of the correlation functions. The calculation is performed by using the background field approach [11]. In the large virtuality region, where $m_b^2 - p_B^2 \gg \Lambda_{QCD}m_b$ and $m_c^2 - q^2 \gg \Lambda_{QCD}m_b$, the operator product expansion is applicable to the correlation functions. In light cone sum rules the method is based on the expansion of the non-local quark–antiquark operators in powers of the deviation from the light cone. In obtaining the expression of the correlation functions the propagator of heavy quark and the matrix elements of the non-local operators $\bar{q}(x_1)\Gamma_i q(x_2)$ and $\bar{q}(x_1)G_{\mu\nu}q(x_2)$ between the vacuum and axial vector meson are needed, where Γ_i are the Dirac matrices (in our case $\gamma_\mu(1-\gamma_5)$ or $\sigma_{\mu\nu}$), and $G_{\mu\nu}$ is the gluon field strength tensor.

The expression of the heavy quark operator is given in [12]

$$S_Q = S_Q^{free}(x) + ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int du\bigg[\frac{\not{k} + m_b}{2(m_b^2 - k^2)^2}G_{\mu\nu}(ux)\sigma^{\mu\nu} + \frac{u}{m_b^2 - k^2}x_\mu G^{\mu\nu}(ux)\gamma_\nu\bigg], \tag{16}$$

where S_Q^{free} is the free quark operator and we adopt the convention for covariant derivative $D_\alpha = \partial_\alpha + ig_s A_\alpha^a\lambda^a/2$.

Two particle distribution amplitudes for the axial vector mesons are presented in [13,14]

$$\begin{aligned}&\langle K_1(p,\lambda)\big|\bar{s}_\alpha(x)q_\delta(0)|0\rangle\\&\quad = -\frac{i}{4}\int du\, e^{iupx}\bigg\{f_{K_1}m_{K_1}\bigg[\not{p}\gamma_5\frac{\varepsilon^{(\lambda)*}x}{px}\phi_\parallel(u) + \bigg(\not{\varepsilon}^{(\lambda)*} - \frac{\varepsilon^{(\lambda)*}x}{px}\not{p}\bigg)\gamma_5 g_\perp^{(a)}(u) - \not{x}\gamma_5\frac{\varepsilon^{(\lambda)*}x}{2(px)^2}m_{K_1}^2\bar{g}_3(u) + \epsilon_{\mu\nu\rho\sigma}\varepsilon^{(\lambda)*\nu}p^\rho x^\sigma\gamma^\mu\frac{g_\perp^{(v)}}{4}\bigg]\\&\qquad + f_\perp^A\bigg[\frac{1}{2}\big(\not{p}\not{\varepsilon}^{(\lambda)*} - \not{\varepsilon}^{(\lambda)*}\not{p}\big)\gamma_5\phi_\perp(u) - \frac{1}{2}(\not{p}\not{x} - \not{x}\not{p})\gamma_5\frac{\varepsilon^{(\lambda)*}x}{(px)^2}m_{K_1}^2\bar{h}_\parallel^{(t)}(u) - \frac{1}{4}\big(\not{\varepsilon}^{(\lambda)*}\not{x} - \not{x}\not{\varepsilon}^{(\lambda)*}\big)\gamma_5\frac{m_{K_1}^2}{px}\bar{h}_3(u)\\&\qquad + i\big(\varepsilon^{(\lambda)*}x\big)m_{K_1}^2\gamma_5\frac{h_\parallel^{(p)}(u)}{2}\bigg] + \mathcal{O}(x^2)\bigg\}_{\delta\alpha},\end{aligned} \tag{17}$$

where

$$\bar{g}_3(u) = g_3(u) + \phi_\parallel - 2g_\perp^{(a)}(u), \qquad \bar{h}_\parallel^{(t)}(u) = h_\parallel^{(t)}(u) - \frac{1}{2}\phi_\perp(u) - \frac{1}{2}h_3(u), \qquad \bar{h}_3(u) = h_3(u) - \phi_\perp(u), \tag{18}$$

and $\phi_\parallel$, $\phi_\perp$ are the twist-2, $g_\perp^{(a)}$, $g_\perp^{(v)}$, $h_\parallel^{(t)}$ and $h_\parallel^{(p)}$ are twist-3, and g_3 and h_3 are twist-4 functions. The three particle distribution amplitudes are define as

$$\begin{aligned}&\langle K_1(p,\lambda)\big|\bar{s}(x)\gamma_\alpha\gamma_5 g_s G_{\mu\nu}(ux)q(0)|0\rangle = p_\alpha\big(p_\nu\varepsilon_\mu^{(\lambda)*} - p_\mu\varepsilon_\nu^{(\lambda)*}\big)f_{3K_1}^A\mathcal{A} + \cdots,\\&\langle K_1(p,\lambda)\big|\bar{s}(x)\gamma_\alpha g_s\tilde{G}_{\mu\nu}(ux)q(0)|0\rangle = ip_\alpha\big(p_\mu\varepsilon_\nu^{(\lambda)*} - p_\nu\varepsilon_\mu^{(\lambda)*}\big)f_{3K_1}^V\mathcal{V} + \cdots,\end{aligned} \tag{19}$$

where

$$\mathcal{A} = \int \mathcal{D}\alpha\, e^{iPx(\alpha_1+u\alpha_3)} \mathcal{A}(\alpha_1,\alpha_2,\alpha_3), \quad \text{and} \quad \int \mathcal{D}\alpha = \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \int_0^1 d\alpha_3\, \delta(1-\alpha_1-\alpha_2-\alpha_3).$$

Here α_1, α_2, and α_3 are the respective momentum fractions carried by s, $\bar{q}$ quarks and gluon in the meson. Using these definitions, and after lengthy calculations for the theoretical parts of the correlation functions, we obtain

$$\begin{aligned}
\textit{Correlation function} = {} & \frac{1}{4}\int du \Bigg\{ f_{K_i} m_{K_i} \Bigg[\varepsilon_\alpha^{(\lambda)*} \Phi_a^{(i)} \frac{\partial}{\partial Q_\alpha} \mathrm{Tr}(\Gamma \not{p} S_Q) - g_\perp^a \,\mathrm{Tr}\big(\Gamma S_Q \not{\varepsilon}^{(\lambda)*}\big) \\
& - \frac{1}{2} m_{K_i}^2 \bar{g}_3^{(ii)} \varepsilon_\alpha^{(\lambda)*} \frac{\partial}{\partial Q_\alpha}\frac{\partial}{\partial Q_\beta} \mathrm{Tr}(\Gamma S_Q \gamma_\beta) + i\epsilon_{\alpha\beta\rho\sigma} \frac{g_\perp^v}{4} \varepsilon_\beta^{(\lambda)*} p^\rho \frac{\partial}{\partial Q_\sigma} \mathrm{Tr}(\Gamma S_Q \gamma_\alpha \gamma_5) \Bigg] \\
& + f_{K_i}^\perp \Bigg[\frac{1}{2}\phi_\perp(u)\,\mathrm{Tr}\big[\Gamma S_Q \big(\not{p}\not{\varepsilon}^{(\lambda)*} - \not{\varepsilon}^{(\lambda)*}\not{p}\big)\big] - \frac{1}{2} m_{K_i}^2 \bar{h}_\parallel^{(t)(ii)} \varepsilon_\alpha^{(\lambda)*} \frac{\partial}{\partial Q_\alpha}\frac{\partial}{\partial Q_\beta} \mathrm{Tr}\big[\Gamma S_Q (\not{p}\gamma_\beta - \gamma_\beta \not{p})\big] \\
& + \frac{h_3^{(i)}}{4} m_{K_i}^2 \frac{\partial}{\partial Q_\alpha} \mathrm{Tr}\big[\Gamma S_Q \gamma_5 \big(\not{\varepsilon}^{(\lambda)*}\gamma_\alpha - \gamma_\alpha \not{\varepsilon}^{(\lambda)*}\big)\big] + \frac{1}{2} h_\parallel^{(p)} m_{K_i}^2 \varepsilon_\alpha^{(\lambda)*} \frac{\partial}{\partial Q_\alpha} \mathrm{Tr}[\Gamma S_Q] \Bigg] \Bigg\} \\
& + \frac{1}{4}\int dv \int \mathcal{D}\alpha_i \frac{1}{\{m_b^2 - [q + (\alpha_1 + v\alpha_3)p]^2\}^2} \big\{ 2vpq\big[f_{3i}^A \mathcal{A}(\alpha_i) + f_{3i}^V \mathcal{V}(\alpha_i)\big] \mathrm{Tr}\big(\Gamma \not{\varepsilon}^{(\lambda)*} \not{p}\big)\big\},
\end{aligned} \tag{20}$$

where

$$S_Q = \frac{m_b + \not{Q}}{m_b^2 - Q^2}, \quad \text{with } Q = q + pu,$$

$$\Phi_a^{(i)} = \int_0^u \big[\phi_\parallel^{(v)} - g_\perp^a(v)\big] dv, \qquad f^{(i)} = \int_0^u f(v)\, dv, \qquad f^{(ii)} = \int_0^u dv \int_0^v dv'\, f\big(v'\big),$$

and $i = 1$ (2) correspond to K_{1A} (K_{1B}), respectively, Γ is equal to $\gamma_\mu(1-\gamma_5)$ or $\sigma_{\mu\nu}$. After taking derivatives and traces, equating expressions of correlation functions (14), (15) and (20), and performing Borel transformation with respect to the variable $-(p+q)^2$ in order to suppress the higher states and continuum contributions, one can obtain the sum rules for the transition form factors. Here we present the sum rules only for the tensor form factors, since $V_1^{K_1}$, $V_2^{K_1}$, $V_0^{K_1}$ and A^{K_1} are calculated within the same framework in [15]:

$$\begin{aligned}
A_i\big(q^2\big) = {} & -\frac{m_b^2 f_{\perp i}}{2m_B^2 f_B} e^{m_B^2/M^2} \int_0^1 du \Bigg\{ \frac{1}{u} e^{-s(u)/M^2} \theta\big[s_0 - s(u)\big] \Bigg[\phi_\perp(u) - \frac{m_i f_i}{m_b f_{\perp i}} \bigg(u g_\perp^{(a)}(u) + \phi_a^{(i)} + \frac{g_\perp^v(u)}{4} \bigg) \Bigg] \\
& - \frac{1}{4} \frac{e^{-s(u)/M^2} m_i f_i}{u m_b f_{\perp i}} \big(m_b^2 + q^2\big) g_\perp^v(u) \bigg(\frac{\theta[s_0 - s(u)]}{uM^2} + \frac{\delta[u - u_0]}{s_0 - q^2} \bigg) \Bigg\} \\
& - \frac{m_b}{2m_B^2 f_B} e^{m_B^2/M^2} \int_0^1 v\, dv \int e^{-s(k)/M^2} d\alpha_1\, d\alpha_3 \frac{f_{3i}^A \mathcal{A}(\alpha_i) + f_{3i}^V \mathcal{V}(\alpha_i)}{(\alpha_1 + v\alpha_3)^2} \\
& \times \bigg\{ \theta\big[s_0 - s(k)\big] - \big(m_b^2 - q^2\big) \bigg(\frac{\theta[s_0 - s(k)]}{(\alpha_1 + v\alpha_3)M^2} + \frac{\delta[k - u_0]}{s_0 - q^2} \bigg) \bigg\},
\end{aligned}$$

$$\begin{aligned}
B_i\big(q^2\big) = {} & -\frac{m_b^2 f_{\perp i}}{2m_B^2 f_B} e^{m_B^2/M^2} \int_0^1 du \Bigg\{ \frac{1}{u} e^{-s(u)/M^2} \theta\big[s_0 - s(u)\big] \Bigg[\phi_\perp(u) - \frac{m_i f_i}{m_b f_{\perp i}} \bigg(-(2-u) g_\perp^{(a)}(u) + \phi_a^{(i)} + \bigg(1 - \frac{2}{u}\bigg) \frac{g_\perp^v(u)}{4} \bigg) \Bigg] \\
& - \frac{1}{u} e^{-s(u)/M^2} \frac{1}{4} \frac{m_i f_i}{m_b f_{\perp i}} \bigg[2m_b^2 - \big(m_b^2 - q^2\big)\bigg(1 - \frac{2}{u}\bigg) \bigg] g_\perp^{(v)}(u) \bigg(\frac{\theta[s_0 - s(u)]}{uM^2} + \frac{\delta[u - u_0]}{s_0 - q^2} \bigg) \Bigg\} \\
& - \frac{m_b}{2m_B^2 f_B} e^{m_B^2/M^2} \int_0^1 v\, dv \int e^{-s(k)/M^2} d\alpha_1 d\alpha_3 \frac{f_{3i}^A \mathcal{A}(\alpha_i) + f_{3i}^V \mathcal{V}(\alpha_i)}{(\alpha_1 + v\alpha_3)^2} \\
& \times \bigg\{ \theta\big[s_0 - s(k)\big] - \big(m_b^2 - q^2\big) \bigg(\frac{\theta[s_0 - s(k)]}{(\alpha_1 + v\alpha_3)M^2} + \frac{\delta[k - u_0]}{s_0 - q^2} \bigg) \bigg\},
\end{aligned}$$

$$C_i\big(q^2\big) = \frac{m_b m_i f_i}{2f_B} e^{m_B^2/M^2} \int_0^1 du \Bigg\{ \frac{1}{u} e^{-s(u)/M^2} \bigg(2\phi_a^{(i)}(u) - \frac{g_\perp^{(v)}(u)}{2} \bigg) \bigg(\frac{\theta[s_0 - s(u)]}{uM^2} + \frac{\delta[u - u_0]}{s_0 - q^2} \bigg) \Bigg\}, \tag{21}$$

where $i = A$ or B, and

$$s(n) = \frac{m_b^2 - (1-n)q^2}{n}, \qquad k \equiv \alpha_1 + v\alpha_3, \quad \text{and} \quad u_0 = \frac{m_b^2 - q^2}{s_0 - q^2}.$$

In these expressions, we neglect terms $\sim m_{K_i}^2$. Using Eq. (13) one can easily obtain the corresponding sum rules for T_1, T_2 and T_3 tensor form factors.

A few words about the form factors responsible for the $B \to K_1$ transition, in the large recoil region in the heavy quark limit, are in order. It can be shown that, similar to the $B \to V$ (vector meson) case, all seven form factors responsible for the $B \to K_1$ transition can be expressed in terms of the independent functions $\xi_\perp^{K_1}(q^2)$ and $\xi_\parallel^{K_1}(q^2)$, in the large recoil region and in the heavy quark limit. Indeed we find that, for to the $B \to K_1$ transition these form factors can be written in terms of $\xi_\perp^{K_1}(q^2)$ and $\xi_\parallel^{K_1}(q^2)$ as:

$$\begin{aligned}
&V_0^{K_1}(q^2) = \left(1 - \frac{m_{K_1}^2}{m_B E}\right)\xi_\parallel^{K_1}(q^2) + \frac{m_{K_1}}{m_B}\xi_\perp^{K_1}(q^2),\\
&V_1^{K_1}(q^2) = \left(\frac{2E}{m_B + m_{K_1}}\right)\xi_\perp^{K_1}(q^2), \qquad V_2^{K_1}(q^2) = \left(1 + \frac{m_{K_1}}{m_B}\right)\xi_\perp^{K_1}(q^2) - \frac{m_{K_1}}{E}\xi_\parallel^{K_1}(q^2),\\
&A^{K_1}(q^2) = \left(1 + \frac{m_{K_1}}{m_B}\right)\xi_\perp^{K_1}(q^2), \qquad T_1^{K_1}(q^2) = \xi_\perp^{K_1}(q^2), \qquad T_2^{K_1}(q^2) = \left(1 - \frac{q^2}{m_B^2 - m_{K_1}^2}\right)\xi_\perp^{K_1}(q^2),\\
&T_3^{K_1}(q^2) = \xi_\perp^{K_1}(q^2) - \left(1 - \frac{m_{K_1}^2}{m_B^2}\right)\frac{m_{K_1}}{E}\xi_\parallel^{K_1},
\end{aligned} \tag{22}$$

where

$$E = \frac{m_B^2 + m_{K_1}^2 - q^2}{2m_B},$$

is the energy of K_1 meson.

Explicit expressions of the functions $\xi_\perp^{K_1}(q^2)$ and $\xi_\parallel^{K_1}(q^2)$ can be obtained following the same steps of calculation as is given in [16]. These expressions are quite lengthy, and therefore we do not present them here in this work.

Our final remark in this section is as follows. The physical $K_1(1270)$ and $K_1(1400)$ are the mixing states of K_{1A} and K_{1B}, and the form factors for the $B \to K_1(1270)$ and $B \to K_1(1400)$ transitions can be obtained from $B \to K_{1A}$ and $B \to K_{1B}$ transition form factors with the help of following transformations,

$$\begin{aligned}
&\begin{pmatrix} \langle \bar{K}_1(1270)|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle \\ \langle \bar{K}_1(1400)|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle \end{pmatrix} = \mathcal{M}\begin{pmatrix} \langle \bar{K}_{1A}|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle \\ \langle \bar{K}_{1B}|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle \end{pmatrix},\\
&\begin{pmatrix} \langle \bar{K}_1(1270)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B\rangle \\ \langle \bar{K}_1(1400)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B\rangle \end{pmatrix} = \mathcal{M}\begin{pmatrix} \langle \bar{K}_{1A}|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B\rangle \\ \langle \bar{K}_{1B}|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B\rangle \end{pmatrix},
\end{aligned}$$

where

$$\mathcal{M} = \begin{pmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix}.$$

From the analysis of $B \to K_1\gamma$ and $\tau^- \to K_1(1270)\nu_\tau$ decays, the mixing angle θ is obtained to have the value $\theta = -(34^\circ \pm 13^\circ)$, where the minus sign is related to the relative phases of $|\bar{K}_{1A}\rangle$ and $|\bar{K}_{1B}\rangle$. The phases are fixed by adopting the conventions $f_{K_{1A}} > 0$ and $f_{K_{1B}} > 0$.

3. Numerical analysis

In this section we present our numerical analysis of the sum rules for the form factors. The main parameters entering to the sum rules for the $B \to K_1$ transition form factors are the leptonic decay constant of the B_d meson, DAs of the K_1 meson, the mass of the b-quark, Borel parameter M^2 and the continuum threshold s_0. The explicit expressions of the DAs for K_{1A} and K_{1B} mesons, as well as the parameters in the DAs are given in [13,14] and their properties are presented them in Appendix A.

Few words about the value of the leptonic decay constant f_B are in order. It is shown in [17] that the pole mass of the b-quark produces rather large higher-order radiative NLO corrections in the results for f_B. Moreover, it is shown in this work that in the $\overline{MS}$ scheme the higher order corrections are under control, and therefore, the predictions for f_B is more reliable. For this reason in further numerical analysis we use the value $f_B = 210 \pm 19$ MeV as is obtained in [17] within the framework of $\overline{MS}$ scheme, at $\mu = \overline{m}_b$ scale. For the $\overline{MS}$ mass $\overline{m}_b$ we use $\overline{m}_b(2\ \text{GeV}) = 4.98$ GeV which is obtained from $\overline{m}_b(\overline{m}_b) = 4.2$ GeV [17]. For the strange quark mass we use the value $m_s(2\ \text{GeV}) = 102 \pm 8$ MeV, which is obtained from the analysis of pseudoscalar QCD sum rules [18].

As has already been noted, the sum rules for the form factors also contain two auxiliary parameters: the Borel mass parameter M^2 and the continuum threshold s_0, and obviously, any physical quantity should be independent of them. For this reason, we try to find such regions these parameters where physically measurable quantities are independent of them.

In determining the value of the continuum threshold s_0, we require that the prediction of the mass sum rules for the B meson coincides with the experimental data. We also require that s_0 must not be far from the "reliable" region, i.e., it should be below the next resonance in this channel. These conditions lead to the result that, the expected values of s_0 lie in the interval $33\ \text{GeV}^2 \leqslant s_0 \leqslant 38\ \text{GeV}^2$. The upper bound of M^2 is determined by demanding that the total contributions of higher states and continuum threshold should be less than half of the dispersion integral. The lower limit of M^2 is found if we require that the highest power $1/M^2$ term contributes less than, say, 25% of the sum rules. Both these conditions are satisfied when M^2 varies in the region $6\ \mathrm{GeV}^2 \leqslant M^2 \leqslant 16\ \mathrm{GeV}^2$. In numerical calculations we have used $M^2 = 10\ \mathrm{GeV}^2$ and $s_0 = 34\ \mathrm{GeV}^2$.

Unfortunately, the sum rules cannot predict the dependence of the form factors on q^2 in the relevant physical region $4m_\ell^2 \leqslant q^2 \leqslant (m_B - m_{K_1})^2$. The sum rules results are not reliable when $q^2 > 10\ \mathrm{GeV}^2$. In order to extend the results for the form factors coming from QCD sum rules predictions to cover the whole physical region, we look for a parametrization of the form factors in such a way that, the results obtained for the region $4m_\ell^2 \leqslant q^2 \leqslant 10\ \mathrm{GeV}^2$ can be extrapolated to whole physical region.

To extend our calculations to whole physical region, we use the z-series parametrization (for more about this parametrization, see [19] and references therein), which is based on the analyticity of the form factors on q^2. Before presenting the series expansion of the tensor form factors of the $B \to K_1$ transition, following the work [19], we present the helicity amplitudes as the linear combination of the form factors as are defined in Eq. (4), which are more convenient for the analysis. After a simple calculation we obtain the following helicity amplitudes for the $B \to K_1$ transition (see also [19])

$$H_0(q^2) = \frac{\sqrt{q^2}(m_B^2 + 3m_{K_1}^2 - q^2)}{2m_{K_1}\sqrt{\lambda}} T_2(q^2) - \frac{\sqrt{q^2\lambda}}{2m_{K_1}(m_B^2 - m_{K_1}^2)} T_3(q^2),$$
$$H_1(q^2) = \sqrt{2}T_1(q^2), \qquad H_2(q^2) = \frac{\sqrt{2}}{\sqrt{\lambda}}(m_B^2 - m_{K_1}^2)T_2(q^2), \tag{23}$$

where $\lambda(m_B^2, m_{K_1}^2, q^2) = m_B^4 + m_{K_1}^4 + q^4 - 2m_B^2 m_{K_!}^2 - 2m_B^2 q^2 - 2m_{K_1}^2 q^2$, and subscripts 0 and 1, 2 correspond to the longitudinal and linear combinations of the transversal polarizations,

$$\varepsilon_{1,2}^\mu = \frac{1}{\sqrt{2}}\left[\varepsilon_-^\mu(q) \mp \varepsilon_+^\mu(q)\right],$$

of the virtual axial meson, respectively.

We define the following parametrization for the tensor form factors based on the z-series expansion,

$$H_0(q^2) = \frac{\sqrt{-z(q^2,0)}}{B(q^2)\sqrt{z(q^2,q_-^2)}\phi(q^2)} \sum_{k=0}^{N-1} \beta_k^{(0)} z^k,$$
$$H_1(q^2) = \frac{1}{B(q^2)\phi(q^2)} \sum_{k=0}^{N-1} \beta_k^{(1)} z^k, \qquad H_2(q^2) = \frac{1}{B(q^2)\sqrt{z(q^2,q_-^2)}\phi(q^2)} \sum_{k=0}^{N-1} \beta_k^{(2)} z^k, \tag{24}$$

where $z = z(q^2, q_0^2)$ is defined as,

$$z(q^2, q_0^2) = \frac{\sqrt{q_+^2 - q^2} - \sqrt{q_+^2 - q_0^2}}{\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2}}, \tag{25}$$

and $q_\pm^2 = (m_B \pm m_{K_1})^2$ and $B(q^2) = z(q^2, m_{res}^2)$. The parameter q_0^2 is chosen from the solution of the equation $z(0, q_0^2) = -z(q_-^2, q_0^2)$, which gives $q_0^2 = 10.55\ \mathrm{GeV}^2$. The mass of the resonances entering to the factor $B(q^2)$ are $m_B^*(1^-) = 5.41$ GeV for $H_0(q^2)$, $H_2(q^2)$, and $m_B^*(1^-) = 5.83$ GeV for $H_1(q^2)$, respectively (see [19]). The function $\phi(q^2)$ is given in Eq. (39) of the work [19]. In order to perform the numerical analysis for the helicity amplitudes, and derive the unitary bound one needs to calculate the two-point correlation function of the tensor current. This calculation is done in [19] and we will use the results of this work. For the z-series expansion parametrization, the unitarity constraint leads to the result

$$\sum_{k=0}^{N-1} \left\{\beta_k^{(0)2} + \beta_k^{(1)2} + \beta_k^{(2)2}\right\} \leqslant 1. \tag{26}$$

In Figs. 1–3 we present the fits of the series expansion parametrization to LCSR results for the helicity amplitudes $H_0(q^2)$, $H_1(q^2)$ and $H_2(q^2)$, respectively. In these figures, we take into account the uncertainties coming from Gegenbauer moments of the axial vector meson, mass of b and s quark, Borel mass M^2 and the threshold parameter s_0.

In Table 1 we present the values of the coefficients $\beta_k^{(0)K_{1A}}$ and $\beta_k^{(1)K_{1A}}$ $(k = 0, 1)$, entering to the series expansion of the helicity amplitudes $H_0(q^2)$, $H_1(q^2)$ and $H_2(q^2)$, and the unitarity constraint $\sum_{k=0}^{1}(\beta_k^{K_{1A}})^2$ given in Eq. (26). Due to the large cancellation among different terms in z-series expansion, we could not fix all coefficients β_k from the fit. Therefore, we keep only the first two terms in the expansion, i.e., we take $N = 2$ (see also [19]).

Our final remark is as follows: As has already been noted, in the numerical calculations we use $f_B = 210 \pm 19$ MeV. If $f_B = 145$ MeV [20] had been used the values of all form factors presented in this work increase by a factor 1.4.

In summary, we calculate the tensor form factors of B decays into P-wave axial vector meson. These form factors are relevant to the studies for the exclusive FCNC transitions. The sum rules obtained could further be improved by including the $\mathcal{O}(\alpha_s)$ corrections, as well as, improving the values of the input parameters involving the DAs.

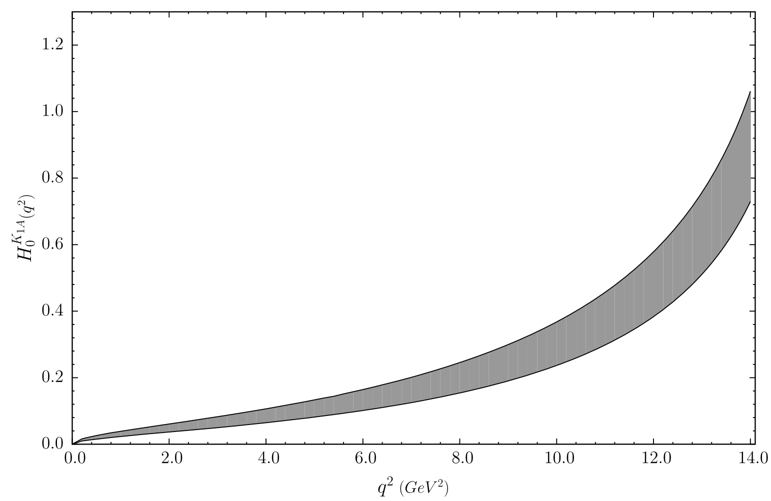

Fig. 1. The dependence of the helicity amplitude $H_0(q^2)$ on q^2 for the z–series expansion parametrization fitted to the LCSR prediction for the $B \to K_{1A}$ transition.

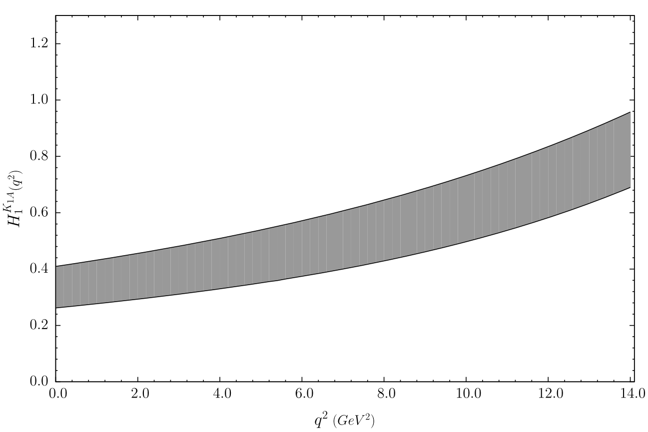

Fig. 2. The same as in Fig. 1, but for the helicity amplitude $H_1(q^2)$.

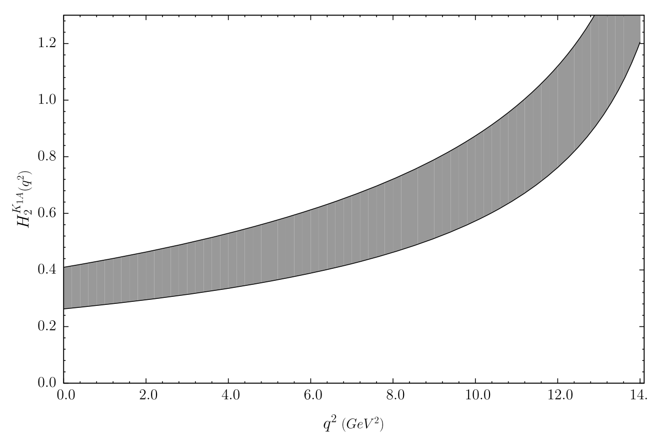

Fig. 3. The same as in Fig. 1, but for the helicity amplitude $H_2(q^2)$.

Table 1
The values of the coefficients $\beta_k^{K_{1A}}$ of the series expansion parametrization of the helicity amplitudes $H_0(q^2)$, $H_1(q^2)$ and $H_2(q^2)$.

	$\beta_0^{K_{1A}}$	$\beta_1^{K_{1A}}$	$\sum_{k=0}^{1}(\beta_k^{K_{1A}})^2$
H_0	3.7×10^{-5}	-1.3×10^{-3}	1.7×10^{-6}
H_1	8.4×10^{-5}	-3.0×10^{-3}	9.0×10^{-6}
H_2	2.1×10^{-5}	-6.4×10^{-4}	4.1×10^{-7}

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Appendix A. Distribution amplitudes

The two-parton chiral-even LCDAs are given by

$$\begin{aligned}&\langle K_1(p,\lambda)\big|\bar{s}(x)\gamma_\mu\gamma_5\psi(0)|0\rangle\\&\quad= if_{K_1}m_{K_1}\int_0^1 du\, e^{iupx}\left\{p_\mu\frac{\varepsilon^{(\lambda)*}x}{px}\phi_\parallel(u)+\left(\varepsilon_\mu^{(\lambda)*}-p_\mu\frac{\varepsilon^{(\lambda)*}x}{px}\right)g_\perp^{(a)}(u)-\frac{1}{2}x_\mu\frac{\epsilon^{*(\lambda)}x}{(px)^2}m_{K_1}^2\bar{g}_3(u)+\mathcal{O}\big(x^2\big)\right\},\end{aligned} \tag{A.1}$$

$$\langle K_1(p,\lambda)\big|\bar{s}(x)\gamma_\mu\psi(0)|0\rangle = -if_{K_1}m_{K_1}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{(\lambda)*\nu}p^\rho x^\sigma\int_0^1 du\, e^{iupx}\left(\frac{g_\perp^{(v)}(u)}{4}+\mathcal{O}\big(x^2\big)\right), \tag{A.2}$$

where $\psi \equiv u$ (or d), $x^2 \neq 0$, and u is the momentum fraction carried by the s in the $K_{1A(B)}$ meson. The two-parton chiral-odd LCDAs are defined by

$$\langle K_1(p,\lambda)|\bar{s}(x)\sigma_{\mu\nu}\gamma_5\psi(0)|0\rangle = f_{K_1}^{\perp}\int_0^1 du\, e^{iupx}\left\{\left(\varepsilon_\mu^{(\lambda)*}p_\nu - \varepsilon_\nu^{(\lambda)*}p_\mu\right)\phi_\perp(u) + \frac{m_{K_1}^2\varepsilon^{(\lambda)*}x}{(px)^2}(p_\mu x_\nu - p_\nu x_\mu)\bar{h}_\parallel^{(t)}(u) + \frac{1}{2}\left(\varepsilon_\mu^{*(\lambda)}x_\nu - \varepsilon_\nu^{*(\lambda)}x_\mu\right)\frac{m_{K_1}^2}{px}\bar{h}_3(u) + \mathcal{O}\left(x^2\right)\right\}, \tag{A.3}$$

$$\langle K_1(p,\lambda)|\bar{s}(x)\gamma_5\psi(0)|0\rangle = f_{K_1}^{\perp}m_{K_1}^2\left(\varepsilon^{*(\lambda)}x\right)\int_0^1 du\, e^{iupx}\left(\frac{h_\parallel^{(p)}(u)}{2} + \mathcal{O}\left(x^2\right)\right), \tag{A.4}$$

where the functions $\bar{g}_3(u)$, $\bar{h}_\parallel^{(t)}$ and $\bar{h}_3(u)$ are given in Eq. (19). In $SU(3)$ limit, due to G-parity, $\phi_\parallel$, $g_\perp^{(a)}$, $g_\perp^{(v)}$, and g_3 are symmetric (antisymmetric) under the replacement $u \to 1-u$ for the 1^3P_1 (1^1P_1) states, whereas $\phi_\perp$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$, and h_3 are antisymmetric (symmetric). Up to twist-3, we adopt the following normalization conventions [14],

$$\int_0^1 du\,\phi_\parallel(u) = \int_0^1 du\, g_\perp^{(a)}(u) = \int_0^1 du\, g_\perp^{(v)}(u) = 1, \qquad \int_0^1 du\,\phi_\perp(u) = \int_0^1 du\, h_\parallel^{(t)}(u) = a_0^\perp, \qquad \int_0^1 du\, h_\parallel^{(p)}(u) = a_0^\perp + \delta_-, \tag{A.5}$$

for K_{1A}, but becomes

$$\int_0^1 du\,\phi_\parallel(u) = \int_0^1 du\, g_\perp^{(a)}(u) = a_0^\parallel, \qquad \int_0^1 du\, g_\perp^{(v)}(u) = a_0^\parallel + \tilde{\delta}_-, \qquad \int_0^1 du\,\phi_\perp(u) = \int_0^1 du\, h_\parallel^{(t)}(u) = \int_0^1 du\, h_\parallel^{(p)}(u) = 1, \tag{A.6}$$

for K_{1B}, where

$$\tilde{\delta}_- = -\frac{f_{K_1}}{f_{K_1}^\perp}\frac{m_s}{m_{K_1}}, \qquad \tilde{\delta}_- = -\frac{f_{K_1}^\perp}{f_{K_1}}\frac{m_s}{m_{K_1}}, \tag{A.7}$$

and $a_0^{\parallel,\perp}$ are defined through

$$\langle K_{1A}(p,\lambda)|\bar{s}(0)\sigma_{\mu\nu}\gamma_5\psi(0)|0\rangle = f_{K_{1A}}a_0^{\perp,K_{1A}}\left(\varepsilon_\mu^{(\lambda)*}p_\nu - \varepsilon_\nu^{(\lambda)*}p_\mu\right), \tag{A.8}$$

$$\langle K_{1B}(p,\lambda)|\bar{s}(0)\gamma_\mu\gamma_5\psi(0)|0\rangle = if_{K_{1B}}^\perp(1\ \text{GeV})a_0^{\parallel,K_{1B}}m_{K_{1B}}\varepsilon_\mu^{(\lambda)*}, \tag{A.9}$$

with $f_{K_{1A}}^\perp = f_{K_{1A}}$ and $f_{^1P_1} = f_{^1P_1}^\perp(\mu = 1\ \text{GeV})$. $a_0^{\perp,K_{1A}}$ and $a_0^{\parallel,K_{1B}}$ are the G-parity violating zeroth Gegenbauer moments and vanish in the $SU(3)$ limit.

We use the twist-2 distributions [14]

$$\phi_\parallel(u) = 6u\bar{u}\left[1 + 3a_1^\parallel\xi + a_2^\parallel\frac{3}{2}\left(5\xi^2 - 1\right)\right], \tag{A.10}$$

$$\phi_\perp(u) = 6u\bar{u}\left[a_0^\perp + 3a_1^\perp\xi + a_2^\perp\frac{3}{2}\left(5\xi^2 - 1\right)\right], \tag{A.11}$$

for the K_{1A} and

$$\phi_\parallel(u) = 6u\bar{u}\left[a_0^\parallel + 3a_1^\parallel\xi + a_2^\parallel\frac{3}{2}\left(5\xi^2 - 1\right)\right], \tag{A.12}$$

$$\phi_\perp(u) = 6u\bar{u}\left[1 + 3a_1^\perp\xi + a_2^\perp\frac{3}{2}\left(5\xi^2 - 1\right)\right], \tag{A.13}$$

for the K_{1B}, where $\xi = 2u - 1$. For the relevant three-parton twist-3 chiral-even LCDAs, we use the contributions up to terms of conformal spin 9/2 and take into account the corrections arising from the strange quark mass:

$$\mathcal{A} = 5040(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3^2 + 360\alpha_1\alpha_2\alpha_3^2\left[\lambda_{K_{1A}}^A + \sigma_{K_{1A}}^A\frac{1}{2}(7\alpha_3 - 3)\right], \tag{A.14}$$

$$\mathcal{V} = 360\alpha_1\alpha_2\alpha_3^2\left[1 + \omega_{K_{1A}}^V\frac{1}{2}(7\alpha_3 - 3)\right] + 5040(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3^2\sigma_{K_{1A}}^V, \tag{A.15}$$

for the K_{1A}, and

Table 2
Gegenbauer moments of twist-2 and twist-3 LCDAs at the scale 2.2 GeV [14]. The G-parity violating parameters are updated using new values for $a_0^{\perp,K_{1A}}$ and $a_0^{\parallel,K_{1B}}$ given in Ref. [15].

Gegenbauer moments for leading-twist LCDAs						
	$a_0^{\parallel}$	$a_1^{\parallel}$	$a_2^{\parallel}$	$a_0^{\perp}$	$a_1^{\perp}$	$a_2^{\perp}$
K_{1A}	1	$-0.25^{+0.00}_{-0.17}$	-0.04 ± 0.02	$0.25^{+0.03}_{-0.16}$	-0.88 ± 0.39	0.01 ± 0.15
K_{1B}	-0.19 ± 0.07	-1.57 ± 0.37	$0.07^{+0.11}_{-0.14}$	1	$0.24^{+0.00}_{-0.27}$	-0.02 ± 0.17

G-parity conserving parameters of twist-3 3-parton LCDAs			
	$f^V_{3,{}^3P_1}$ [GeV^2]	$\omega^V_{{}^3P_1}$	$f^A_{3,{}^3P_1}$ [GeV^2]
K_{1A}	0.0034 ± 0.0018	-3.1 ± 1.1	0.0014 ± 0.0007
	$f^A_{3,{}^1P_1}$ [GeV^2]	$\omega^A_{{}^1P_1}$	$f^V_{3,{}^1P_1}$ [GeV^2]
K_{1B}	-0.0041 ± 0.0018	-1.7 ± 0.4	0.0029 ± 0.0012

G-parity violating parameters of twist-3 3-parton LCDAs of the K_{1A}		
$\sigma^V_{K_{1A}}$	$\lambda^A_{K_{1A}}$	$\sigma^A_{K_{1A}}$
0.01 ± 0.04	-0.12 ± 0.22	-1.9 ± 1.1

G-parity violating parameters of twist-3 3-parton LCDAs of the K_{1B}		
$\lambda^V_{K_{1B}}$	$\sigma^V_{K_{1B}}$	$\sigma^A_{K_{1B}}$
-0.23 ± 0.18	1.3 ± 0.8	0.03 ± 0.03

$$\mathcal{A}=360\alpha_1\alpha_2\alpha_3^2\left[1+\omega^A_{K_{1B}}\frac{1}{2}(7\alpha_3-3)\right]+5040(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3^2\sigma^A_{K_{1B}}, \tag{A.16}$$

$$\mathcal{V}=5040(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3^2+360\alpha_1\alpha_2\alpha_3^2\left[\lambda^V_{K_{1B}}+\sigma^V_{K_{1B}}\frac{1}{2}(7\alpha_3-3)\right], \tag{A.17}$$

for the K_{1B}, where λ's correspond to conformal spin 7/2, while ω's and σ's are parameters with conformal spin 9/2. As the $SU(3)$-symmetry is restored, we have $\lambda\text{'s}=\sigma\text{'s}=0$.

For the relevant two-parton twist-3 chiral-even LCDAs, we take the approximate expressions up to conformal spin 9/2 and of order m_s [14]:

$$\begin{aligned}g_\perp^{(a)}(u)={}&\frac{3}{4}\left(1+\xi^2\right)+\frac{3}{2}a_1^{\parallel}\xi^3+\left(\frac{3}{7}a_2^{\parallel}+5\zeta^V_{3,K_{1A}}\right)\left(3\xi^2-1\right)+\left(\frac{9}{112}a_2^{\parallel}+\frac{105}{16}\zeta^A_{3,K_{1A}}-\frac{15}{64}\zeta^V_{3,K_{1A}}\omega^V_{K_{1A}}\right)\left(35\xi^4-30\xi^2+3\right)\\&+5\left[\frac{21}{4}\zeta^V_{3,K_{1A}}\sigma^V_{K_{1A}}+\zeta^A_{3,K_{1A}}\left(\lambda^A_{K_{1A}}-\frac{3}{16}\sigma^A_{K_{1A}}\right)\right]\xi\left(5\xi^2-3\right)\\&-\frac{9}{2}a_1^{\perp}\tilde{\delta}_+\left(\frac{3}{2}+\frac{3}{2}\xi^2+\ln u+\ln\bar{u}\right)-\frac{9}{2}a_1^{\perp}\tilde{\delta}_-(3\xi+\ln\bar{u}-\ln u),\end{aligned} \tag{A.18}$$

$$\begin{aligned}g_\perp^{(v)}(u)={}&6u\bar{u}\left\{1+\left(a_1^{\parallel}+\frac{20}{3}\zeta^A_{3,K_{1A}}\lambda^A_{K_{1A}}\right)\xi+\left[\frac{1}{4}a_2^{\parallel}+\frac{5}{3}\zeta^V_{3,K_{1A}}\left(1-\frac{3}{16}\omega^V_{K_{1A}}\right)+\frac{35}{4}\zeta^A_{3,K_{1A}}\right]\left(5\xi^2-1\right)\right.\\&\left.+\frac{35}{4}\left(\zeta^V_{3,K_{1A}}\sigma^V_{K_{1A}}-\frac{1}{28}\zeta^A_{3,K_{1A}}\sigma^A_{K_{1A}}\right)\xi\left(7\xi^2-3\right)\right\}\\&-18a_1^{\perp}\tilde{\delta}_+(3u\bar{u}+\bar{u}\ln\bar{u}+u\ln u)-18a_1^{\perp}\tilde{\delta}_-(u\bar{u}\xi+\bar{u}\ln\bar{u}-u\ln u),\end{aligned} \tag{A.19}$$

for the K_{1A}, and

$$\begin{aligned}g_\perp^{(a)}(u)={}&\frac{3}{4}a_0^{\parallel}\left(1+\xi^2\right)+\frac{3}{2}a_1^{\parallel}\xi^3+5\left[\frac{21}{4}\zeta^V_{3,K_{1B}}+\zeta^A_{3,K_{1B}}\left(1-\frac{3}{16}\omega^A_{K_{1B}}\right)\right]\xi\left(5\xi^2-3\right)\\&+\frac{3}{16}a_2^{\parallel}\left(15\xi^4-6\xi^2-1\right)+5\zeta^V_{3,K_{1B}}\lambda^V_{K_{1B}}\left(3\xi^2-1\right)+\frac{105}{16}\left(\zeta^A_{3,K_{1B}}\sigma^A_{K_{1B}}-\frac{1}{28}\zeta^V_{K_{1B}}\sigma^V_{K_{1B}}\right)\left(35\xi^4-30\xi^2+3\right)\\&-15a_2^{\perp}\left[\tilde{\delta}_+\xi^3+\frac{1}{2}\tilde{\delta}_-\left(3\xi^2-1\right)\right]-\frac{3}{2}\left[\tilde{\delta}_+(2\xi+\ln\bar{u}-\ln u)+\tilde{\delta}_-(2+\ln u+\ln\bar{u})\right]\left(1+6a_2^{\perp}\right),\end{aligned} \tag{A.20}$$

$$\begin{aligned}g_\perp^{(v)}(u)={}&6u\bar{u}\left\{a_0^{\parallel}+a_1^{\parallel}\xi+\left[\frac{1}{4}a_2^{\parallel}+\frac{5}{3}\zeta^V_{3,K_{1B}}\left(\lambda^V_{K_{1B}}-\frac{3}{16}\sigma^V_{K_{1B}}\right)+\frac{35}{4}\zeta^A_{3,K_{1B}}\sigma^A_{K_{1B}}\right]\left(5\xi^2-1\right)\right.\\&\left.+\frac{20}{3}\xi\left[\zeta^A_{3,K_{1B}}+\frac{21}{16}\left(\zeta^V_{3,K_{1B}}-\frac{1}{28}\zeta^A_{3,K_{1B}}\omega^A_{K_{1B}}\right)\left(7\xi^2-3\right)\right]-5a_2^{\perp}\left[2\tilde{\delta}_+\xi+\tilde{\delta}_-\left(1+\xi^2\right)\right]\right\}\\&-6\left[\tilde{\delta}_+(\bar{u}\ln\bar{u}-u\ln u)+\tilde{\delta}_-(2u\bar{u}+\bar{u}\ln\bar{u}+u\ln u)\right]\left(1+6a_2^{\perp}\right),\end{aligned} \tag{A.21}$$

for the K_{1B}, where

$$\tilde{\delta}_\pm = \pm \frac{f_{K_1}^\perp}{f_{K_1}} \frac{m_s}{m_{K_1}}, \qquad \zeta_{3,K_1}^{V(A)} = \frac{f_{3K_1}^{V(A)}}{f_{K_1} m_{K_1}}. \tag{A.22}$$

The relevant parameters entering to the expressions of DAs are listed in Table 2.

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