



# Intrinsic deviation from the tri-bimaximal neutrino mixing in a class of $A_4$ flavor models

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## ABSTRACT

It is well known that the tri-bimaximal neutrino mixing pattern  $V_0$  can be derived from a class of flavor models with the non-Abelian  $A_4$  symmetry. We point out that small corrections to  $V_0$ , which are inherent in the  $A_4$  models and arise from both the charged-lepton and neutrino sectors, have been omitted in the previous works. We show that such corrections may lead the  $3 \times 3$  neutrino mixing matrix  $V$  to a non-unitary deviation from  $V_0$ , but they cannot result in a nonzero value of  $\theta_{13}$  or any new CP-violating phases. Current experimental constraints on the unitarity of  $V$  allow us to constrain the model parameters to some extent.

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## 1. Introduction

Thanks to a number of convincing neutrino oscillation experiments [1], we have known two neutrino mass-squared differences ( $\Delta m_{21}^2$  and  $|\Delta m_{31}^2|$ ) and two neutrino mixing angles ( $\theta_{12}$  and  $\theta_{23}$ ) to a good degree of accuracy [2]. The smallest neutrino mixing angle  $\theta_{13}$  remains unknown, but there are some preliminary hints that it might not be very small (e.g.,  $\theta_{13} \sim 7^\circ$  [2–4]). Nevertheless, current experimental data are consistent very well with a constant neutrino mixing matrix—the so-called tri-bimaximal mixing pattern [5]

$$V_0 = U_\omega^T U_\nu^* = \frac{1}{\sqrt{6}} Q_l \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix} Q_\nu, \quad (1)$$

where

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (2)$$

$$U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix},$$

$\omega = e^{i2\pi/3}$ ,  $Q_l = \text{Diag}\{1, \omega, -\omega^2\}$  and  $Q_\nu = \text{Diag}\{1, 1, i\}$  [6]. The diagonal phase matrix  $Q_l$  can be rotated away by redefining the

phases of three charged-lepton fields, but  $Q_\nu$  may affect the neutrinoless double-beta decay if neutrinos are the Majorana particles. Given the standard parametrization of the Maki–Nakagawa–Sakata–Pontecorvo (MNSP) neutrino mixing matrix [7],  $V_0$  corresponds to  $\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ ,  $\theta_{13} = 0^\circ$  and  $\theta_{23} = 45^\circ$ . A more realistic form of the MNSP matrix  $V$  is expected to slightly deviate from  $V_0$  due to some nontrivial perturbations, such that both nonzero  $\theta_{13}$  and CP violation can emerge.

It is possible to derive the tri-bimaximal mixing pattern  $V_0$  from some neutrino mass models with certain flavor symmetries [8]. In this connection the earliest and most popular application is the non-Abelian discrete  $A_4$  symmetry (see, e.g., Refs. [9–11]). But the neutrino mixing matrix derived from a specific  $A_4$  model is in general not equal to  $V_0$  unless some approximations are made. In other words, small corrections to  $V_0$  are generally inherent in the  $A_4$  models and can arise both from the charged-lepton sector and from the neutrino sector. This observation is particularly interesting for an  $A_4$  model built in the vicinity of the TeV scale, because the resultant corrections to  $V_0$  may not be strongly suppressed. We show that such corrections can lead the  $3 \times 3$  neutrino mixing matrix  $V$  to a non-unitary deviation from  $V_0$ , although they cannot give rise to a nonzero value of  $\theta_{13}$  or any new CP-violating phases. We find that current experimental constraints on the unitarity of  $V$  allow us to constrain the parameters of an  $A_4$  model to some extent.

The remaining part of this Letter is organized as follows. In Section 2 we first outline the salient features of a typical  $A_4$  model and then diagonalize the  $6 \times 6$  mass matrices of charged leptons and neutrinos. We show that both  $U_\omega$  and  $U_\nu$  in Eq. (2) get modified in this framework. In Section 3 we work out the non-

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**Table 1**

The particle content and charge assignments of the model [11], where the subscript  $i$  (for  $i = 1, 2, 3$ ) stands for the family index.

	$Q_i$	$d_i^c$	$u_i^c$	$L_i$	$e_1^c, e_2^c, e_3^c$	$\nu_i^c$	$E_i$	$E_i^c$	$H_u$	$H_d$	$\chi_i$	$\chi_i'$	$S_{a,b}$
$SU(2)_L$	2	1	1	2	1	1	1	1	2	2	1	1	1
$U(1)_Y$	1/3	2/3	-4/3	-1	2	0	-2	2	1	-1	0	0	0
$A_4$	1	1	1	3	1, 1', 1''	3	3	3	1	1	3	3	1
$Z_4$	1	1	0	1	3	0	1	1	1	0	2	2	2
$Z_3$	1	2	0	0	0	1	0	0	2	0	0	1	1

unitary departure of the resultant  $3 \times 3$  MNSP matrix  $V$  from the tri-bimaximal mixing pattern  $V_0 = U_\omega^T U_\nu^*$ . We also constrain the model parameters to some extent by taking account of current experimental constraints on the unitarity of  $V$ . Section 4 is devoted to a summary and some concluding remarks.

## 2. Corrections to $U_\omega$ and $U_\nu$ in a typical $A_4$ model

Let us consider a simple but typical  $A_4$  model proposed by Babu and He in Ref. [11]. The model is an extension of the standard electroweak  $SU(2)_L \times U(1)_Y$  model with some additional particles, and it is supersymmetric and  $(A_4 \times Z_4 \times Z_3)$ -invariant. The particle content and charge assignments are summarized in Table 1. The discrete symmetries force the superpotentials of quarks and leptons to have the following forms:

$$\begin{aligned}
 W_q &= y_{ij}^d Q_i d_j^c H_d + y_{ij}^u Q_i u_j^c H_u, \\
 W_\ell &= M_E E_i E_i^c + f_\ell L_i E_i^c H_d + h_e (E_1 \chi_1 + E_2 \chi_2 + E_3 \chi_3) e_1^c \\
 &\quad + h_\mu (E_1 \chi_1 + \omega E_2 \chi_2 + \omega^2 E_3 \chi_3) e_2^c \\
 &\quad + h_\tau (E_1 \chi_1 + \omega^2 E_2 \chi_2 + \omega E_3 \chi_3) e_3^c, \\
 W_\nu &= f_\nu L_i \nu_i^c H_u + \frac{1}{2} f_{S_a} \nu_i^c \nu_i^c S_a + \frac{1}{2} f_{S_b} \nu_i^c \nu_i^c S_b \\
 &\quad + \frac{1}{2} f_{\chi'} [(v_2^c v_3^c + v_3^c v_2^c) \chi_1' + (v_1^c v_3^c + v_3^c v_1^c) \chi_2' \\
 &\quad + (v_2^c v_1^c + v_1^c v_2^c) \chi_3'], \quad (3)
 \end{aligned}$$

where the notations are self-explanatory [11]. Note that the quark sector is completely the same as that in the minimal supersymmetric standard model, and the  $Z_4$  symmetry works as an R-parity such that the superpotentials possess two units of charge. Thanks to the supersymmetry and new scalars in Eq. (3), it is possible to obtain the vacuum expectation values [11]

$$\begin{aligned}
 \langle S_a \rangle &= 0, & \langle S_b \rangle &= v_s, & \langle H_u \rangle &= v_u, & \langle H_d \rangle &= v_d, \\
 \langle \chi \rangle &= (v_\chi, v_\chi, v_\chi), & \langle \chi' \rangle &= (0, v_{\chi'}, 0), \quad (4)
 \end{aligned}$$

where  $v_u^2 + v_d^2 = v^2$  with  $v \simeq 174$  GeV. Thus the  $A_4$  symmetry is broken after  $\chi$  and  $\chi'$  develop their vacuum expectation values.

In the basis of  $(e, E)$  versus  $(e^c, E^c)^T$ , we obtain the  $6 \times 6$  mass matrix of charged leptons from Eqs. (3) and (4):

$$\mathcal{M}_{\ell E} = \begin{pmatrix} \mathbf{0} & f_\ell v_d \mathbf{1} \\ H & M_E \mathbf{1} \end{pmatrix}, \quad (5)$$

where  $\mathbf{1}$  denotes the  $3 \times 3$  identity matrix, and

$$H = \begin{pmatrix} h_e & h_\mu & h_\tau \\ h_e & \omega h_\mu & \omega^2 h_\tau \\ h_e & \omega^2 h_\mu & \omega h_\tau \end{pmatrix} v_\chi = \sqrt{3} U_\omega \begin{pmatrix} h_e & 0 & 0 \\ 0 & h_\mu & 0 \\ 0 & 0 & h_\tau \end{pmatrix} v_\chi. \quad (6)$$

Note that  $f_\ell$ ,  $M_E$  and  $h_\alpha$  (for  $\alpha = e, \mu, \tau$ ) can all be arranged to be real in a suitable phase convention, and the mass scale  $M_E$  is assumed to be extremely large in comparison with the magnitudes of  $f_\ell v_d$  and  $h_\alpha v_\chi$ . The  $6 \times 6$  Hermitian matrix  $\mathcal{M}_{\ell E} \mathcal{M}_{\ell E}^\dagger$  can be

diagonalized via the unitary transformation  $V_l^\dagger \mathcal{M}_{\ell E} \mathcal{M}_{\ell E}^\dagger V_l$ , where  $V_l$  is given by

$$V_l \simeq \begin{pmatrix} \mathbf{1} + \frac{HH^\dagger}{M_E^2} & \frac{f_\ell v_d}{M_E} \mathbf{1} \\ -\frac{f_\ell v_d}{M_E} \mathbf{1} & \mathbf{1} + \frac{HH^\dagger}{M_E^2} \end{pmatrix} \begin{pmatrix} U_\omega & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \quad (7)$$

as a good approximation. The masses of three standard charged leptons turn out to be

$$m_\alpha \simeq \sqrt{3} \frac{f_\ell v_d}{M_E} v_\chi h_\alpha, \quad (8)$$

where  $\alpha$  runs over  $e, \mu$  and  $\tau$ . Eq. (7) shows that  $U_\omega$  receives a small correction:

$$U_\omega \rightarrow U'_\omega = \left( \mathbf{1} + \frac{HH^\dagger}{M_E^2} \right) U_\omega. \quad (9)$$

It is actually  $U'_\omega$  that characterizes the contribution of charged leptons to the lepton flavor mixing in this  $A_4$  model.

Now we turn to the neutrino sector. The type-I seesaw mechanism [12] is implemented in the  $A_4$  model under consideration, and thus the overall neutrino mass matrix is a symmetric  $6 \times 6$  matrix:

$$\mathcal{M}_{\nu\nu^c} = \begin{pmatrix} \mathbf{0} & f_\nu v_u \mathbf{1} \\ f_\nu v_u \mathbf{1} & M_R \end{pmatrix}, \quad (10)$$

where  $M_R$  takes the form

$$M_R = \begin{pmatrix} f_{S_b} v_s & 0 & f_{\chi'} v_{\chi'} \\ 0 & f_{S_b} v_s & 0 \\ f_{\chi'} v_{\chi'} & 0 & f_{S_b} v_s \end{pmatrix}. \quad (11)$$

The symmetric neutrino mass matrix in Eq. (10) can be diagonalized via the orthogonal transformation  $V_\nu^T \mathcal{M}_{\nu\nu^c} V_\nu$ , where the unitary matrix  $V_\nu$  is given by

$$V_\nu \simeq \begin{pmatrix} \mathbf{1} - \frac{1}{2} \cdot \frac{|f_\nu|^2 v_u^2}{M_R^* M_R} & \frac{f_\nu^* v_u}{M_R} \\ -\frac{f_\nu v_u}{M_R} & \mathbf{1} - \frac{1}{2} \cdot \frac{|f_\nu|^2 v_u^2}{M_R^* M_R} \end{pmatrix} \begin{pmatrix} U_\nu P_\nu & \mathbf{0} \\ \mathbf{0} & U_R \end{pmatrix} \quad (12)$$

to a good degree of accuracy. In this expression  $U_\nu$  has been given in Eq. (2),  $P_\nu$  denotes a diagonal phase matrix [11], and  $U_R$  is a unitary matrix responsible for the diagonalization of  $M_R$ . The masses of three light (active) neutrinos turn out to be  $m_1 \simeq |m_0(1+x)|$ ,  $m_2 \simeq |m_0(1+x)(1-x)|$  and  $m_3 \simeq |m_0(1-x)|$ , where

$$m_0 = \frac{f_\nu^2 v_u^2 f_{S_b} v_s}{f_{S_b}^2 v_s^2 - f_{\chi'}^2 v_{\chi'}^2}, \quad x = -\frac{f_{\chi'} v_{\chi'}}{f_{S_b} v_s}. \quad (13)$$

Because both  $m_0$  and  $x$  are complex, it is possible to adjust their magnitudes and phases such that the resultant values of  $m_i$  (for  $i = 1, 2, 3$ ) satisfy current experimental data on the neutrino mass spectrum [11]. Eq. (12) shows that  $U_\nu P_\nu$ , which signifies the contribution of neutrinos to the lepton flavor mixing, receives a small correction:

$$U_\nu P_\nu \rightarrow U'_\nu P_\nu = \left( \mathbf{1} - \frac{1}{2} \cdot \frac{|f_\nu|^2 v_u^2}{M_R^* M_R} \right) U_\nu P_\nu. \quad (14)$$

In other words,  $U'_\nu$  is not exactly unitary and its departure from  $U_\nu$  is in general an unavoidable consequence in the type-I seesaw mechanism [13].

### 3. Non-unitary corrections to $V_0$

With the help of the results obtained in Eqs. (9) and (14), we are able to calculate the MNSP matrix  $V = U'_\omega{}^T (U'_\nu P_\nu)^*$  and demonstrate its non-unitary deviation from the tri-bimaximal mixing pattern  $V_0$ . We find

$$\begin{aligned} V &= U_\omega^T \left( \mathbf{1} + \frac{H^* H^T}{M_E^2} \right) \left( \mathbf{1} - \frac{1}{2} \cdot \frac{|f_\nu|^2 v_u^2}{M_R M_R^\dagger} \right) U'_\nu P_\nu^* \\ &\simeq V_0 P_\nu^* + \frac{1}{f_\ell^2 v_d^2} \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} V_0 P_\nu^* \\ &\quad - \frac{1}{2} \cdot \frac{1}{|f_\nu|^2 v_u^2} V_0 \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} P_\nu^* \\ &\simeq Q_l \left[ \mathbf{1} + \frac{1}{f_\ell^2 v_d^2} \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} \right. \\ &\quad \left. - \frac{1}{12} \cdot \frac{1}{|f_\nu|^2 v_u^2} \begin{pmatrix} 2(m_1^2 + m_2^2) & 2(m_2^2 - m_1^2) & 2(m_1^2 - m_2^2) \\ 2(m_2^2 - m_1^2) & m_1^2 + 2m_2^2 + 3m_3^2 & 3m_3^2 - m_1^2 - 2m_2^2 \\ 2(m_1^2 - m_2^2) & 3m_3^2 - m_1^2 - 2m_2^2 & m_1^2 + 2m_2^2 + 3m_3^2 \end{pmatrix} \right] V_0' P_\nu^*, \end{aligned} \quad (15)$$

where

$$V_0' = Q_l^* V_0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix} Q_\nu, \quad (16)$$

and  $Q_l$  and  $Q_\nu$  have been given below Eq. (2). In obtaining Eq. (15) we have omitted the higher-order and much smaller corrections. Because of  $v_u = v \sin \beta$  and  $v_d = v \cos \beta$  in the supersymmetric  $A_4$  model under consideration,  $v_d \ll v_u$  might hold for a very large value of  $\tan \beta$ . Depending on the magnitudes of  $f_\ell^2$  and  $|f_\nu|^2$ , the term proportional to  $1/(f_\ell^2 v_d^2)$  or  $1/(|f_\nu|^2 v_u^2)$  in Eq. (15) might not be negligibly small. These two terms, which are inherent in the model itself, measure the non-unitary contribution to  $V$  or the departure of  $V$  from  $V_0' P_\nu^*$ . This observation makes sense since it indicates that the exact tri-bimaximal neutrino mixing pattern  $V_0$  is not an exact consequence of a class of  $A_4$  flavor models.

One may parametrize the analytical result obtained in Eq. (15) as follows:

$$V = Q_l (\mathbf{1} - \eta) V_0' P_\nu^* = V_0 P_\nu^* - Q_l \eta V_0' P_\nu^*, \quad (17)$$

where the Hermitian matrix  $\eta$  signifies the non-unitary deviation of  $V$  from  $V_0 P_\nu^*$ . Note that the diagonal phase matrix  $Q_l$  in  $V$  can always be rotated away through a redefinition of the phases of three charged leptons, and the diagonal phase matrices  $Q_\nu$  and  $P_\nu^*$  in  $V$  only provide us with the Majorana phases which have nothing to do with leptonic CP violation in neutrino oscillations. Note also that  $\eta$  itself is real in this  $A_4$  model, as one can easily see from Eq. (15), and thus the unitarity violation of  $V$  does not give rise to any new CP-violating phases. Moreover, it is impossible to obtain nonzero  $V_{e3}$  or  $\theta_{13}$  from this typical  $A_4$  model, simply because  $\eta_{e\mu} = -\eta_{e\tau}$  holds. Such a disappointing observation implies that the residual flavor symmetry remains powerful to keep  $V_{e3}$

or  $\theta_{13}$  vanishing and forbid CP violation, even though the MNSP matrix  $V$  is not exactly unitary.

Current experimental data allow us to constrain the matrix elements of  $\eta$  and then constrain the model parameters to some extent. A recent analysis yields [14]

$$|\eta| < \begin{pmatrix} 2.0 \times 10^{-3} & 6.0 \times 10^{-5} & 1.6 \times 10^{-3} \\ 6.0 \times 10^{-5} & 8.0 \times 10^{-4} & 1.1 \times 10^{-3} \\ 1.6 \times 10^{-3} & 1.1 \times 10^{-3} & 2.7 \times 10^{-3} \end{pmatrix}. \quad (18)$$

In view of Eqs. (15) and (16), we immediately obtain

$$\begin{aligned} \eta_{e\mu} &= -\eta_{e\tau} = \frac{\Delta m_{21}^2}{6|f_\nu|^2 v_u^2} = \frac{\Delta m_{21}^2}{6|f_\nu|^2 v^2 \sin^2 \beta}, \\ \eta_{\mu\tau} &= \frac{\Delta m_{31}^2 + 2\Delta m_{32}^2}{12|f_\nu|^2 v_u^2} \simeq \frac{\Delta m_{31}^2}{4|f_\nu|^2 v^2 \sin^2 \beta}, \end{aligned} \quad (19)$$

where  $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2$  and  $\Delta m_{31}^2 \equiv m_3^2 - m_1^2 \simeq m_3^2 - m_2^2 \equiv \Delta m_{32}^2 \simeq \pm 2.4 \times 10^{-3} \text{ eV}^2$  [2]. Eq. (19) leads us to a simple but instructive relation for three off-diagonal matrix elements of  $\eta$ :

$$\frac{\eta_{e\mu}}{\eta_{\mu\tau}} = -\frac{\eta_{e\tau}}{\eta_{\mu\tau}} \simeq \frac{2}{3} \cdot \frac{\Delta m_{21}^2}{\Delta m_{31}^2}. \quad (20)$$

Therefore,  $|\eta_{e\mu}|/|\eta_{\mu\tau}| = |\eta_{e\tau}|/|\eta_{\mu\tau}| \simeq 2.1 \times 10^{-2}$ . Comparing this prediction with Eq. (18), one may self-consistently get  $|\eta_{e\mu}| = |\eta_{e\tau}| < 2.3 \times 10^{-5}$  by taking  $|\eta_{\mu\tau}| < 1.1 \times 10^{-3}$ . So it is more appropriate to use the upper bound of  $|\eta_{\mu\tau}|$  to constrain the lower bound of  $|f_\nu|$  by means of Eq. (19). We arrive at

$$|f_\nu| = \frac{1}{2v \sin \beta} \cdot \frac{\sqrt{|\Delta m_{31}^2|}}{\sqrt{|\eta_{\mu\tau}|}} > \frac{4.2}{\sin \beta} \times 10^{-12}. \quad (21)$$

This result, which depends on the value of  $\tan \beta$  in the supersymmetric  $A_4$  model, implies that the Yukawa coupling of neutrinos should not be too small in order to preserve the unitarity of  $V$  at an experimentally-allowed level. It clearly indicates that an arbitrary choice of  $f_\nu$  in the neglect of small unitarity violation of  $V$  is inappropriate for model building, because the correlation between  $f_\nu$  and the deviation of  $V$  from the tri-bimaximal mixing pattern is an intrinsic property of a class of  $A_4$  models.

The diagonal matrix elements of  $\eta$  consist of the contributions from both the charged-lepton sector and the neutrino sector, as shown in Eq. (15). Their competition depends on the sizes of  $f_\ell$ ,  $f_\nu$  and  $\tan \beta$ . For simplicity, here we assume that the charged-lepton contribution to  $\eta_{\alpha\alpha}$  (for  $\alpha = e, \mu, \tau$ ) is dominant. Then it is straightforward to obtain

$$\eta_{\alpha\alpha} \simeq -\frac{m_\alpha^2}{f_\ell^2 v_d^2} = -\frac{m_\alpha^2}{f_\ell^2 v^2 \cos^2 \beta}. \quad (22)$$

As a result,

$$\eta_{ee} : \eta_{\mu\mu} : \eta_{\tau\tau} \simeq m_e^2 : m_\mu^2 : m_\tau^2 \simeq 1 : 44\,566 : 12\,880\,040, \quad (23)$$

where we have input the central values of three charged-lepton masses at the electroweak scale [15]. Comparing this prediction with Eq. (18), one may self-consistently arrive at  $|\eta_{ee}| < 2.1 \times 10^{-10}$  and  $|\eta_{\mu\mu}| < 9.3 \times 10^{-6}$  by taking  $|\eta_{\tau\tau}| < 2.7 \times 10^{-3}$ . It is therefore more appropriate to use the upper bound of  $|\eta_{\tau\tau}|$  to constrain the lower bound of  $|f_\ell|$  with the help of Eq. (22). We find

$$|f_\ell| \simeq \frac{m_\tau}{v \cos \beta \sqrt{|\eta_{\tau\tau}|}} > \frac{0.19}{\cos \beta}, \quad (24)$$

where  $m_\tau \simeq 1746.24$  MeV has been input at the electroweak scale [15]. This result, which also depends on the value of  $\tan \beta$  in the supersymmetric  $A_4$  model, shows that the Yukawa coupling of charged leptons should be relatively large in order to preserve the unitarity of  $V$  as constrained by current measurements. We stress that an arbitrary choice of either  $f_\ell$  or  $f_\nu$  in the neglect of small unitarity violation of  $V$  might be problematic for model building, simply because they receive constraints both from the model itself and from the experimental data. In this sense one must be cautious to claim that an  $A_4$  flavor model can predict the tri-bimaximal neutrino mixing pattern whose matrix elements are constant and thus have nothing to do with the model parameters [16]. In fact, the slight (non-unitary) deviation of  $V$  from the tri-bimaximal mixing pattern is likely to impose a strong restriction on some model parameters like  $f_\ell$ ,  $f_\nu$  and  $\tan \beta$ .

#### 4. Summary

We have examined a class of  $A_4$  flavor models to see whether the tri-bimaximal neutrino mixing pattern  $V_0$  is an exact consequence of such models. We find that small corrections to  $V_0$  are actually inherent in the  $A_4$  models and may arise from both the charged-lepton and neutrino sectors. We have demonstrated that such corrections may lead the MNSP matrix  $V$  to a non-unitary deviation from  $V_0$ , but they cannot result in a nonzero  $V_{e3}$  (or  $\theta_{13}$ ) or any new CP-violating phases. In particular, the slight unitarity violation of  $V$  is sensitive to several model parameters, including the Yukawa couplings of charged leptons and neutrinos. We have shown that current experimental constraints on the unitarity of  $V$  allow us to constrain the model parameters to some extent.

We stress that the departure of  $V$  from  $V_0$  explored in this work is an intrinsic property of a class of flavor models with the non-Abelian  $A_4$  symmetry. Different departures may result either from the vacuum-expectation-value misalignments in a certain  $A_4$  model or from some purely phenomenological perturbations [17]. The non-unitary deviation of  $V$  from  $V_0$  is in some sense more interesting because it might give rise to new CP-violating effects in a variety of long-baseline neutrino oscillation experiments [18]. Since a lot of attention has been paid to how to derive the tri-bimaximal neutrino mixing pattern  $V_0$ , the points revealed in our Letter should be taken into account when one attempts to build specific flavor models with discrete family symmetries.

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