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QED collinear radiation factors in the next-to-leading logarithmic approximation

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Abstract

The effect of collinear photon radiation by charged particles is considered in the second order of the perturbation theory. Double and single photon radiation is evaluated. The corresponding radiation factors are obtained. The QED renormalization group approach is exploited in the next-to-leading order. The results are suited to perform a systematic treatment of the second order next-to-leading logarithmic radiative corrections to various processes either analytically or numerically.
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1. Introduction

The modern high energy physics experiments with advanced techniques and high statistics require adequately precise theoretical predictions. Among various effects which have to be taken into account, QED radiative corrections (RC) give important contributions to the predictions. At high energies they are usually computed with help of the QED perturbation theory. But direct computations of higher order QED corrections to complicated processes can be rather cumbersome. For this reason certain methods were developed to evaluate first the numerically most important contributions. In particular, besides the expansion in the powers of the fine structure constant α, one can use an expansion in powers of the so-called large logarithm, $L = \ln(M^2/m^2)$, where M is a large energy scale, and m is a charged particle mass, $m \ll M$.

In this Letter we present the derivation of a particular contribution of QED RC of the order $\mathcal{O}(\alpha^2 L^{2,1})$. It is well known, that the angular distribution of a photon emitted by a high-energy particle is peaked in the forward direction. Moreover, it is easy to show starting from the matrix element, that a process with emission of a collinear photon can be represented in a factorized form (see, e.g., Ref. [1]). As usually the factorization appears if it is possible to separate the long-distance sub-process of collinear photon emission and the main short-distance sub-process. In other words, we assume that the experimental conditions of the particle registration allow to neglect the effects of small changes of transverse momenta arising from emission of the photon at a small angle with respect to its parent particle: $\vartheta_\gamma \ll 1$. So the cross section (or the decay width) of the process with hard collinear photon emission can be represented as a convolution of the radiation factor R and the distribution of the radiation-less processes $\mathrm{d}\hat{\sigma}$ (in example of the $2 \to 2$ type):

$$\mathrm{d}\sigma\left[a(p_1)+b(p_2)\to c(q_1)+d(q_2)+\gamma\big(k\sim(1-z)p_1\big)\right]=\mathrm{d}\hat{\sigma}\left[a(zp_1)+b(p_2)\to c(q_1)+d(q_2)\right]\otimes R_{\mathrm{H}}^{\mathrm{ISR}}(z),$$

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$$\mathrm{d}\sigma\left[a(p_1)+b(p_2)\to c(q_1)+d(q_2)+\gamma\big(k\sim(1-z)q_1\big)\right]=\mathrm{d}\hat{\sigma}\left[a(p_1)+b(p_2)\to c(q_1)+d(q_2)\right]R_{\mathrm{H}}^{\mathrm{FSR}}(z), \tag{1}$$

where $z=E'/E$ is the energy fraction of the particle emitted the photon, E and E' are the charged particle energy *before* and *after* radiation of the photon, respectively. In the case of the final state radiation (FSR), we observed the energy of particle c being equal to zq_1^0, and we have a direct product of the two factors. In the case of the initial state radiation (ISR), we usually compute the kernel cross section in the center-of-mass reference frame of particles $a(zp_1)$ and $b(p_2)$ and then perform a relativistic boost to the laboratory reference frame.

The aim of this Letter is to derive explicit expressions for the radiation factors which describe single and double collinear photon emission in the $\mathcal{O}(\alpha^2L^{2,1})$ order.

This Letter is organized as follows. In Section 2 we re-call the known results for the first order collinear radiation factors. In Sections 3 and 4 we present our results for the second order radiation factors for double and single hard photon radiation, respectively. Possible applications of the results are discussed in Conclusions.

2. The first order approximation

The derivation of the collinear radiation factors due to an emission of a single hard photon in $\mathcal{O}(\alpha)$ can be found in Ref. [1]. The factors read

$$R_{\mathrm{H}}^{\mathrm{ISR}}(z)=\frac{\alpha}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{E^2}{m^2}-1+l_0\right)+1-z+\mathcal{O}\left(\frac{m^2}{E^2}\right)+\mathcal{O}\big(\vartheta_0^2\big)\right], \tag{2}$$

$$R_{\mathrm{H}}^{\mathrm{FSR}}(z)=\frac{\alpha}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{E^2}{m^2}-1+l_0+2\ln z\right)+1-z+\mathcal{O}\left(\frac{m^2}{E^2}\right)+\mathcal{O}\big(\vartheta_0^2\big)\right]. \tag{3}$$

The mass of the particle, m, is assumed to be small compared with the energy, and terms suppressed by the factor m^2/E^2 are omitted. The photon emission angle with respect to its parent particle is restricted by the condition[1]

$$\vartheta_\gamma<\vartheta_0,\qquad \frac{m}{E}\ll\vartheta_0\ll 1,\qquad l_0=\ln\frac{\vartheta_0^2}{4}. \tag{4}$$

The energy of the emitted photon is assumed to be above a certain threshold, $E_\gamma>\Delta E$. The parameters ϑ_0 and Δ either might be related to concrete experimental conditions, or serve as auxiliary quantities. In the latter case they should cancel out after summing up the contributions due to emission of the collinear hard photons with the ones of non-collinear hard photons and of soft photons. These $\mathcal{O}(\alpha)$ radiation factors are universal and describe collinear single photon emission for various high-energy processes [1].

3. Double hard photon radiation

In paper [2] the effect of double hard photon radiation in Bhabha scattering was considered. In particular the effect of double photon emission inside a collinear cone along the direction of motion of any of the 4 charged particles in this process was presented in a form being differential in the energy fraction of both the photons. So to get the collinear radiation factor, we have just to integrate over one of the energy fractions keeping their sum fixed. The lower limit of the integral over the photon energy fraction is chosen to be equal to the parameter Δ because both the photons should be hard and have therefore energy above ΔE. In this way for the case of the initial state radiation we got

$$\begin{aligned}R_{\mathrm{HH}}^{\mathrm{ISR}}(z)=\left(\frac{\alpha}{2\pi}\right)^2L\bigg\{&(L+2l_0)\left(\frac{1+z^2}{1-z}\big(2\ln(1-z)-2\ln\Delta-\ln z\big)+\frac{1+z}{2}\ln z-1+z\right)\\&+\frac{1+z^2}{1-z}\big(\ln^2z+2\ln z-4\ln(1-z)+4\ln\Delta\big)+(1-z)\big(2\ln(1-z)-2\ln\Delta-\ln z+3\big)+\frac{1+z}{2}\ln^2z\bigg\},\end{aligned} \tag{5}$$

where z is, as in Eq. (2), the energy fraction of the charged particle *after* the emission of the two photons.

[1] If required, the condition $m/E\ll\vartheta_0$ can be removed by restoring the omitted terms proportional to m^2/E^2.

The corresponding radiation factor for the final state radiation case can be obtained from the ISR one by means of the Gribov–Lipatov relation:

$$\begin{aligned} R_{\mathrm{HH}}^{\mathrm{FSR}}(z) &= -z R_{\mathrm{HH}}^{\mathrm{ISR}}\left(\frac{1}{z}\right)\Big|_{\ln\Delta\to\ln\Delta-\ln z;\ l_0\to l_0+2\ln z} \\ &= \left(\frac{\alpha}{2\pi}\right)^2 L\left\{(L+2l_0)\left[\frac{1+z^2}{1-z}\big(2\ln(1-z)-2\ln\Delta+\ln z\big)+\frac{1+z}{2}\ln z-1+z\right]\right. \\ &\quad + \frac{1+z^2}{1-z}\big(5\ln^2 z-2\ln z-4\ln(1-z)+4\ln\Delta+8\ln z\big(\ln(1-z)-\ln\Delta\big)\big) \\ &\quad \left. + (1-z)\big(2\ln(1-z)-2\ln\Delta-3\ln z+3\big)+\frac{3(1+z)}{2}\ln^2 z\right\}. \end{aligned} \tag{6}$$

Note that the additional interchanges in the above relation applied for $\ln\Delta$ and l_0 appear in our case from the crossing relations of the two channels with the given cuts on the energies of the soft photon and on the photon emission angle.

4. Single hard photon radiation

We have to consider also the process of single hard photon emission accompanied by a one-loop virtual correction or by emission of a soft photon. As concerns soft photon radiation, its contribution does factorize[2] with respect to the collinear hard photon emission:

$$\mathrm{d}\sigma_{\mathrm{HS}} = R_{\mathrm{H}} \otimes \delta_{\mathrm{S}}\,\mathrm{d}\sigma^{(0)}, \qquad \delta_{\mathrm{S}} = \frac{\mathrm{d}\sigma^{(1)}_{\mathrm{Soft}}}{\mathrm{d}\sigma^{(0)}}, \tag{7}$$

where δ_{S} is the one-loop soft photon radiation factor for the given process, computed in the standard way [3]. This quantity has an infra-red divergence, which cancels out after summation with the virtual loop contribution. And $\mathrm{d}\sigma^{(0)}$ is the Born level cross section.

So our aim is to get the radiation factor $R^{\mathrm{ISR}}_{\mathrm{H(S+V)}}(z)$, where both the one-loop virtual correction and soft photon radiation are taken into account. To find this radiation factor, we will exploit the known result for the complete second order next-to-leading order (NLO) QED corrections provided by the renormalization group approach [4,5,7–9]. In analogy to QCD we can write the master formula for the NLO corrected cross section, e.g., for Bhabha scattering in the form (see Ref. [10]):

$$\mathrm{d}\sigma = \int\limits_{\bar z_1}^{1}\mathrm{d}z_1\int\limits_{\bar z_2}^{1}\mathrm{d}z_2\,\mathcal{D}^{\mathrm{str}}_{ee}(z_1)\mathcal{D}^{\mathrm{str}}_{ee}(z_2)\big(\mathrm{d}\sigma^{(0)}\ (z_1,z_2)+\mathrm{d}\bar\sigma^{(1)}\ (z_1,z_2)+\mathcal{O}\big(\alpha^2L^0\big)\big)\int\limits_{\bar y_1}^{1}\frac{\mathrm{d}y_1}{Y_1}\int\limits_{\bar y_2}^{1}\frac{\mathrm{d}y_2}{Y_2}\mathcal{D}^{\mathrm{frg}}_{ee}\left(\frac{y_1}{Y_1}\right)\mathcal{D}^{\mathrm{frg}}_{ee}\left(\frac{y_2}{Y_2}\right), \tag{8}$$

where $\mathrm{d}\bar\sigma^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the massless scattering, calculated using the $\overline{\mathrm{MS}}$ scheme to subtract the mass singularities. The energy fractions of the incoming partons are $z_{1,2}$, and $Y_{1,2}$ are the energy fractions of the outgoing electron and positron. $\mathcal{D}^{\mathrm{str(frg)}}_{ee}$ are the structure (fragmentation) functions of an electron. Here we consider only the photonic contributions to the non-singlet part of these functions. The radiation factors corresponding to the collinear emission of light pairs were evaluated in Ref. [11]. With help of the master formula we can find the most important contributions reinforced by the large logarithm L in radiative corrections to a wide class of other processes as well.

The structure (fragmentation) functions describe the probability to find a massless (massive) electron with energy fraction z in the given massive (massless) electron. In our case with the next-to-leading accuracy we have

$$\begin{aligned} \mathcal{D}^{\mathrm{str,frg}}_{ee}(z) &= \delta(1-z)+\frac{\alpha}{2\pi}d^{(1)}(z,\mu_0,m_e)+\frac{\alpha}{2\pi}LP^{(0)}(z) \\ &\quad + \left(\frac{\alpha}{2\pi}\right)^2\left(\frac{1}{2}L^2P^{(0)}\otimes P^{(0)}(z)+LP^{(0)}\otimes d^{(1)}(z,\mu_0,m_e)+LP^{(1,\gamma)\mathrm{str,frg}}(z)\right)+\mathcal{O}\big(\alpha^2L^0,\alpha^3\big). \end{aligned} \tag{9}$$

The leading order QED splitting functions read

$$P^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+ = \lim_{\Delta\to 0}\big\{\delta(1-z)P^{(0)}_{\Delta}+\Theta(1-z-\Delta)P^{(0)}_{\Theta}(z)\big\}, \quad P^{(0)}_{\Delta} = 2\ln\Delta+\frac{3}{2},\ P^{(0)}_{\Theta}(z) = \frac{1+z^2}{1-z}.$$

[2] This is a non-trivial factorization property, since both the processes of soft and collinear hard photon emission have *long distance* scales. Nevertheless, our direct calculations confirmed this property, contrary to the case of virtual corrections to hard collinear photon emission, where there is no such a factorization.

$$P^{(0)} \otimes P^{(0)}(z) = \lim_{\Delta\to 0}\Biggl\{\delta(1-z)\biggl[\biggl(2\ln\Delta+\frac{3}{2}\biggr)^2-4\zeta(2)\biggr] + \Theta(1-z-\Delta)2\biggl[\frac{1+z^2}{1-z}\biggl(2\ln(1-z)-\ln z+\frac{3}{2}\biggr)+\frac{1+z}{2}\ln z-1+z\biggr]\Biggr\}, \tag{10}$$

where symbols δ and Θ denote the Dirac δ-function and the step function, respectively.

The space-like (ISR) and time-like (FSR) next-to-leading terms of the QED splitting functions for photonic corrections can be cast as [5,9]

$$\begin{aligned} P^{(1,\gamma)\mathrm{str}}(z) &= \delta(1-z)\biggl(\frac{3}{8}-3\zeta_2+6\zeta_3\biggr)+\frac{1+z^2}{1-z}\bigl(-2\ln z\ln(1-z)+\ln^2 z+2\,\mathrm{Li}_2(1-z)\bigr) \\ &\quad -\frac{1}{2}(1+z)\ln^2 z+2\ln z-2z+3, \\ P^{(1,\gamma)\mathrm{frg}}(z) &= \delta(1-z)\biggl(\frac{3}{8}-3\zeta_2+6\zeta_3\biggr)+\frac{1+z^2}{1-z}\bigl(2\ln z\ln(1-z)-2\ln^2 z-2\,\mathrm{Li}_2(1-z)\bigr) \\ &\quad +\frac{1}{2}(1+z)\ln^2 z+2z\ln z-3z+2. \end{aligned} \tag{11}$$

The dilogarithm and the Riemann zeta-function are defined as usually:

$$\mathrm{Li}_2(x) = -\int_0^1 \mathrm{d}y\frac{\ln(1-xy)}{y}, \qquad \zeta(n)=\sum_{k=1}^{\infty}\frac{1}{k^n}. \tag{12}$$

In the next-to-leading calculations we need also the initial condition for the structure and fragmentation functions at a certain scale μ_0:

$$d^{(1)}(z,\mu_0,m_e) \equiv d^{(1)}(z) = \lim_{\Delta\to 0}\Biggl\{\delta(1-z)\biggl(\ln\frac{\mu_0^2}{m_e^2}-1\biggr)\biggl(2\ln\Delta+\frac{3}{2}\biggr) + \Theta(1-z-\Delta)\frac{1+z^2}{1-z}\biggl[\ln\frac{\mu_0^2}{m_e^2}-1-2\ln(1-z)\biggr]\Biggr\}. \tag{13}$$

The modified minimal subtraction scheme $\overline{\mathrm{MS}}$ is used, while the final results are independent on the scheme choice. We take the factorization scale equal to the particle energy E. The renormalization scale μ_0 will be taken equal to m_e. More details on the application of the approach to the calculation of second order next-to-leading QED corrections can be found in Refs. [5,6,9].

Let us consider the $\mathcal{O}(\alpha^2 L^n)$ $(n>0)$ radiative corrections to a given process, which are related to at least one hard photon emission. They can be separated into four parts according to their kinematics:

$$\delta^{(2)\mathrm{NLO}}_{\mathrm{Hard}} = \delta^{(2)}_{\mathrm{HH(coll)}}+\delta^{(2)}_{\mathrm{HH(s\text{-}coll)}}+\delta^{(2)}_{\mathrm{(S+V)H(n\text{-}coll)}}+\delta^{(2)}_{\mathrm{(S+V)H(coll)}}, \tag{14}$$

where $\delta^{(2)}_{\mathrm{HH(coll)}}$ gives the contribution of double hard photon emission considered in the previous section. The case when one of the photons is emitted at large angle ($\vartheta_\gamma > \vartheta_0$) and the other one is collinear is denoted $\delta^{(2)}_{\mathrm{HH(s\text{-}coll)}}$, where "s-coll" means a semi-collinear kinematics, see Ref. [2] for details. The term $\delta^{(2)}_{\mathrm{(S+V)H(n\text{-}coll)}}$ corresponds to single hard non-collinear ($\vartheta_\gamma > \vartheta_0$) photon emission accompanied by the $\mathcal{O}(\alpha)$ soft and virtual photonic corrections. Note that since the non-collinear photon emission does not give rise to the large logarithm, we can keep in $\delta^{(2)}_{\mathrm{(S+V)H(n\text{-}coll)}}$ only the leading logarithmic terms in the sum of soft and virtual corrections. And the last term is the contribution that we are looking for: the one due to single hard collinear photon emission accompanied by $\mathcal{O}(\alpha)$ soft and virtual corrections.

On the other hand, the same quantity can be found in the master formula (8):

$$\begin{aligned} \delta^{(2)\mathrm{NLO}}_{\mathrm{Hard}} &= \frac{\alpha}{2\pi}LP^{(0)}_{\Theta}\otimes\mathrm{d}\bar\sigma^{(1)}_{\Theta}+\frac{\alpha}{2\pi}LP^{(0)}_{\Delta}\otimes\mathrm{d}\bar\sigma^{(1)}_{\Theta}+\frac{\alpha}{2\pi}\bigl(LP^{(0)}_{\Theta}+d^{(1)}_{\Theta}\bigr)\otimes\mathrm{d}\bar\sigma^{(1)}_{\Delta} \\ &\quad +\biggl(\frac{\alpha}{2\pi}\biggr)^2\biggl(\frac{L^2}{2}P^{(0)}\otimes P^{(0)}+LP^{(0)}\otimes d^{(1)}+LP^{(1,\gamma)\mathrm{str}}\biggr)_{\Theta}\otimes\mathrm{d}\sigma^{(0)}, \end{aligned} \tag{15}$$

where we left out the splitting functions arguments for short. Here $\mathrm{d}\bar\sigma^{(1)}_{\Theta}$ is the contribution of single hard photon emission and $\mathrm{d}\bar\sigma^{(1)}_{\Delta}$ is the soft-virtual contribution (in the $\overline{\mathrm{MS}}$ scheme with massless electrons). Lower indexes Θ and Δ mean here the parts of the corresponding functions related to hard and soft plus virtual radiation, respectively. Again we kept in the above equation only the terms reinforced by the large logarithm.

Comparing the two expression (14) and (15) we get

$$\delta^{(2)}_{\mathrm{(S+V)H(coll)}} = R^{\mathrm{ISR}}_{\mathrm{H(S+V)}}(z)\otimes \mathrm{d}\hat{\sigma}(z) = \frac{\alpha}{2\pi}LP^{(0)}\otimes \mathrm{d}\bar{\sigma}^{(1)}_{\Theta} + \frac{\alpha}{2\pi}LP^{(0)}_{\Theta}\otimes \mathrm{d}\bar{\sigma}^{(1)}_{\Delta}$$
$$+\left(\frac{\alpha}{2\pi}\right)^2\left(\frac{L^2}{2}P^{(0)}\otimes P^{(0)} + LP^{(0)}\otimes d^{(1)} + LP^{(1,\gamma)\mathrm{str}}\right)_{\Theta}\otimes \mathrm{d}\sigma^{(0)}$$
$$-\frac{\alpha}{2\pi}R^{(0)}_{\Theta}\otimes \mathrm{d}\sigma^{(1)}_{\Theta}\bigg|_{\vartheta_\gamma\geqslant\vartheta_0} - \frac{\alpha}{2\pi}LP^{(0)}_{\Delta}\otimes \mathrm{d}\sigma^{(1)}_{\Theta}\bigg|_{\vartheta_\gamma\geqslant\vartheta_0} - \left(\frac{\alpha}{2\pi}\right)^2 R^{\mathrm{ISR}}_{\mathrm{HH}}\otimes \mathrm{d}\sigma^{(0)}. \tag{16}$$

The $\overline{\mathrm{MS}}$ subtraction leads to the following relations:

$$\mathrm{d}\bar{\sigma}^{(1)}_{\Delta} = \mathrm{d}\sigma^{(1)}_{\mathrm{Soft}} + \mathrm{d}\sigma^{(1)}_{\mathrm{Virt}} - \frac{\alpha}{2\pi}\left(LP^{(0)}_{\Delta} + d^{(1)}_{\Delta}\right)\mathrm{d}\sigma^{(0)}, \tag{17}$$
$$\mathrm{d}\bar{\sigma}^{(1)}_{\Theta} = \mathrm{d}\sigma^{(1)}_{\Theta} - \frac{\alpha}{2\pi}\left(LP^{(0)}_{\Theta} + d^{(1)}_{\Theta}\right)\otimes \mathrm{d}\sigma^{(0)}. \tag{18}$$

Summing up the parts in (16) proportional to $\sigma^{(1)}_{\Theta}$ with help of (18) we arrive at

$$\frac{\alpha}{2\pi}LP^{(0)}_{\Theta}\otimes \mathrm{d}\sigma^{(1)}_{\Theta}\big|_{\vartheta_\gamma<\vartheta_0} = \frac{\alpha}{2\pi}LP^{(0)}_{\Theta}\otimes \frac{\alpha}{2\pi}R^{\mathrm{ISR}}_{\mathrm{H}}\otimes \mathrm{d}\sigma^{(0)}. \tag{19}$$

After substitution (17) and (19) to (16) we get the quantity, we are looking for, expressed via the known objects:

$$R^{\mathrm{ISR}}_{\mathrm{H(S+V)}}(z)\otimes \mathrm{d}\hat{\sigma}(z) = \left(\frac{\alpha}{2\pi}\right)^2 LP^{(0)}\otimes R^{\mathrm{ISR}}_{\mathrm{H}}\otimes \mathrm{d}\sigma^{(0)} - \left(\frac{\alpha}{2\pi}\right)^2 LP^{(0)}\otimes\left(LP^{(0)}_{\Theta} + d^{(1)}_{\Theta}\right)\otimes \mathrm{d}\sigma^{(0)} - \left(\frac{\alpha}{2\pi}\right)^2 R^{\mathrm{ISR}}_{\mathrm{HH}}\otimes \mathrm{d}\sigma^{(0)}$$
$$+\frac{\alpha}{2\pi}LP^{(0)}_{\Theta}\otimes\left(\mathrm{d}\sigma^{(1)}_{\mathrm{Soft}} + \mathrm{d}\sigma^{(1)}_{\mathrm{Virt}} - \frac{\alpha}{2\pi}\left(LP^{(0)}_{\Delta} + d^{(1)}_{\Delta}\right)\mathrm{d}\sigma^{(0)}\right)$$
$$+\left(\frac{\alpha}{2\pi}\right)^2\left(\frac{L^2}{2}P^{(0)}\otimes P^{(0)} + LP^{(0)}\otimes d^{(1)} + LP^{(1,\gamma)\mathrm{str}}\right)_{\Theta}\otimes \mathrm{d}\sigma^{(0)}. \tag{20}$$

Using the tables of convolution integrals [12] we obtain the final answer for the ISR factor

$$R^{\mathrm{ISR}}_{\mathrm{H(S+V)}}(z)\otimes \mathrm{d}\hat{\sigma}(z) = \delta^{(1)}_{\mathrm{(S+V)}}R^{\mathrm{ISR}}_{\mathrm{H}}(z)\otimes \mathrm{d}\sigma^{(0)}(z) + \left(\frac{\alpha}{2\pi}\right)^2 L\bigg[2\frac{1+z^2}{1-z}\left(\mathrm{Li}_2(1-z) - \ln(1-z)\ln z\right)$$
$$-(1+z)\ln^2 z + (1-z)\ln z + z\bigg]\otimes \mathrm{d}\sigma^{(0)}(z),$$
$$\delta^{(1)}_{\mathrm{(S+V)}} = \frac{\mathrm{d}\sigma^{(1)}_{\mathrm{Soft}} + \mathrm{d}\sigma^{(1)}_{\mathrm{Virt}}}{\mathrm{d}\sigma^{(0)}}. \tag{21}$$

For the final state radiation factor we use again the Gribov–Lipatov relation and get

$$R^{\mathrm{FSR}}_{\mathrm{H(S+V)}}(z)\,\mathrm{d}\hat{\sigma} = \delta^{(1)}_{\mathrm{(S+V)}}R^{\mathrm{FSR}}_{\mathrm{H}}(z)\,\mathrm{d}\sigma^{(0)} + \left(\frac{\alpha}{2\pi}\right)^2 L\bigg[\frac{1+z^2}{1-z}\left(-2\,\mathrm{Li}_2(1-z) - 3\ln^2 z + 2\ln(1-z)\ln z\right)$$
$$+(1+z)\ln^2 z - (1-z)\ln z - 1\bigg]\mathrm{d}\sigma^{(0)}. \tag{22}$$

5. Conclusions

In this way, we received the explicit expressions for the radiation factors (5), (6), (21), (22), which describe hard collinear photon emission in the second order of the perturbation theory within the next-to-leading logarithmic approximation. The double hard photon contribution (5) was obtained by analytical one-fold integration of the corresponding expression from Ref. [2]. For the contribution of single hard photon accompanied by a virtual loop or soft photonic correction we used the QED NLO renormalization group master formula and subtracted from it all, but the quantity we were looking for. The final state radiation factors (6), (22), were found with help of the Gribov–Lipatov relation applied to the ISR ones.

The factors are universal. Together with other relevant contributions, they can be used in analytic and numeric calculations of QED radiative corrections to a wide range of processes. The applicability condition consists in the possibility to factorize the collinear radiation with respect to the hard sub-process distribution, as described above. In particular, we are going to implement the factors into the Monte Carlo event generators `LABSMC` [13], `SAMBHA` [14], and `MCGPJ` [15] for several high energy processes. Our results can be exploited also to provide advanced theoretical predictions for experimental observables with so-called tagged photons, when hard photons emitted at zero (small) angles with respect to colliding charged particles are detected [16–18].

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