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Softness of brane-localized supersymmetry breaking on orbifolds

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Abstract

We consider the brane-localized supersymmetry breaking in 5D compactified on S^1/Z_2. In case of a bulk gaugino with arbitrary brane masses for its even and odd modes, we find the mass spectrum and the wave functions of gaugino. We show that the gaugino masses at the distant brane are soft in the usual sense in the effective field theory with zero modes of bulk gauge fields and they are also extremely soft in view of the one-loop finite mass of a brane scalar in the KK regularization.
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1. Introduction

Orbifold compactification of extra dimensions is necessary to get a chiral fermion and a lower supersymmetry as zero modes from higher dimensions [1]. Moreover, in recent works on GUT orbifolds, Scherk–Schwarz twists [2] have been also used to break the GUT symmetry in higher dimensions into the SM gauge group and break further the remaining supersymmetry after orbifolding. It is noticeable that as far as the mass spectrum and the mode functions are concerned, a Scherk–Schwarz (SS) breaking in orbifolds represented by a local symmetry in the Lagrangian is equivalent to a Wilson-line breaking along extra dimensions [3,4]. For instance, a SS twist for gauge symmetry breaking in orbifolds corresponds to a Wilson line of $\langle A_5 \rangle \neq 0$ of the 5D gauge field, and a SS twist for supersymmetry breaking in orbifolds corresponds to a Wilson line of $\langle V_5^1 + i V_5^2 \rangle \neq 0$ of the $SU(2)_R$ gauge fields in the 5D off-shell supergravity [5], which is the nonzero F term of the radion multiplet [6]. There has been a lot of discussion on the softness of SS breaking of supersymmetry in 5D compactified on the orbifold in view of the one-loop corrections for the zero mode of a bulk scalar [7–12]. It has been shown that the one-loop finiteness of SS breaking mainly comes from the so-called KK regularization [7,8,11,13].

As an alternative to the Scherk–Schwarz breaking of supersymmetry, in this Letter, we consider the brane-localized supersymmetry breaking [14–23]. For simplicity, we consider the 5D SUSY $U(1)$ gauge theories on

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S^1/Z_2 where the brane-localized supersymmetry breaking is parametrized by general brane mass terms for gaugino. When one introduces brane mass terms for gaugino, it is likely to simply drop the mass term for the odd mode of gaugino [6,19]. However, in case that the wave functions of odd modes have a discontinuity on the branes, the odd mass term also contributes to the equations of motion so that it makes the wave functions of even modes discontinuous on the branes [20–22]. Then, the brane coupling of the even modes are determined from the careful integration of the brane action, but not from the equations of motion.

In this Letter, with general brane mass terms for gaugino, we find the mass spectrum and the wave functions of gaugino. While the mass spectrum is the same as the case with a specific Scherk–Schwarz parameter, the wave functions of gaugino are modified due to the brane mass terms. Therefore, we find that the generic brane mass terms are not soft even in the usual sense in the effective field theory with zero modes of gauge fields. We also show that for the same brane couplings of gauge boson and gaugino, the one-loop finiteness of a brane scalar mass in our model is guaranteed in the KK regularization scheme. We find that this is the case with distant breaking of supersymmetry [15–18], i.e., brane matters at one brane and only brane masses of gaugino at the other brane. The one-loop finiteness in our model is due to the distant supersymmetry breaking which is necessary for the 4D supersymmetric gauge coupling at the brane where matter fields are located.

This Letter is organized as follows. For comparison with our brane-localized supersymmetry breaking, we first give a brief review on the Scherk–Schwarz boundary condition in 5D compactified on S^1/Z_2. In Section 3, we consider the general brane-localized supersymmetry breaking in the gauge sector and show the wave functions and the mass spectrum of the bulk gaugino. Then, in Section 4, we present the one-loop KK gauge corrections to a massless scalar located at the brane and discuss its finiteness in the context of the distant supersymmetry breaking. In Section 5, the conclusion is drawn.

2. Scherk–Schwarz boundary conditions

Let us first give a review on the Scherk–Schwarz breaking on orbifolds. One can impose a general SS boundary condition on a bulk field $\Phi(x, y)$ living in S^1 with the radius R as

$$\Phi(x, y + 2\pi R) = e^{2\pi i \omega} \Phi(x, y), \tag{1}$$

where x, y denotes 4D and extra dimension coordinates respectively and ω is the SS parameter. Then, one gets a mode expansion of the bulk field as

$$\Phi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{i(n+\omega)y/R} \Phi^{(n)}(x). \tag{2}$$

After the Z_2 orbifolding, which identifies y with $-y$ in S^1, the bulk field becomes even or odd under Z_2 as follows

$$\Phi_+(x, y) = \frac{1}{2} \big(\Phi(x, y) + \Phi(x, -y) \big) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \cos\big((n+\omega)y/R\big) \Phi^{(n)}(x), \tag{3}$$

$$\Phi_-(x, y) = \frac{1}{2i} \big(\Phi(x, y) - \Phi(x, -y) \big) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \sin\big((n+\omega)y/R\big) \Phi^{(n)}(x) \tag{4}$$

with the mass spectrum

$$M_n^2 = \frac{(n+\omega)^2}{R^2}, \quad n = \text{integer}. \tag{5}$$

Therefore, from Eqs. (3) and (4), we can rewrite the SS boundary conditions on S^1/Z_2 as

$$\begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix}(x, y + 2\pi R) = \begin{pmatrix} \cos(2\pi\omega) & -\sin(2\pi\omega) \\ \sin(2\pi\omega) & \cos(2\pi\omega) \end{pmatrix} \begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix}(x, y). \tag{6}$$

For instance, in 5D SUSY $U(1)$ gauge theories compactified on S^1/Z_2, the bulk gaugino is composed of two Weyl spinors λ_1 and λ_2, which are even and odd under Z_2, respectively. Then, performing a SS twist of the bulk gaugino and replacing the twisted gaugino by the untwisted bulk gaugino $(\tilde{\lambda}_1, \tilde{\lambda}_2)$ as

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}(x, y) = \begin{pmatrix} \cos(\omega y/R) & -\sin(\omega y/R) \\ \sin(\omega y/R) & \cos(\omega y/R) \end{pmatrix} \begin{pmatrix} \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \end{pmatrix}(x, y), \tag{7}$$

one can show that the twisted bulk gaugino without mass terms is equivalent to the untwisted bulk gaugino with constant bulk mass terms such as

$$-\frac{1}{2}(\lambda_1 \partial_y \lambda_2 - \lambda_2 \partial_y \lambda_1) = -\frac{1}{2}\frac{\omega}{R}(\tilde{\lambda}_1 \tilde{\lambda}_1 + \tilde{\lambda}_2 \tilde{\lambda}_2). \tag{8}$$

3. Brane-localized supersymmetry breaking

Now we are in a position to consider the brane-localized supersymmetry breaking. We consider a 5D SUSY $U(1)$ model compactified on S^1/Z_2 with the radius of R. After orbifolding, there appear two fixed points at $y = 0$ and $y = \pi R$ where brane matters can be located. The 5D action for the bulk gaugino we are considering is

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \left[\bar{\lambda}_1 i \bar{\sigma}^\mu \partial_\mu \lambda_1 + \bar{\lambda}_2 i \bar{\sigma}^\mu \partial_\mu \lambda_2 - \frac{1}{2}(\lambda_1 \partial_y \lambda_2 - \lambda_2 \partial_y \lambda_1) + \text{h.c.} \right.$$
$$\left. - \varepsilon_0(\lambda_1 \lambda_1 + \rho_0 \lambda_2 \lambda_2)\delta(y) - \varepsilon_\pi(\lambda_1 \lambda_1 + \rho_\pi \lambda_2 \lambda_2)\delta(y - \pi R) + \text{h.c.} \right], \tag{9}$$

where $\varepsilon_{0,\pi}$ are the dimensionless parameters of brane mass terms for gauginos and $\rho_{0,\pi}$ are the ratios between brane mass parameters of even and odd modes of gaugino at each brane. The brane mass terms have been also considered only at one fixed point in S^1/Z_2 in the presence of the Scherk–Schwarz breaking [22]. In our case, we consider a more general situation where brane mass terms exist at both two fixed points in S^1/Z_2.

We have chosen two Weyl components of the bulk gaugino, λ_1 and λ_2, to be even and odd under Z_2 respectively as the following

$$\lambda_1(-y) = \lambda_1(y), \qquad \lambda_2(-y) = -\lambda_2(y). \tag{10}$$

Then, when we make a KK reduction of the gaugino as

$$\begin{pmatrix} \lambda_1(x, y) \\ \lambda_2(x, y) \end{pmatrix} = \sum_n N_n \begin{pmatrix} u_1^{(n)}(y) \\ u_2^{(n)}(y) \end{pmatrix} \lambda^{(n)}(x), \tag{11}$$

where $i\bar{\sigma}^\mu \partial_\mu \lambda^{(n)} = M_n \bar{\lambda}^{(n)}$ with the KK mass M_n and N_n is the normalization constant, the equations of motion for the gaugino become

$$\partial_y u_1^{(n)} + \left(M_n - 2\rho_0 \varepsilon_0 \delta(y) - 2\rho_\pi \varepsilon_\pi \delta(y - \pi R) \right) u_2^{(n)} = 0, \tag{12}$$
$$-\partial_y u_2^{(n)} + \left(M_n - 2\varepsilon_0 \delta(y) - 2\varepsilon_\pi \delta(y - \pi R) \right) u_1^{(n)} = 0. \tag{13}$$

Now one can find it easy to solve the equation for the ratio $t_n \equiv u_2^{(n)}/u_1^{(n)}$ as follows

$$\frac{\partial t_n}{\partial y} = M_n(1 + t_n^2) - 2\varepsilon_0(1 + \rho_0 t_n^2)\delta(y) - 2\varepsilon_\pi(1 + \rho_\pi t_n^2)\delta(y - \pi R). \tag{14}$$

Thus, after integrating both sides of the above equation over an infinitesimal interval around the branes, we obtain the following limiting values of t_n on the boundaries

$$\frac{1}{\sqrt{\rho_0}} \arctan(\sqrt{\rho_0}\, t_n)\Big|_{y=0^+} = -\varepsilon_0, \tag{15}$$

$$\frac{1}{\sqrt{\rho_\pi}} \arctan(\sqrt{\rho_\pi}\, t_n)\Big|_{y=\pi R^-} = \varepsilon_\pi. \tag{16}$$

Then, we get solutions for t_n as

$$t_n = \begin{cases} \tan[M_n y - \arctan\alpha(\rho_0, \varepsilon_0\epsilon(y))], & -\pi R < y < \pi R, \\ \tan[M_n(y - \pi R) - \arctan\alpha(\rho_\pi, \varepsilon_\pi\epsilon(y - \pi R))], & 0 < y < 2\pi R, \end{cases} \tag{17}$$

where

$$\alpha(\rho_0, \varepsilon_0\epsilon(y)) = \frac{1}{\sqrt{\rho_0}} \tan(\sqrt{\rho_0}\, \varepsilon_0\epsilon(y)), \tag{18}$$

$$\alpha(\rho_0, \varepsilon_\pi\epsilon(y - \pi R)) = \frac{1}{\sqrt{\rho_\pi}} \tan(\sqrt{\rho_\pi}\, \varepsilon_\pi\epsilon(y - \pi R)), \tag{19}$$

with $\epsilon(y)$ being the step function of periodicity $2\pi R$ given by

$$\epsilon(y) = \begin{cases} +1, & 0 < y < \pi R, \\ 0, & y = 0, \\ -1, & -\pi R < y < 0. \end{cases} \tag{20}$$

Here we note $\alpha(\rho_0, \varepsilon_0) = \tanh(\sqrt{|\rho_0|}\,\varepsilon_0)/\sqrt{|\rho_0|}$ for $\rho_0 < 0$ and $\alpha(\rho_\pi, \varepsilon_\pi) = \tanh(\sqrt{|\rho_\pi|}\,\varepsilon_\pi)/\sqrt{|\rho_\pi|}$ for $\rho_\pi < 0$. We also find the mass spectrum of the gaugino as

$$M_n = \frac{n}{R} + \frac{1}{\pi R}\big(\arctan\alpha(\rho_0, \varepsilon_0) + \arctan\alpha(\rho_\pi, \varepsilon_\pi)\big), \tag{21}$$

where n is an integer. The mass spectrum with $\alpha(\rho_\pi, \varepsilon_\pi) = 0$, i.e., $\varepsilon_\pi = 0$, is the same as the result in Ref. [22]. Thus, we find that the mass spectrum of gaugino is shifted by the amount given in terms of the brane mass parameters. This is equivalent to the one from a Scherk–Schwarz breaking of parameter

$$\omega = \frac{1}{\pi}\big[\arctan\alpha(\rho_0, \varepsilon_0) + \arctan\alpha(\rho_\pi, \varepsilon_\pi)\big]. \tag{22}$$

Particularly, for $\alpha(\rho_0, \varepsilon_0) = -\alpha(\rho_\pi, \varepsilon_\pi)$, we have the remaining supersymmetry restored. This would be the case with two fine-tunings of $\varepsilon_0 = -\varepsilon_\pi$ and $\rho_0 = \rho_\pi$.

For the strong supersymmetry breaking, $\varepsilon_0 \gg 1$ and/or $\varepsilon_\pi \gg 1$, the mass spectrum depends on the sign of ρ_0 and ρ_π. For positive sign of odd-mode mass parameters, depending on the large even-mode mass parameters, the zero-mode gaugino mass oscillates between two values: $M_0 \simeq \pm 1/R$ for $\rho_0 > 0$ and $\rho_\pi > 0$ in the case with strong supersymmetry breaking on both branes while $M_0 \simeq (\pm\frac{1}{2} + \frac{1}{\pi}\arctan\alpha(\rho_{\pi(0)}, \varepsilon_{\pi(0)}))/R$ for $\rho_{0(\pi)} > 0$ in the case with strong supersymmetry breaking on either brane. On the other hand, for negative sign of odd-mode mass parameters, the leading mass spectrum becomes independent of the large even-mode mass parameter but the still depends on ρ_0 and/or ρ_π: $M_0 \simeq [\arctan(1/\sqrt{|\rho_0|}) + \arctan(1/\sqrt{|\rho_\pi|})]/(\pi R)$ for $\rho_0 < 0$ and $\rho_\pi < 0$ in the case with strong supersymmetry breaking on both branes while $M_0 \simeq [\arctan(1/\sqrt{|\rho_{0(\pi)}|}) + \arctan\alpha(\rho_{\pi(0)}, \varepsilon_{\pi(0)})]/(\pi R)$ for $\rho_{0(\pi)} < 0$ in the case with strong supersymmetry breaking on either brane.

Moreover, from Eqs. (12) and (13), we get the eigenmodes for the gaugino for $-\pi R < y < \pi R$ as follows

$$
\begin{pmatrix} u_1^{(n)}(y) \\ u_2^{(n)}(y) \end{pmatrix} = A\big(\rho_0, \varepsilon_0\epsilon(y)\big) \begin{pmatrix} \cos\big[M_n y - \arctan\alpha\big(\rho_0, \varepsilon_0\epsilon(y)\big)\big] \\ \sin\big[M_n y - \arctan\alpha\big(\rho_0, \varepsilon_0\epsilon(y)\big)\big] \end{pmatrix},
\tag{23}
$$

where

$$
A\big(\rho_0, \varepsilon_0\epsilon(y)\big) \equiv \left(\frac{1 + \alpha^2(\rho_0, \varepsilon_0\epsilon(y))}{1 + \rho_0\alpha^2(\rho_0, \varepsilon_0\epsilon(y))} \right)^{1/2}.
\tag{24}
$$

The prefactor $A(\rho_0, \varepsilon_0\epsilon(y))$ has been already found in Ref. [22]. However, for the analysis of brane couplings of gaugino, we need to know the correct normalization constant which is obtained by inserting the equations of motion in the action as

$$
N_n = \left(\int_{-\pi R}^{\pi R} dy \left[\left(u_1^{(n)}\right)^2 + \left(u_2^{(n)}\right)^2 \right] \right)^{-1/2} = \frac{1}{\sqrt{2\pi R}} \frac{1}{A(\rho_0, \varepsilon_0)}.
\tag{25}
$$

Likewise, we get the eigenmodes for the gaugino for $0 < y < 2\pi R$ as follows

$$
\begin{pmatrix} u_1^{(n)}(y) \\ u_2^{(n)}(y) \end{pmatrix} = (-1)^n A\big(\rho_\pi, \varepsilon_\pi\epsilon(y - \pi R)\big) \begin{pmatrix} \cos\big[M_n(y - \pi R) - \arctan\alpha\big(\rho_\pi, \varepsilon_\pi\epsilon(y - \pi R)\big)\big] \\ \sin\big[M_n(y - \pi R) - \arctan\alpha\big(\rho_\pi, \varepsilon_\pi\epsilon(y - \pi R)\big)\big] \end{pmatrix}
\tag{26}
$$

with the normalization constant

$$
N_n = \left(\int_{0}^{2\pi R} dy \left[\left(u_1^{(n)}\right)^2 + \left(u_2^{(n)}\right)^2 \right] \right)^{-1/2} = \frac{1}{\sqrt{2\pi R}} \frac{1}{A(\rho_\pi, \varepsilon_\pi)},
\tag{27}
$$

where we inserted $(-1)^n$ in comparison with the previous solutions for $0 < y < \pi R$.

Then, the values of even and odd mode functions of gaugino at the branes are given by the definition of $\epsilon(y)$ as $u_1^{(n)}(0) = 1$, $u_1^{(n)}(\pi R) = (-1)^n$ and $u_2^{(n)}(0) = u_2^{(n)}(\pi R) = 0$ in any case. However, one should be careful in finding the *real* brane coupling of gaugino with the integration of the product of a discontinuous mode function and a delta function. The brane coupling of the nth (n is a nonnegative integer) KK mode of the bulk gauge boson[1] is given as $\sqrt{2^{(1-\delta_{n,0})}} g_4$ at $y = 0$ and $(-1)^n \sqrt{2^{(1-\delta_{n,0})}} g_4$ at $y = \pi R$ where $g_4 = g_5/\sqrt{2\pi R}$. On the other hand, the brane couplings of the nth (n is an integer) even mode of gaugino at $y = 0$ and $y = \pi R$ are given from the integrations of the brane action, respectively,

$$
g_0 \equiv g_5 \int dy\, \delta(y) N_n u_1^{(n)}(y) = g_4 A_0^{-1} \frac{\sin(\sqrt{\rho_0}\,\varepsilon_0)}{\sqrt{\rho_0}\,\varepsilon_0}
\tag{28}
$$

and

$$
g_\pi \equiv g_5 \int dy\, \delta(y - \pi R) N_n u_1^{(n)}(y) = g_4 (-1)^n A_\pi^{-1} \frac{\sin(\sqrt{\rho_\pi}\,\varepsilon_\pi)}{\sqrt{\rho_\pi}\,\varepsilon_\pi},
\tag{29}
$$

where $A_0 \equiv A(\rho_0, \varepsilon_0)$ and $A_\pi \equiv A(\rho_\pi, \varepsilon_\pi)$. Of course, the brane couplings of the odd modes of gaugino turn out to be zero after the integration of the brane action.

For generic $\rho_{0,\pi}$ and $\varepsilon_{0,\pi}$, the brane coupling squared of the gaugino is different from that of the gauge boson. Henceforth let us use the word of *brane coupling* for *brane coupling squared* without confusion. Irrespective of the mass spectrum of gaugino, the same brane coupling of gauge boson and gaugino is necessary for no

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[1] The loop correction coming from each massive KK mode with the brane coupling squared of $2g_4^2$ corresponds to those from two extra momentum states with the brane coupling squared of g_4^2.

quadratic divergence, i.e., softly broken supersymmetry in usual sense, for a brane scalar which is located at either brane [23]. However, since our mass spectrum of gaugino is given as that of a Scherk–Schwarz twist, the same brane coupling of gauge boson and gaugino would give rise to one-loop finiteness, i.e., *extreme* softness of brane-localized supersymmetry breaking, which is the case with the distant supersymmetry breaking as will be seen in the next section.

Particularly, for $\rho_0 = \rho_\pi = 0$, which is the usual assumption in the literature [6,19], the brane couplings at $y = 0$ and $y = \pi R$ are proportional to $1/(1 + \varepsilon_0^2)$ and $1/(1 + \varepsilon_\pi^2)$, respectively. In this case, the mass spectrum of gaugino is $M_n = n/R + (\arctan \varepsilon_0 + \arctan \varepsilon_\pi)/(\pi R)$, which is the same result as in [23]. Then, imposing the additional condition $\varepsilon_0 = 0$ or $\varepsilon_\pi = 0$ is necessary for the same coupling at either brane. On the other hand, for the equal masses of even and odd modes, i.e., $\rho_0 = \rho_\pi = 1$ [21], the brane couplings at $y = 0$ and $y = \pi R$ are proportional to $(\sin \varepsilon_0)^2/\varepsilon_0^2$ and $(\sin \varepsilon_\pi)^2/\varepsilon_\pi^2$, respectively. In this case, the mass spectrum of gaugino is given by $M_n = n/R + (\varepsilon_0 + \varepsilon_\pi)/(\pi R)$, which is different from the case with vanishing odd mass terms. For the same brane coupling of gauge boson and gaugino at either brane, we need the condition $\varepsilon_0 = 0$ or $\varepsilon_\pi = 0$ again. Thus, for general ρ_0 and ρ_π, which then contributes to the shape of wave functions and the mass spectrum, we can show that with the local supersymmetry breaking at the distant brane, the brane couplings of gauge boson and gaugino are the same at the other brane.

4. One-loop corrections at the brane

As far as the gauge interaction with brane matters is concerned, the only difference between the brane-localized breaking and the Scherk–Schwarz breaking comes from the brane scalar-gaugino-brane fermion vertices. After reducing the relevant brane interaction of gaugino, we get in the mass eigenstates

$$
\mathcal{L}_5 \supset \int dy\, g_5 \big[-\sqrt{2}\, i q_0 \phi_0^\dagger \lambda_1 \psi_0 \delta(y) - \sqrt{2}\, i q_\pi \phi_\pi^\dagger \lambda_1 \psi_\pi \delta(y - \pi R) + \text{h.c.} \big]
$$

$$
= \sum_{n=-\infty}^{\infty} \big[-g_0 q_0 \sqrt{2}\, i \phi_0^\dagger \lambda^{(n)} \psi_0 - g_\pi q_\pi \sqrt{2}\, i \phi_\pi^\dagger \lambda^{(n)} \psi_\pi + \text{h.c.} \big], \tag{30}
$$

where (ϕ_0, ψ_0) and (ϕ_π, ψ_π) are brane matter multiplets at $y = 0$ and $y = \pi R$, respectively, and $g_{0,\pi}$ given by Eqs. (28) and (29) are brane couplings of gaugino and $q_{0,\pi}$ denote $U(1)$ charges of brane matters. For comparison, in the case with a Scherk–Schwarz twist, $g_0 = g_4$ at $y = 0$ which was used to show the one-loop finiteness of the mass of a brane scalar at $y = 0$ from the infinite sum of KK modes [7].

Thus, due to brane masses of gaugino, the one-loop correction to the mass of a massless scalar ϕ_0 at $y = 0$ [7, 15,18,23] becomes nonzero as

$$
-i m_{\phi_0}^2 = 4 g_4^2 q_0^2 \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \left[\frac{1}{p^2 - (n/R)^2} - \frac{r_0}{p^2 - (n+\omega)^2/R^2} \right]
$$

$$
= i \frac{g_4^2 q_0^2}{2\pi^2 R^2} \sum_{n=-\infty}^{\infty} \int_0^\infty dx\, x^3 \left[-\frac{1}{x^2 + n^2} + \frac{r_0}{x^2 + (n+\omega)^2} \right], \tag{31}
$$

where $r_0 \equiv g_0^2/g_4^2$, and ω given by Eq. (22) corresponds to a sort of SS parameter and in the second line, we changed to the variable $x = p_E R$ with the Euclidean momentum p_E. Likewise, the one-loop correction to the mass of a massless scalar ϕ_π at $y = \pi R$ is given by $m_{\phi_0}^2$ with $(g_0, q_0, r_0) \to (g_\pi, q_\pi, r_\pi = g_\pi^2/g_4^2)$. Then, with the Λ cutoff regularization for the 4D loop integral at each KK level and the cutoff of the number of KK modes

$N = [\Lambda R]$ [23], we get the one-loop scalar mass as

$$m_{\phi_0}^2 = \frac{g_4^2 q_0^2}{4\pi^2 R^2} \sum_{n=-N}^{N} \left[(1 - r_0)(\Lambda R)^2 - n^2 \ln \frac{(\Lambda R)^2 + n^2}{n^2} + r_0(n + \omega)^2 \ln \frac{(\Lambda R)^2 + (n + \omega)^2}{(n + \omega)^2} \right]. \tag{32}$$

Thus, for $r_0 \neq 1$, the one-loop scalar mass at $y = 0$ has a quadratic divergence as well as a log divergence at each KK level. In fact, $r_0 \neq 1$ is not the supersymmetric gauge coupling in the 4D effective field theory with softly broken supersymmetry. For the small brane mass parameters, $\varepsilon_0 \ll 1$, we get $r_0 \simeq 1 + (\frac{2}{3}\rho_0 - 1)\varepsilon_0^2 + \mathcal{O}(\varepsilon_0^4)$ from Eq. (28), which gives rise to the reduction of the sum of quadratic divergences with the cutoff of the number of KK modes [23].

Now let us take a different regularization scheme for the loop divergence. When we can rewrite the one-loop scalar mass at $y = 0$ as

$$m_{\phi_0}^2 = \frac{g_4^2 q_0^2}{2\pi^2 R^2} \big(C(0) - r_0 C(\omega) \big), \tag{33}$$

where

$$C(\omega) = \sum_{n=-\infty}^{\infty} \int_0^{\infty} dx \, \frac{x^3}{x^2 + (n + \omega)^2}, \tag{34}$$

and change the infinite sum of KK modes in $C(\omega)$ into the contour integral [7,15], we get

$$C(0) - r_0 C(\omega) = \frac{\pi}{2} \int_0^{\infty} dx \, x^2 \big[\coth(\pi x) - r_0 \coth\big(\pi(x + i\omega)\big) + \text{h.c.} \big]$$

$$= \pi(1 - r_0) \int_0^{\infty} dx \, x^2 + \frac{1}{4\pi^2} \big[2\zeta(3) - r_0 \big(\text{Li}_3\big(e^{-2i\pi\omega}\big) + \text{Li}_3\big(e^{2i\pi\omega}\big) \big) \big], \tag{35}$$

where $\zeta(3)$ is the Riemann's zeta function and $\text{Li}_3(x)$ is the trilogarithm as

$$\text{Li}_3(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^3}. \tag{36}$$

Therefore, for $r_0 \neq 1$, there would still appear a cubic divergent one-loop mass, which corresponds to the sum of quadratic divergences coming from KK modes. However, there is no other divergence in this regularization. For no cubic divergence in this regularization, we must take $r_0 = 1$, i.e., $\varepsilon_0 = 0$, for which the SS parameter is given by $\omega = \arctan(\alpha(\rho_\pi, \varepsilon_\pi))/\pi$. This is the case with gaugino mediation of supersymmetry breaking at the distant brane [17]. In this case, the one-loop radiative mass squared for a brane scalar is positive and finite, which means that the brane-localized supersymmetry breaking is extremely soft in the so-called KK regularization scheme. This infinite sum of KK modes was advocated from the mixed position-momentum propagator of the bulk field [18]. Likewise, a massless scalar ϕ_π at $y = \pi R$ also gets a similar finite one-loop mass for $g_\pi = g_4$, i.e., $\varepsilon_\pi = 0$, for which the corresponding SS parameter is given by $\omega = \arctan(\alpha(\rho_0, \varepsilon_0))/\pi$.

This result also sheds light on the aspect of supersymmetric flavor problem. In the presence of distant supersymmetry breaking in the gauge sector, we can generalize the result to the case with a bulk non-Abelian group. Thus, we find that radiative soft masses of brane scalars are to be finite and flavor diagonal as

$$\big(m_{\phi_0}^2\big)_j^i = \delta_j^i \frac{g_4^2 C_2(\phi)}{4\pi^4 R^2} \sum_{k=1}^{\infty} \frac{(1 - \cos(2\pi k\omega))}{k^3} \simeq \delta_j^i \frac{g_4^2 C_2(\phi)}{\pi^2} \left(\frac{\omega}{R} \right)^2 \left[\frac{3}{4} - \frac{1}{2} \ln(2\pi\omega) \right], \tag{37}$$

where we picked up the leading term in powers of ω^2 and $C_2(\phi)$ is the quadratic Casimir of the ϕ-representation under the gauge group.

5. Conclusion

To conclude, we considered the brane-localized supersymmetry breaking on S^1/Z_2 by introducing brane mass terms for the bulk gaugino. We have found that the brane mass terms for the odd mode of gaugino play a role in modifying the mass spectrum of gaugino and determining the brane coupling of the even mode of gaugino. We showed that in the presence of brane gaugino mass terms, the mass spectrum of gaugino is shifted by the amount given by brane mass parameters. For the local supersymmetry breaking at the distant brane, we found that KK gauge corrections to the self-energy of a brane scalar is soft and flavor diagonal at one-loop order.

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