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Is vertical disparity used to determine azimuth?

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Abstract

The azimuth of a stimulus relative to the head can be determined from an extra-retinal, eye-position signal plus an estimate of the retinal eccentricity of the image. Alternatively, azimuth could be determined from retinal-image information alone. Specifically, stimulus azimuth could be estimated from two derivatives of vertical disparity: vertical size ratio (which varies with azimuth), and the horizontal gradient of vertical size ratio (a measure of distance). Here we examine the determinants of perceived azimuth in viewing conditions that, theoretically, should favor the use of vertical disparity. We find no evidence that vertical disparity is used. Perceived azimuth was determined completely by felt eye position and the retinal eccentricity of the image.

Keywords: Binocular vision; Space perception; Stereopsis; Vertical disparity; Visual direction

1. Introduction

This paper concerns the means by which human observers determine the location of an object relative to the head. Consider the viewing situation in Fig. 1. The observer fixates point P on a surface. P and the eyes lie in the visual plane and all angles and distances we discuss are in this plane. Relative to straight ahead, the left eye is turned through angle α_L and the right eye through α_R. Measured from the cyclopean eye, the horizontal eccentricity or azimuth of P is given by the average of α_L and α_R; [1] this quantity is called the horizontal version of the eyes, γ. Thus, an observer can in principle estimate a fixated object's azimuth, a, from γ. If the observer is not fixating the object, the azimuth is the sum of the retinal image eccentricity (r; which is the average of the retinal eccentricities in the two eyes) and the version: $a = r + \gamma$. Azimuth is, therefore, given by:

$$\hat{a} = \hat{r} + \hat{\gamma} \tag{1}$$

where the hats signify measurements of the relevant quantities.

Theoretically, one can also estimate an object's azimuth from retinal-image information alone. If the object is placed to the left of the head's median plane as in Fig. 1, the retinal image will be taller in the left than in the right eye. We can represent the vertical disparity with the vertical size ratio (VSR) which is the ratio of vertical angles the object subtends at the two eyes (Gillam & Lawergren, 1983; Rogers & Bradshaw, 1993). [2] The black circles in Fig. 2 show regions in space for which VSR is constant. Notice that VSR is greater than 1 for objects to the left of straight ahead and less than 1 for objects to the right. To estimate azimuth, we also need an estimate of distance (or a related quantity). For example, one could use stimulus vergence, the angle between lines from the eyes to the point of interest. The gray circles in Fig. 2 represent regions in space for which vergence (μ in Fig. 1) is constant. The stimulus vergence could in principle be determined from retinal-image information alone (see Eq. (4) in Backus, Banks, van Ee, & Crowell, 1999) or from extra-retinal, eye-position signals and the difference in retinal eccentricities in the two eyes (Backus et al., 1999; Rogers & Bradshaw, 1995). To close approximation:

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[1] For this statement to be quantitatively precise, the cyclopean eye must lie on the Vieth–Muller circle instead of on the interocular axis.

[2] VSR describes the environmental signal of import. It might plausibly be measured as the horizontal gradient of vertical disparity by the nervous system.

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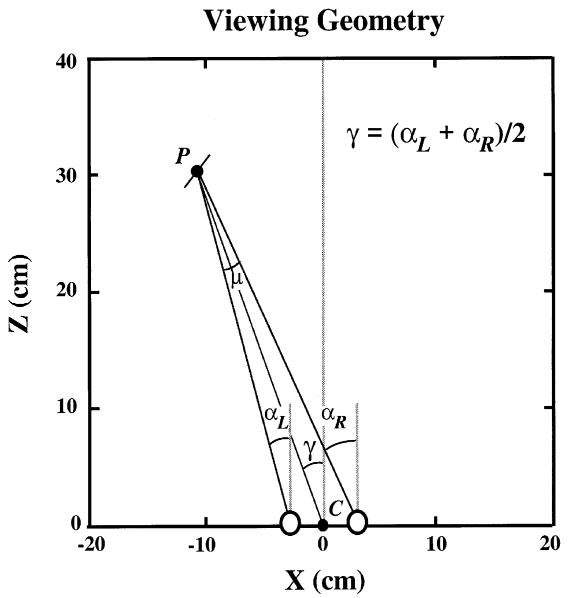

Fig. 1. Binocular viewing geometry. Overhead view of a binocular observer fixating point P on a surface. The left and right eyes are rotated through angles α_L and α_R relative to straight ahead. The angle between the lines of sight—the vergence—is represented by μ. The cyclopean eye is represented by C. The version γ is the average of α_L and α_R and corresponds to the angle between straight ahead and a line segment from the cyclopean eye to P.

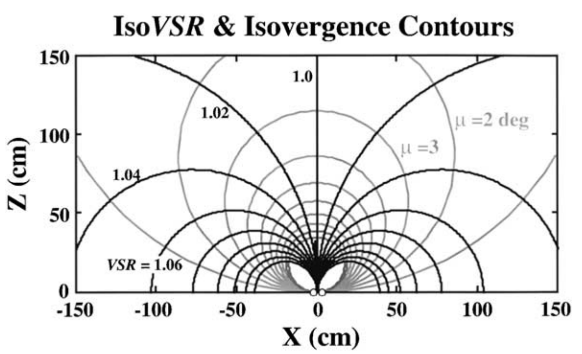

Fig. 2. Iso-VSR and iso-vergence contours. Overhead view of a binocular observer. The black circles represent contours along which VSR is constant. VSR is defined as the ratio of elevation angles (β_L/β_R) in the two eyes. The circles to the left of straight ahead have VSRs greater than 1 and those to the right have VSRs smaller than 1. The gray circles represent contours along which vergence is constant. The larger circles correspond to smaller vergences (and greater distances).

$$a \approx \tan^{-1}\left(\frac{\ln \mathrm{VSR}}{\mu}\right) \tag{2}$$

(Backus et al., 1999). If μ is measured from retinal-image information alone, Eq. (2) represents a method for estimating azimuth without extra-retinal, eye-position signals.

In summary, there are at least two ways to estimate an object's azimuth: from the eye position-specified azimuth (Eq. (1)) and from the disparity-specified azimuth (Eq. (2)). The former requires an estimate of the eyes' version (plus an estimate of the retinal eccentricity of the object). The latter requires estimates of VSR and distance.

In the research presented here, we looked for an effect of vertical size manipulation on perceived azimuth. To do so, one needs to determine the conditions under which such an effect is most likely to be observed. To this end, we conducted a Monte Carlo simulation to determine the expected variability of azimuth estimates based on Eq. (2). We assumed additive Gaussian noise (mean = 0) in the measurements of VSR and μ. The assumed standard deviations of the noises were 0.0013 for ln VSR and 0.5° for μ. These are the same values we have used in previous simulations (Backus & Banks, 1999; Banks, Hooge, & and Backus, 2001). In the simulation, the azimuth to the object varied from −20 to 20° and the distance varied from 20 to 180 cm. For each viewing condition considered, the simulation drew a value from each signal measurement distribution and calculated an azimuth estimate. From 10,000 simulation trials, we determined the mean and standard deviation of the distribution of estimates. The means revealed little bias (the largest biases were ~0.25°, which confirms that Eq. (2) is an excellent approximation). Fig. 3 shows the standard deviations of the azimuth estimates as a function of the true azimuth and distance. The surface is concave from the reader's station point. There are two obvious effects. First, the variability of azimuth estimates increases roughly in proportion to stimulus distance. This effect is primarily due to increasing variability in the distance estimate (given the assumption of additive noise in μ) as distance increases. Second, the variability of azimuth estimates is lowest when the stimulus is straight ahead and highest when the stimulus is way to the left or right. This effect is explained in Fig. 4. The figure illustrates two conditions: one in which the true VSR is 1 and μ is 3° (straight ahead) and another in which VSR is 1.04 and μ is again 3° (to the left). In both cases, the uncertainties (±1 standard deviation in the Gaussian distributions) in VSR and μ are shown. The regions of solutions consistent with those uncertainties are represented by the gray shaded ovals. The span of possible azimuths in the two cases is represented by the black lines. The error in the azimuth estimate is greater for eccentric stimulus positions in part because azimuth lines are nearly tangent to iso-VSR contours when the object is straight ahead VSR = 1 and are not tangent to iso-VSR contours when the object is eccentric (VSR = 1.04). If we assume that the visual system adopts the more accurate of the two methods of azimuth estimation (Eqs. (1) and (2)), the simulation results suggest that we will be most likely to observe an effect of vertical disparity manipulation when the stimulus is nearly straight ahead at close range.

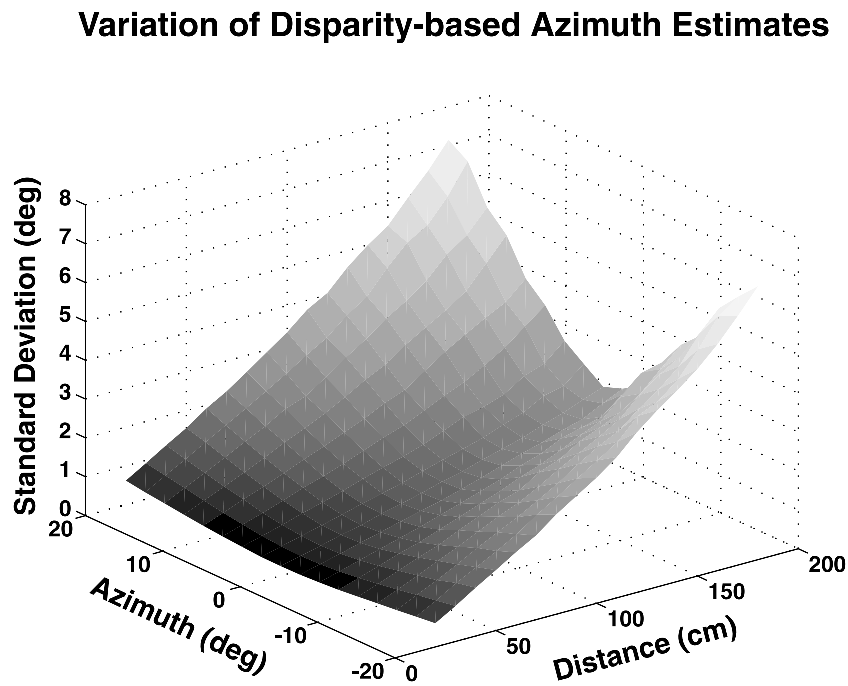

Fig. 3. Variation of azimuth estimates based on Eq. (2). The standard deviations of azimuth estimates (Eq. (2)) are plotted as a function of the true azimuth and distance of the object. These variability measures were obtained in a Monte Carlo simulation. For each azimuth and distance considered, the simulation drew a value from each signal measurement (ln VSR and μ) and calculated an azimuth estimate. From 10,000 simulation trials, we determined the mean and standard deviation of the distribution of estimates.

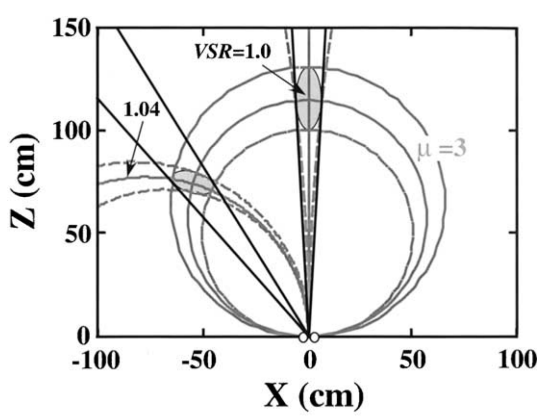

Fig. 4. Explanation for the azimuth effect in Fig. 3. Two viewing conditions are shown. For one, the stimulus is straight ahead: VSR = 1 and $\mu = 3$ °. For the other, the stimulus is to the left: VSR = 1.04 and $\mu = 3$. The uncertainties associated with the VSR and μ measurements are represented by the dashed lines. They are the standard deviations of the Gaussian distributions as described in the text. The gray shaded ovals represent the areas contained by these uncertainties. The solid black lines represent the most leftward and most rightward azimuths associated with the solution areas. Of course, the actual distributions of azimuth estimates will be complicated, but this geometric argument illustrates the primary cause of the azimuth effect in Fig. 3.

Is there empirical evidence for use of vertical disparity in azimuth estimation? To our knowledge, the only work concerns the consequence of vertical magnification of one eye's image (using a unilateral, afocal magnifier). When one image is magnified vertically, observers report that an objectively frontoparallel surface appears to be slanted toward the magnified eye (Ogle, 1950). This consequence of altering vertical disparity is the induced effect (Backus & Banks, 1999; Banks & Backus, 1998; Gillam & Lawergren, 1983; Gillam, Chambers, & Lawergren, 1988; Ogle, 1938; Stenton, Frisby, & Mayhew, 1984). The induced slant is consistent with current stereoscopic theory (Backus et al., 1999; Gårding, Porrill, Mayhew, & Frisby, 1995; Howard & Rogers, 1995; Longuet-Higgins, 1982). If vertical disparity is used in the estimation of azimuth, the apparent azimuth of a surface ought to change when vertical magnification is applied to one eye (Eq. (2)). Several investigators have reported anecdotally that no such change in apparent azimuth occurs (Frisby, 1984; Gillam & Lawergren, 1983; Ogle, 1950). As far as we can tell from those reports, the observations were conducted in well-illuminated environments, so the observers could see facial features (such as the nose) that clearly indicated the head-centric azimuth of the viewed surface. Thus, no one has rigorously tested the hypothesis that vertical disparity is a signal used to estimate azimuth. [3] Here we report such a test.

[3] We learned while preparing this manuscript that Berends (2001) has also examined this question experimentally. It appears that her observers were able to see facial features (including their spectacles in some cases) and the edges of the stimulus display.

2. Methods

2.1. Observers

Four observers participated in the experiments. MSB, RSB, and JO have normal vision. BTB is a 7-diopter myope and wore correcting contact lenses during the experimental measurements. RSB and JO were unaware of the experimental hypotheses at the time the data were collected. MSB, RSB, and BTB did the experiment with a simulated viewing distance of 57 cm and RSB and JO did the experiment with a simulated distance of 19 cm.

2.2. Apparatus

Stimuli were displayed on a haploscope consisting of two 58-cm monochrome CRTs, one seen by the left eye in a mirror placed near that eye and the other seen by the right eye in a mirror placed near that eye. Each mirror and CRT was attached to an armature that rotated about a vertical axis. The observer was positioned such that the rotation axes of the two armatures were co-linear with the vertical rotation axes of the eyes. Once adjusted correctly, head position was fixed with a bite bar. Natural pupils were used. The distance to the CRTs was fixed at 42 cm.

A Macintosh 840/AV generated the stimuli and collected the responses. Each CRT displayed 1280×1024 pixels at a refresh rate of 75 Hz. Angular subtense of a pixel was $\sim 2.5'$ at screen center. Despite the short viewing distance, the visual locations of the dots and lines in our displays were specified to within $\sim 30''$. This high level of spatial precision was achieved by anti-aliasing and spatial calibration (see Backus et al., 1999 for details).

The experimental stimuli were sparse random-dot displays that simulated a plane. The dots were randomly distributed within an ellipse subtending $25 \times 30°$ at the cyclopean eye. [4] The orientation of the ellipse's major axis varied randomly from trial to trial. There were ~ 300 dots in each stimulus. The slant of the plane varied randomly from -30 to $30°$. [5] The slant axis was always vertical (tilt $= 0°$). The simulated distance to the midpoint of the stimulus was 57 or 19 cm from the cyclopean eye. A fixation target containing a central binocular point and nonius lines above and below that point served as the fixation aid. The fixation point was always in the center of the dot display. Observers were instructed to fixate this target during stimulus presentations.

We independently manipulated the eye position-specified azimuth and the disparity-specified azimuth. (Thus, we used a cue-conflict paradigm to examine the relative influences of the two cues.) The eye position azimuth varied from -10 to $10°$ in steps of 2.5°. This was accomplished by placing the stimuli at different positions on the CRTs. The disparity azimuth varied from -20 to $20°$ in steps of 10°. This was accomplished by varying the vertical disparity field (primarily the vertical magnification). [6] The eye position-specified and disparity-specified azimuths were varied randomly from trial to trial. Each combination of eye position and disparity azimuth was presented five times. The data figures show the averages of the five settings.

On each trial, a stimulus appeared and the observer indicated its perceived direction with an unseen pointer that was underneath the haploscope table. The pointer pivoted about a point near the chest. Observers held its near end with one hand and its far end with the other. After they had oriented the pointer in the desired direction, they pressed a button, the pointer position was recorded, and the stimulus was extinguished. No feedback was given. The azimuth of the pointer was determined with a 10-turn potentiometer whose output was read through an A/D converter. The relationship between the pointer azimuth and voltage was calibrated before each experimental run. The precision of the device was better than 1°.

To test the hypothesis that vertical disparity is used in the estimation of azimuth, it is crucial to eliminate other cues to azimuth. We eliminated these unwanted cues by making sure that observers could not see the apparatus or room (including the edges of the CRTs) or parts of their own torso, hands, or face (including the bridge of the nose). To accomplish this, the room was completely dark except for the dim dots and lines in the stimuli. Moreover, the observers were light adapted periodically by fixating a diffuse light. None of the observers was able to see anything but the stimulus during data collection.

3. Results and discussion

The results when viewing distance was 57 cm are plotted in Figs. 5 and 6. Each panel represents the data from a different observer. In Fig. 5, the average azimuth

[4] Vertical disparity effects usually increase as display size increases up to a point (Backus et al., 1999; Bradshaw, Glennerster, & Rogers, 1996; Rogers & Bradshaw, 1993, 1995). It is thus possible that VSR could affect perceived direction in displays larger than 25° displays we used. The fact that we observed no effect of vertical disparity manipulation with our medium-size displays (while we and others have observed strong effects on perceived slant and curvature; Backus & Banks, 1999; Rogers & Bradshaw, 1995) makes it unlikely that we would observe an effect with an even larger display.

[5] The slant was randomized in order to eliminate perceived slant as a cue to azimuth.

[6] The disparity patterns presented to the eyes were veridical for the azimuth and distance of the simulated surface.

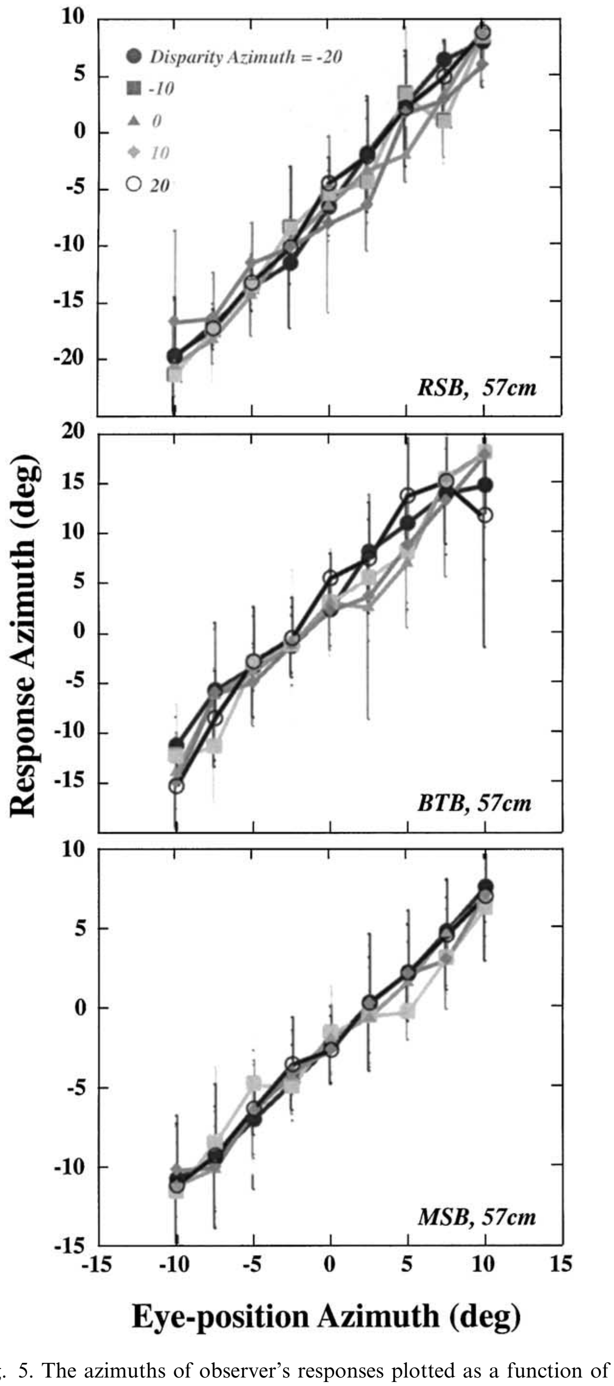

Fig. 5. The azimuths of observer's responses plotted as a function of eye position-specified azimuth. Each panel shows the data from one observer. Different disparity-specified azimuths are represented by different symbols: $-20°$ by filled circles, $-10°$ by filled squares, $0°$ by filled triangles, $10°$ by filled diamonds, and $20°$ by unfilled circles. Each data point represents the average of five settings. The simulated distance (the distance indicated by vergence and the horizontal gradient of VSR) was 57 cm. The error bars indicate ± 1 standard deviation. Note the differences in the cordinates for the three observers.

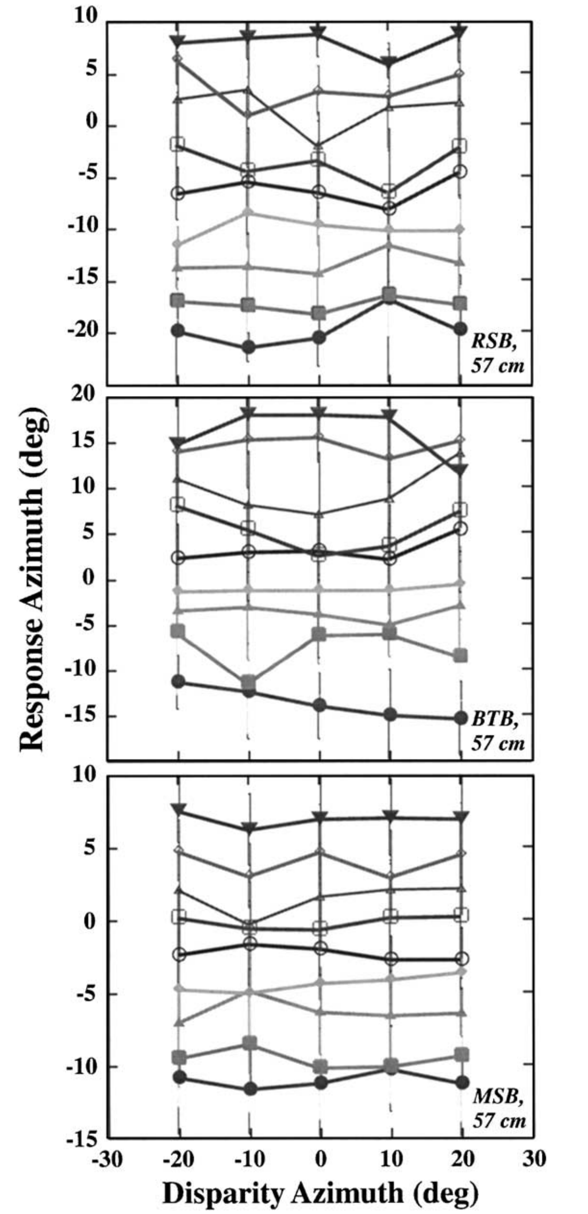

Fig. 6. Response azimuth as a function of disparity-specified azimuth. The data have been replotted from Fig. 5. The azimuths of the observer's responses are plotted as a function of the disparity-specified azimuth. Different eye position-specified azimuths are represented by different symbols: $-10°$ by filled circles, $-7.5°$ by filled squares, $-5°$ by filled triangles, $-2.5°$ by filled diamonds, $0°$ by unfilled circles, $2.5°$ by unfilled squares, $5°$ by unfilled triangles, $7.5°$ by unfilled diamonds, and $10°$ by inverted triangles. Each data point represents the average of five settings. The simulated distance (the distance indicated by vergence and the horizontal gradient of VSR) was 57 cm. The error bars indicate ± 1 standard deviation.

of the pointing response is plotted as a function of the eye position-specified azimuth. Different symbols represent different disparity-specified azimuths. There was a large and consistent effect of eye position-specified azimuth on the pointing responses. In other words, when the observers had to turn their eyes to the left to view the stimulus, they pointed leftward and when they had to turn their eyes to the right, they pointed rightward. There appears to be no effect of the disparity-specified azimuth. Specifically, when the VSR of the stimulus was

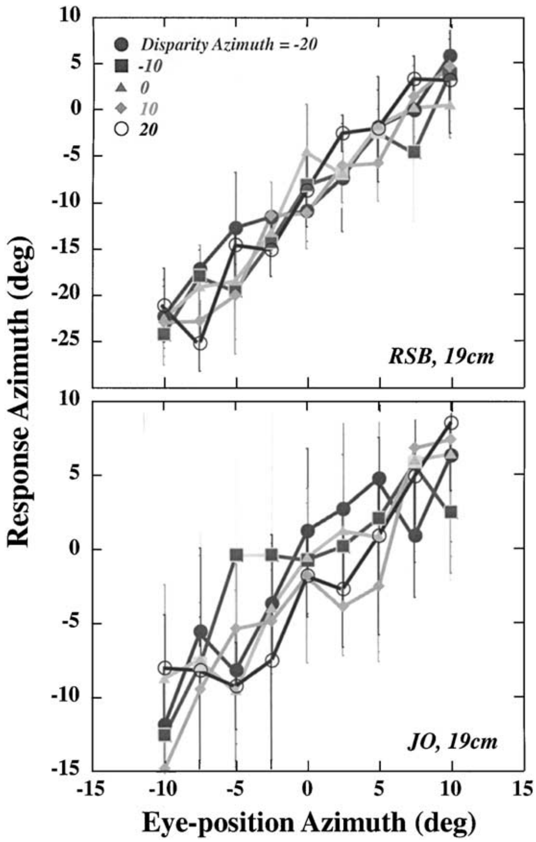

Fig. 7. Response azimuth as a function of eye position-specified azimuth for a viewing distance of 19 cm. Each panel shows the data from one observer. The azimuths of the observer's responses are plotted as a function of the eye position-specified azimuth. Different disparity-specified azimuths are represented by different symbols as in Fig. 5. The error bars indicate ±1 standard deviation.

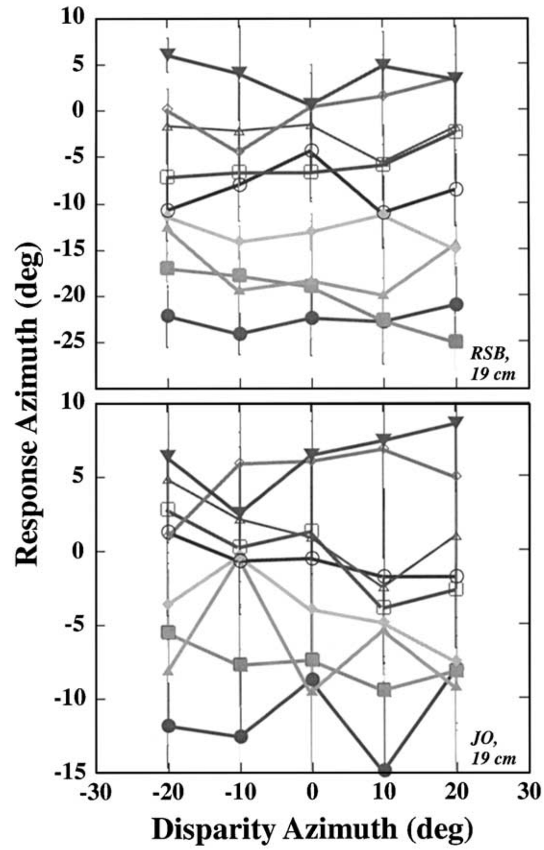

Fig. 8. Response azimuth as a function of disparity-specified azimuth for a viewing distance of 19 cm. The data have been replotted from Fig. 7. Different eye position-specified azimuths are represented by different symbols as in Fig. 6. The error bars indicate ±1 standard deviation.

varied (with the eyes in constant position), the pointing response did not change.

The same data are plotted in terms of the disparity-specified azimuth in Fig. 6. If there were an effect of vertical disparity, the data would have a positive slope. Clearly they do not.

The data of Figs. 5 and 6 show that vertical disparity is not used at 57 cm in the calculation of stimulus azimuth. The determinants of perceived azimuth appear to be felt eye position and the retinal eccentricity of the stimulus (which in this case is 0°).

The simulation results (Fig. 3) suggest that disparity-based azimuth estimation might be most accurate (and therefore most likely to be observed) when the stimulus distance is short. We, therefore, conducted the same experiment at a viewing distance of 19 cm. Figs. 7 and 8 show the data. In Fig. 7, the azimuth of the responses is again plotted as a function of the eye position-specified azimuth. Different symbols represent different disparity-specified azimuths. As in Fig. 5, there is a large and systematic effect of eye position azimuth and there appears to be no effect of disparity-based azimuth. Fig. 8 shows the same data when plotted as a function of the disparity-specified azimuth. Again there appears to be no effect of changes in the disparity-specified azimuth. Thus, vertical disparity does not seem to be used in the calculation of azimuth even at short viewing distances.

4. Conclusion

Recent work in stereopsis has found that vertical disparity is used to estimate surface orientation, shape, and distance (Howard & Rogers, 1995). In principle, vertical disparity could also be used in the estimation of stimulus azimuth. We find, however, that vertical disparity has no discernible effect on perceived azimuth even when conflicting cues, such as the apparent position of the nose, are eliminated.

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References

Backus, B. T., & Banks, M. S. (1999). Estimator reliability and distance scaling in stereoscopic slant perception. *Perception*, *28*, 217–242.

Backus, B. T., Banks, M. S., van Ee, R., & Crowell, J. A. (1999). Horizontal and vertical disparity, eye position, and stereoscopic slant perception. *Vision Research*, *39*, 1143–1170.

Banks, M. S., & Backus, B. T. (1998). Extra-retinal and perspective cues cause the small range of the induced effect. *Vision Research*, *38*, 187–194.

Banks, M.S., Hooge, I.T.C., and Backus, B.T. (2001). Perceiving slant about a horizontal axis from stereopsis. *Journal of Vision*, *1*, 55–79.

Berends, E.M. (2001). Depth perception and adaptation to disparity. PhD thesis, Utrecht University.

Bradshaw, M. F., Glennerster, A., & Rogers, B. J. (1996). The effect of display size on disparity scaling from differential perspective and vergence cues. *Vision Research*, *36*, 1255–1264.

Frisby, J. P. (1984). An old illusion and a new theory of stereoscopic depth perception. *Nature*, *307*, 592–593.

Gårding, J., Porrill, J., Mayhew, J. E. W., & Frisby, J. P. (1995). Stereopsis, vertical disparity and relief transformations. *Vision Research*, *35*, 703–722.

Gillam, B., & Lawergren, B. (1983). The induced effect, vertical disparity, and stereoscopic theory. *Perception and Psychophysics*, *34*, 121–130.

Gillam, B., Chambers, D., & Lawergren, B. (1988). The role of vertical disparity in the scaling of stereoscopic depth perception: An empirical and theoretical study. *Perception and Psychophysics*, *44*, 473–483.

Howard, I. P., & Rogers, B. J. (1995). *Binocular vision and stereopsis*. Oxford: Oxford University.

Longuet-Higgins, H. C. (1982). The role of the vertical dimension in stereoscopic vision. *Perception*, *11*, 371–376.

Ogle, K. N. (1938). Induced size effect. I. A new phenomenon in binocular space perception associated with the relative size of the images of the two eyes.. *Archives of Ophthalmology*, *20*, 604–623.

Ogle, K. N. (1950). *Researches in binocular vision*. New York: Hafner.

Rogers, B. J., & Bradshaw, M. F. (1993). Vertical disparities differential perspective and binocular stereopsis. *Nature*, *361*, 253–255.

Rogers, B. J., & Bradshaw, M. F. (1995). Disparity scaling and the perception of frontoparallel surfaces. *Perception*, *24*, 155–179.

Stenton, S. P., Frisby, J. P., & Mayhew, J. E. W. (1984). Vertical disparity pooling and the induced effect. *Nature*, *309*, 622–624.