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Electromagnetic mean squared radii of $\Lambda(1405)$ in chiral dynamics

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ABSTRACT

The electromagnetic mean squared radii, $\langle r^2\rangle_{\mathrm{E}}$ and $\langle r^2\rangle_{\mathrm{M}}$, of $\Lambda(1405)$ are calculated in the chiral unitary model. We describe the excited baryons as dynamically generated resonances in the octet meson and octet baryon scattering. We evaluate values of $\langle r^2\rangle_{\mathrm{E}}$ and $\langle r^2\rangle_{\mathrm{M}}$ for the $\Lambda(1405)$ on the resonance pole and obtain their complex values. We also consider $\Lambda(1405)$ obtained by neglecting decay channels. For the latter case, we obtain negative and larger absolute electric mean squared radius than that of typical ground state baryons. This implies that $\Lambda(1405)$ has structure that K^- is widely spread around p.

1. Introduction

The structure of the $\Lambda(1405)$ has been a long-standing issue. The $\Lambda(1405)$ has been considered as a quasi-bound state of anti-kaon and nucleon ($\bar{K}N$) system [1–3], before QCD was established. Recent theoretical investigations have also suggested that the $\Lambda(1405)$ is well described as a dynamically generated resonant state of meson–baryon scattering with $I=0$ and $S=-1$ based on chiral dynamics in coupled channel approach [4–10], which is so-called chiral unitary model (ChUM). This model has successfully reproduced cross sections of K^-p to various channels and also the mass spectrum of the $\Lambda(1405)$ resonance below the $\bar{K}N$ threshold [4–6,8,11], giving two states for the $\Lambda(1405)$ found as poles of the scattering amplitude in the complex energy plane [6,9,12]. Photo-properties of $\Lambda(1405)$ have been investigated in the chiral unitary approach in Refs. [13–16].

Most of excited baryons can be described by simple constituent quark models [17]. The $\Lambda(1405)$ is well known as one of the exceptions, so the success of the hadronic molecule picture is in this sense reasonable. Recent study of the $\Lambda(1405)$ based on the N_c scaling in ChUM [18] indicates the dominance of the non-qqq component in the $\Lambda(1405)$, and Ref. [10] also suggests that the $\Lambda(1405)$ is described predominantly by meson–baryon dynamics. The understanding of the structure of the $\Lambda(1405)$ is also relevant for the $\bar{K}N$ phenomenology, since binding energy of $\bar{K}N$ system in the $\Lambda(1405)$ plays an important role for the study of the kaonic nuclei [19–22]. Toward experimental verification, it is desirable to study some quantity which characterizes the molecule structure of $\Lambda(1405)$ [15].

One expects that the $\Lambda(1405)$ as meson–baryon quasi-bound molecule with small binding energy has a larger size than typical ground state baryons dominated by a genuine quark state. If this is the case, the form factor of the $\Lambda(1405)$ falls off more rapidly than that of the nucleon and the production cross section of the $\Lambda(1405)$ has large energy dependence. In this work, we estimate the electromagnetic mean squared radii of the $\Lambda(1405)$ based on the ChUM. Such electromagnetic properties of the $\Lambda(1405)$ could be obtained in photon-induced production experiments [23].

2. $\Lambda(1405)$ in chiral unitary model

In ChUM, the $\Lambda(1405)$ is described in s-wave meson–baryon scattering amplitudes with the strangeness $S=-1$ and charge $Q=0$ obtained by solving the Bethe–Salpeter (BS) equation,

$$T_{ij}(\sqrt{s}) = V_{ij} + \sum_k V_{ik} G_k T_{kj}, \tag{1}$$

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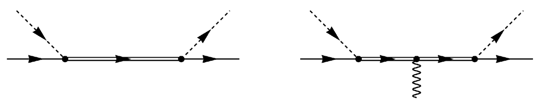

(a) T_{ij} (b) $T^{\mu}_{\gamma ij}$

Fig. 1. Scattering amplitudes close to the pole of the excited baryon. The double lines correspond to the excited baryon.

in which the on-shell factorization leads to an algebraic solution [6]. In Eq. (1), V_{ij} is s-wave meson–baryon interaction kernel and G_k is loop integral of the meson–baryon system. They are functions of the center-of-mass energy, $\sqrt{s}$, in matrix form of the meson–baryon channel ($i, j = K^- p$, $\bar{K}^0 n$, $\pi^0 \Lambda$, $\pi^0 \Sigma^0$, $\eta \Lambda$, $\eta \Sigma^0$, $\pi^+ \Sigma^-$, $\pi^- \Sigma^+$, $K^+ \Xi^-$ and $K^0 \Xi^0$). The interaction kernel V_{ij} is given by the leading order chiral Lagrangian [24,25], which is known as Weinberg–Tomozawa term

$$V_{ij} = -\frac{C_{ij}}{4f^2}(2\sqrt{s} - M_i - M_j), \tag{2}$$

with an averaged meson decay constant $f = 1.123 f_\pi$, $f_\pi = 93.0$ MeV. This expression is obtained by applying the non-relativistic reduction for baryons. The coefficient C_{ij} is fixed by the SU(3) group structure of the interaction and its explicit value is given in [5]. The loop integral $G_k(\sqrt{s})$ is evaluated using the dimensional regularization:

$$\begin{aligned} G_k(\sqrt{s}) &= 2iM_k \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{q_1^2 - m_k^2} \frac{1}{(P - q_1)^2 - M_k^2} \\ &= \frac{2M_k}{16\pi^2}\bigg[a_k(\mu) + \ln\frac{M_k^2}{\mu^2} + \frac{m_k^2 - M_k^2 + s}{2s}\ln\frac{m_k^2}{M_k^2} \\ &\quad + \frac{q_k}{\sqrt{s}}\big(\ln\big(s - M_k^2 + m_k^2 + 2q_k\sqrt{s}\big) \\ &\quad + \ln\big(s + M_k^2 - m_k^2 + 2q_k\sqrt{s}\big) \\ &\quad - \ln\big(-s + M_k^2 - m_k^2 + 2q_k\sqrt{s}\big) \\ &\quad - \ln\big(-s - M_k^2 + m_k^2 + 2q_k\sqrt{s}\big)\big)\bigg], \end{aligned} \tag{3}$$

where the center-of-mass momentum of the two-body system is given by

$$q_k \equiv \sqrt{\frac{(s - M_k^2 + m_k^2)^2 - 4sm_k^2}{4s}}. \tag{4}$$

The subtraction constants $a_k(\mu)$ in Eq. (3) with the regularization scale μ are free parameters in this model. These constants are phenomenologically fixed so as to reproduce the threshold behavior of the scattering amplitudes [7]:

$$\begin{aligned} &a_{\bar{K}N} = -1.84, \quad a_{\pi\Sigma} = -2.00, \quad a_{\pi\Lambda} = -1.83, \\ &a_{\eta\Lambda} = -2.25, \quad a_{\eta\Sigma} = -2.38, \quad a_{K\Xi} = -2.67, \end{aligned} \tag{5}$$

with $\mu = 630$ MeV. In the present model, the $\Lambda(1405)$ is dynamically generated in the obtained BS scattering amplitude without introducing any explicit pole terms in the interaction kernel.

The excited baryon is expressed by a pole of the scattering amplitudes in complex energy plane. The s-wave scattering amplitude can be approximated as,

$$-iT_{ij} \approx (-ig_i)\frac{i}{\sqrt{s} - z_{\mathrm{H}}}(-ig_j), \tag{6}$$

close to the resonance energy, as shown in Fig. 1(a). The real and imaginary parts of the pole position z_{H} express the mass and the half-width of the excited baryon, respectively, and the residues of the pole, g_i and g_j, represent coupling strengths of the excited baryon to the meson–baryon channels. In the present model, two poles in the BS amplitude are found in energies of the $\Lambda(1405)$ as ($z_1 = 1390 - 66i$ MeV) and ($z_2 = 1426 - 17i$ MeV) [9]. It has been reported in Ref. [26] that the position of the lower pole z_1 is dependent on details of model parameters, whereas that of the higher pole z_2 shows little dependence.

3. Form factors of excited baryons

In this section, we discuss the formulation to evaluate the electromagnetic form factors and the mean squared radii of excited baryons described by the BS amplitudes. First of all, let us define the electromagnetic form factors of an excited baryon with spin 1/2, H^*, as matrix elements of the electromagnetic current J^{μ}_{EM} in the Breit frame [15]:

$$\langle H^* | J^{\mu}_{\mathrm{EM}} | H^* \rangle_{\mathrm{Breit}} \equiv \left(G_{\mathrm{E}}\big(Q^2\big), G_{\mathrm{M}}\big(Q^2\big)\frac{i\boldsymbol{\sigma} \times \boldsymbol{q}}{2M_{\mathrm{p}}}\right), \tag{7}$$

with the electric and magnetic form factors, $G_{\mathrm{E}}(Q^2)$ and $G_{\mathrm{M}}(Q^2)$, the virtual photon momentum q^μ, $Q^2 = -q^2$ and the Pauli matrices σ^a ($a = 1, 2, 3$). The magnetic form factor $G_{\mathrm{M}}(Q^2)$ is normalized as the nuclear magneton $\mu_{\mathrm{N}} = e/(2M_{\mathrm{p}})$ with the proton charge e and mass M_{p}. From these form factors, electromagnetic mean squared radii, $\langle r^2 \rangle_{\mathrm{E}}$ and $\langle r^2 \rangle_{\mathrm{M}}$, are calculated by

$$\langle r^2 \rangle_{\mathrm{E}} \equiv -6\frac{dG_{\mathrm{E}}}{dQ^2}\bigg|_{Q^2=0}, \tag{8}$$

$$\langle r^2 \rangle_{\mathrm{M}} \equiv -\frac{6}{G_{\mathrm{M}}(0)}\frac{dG_{\mathrm{M}}}{dQ^2}\bigg|_{Q^2=0}. \tag{9}$$

To extract the matrix elements of the electromagnetic current from the BS amplitudes, we consider the scattering amplitude for the $MB\gamma^* \to M'B'$ process, $T^{\mu}_{\gamma ij}$, which is microscopically calculated by attaching the photon to every place of the constituent meson–baryon components in the BS amplitude [14,15,27]. The matrix elements of the excited baryon can be expressed as residues of the double pole of the $MB\gamma^* \to M'B'$ amplitude. Close to the pole of the excited baryon, as shown in Fig. 1(b), the amplitude $T^{\mu}_{\gamma ij}$ is parametrized by

$$\begin{aligned} &-iT^{\mu}_{\gamma ij}(\sqrt{s'}, \sqrt{s}) \\ &\quad \approx (-ig_i)\frac{i}{\sqrt{s'} - z_{\mathrm{H}}}\langle H^* | iJ^{\mu}_{\mathrm{EM}} | H^* \rangle \frac{i}{\sqrt{s} - z_{\mathrm{H}}}(-ig_j), \end{aligned} \tag{10}$$

where $s' = P'^{\mu}P'_{\mu}$ and $s = P^{\mu}P_{\mu}$, with incoming and outgoing momenta of excited baryon, P^μ and P'^μ, respectively. Combining Eqs. (6) and (10), the matrix elements can be evaluated as residue of the double pole at $\sqrt{s} = \sqrt{s'} = z_{\mathrm{H}}$ as discussed in [15]:

$$\langle H^* | J^{\mu}_{\mathrm{EM}} | H^* \rangle = \mathrm{Res}\left[-\frac{T^{\mu}_{\gamma ij}(\sqrt{s'}, \sqrt{s})}{T_{ij}(\sqrt{s})}\bigg|_{\sqrt{s} \to z_{\mathrm{H}}}\right]. \tag{11}$$

Here we calculate residue of $\sqrt{s'} = z_{\mathrm{H}}$ by "Res" in the right-hand side. Note that this evaluation is free from the non-resonant background, since the values are calculated just on the pole of the excited baryon.

Thus, we have shown that the electromagnetic mean squared radii of the excited baryon can be obtained, once the scattering amplitude $MB\gamma^* \to M'B'$ is calculated.

4. Evaluation of the form factors

We calculate the scattering amplitude of the $MB\gamma^* \to M'B'$ in the chiral unitary approach, in which the amplitude for the

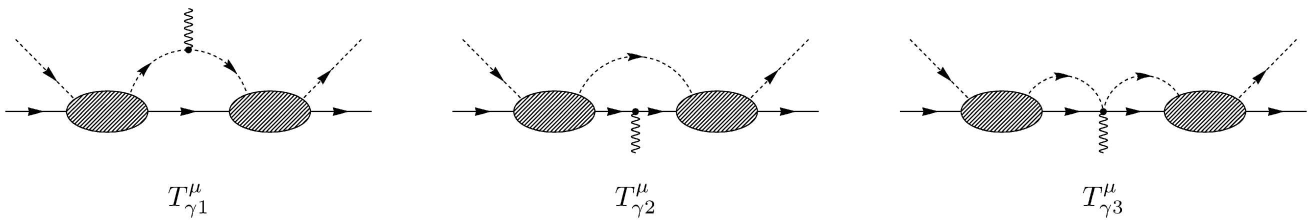

Fig. 2. Diagrams for the form factors of the excited baryon. The shaded ellipses represent the BS amplitude.

$MB \to M'B'$ is given by multiple scattering of the meson and baryon. Thus, the photon couples to the $\Lambda(1405)$ through the constituent mesons and baryons. The calculation should be performed in a gauge-invariant way, which ensures the correct normalization of the electric form factor of the excited baryon, $G_E(Q^2=0)=Q_H$. Following the method of the gauge-invariant calculation for unitarized amplitudes proposed in Ref. [27], we take three relevant diagrams shown in Fig. 2, which have the double pole for the excited baryon. The other diagrams do not contribute to the electromagnetic form factors at the resonance energy $\sqrt{s}=z_H$. Since the pole position is gauge invariant, these three contributions are enough for the gauge-invariant form factors:

$$T^\mu_{\gamma ij} = T^\mu_{\gamma 1ij} + T^\mu_{\gamma 2ij} + T^\mu_{\gamma 3ij}. \tag{12}$$

Using this amplitude $T^\mu_{\gamma ij}$ with photon couplings to mesons and baryons which we will discuss later, we obtain the gauge-invariant form factors through Eq. (11). A proof of the gauge invariance of the form factors is given in Appendix A by using Ward–Takahashi identity.

The elementary couplings of the photon to the meson and baryon are given by imposing gauge invariance to the chiral effective theory in a consistent way with the description of $\Lambda(1405)$. In the present model, the $\Lambda(1405)$ is described by a infinite sum of the loop function G and the s-wave Weinberg–Tomozawa interaction V, with non-relativistic formulation for the baryons. We use the minimal coupling scheme for the photon couplings to the meson and baryon appearing in the BS amplitude. This procedure automatically implements gauge invariance in a consistent way to the original BS amplitude. We also have the anomalous magnetic couplings for the $BB\gamma$ and $\gamma BB'MM'$ couplings. These couplings are given by the chiral perturbation theory as done in Ref. [15]. Finally the elementary electric and magnetic couplings, $V^\mu_{\mathrm{M}_i}$, $V^\mu_{\mathrm{B}_i}$ and Γ^μ_{ij}, for $MM\gamma$, $BB\gamma$ and $\gamma BB'MM'$, respectively, are obtained by sum of the two contributions.

In the minimal coupling scheme, the photon coupling to the meson, $V^\mu_{\mathrm{M}_i}$, is given by

$$-iV^\mu_{\mathrm{M}_i}(k,k') = iQ_{\mathrm{M}_i}(k+k')^\mu, \tag{13}$$

with the incoming and outgoing meson momenta k^μ and k'^μ. The minimal coupling of the photon to the baryon is given by

$$-iV^{(\mathrm{m}),\mu}_{\mathrm{B}_i}(p,p') = \left(iQ_{\mathrm{B}_i}\frac{(p+p')^0}{2M_i}, iQ_{\mathrm{B}_i}\frac{\boldsymbol{p}+\boldsymbol{p}'}{2M_i} + i\mu^{(\mathrm{N})}_i\frac{i\boldsymbol{\sigma}\times\boldsymbol{q}}{2M_{\mathrm{p}}}\right), \tag{14}$$

with the normal magnetic moments $\mu^{(\mathrm{N})}_i$ and the incoming and outgoing baryon momenta p^μ and p'^μ. Here we have performed non-relativistic reduction. These two couplings (13) and (14) are appropriate with the propagators in the loop function (3). The spatial components without the Pauli matrices $\boldsymbol{\sigma}$ in Eqs. (13) and (14) give no contribute to the $\Lambda(1405)$ form factors in the Breit frame. For the $\gamma BB'MM'$ coupling appearing in $T^{\mu=0}_{\gamma 3ij}$, we use the following vertex which is obtained so that Ward–Takahashi identity is satisfied with Eqs. (2), (13) and (14) in tree-level:

$$-i\Gamma^{(\mathrm{m}),\mu}_{ij}(P,P') = i\frac{C_{ij}}{4f^2}\frac{P^\mu+P'^\mu}{\sqrt{s}+\sqrt{s'}}(Q_{\mathrm{M}_i}+Q_{\mathrm{B}_i}+Q_{\mathrm{M}_j}+Q_{\mathrm{B}_j}), \tag{15}$$

with incoming and outgoing meson–baryon momenta, P^μ and P'^μ, respectively. Actually for the neutral excited baryon, this term does not contribute due to $Q_H = Q_B + Q_M = 0$.

For the $BB\gamma$ and $\gamma BB'MM'$ couplings we have also the anomalous coupling terms, which are gauge-invariant by themselves. For these couplings, we use the interaction Lagrangian appearing in the chiral perturbation theory [28]:

$$\mathcal{L}_{\mathrm{int}} = -\frac{i}{4M_p}b_6^{\mathrm{F}}\,\mathrm{Tr}\left(\bar{B}\left[S^\mu,S^\nu\right]\left[F^+_{\mu\nu},B\right]\right) - \frac{i}{4M_p}b_6^{\mathrm{D}}\,\mathrm{Tr}\left(\bar{B}\left[S^\mu,S^\nu\right]\left\{F^+_{\mu\nu},B\right\}\right), \tag{16}$$

with

$$F^+_{\mu\nu} = -e(u^\dagger Q F_{\mu\nu} u + uQF_{\mu\nu}u^\dagger), \tag{17}$$

the electromagnetic field tensor $F_{\mu\nu}$, the charge matrix Q, the spin matrix S_μ and the chiral field $u^2 = U = \exp(i\sqrt{\Phi}/f)$ where Φ is the SU(3) matrix of the Nambu–Goldstone boson field. This interaction Lagrangian gives us spatial components of both the $BB\gamma$ and the $\gamma BB'MM'$ vertices:

$$-iV^{(\mathrm{A}),a}_{\mathrm{B}_i} = i\mu^{(\mathrm{A})}_i\left(\frac{i\boldsymbol{\sigma}\times\boldsymbol{q}}{2M_{\mathrm{p}}}\right)^a, \tag{18}$$

$$-i\Gamma^{(\mathrm{A}),a}_{ij} = A_{ij}\left(\frac{i\boldsymbol{\sigma}\times\boldsymbol{q}}{2M_{\mathrm{p}}}\right)^a, \tag{19}$$

where we have made non-relativistic reduction. $\mu^{(\mathrm{A})}_i$ are the anomalous magnetic moments of the baryons, and the matrix A_{ij} is given as,

$$A_{ij} = i\frac{b_6^{\mathrm{D}}X_{ij} + b_6^{\mathrm{F}}Y_{ij}}{2f^2}, \tag{20}$$

with the coefficients X_{ij}, Y_{ij} fixed only by the flavor SU(3) symmetry and their explicit values are found in Ref. [15]. The anomalous magnetic moments are given by the interaction Lagrangian (16) as $\mu^{(\mathrm{A})}_i = b_6^{\mathrm{D}}d_i + b_6^{\mathrm{F}}f_i$ with the SU(3) coefficients, d_i and f_i. The values of the coefficients b_6^{D} and b_6^{F} are fixed by $b_6^{\mathrm{D}} = 2.40$ and $b_6^{\mathrm{F}} = 0.82$ so as to reproduce the observed $\mu^{(\mathrm{A})}_i$ of the baryons. In the calculation, these values are used for the $\gamma BB'MM'$ vertices (19), while for the baryon magnetic moments we use the experimental values. For the unobserved Σ^0 magnetic moment, we use the SU(3) flavor relation $\mu_{\Sigma^0} = (\mu_{\Sigma^+}+\mu_{\Sigma^-})/2$, which is consistent with the quark model. We neglect the transition magnetic moment $\mu_{\Lambda\Sigma^0}$, because this interaction changes the isospin of the excited baryons 0 to 1.

After obtaining the elementary couplings, we calculate the amplitudes, $T^\mu_{\gamma 1ij}$, $T^\mu_{\gamma 2ij}$ and $T^\mu_{\gamma 3ij}$, accordingly to the Feynman diagrams given in Fig. 2:

$$T^{\mu}_{\gamma 1ij} = \sum_{l} T_{il}(\sqrt{s'}) D^{\mu}_{\mathrm{M}_l} T_{lj}(\sqrt{s}), \tag{21}$$

$$T^{\mu}_{\gamma 2ij} = \sum_{l} T_{il}(\sqrt{s'}) D^{\mu}_{\mathrm{B}_l} T_{lj}(\sqrt{s}), \tag{22}$$

$$T^{\mu}_{\gamma 3ij} = \sum_{k,l} T_{ik}(\sqrt{s'}) G_k(\sqrt{s'}) \Gamma^{\mu}_{kl} G_l(\sqrt{s}) T_{lj}(\sqrt{s}), \tag{23}$$

where the loop integrals are given with the photon couplings to the meson and baryon by

$$D^{\mu}_{\mathrm{M}_l} \equiv i \int \frac{d^4 q_1}{(2\pi)^4} \frac{2M_l}{(P-q_1)^2 - M_l^2} \frac{1}{(q_1+q)^2 - m_l^2} \times \left[V^{\mu}_{\mathrm{M}_l}(q_1, q_1+q)\right] \frac{1}{q_1^2 - m_l^2}, \tag{24}$$

$$D^{\mu}_{\mathrm{B}_l} \equiv i \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{q_1^2 - m_l^2} \frac{2M_l}{(P+q-q_1)^2 - M_l^2} \times \left[V^{\mu}_{\mathrm{B}_l}(P-q_1, P-q_1+q)\right] \frac{2M_l}{(P-q_1)^2 - M_l^2}. \tag{25}$$

The loop function G in Eq. (23) is regularized in the same way as Eq. (3) with the subtraction constants (5). This treatment, in fact, is necessary for the gauge-invariant calculation. The integrals in Eqs. (24) and (25) are convergent and require no regularizations, because $V^{\mu}_{\mathrm{M}_l}$ and $V^{\mu}_{\mathrm{B}_l}$ are not more than second power of the loop momentum.

In the present calculation, we did not introduce the form factors for the ground state mesons and baryons and treat them as point particles, because we are interested in the sizes of the excited baryon generated by meson–baryon dynamics and in estimation of pure dynamical effects. For the qualitative argument or comparison with experiments (if possible), the inclusion of the form factors will be important. In this formulation, inclusion of the form factors for the mesons and baryons is straightforward; we simply multiply the meson and baryon form factors to each vertex. Here, the gauge invariance requires to use a common form factor $F(Q^2)$ to every vertex. The common form factor can be factorized out from the loop integrals since it depends only on the photon momentum, so the correction from the inclusion of the form factor is multiplicative: $\tilde{G}(Q^2) = G(Q^2)F(Q^2)$, where $\tilde{G}(Q^2)$ and $G(Q^2)$ are the form factors of excited baryons with and without the inclusion of the meson and baryon form factors, respectively. It is interesting to note that the electric mean squared radii for neutral excited baryons do *not* depend on the inclusion of the meson and baryon form factor, since

$$\langle r^2 \rangle_{\mathrm{E}} \sim \tilde{G}'_{\mathrm{E}}(0) = G'_{\mathrm{E}}(0)F(0) + G_{\mathrm{E}}(0)F'(0) \tag{26}$$

and $G_{\mathrm{E}}(0) = 0$ for neutral excited baryons and $F(0) = 1$ by definition. Thus, the present results for $\langle r^2 \rangle_{\mathrm{E}}$ of the $\Lambda(1405)$ remain unchanged even with inclusion of the meson and baryon form factor.

5. Numerical results

We show our result for the electromagnetic mean squared radii of the excited baryons in Table 1. As mentioned before, we have two $\Lambda(1405)$ states, z_1 and z_2. The lower state strongly couples to the $\pi\Sigma$ channel, while the higher state dominantly couples to the $\bar{K}N$ channel, as one can see from the analysis of g_i in Ref. [9]. Since the $\pi^+\Sigma^-$ and $\pi^-\Sigma^+$ contribute to the isospin 0 state almost equally, the electric mean squared radius of the lower $\Lambda(1405)$ state z_1 is suppressed.

The electric mean squared radius of the higher $\Lambda(1405)$ state z_2 is more interesting. The higher state has a three times larger absolute value of $\langle r^2 \rangle_{\mathrm{E}}$, 0.32 fm^2, than that of neutron $\sim$−0.12 fm^2. This observation implies that the electric form factor of the

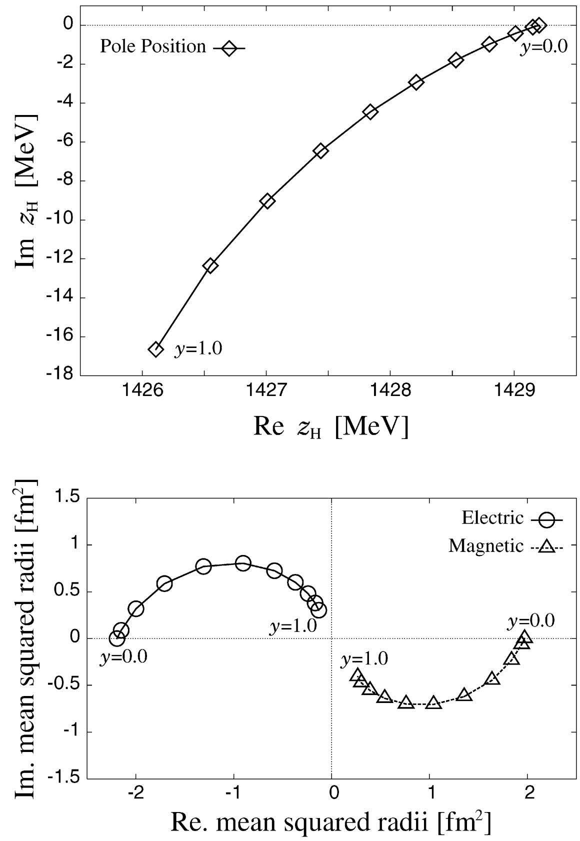

Fig. 3. Upper: Trajectory of the pole position z_2 of the BS amplitude by changing the off-diagonal couplings of $\bar{K}N$ to other channels. Lower: Trajectory of the electromagnetic mean squared radii of the z_2. Both are in step of 0.1 for the parameter y.

$\Lambda(1405)$ is softer, namely has larger energy dependence, than that of the neutron, since the electric form factors for the neutral particles are given by $G_{\mathrm{E}}(Q^2) = -Q^2\langle r^2 \rangle_{\mathrm{E}}/6 + \cdots$ in the expansion of Q^2 due to the neutral charge $G_{\mathrm{E}}(0) = 0$. The softer form factor means that the $\Lambda(1405)$ is more spatially extended. This is because kaon inside the $\Lambda(1405)$ has less "virtuality" than pions surrounding the nucleon [29]. The negative charge radius of the neutron is interpreted as distribution of π^- cloud. For the neutron case, the pion cloud consists of completely virtual pions, since the system needs at least 140 MeV to create a pion. In contrast, for the z_2 pole of the $\Lambda(1405)$, only several MeV is required to make $\bar{K}N$ state which is the dominant component of this resonance according to the analysis of the coupling strengths [9]. Therefore, the K^- inside the $\Lambda(1405)$ can be largely distributed.

These results are free from the non-resonant background since we calculate the mean squared radii on the top of the $\Lambda(1405)$ resonance pole in the complex energy plane. Thus, these values have definite theoretical meanings and are used for comparison with other models. On the other hand, experimental observables may be the ratio of the amplitudes $T^{\mu}_{\gamma ij}/T_{ij}$ in the real energies, which includes the contributions from the non-resonant meson–baryon scattering states [15].

In order to extract the information of the sizes from the form factors, we perform the following analysis. If the decay width of the resonance is small, the imaginary parts of the mean squared radii are small. Thus, it is possible to interpret the mean squared radii as the sizes for the resonant states, since the radii are close to real numbers. For this purpose, we analyze the mean squared radii of a $\Lambda(1405)$ calculated without the $\pi\Sigma$ channel, which is the main decay mode of the actual $\Lambda(1405)$. Neglecting the couplings to the $\pi\Sigma$ channel in C_{ij} and leaving other

Table 1
Electromagnetic mean squared radii, $\langle r^2\rangle_{\mathrm{E}}$ and $\langle r^2\rangle_{\mathrm{M}}$, of $\Lambda(1405)$

State	Pole position [MeV]	Strongly couple to	$\langle r^2\rangle_{\mathrm{E}}$ [fm^2]	$\langle r^2\rangle_{\mathrm{M}}$ [fm^2]
z_1	$1390.43 - 66.21i$	$\pi\Sigma$	$(1.8 - 0.2i) \times 10^{-2}$	$(-1.3 + 2.1i) \times 10^{-2}$
z_2	$1426.11 - 16.65i$	$\bar{K}N$	$-0.131 + 0.303i$	$0.267 - 0.407i$
No decay to $\pi\Sigma$	$1422.34 - 0.02i$	K^-p, $\bar{K}^0n$	$-0.519 - 0.008i$	$0.683 - 0.023i$
$\bar{K}N$ bound state	1429.20	K^-p, $\bar{K}^0n$	-2.193	1.972

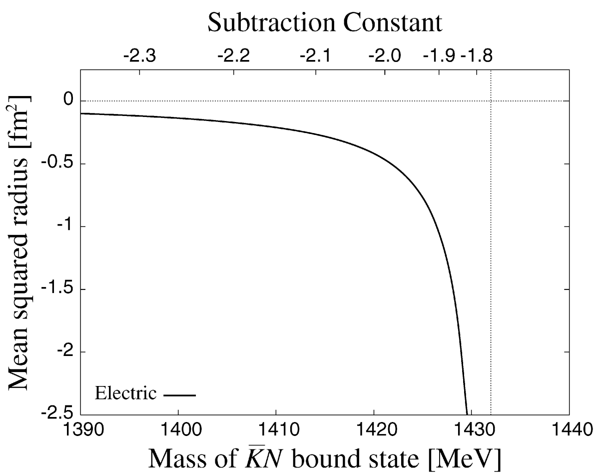

Fig. 4. Electric mean squared radius of the $\bar{K}N$ bound state as a function of the mass. The upper horizontal axis denotes the corresponding values of the subtraction constants $a_{\bar{K}N}$. The vertical dotted line represents position of the K^-p threshold.

parameters as in the calculation of the actual $\Lambda(1405)$, we find the pole position $1422.34 - 0.02i$ MeV, the mean squared radii $\langle r^2\rangle_{\mathrm{E}} = -0.519 - 0.008i$ fm^2 and $\langle r^2\rangle_{\mathrm{M}} = 0.683 - 0.023i$ fm^2. The small imaginary parts of these values come from the coupling to the open $\pi\Lambda$ channel through the tiny isospin breaking in the masses of the constituent baryons and mesons. The absolute values of $\langle r^2\rangle_{\mathrm{E}}$ and $\langle r^2\rangle_{\mathrm{M}}$ are very similar to those obtained by the actual $\Lambda(1405)$ of the z_2 pole.

We also perform analyses for another interesting system of a $\bar{K}N$ bound state, which is generated only by the attractive interaction of the $\bar{K}N$ channel [21]. This bound state is considered to be an origin of the higher state of the actual $\Lambda(1405)$, z_2. To investigate this state, we introduce a parameter y ($0 \leqslant y \leqslant 1$) in front of the off-diagonal couplings of $\bar{K}N$ to the other channels in C_{ij}. The case of $y = 1$ corresponds to the full coupled-channel calculation, whereas $y = 0$ corresponds to the calculation with only the $\bar{K}N$ channels.

The trajectory of the pole position from $y = 1$ to $y = 0$ is shown in Fig. 3. We also show the corresponding electromagnetic mean squared radii in Fig. 3. We thus estimate the size of a meson–baryon resonance by the $\bar{K}N$ bound state which appears at 1429 MeV in our model (see Table 1). The bound state consists of K^-p and $\bar{K}^0n$ components. The electric mean squared radius reflects the charge distribution of the K^-p component, since the $\bar{K}^0n$, in which both hadrons are charge neutral, does not contribute to the electric interactions. The negative sign for the radius implies that the K^- is surrounding around the proton. In addition, with the fact that the electromagnetic size of the proton is roughly 0.9 fm, our result of the electric root mean squared radius $\sqrt{|\langle r^2\rangle_{\mathrm{E}}|} \simeq 1.48$ fm implies that $\Lambda(1405)$ has structure of widely spread K^- clouds around the core of proton with larger size than that of typical ground state baryons. The magnetic mean squared radius, $\langle r^2\rangle_{\mathrm{M}}$, represents distribution of magnetic moments of the core nucleons in the $\bar{K}N$ bound system, because anti-kaons cannot contribute to magnetic interactions. Due to dynamics of the nucleon and anti-kaon, the nucleon is distributed inside of the $\Lambda(1405)$, of which size is seen as the mean squared radii. This implies that the magnetic interaction also suggests a larger size for the $\Lambda(1405)$ than the ordinary baryons.

The results of the mean squared radii of the $\Lambda(1405)$ without the $\pi\Sigma$ decay channel are much smaller than those of the $\bar{K}N$ bound state. This is because the former case provides deeper bound state than the latter case. The resonance energies of these cases are very similar as seen by just 7 MeV difference out of about 1 GeV energy scale, but the binding energies have two times difference. To investigate the relation between the radii and the binding energy, we consider the $\bar{K}N$ bound state and tune the binding energy by changing the subtraction constant $a_{\bar{K}N}$, which is the only parameter of this system. In Fig. 4, we show the electric mean squared radius as a function of the mass of the $\bar{K}N$ bound state. This figure implies that the deeper bound states have the smaller radii. Since the smaller size of the bound state is expected in the deeper bound state, the obtained radius can be interpreted as the size of the $\Lambda(1405)$. Therefore, knowing precise resonance point of the $\Lambda(1405)$ is very important to understand its properties.

Let us examine our result of electric mean squared radius in comparison with theoretical estimates of the size of the $\Lambda(1405)$. In the study of the kaonic nuclei, single-channel $\bar{K}N$ potentials were constructed from the phenomenological interaction [20] and from the chiral coupled-channel approach [21], both of which reproduce experimental data of $\bar{K}N$ phenomenology. The relative mean distance of $\bar{K}$ and N in $\Lambda(1405)$ was estimated to be about 1.36 fm by phenomenological potential [20] and about 1.8 fm by chiral potential [22] where the difference of the results is attributed to the difference of binding energies and strengths of the potentials. Our result, $\sqrt{|\langle r^2\rangle_{\mathrm{E}}|} \simeq 1.48$ fm, is quantitatively similar to the estimated size of the $\Lambda(1405)$, although one should note that we have evaluated the charge radius which cannot be directly compared with the mean distance of $\bar{K}$ and N.

6. Conclusions

We have calculated the electromagnetic mean squared radii of $\Lambda(1405)$ based on the meson–baryon picture in the chiral unitary model. The evaluation of the electromagnetic mean squared radii have been performed in two ways: In the first approach, with full coupled channels for the $\Lambda(1405)$, the mean squared radii in complex numbers are obtained at the poles for the physical resonant states and the absolute values suggest that the form factors for the $\Lambda(1405)$ is softer than those for the neutron. In the second, we describe $\Lambda(1405)$ as a bound state of $\bar{K}N$ by neglecting all the off-diagonal couplings of $\bar{K}N$ to the other channels, in order to estimate the size of the resonant state. As a consequence of the small binding energy in chiral unitary model, our result implies that K^- in $\Lambda(1405)$ is widely spread around p and that the size of the $\Lambda(1405)$ is larger than that of typical ground state baryons.

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Appendix A

In this appendix, we show that the present formulation for the amplitude with electromagnetic coupling developed in Sections 3 and 4 satisfies Ward–Takahashi identity at the resonance point of the excited baryon with charge $Q_{\rm H}$.

The photon coupling amplitude in the present formulation is given by three contributions:

$$T^{\mu}_{\gamma} = T^{\mu}_{\gamma 1} + T^{\mu}_{\gamma 2} + T^{\mu}_{\gamma 3}. \tag{A.1}$$

The definition of each term is, as given in Eqs. (21), (22) and (23),

$$\begin{aligned} T^{\mu}_{\gamma 1 ij} &= \sum_{l} T_{il}(\sqrt{s'}) D^{\mu}_{{\rm M}_l} T_{lj}(\sqrt{s}), \\ T^{\mu}_{\gamma 2 ij} &= \sum_{l} T_{il}(\sqrt{s'}) D^{\mu}_{{\rm B}_l} T_{lj}(\sqrt{s}), \\ T^{\mu}_{\gamma 3 ij} &= \sum_{k,l} T_{ik}(\sqrt{s'}) G_k(\sqrt{s'}) \Gamma^{\mu}_{kl} G_l(\sqrt{s}) T_{lj}(\sqrt{s}), \end{aligned} \tag{A.2}$$

where $s \equiv P^{\mu}P_{\mu}$, $s' \equiv P'^{\mu}P'_{\mu}$ and $P'^{\mu} = P^{\mu} + q^{\mu}$ with the photon momentum q_{μ}. Let us also recall the photon couplings appearing these three contributions:

$$\begin{aligned} -iV^{\mu}_{{\rm M}_i}(k,k') &= iQ_{{\rm M}_i}(k+k')^{\mu}, \\ -iV^{\mu}_{{\rm B}_i}(p,\ p') &= iQ_{{\rm B}_i}(p+p')^{\mu}, \\ -i\Gamma^{\mu}_{ij}(P,\ P') &= iQ_{\rm H}\frac{C_{ij}}{2f^2}\frac{P^{\mu}+P'^{\mu}}{\sqrt{s}+\sqrt{s'}}, \end{aligned} \tag{A.3}$$

with incoming and outgoing momenta of mesons k^{μ}, k'^{μ} and baryons p^{μ}, p'^{μ}, charge of mesons $Q_{{\rm M}_i}$ and baryons $Q_{{\rm B}_i}$, and $P^{\mu} = k^{\mu} + p^{\mu}$, $P'^{\mu} = k'^{\mu} + p'^{\mu}$, $Q_{\rm H} = Q_{\rm M} + Q_{\rm B}$. Here we have omitted the terms with the Pauli matrices $\boldsymbol{\sigma}$, which vanish when q^{μ} is multiplied.

With the aid of the following identity,

$$\begin{aligned} & q_{\mu}\frac{1}{(q_1+q)^2-m^2}(2q_1+q)^{\mu}\frac{1}{q_1^2-m^2} \\ &\quad = \frac{1}{q_1^2-m^2} - \frac{1}{(q_1+q)^2-m^2}, \end{aligned} \tag{A.4}$$

we can obtain the loop integrals in Eqs. (24) and (25) in terms of the loop function G:

$$\begin{aligned} q_{\mu}D^{\mu}_{{\rm M}_l} &= Q_{{\rm M}_l}\big[G_l(\sqrt{s'}) - G_l(\sqrt{s})\big], \\ q_{\mu}D^{\mu}_{{\rm B}_l} &= Q_{{\rm B}_l}\big[G_l(\sqrt{s'}) - G_l(\sqrt{s})\big]. \end{aligned} \tag{A.5}$$

Therefore, multiplying q_{μ} to $T^{\mu}_{\gamma 1ij} + T^{\mu}_{\gamma 2ij}$, we obtain

$$q_{\mu}\big(T^{\mu}_{\gamma 1ij} + T^{\mu}_{\gamma 2ij}\big) = Q_{\rm H}\sum_{l} T'_{il}[G'_l - G_l]T_{lj}. \tag{A.6}$$

Here G' and T' mean $G(\sqrt{s'})$ and $T(\sqrt{s'})$, respectively.

The coupling Γ^{μ}_{ij} satisfies

$$\begin{aligned} q_{\mu}\Gamma^{\mu}_{ij}(P,P') &= -Q_{\rm H}\frac{C_{ij}}{2f^2}(\sqrt{s'}-\sqrt{s}) \\ &= Q_{\rm H}\big[V'_{ij} - V_{ij}\big], \end{aligned} \tag{A.7}$$

with the Weinberg–Tomozawa interaction $V'_{ij} \equiv V_{ij}(\sqrt{s'})$ and $V_{ij} \equiv V_{ij}(\sqrt{s})$. Thus,

$$q_{\mu}T^{\mu}_{\gamma 3ij} = Q_{\rm H}\sum_{k,l} T'_{ik}G'_k\big[V'_{kl} - V_{kl}\big]G_l T_{lj}. \tag{A.8}$$

Collecting all the terms and using BS equation (1), we obtain

$$\begin{aligned} q_{\mu}T^{\mu}_{\gamma} &= Q_{\rm H}\big[T'G'(T - VGT) - (T' - T'G'V')GT\big] \\ &= Q_{\rm H}[T'G'V - V'GT], \end{aligned} \tag{A.9}$$

in matrix form. This equation has only single pole terms in the right-hand side. For calculation of the form factor of the baryon resonances, we extract residues of double pole of the amplitude in the Breit frame and the right-hand side of this equation does not contribute to the residues. Therefore, the present formulation satisfies Ward–Takahashi identity at the resonance position and the form factors evaluated by T^{μ}_{γ} are gauge-invariant. It is also seen in the above argument that, off the pole position, it is necessary to include the other terms shown in Ref. [27] for gauge invariance.

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