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Overall feature of CP dependence for neutrino oscillation probability in arbitrary matter profile

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Abstract

We study the CP dependence of neutrino oscillation probability for all channels in arbitrary matter profile within three generations. We show that an oscillation probability for $\nu_e \to \nu_\mu$ can be written in the form $P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B_{e\mu}\sin\delta + C_{e\mu}$ without any approximation using the CP phase δ. This result holds not only in constant matter but also in arbitrary matter. Another probability for $\nu_\mu \to \nu_\tau$ can be written in the form $P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta + E_{\mu\tau}\sin 2\delta$. The term which is proportional to $\sin 2\delta$ disappear, namely $E_{\mu\tau} = 0$, in symmetric matter. It means that the probability reduces to the same form as in constant matter. As for other channels, probabilities in arbitrary matter are at most the quadratic polynomials of $\sin\delta$ and $\cos\delta$ as in the above two channels. In symmetric matter, the oscillation probability for each channel reduces to the same form with respect to δ as that in constant matter.

1. Introduction

In solar and atmospheric neutrino experiments, the ν_e deficit [1] and the ν_μ anomaly [2] have been observed. These results strongly suggest the finite mixing angles θ_{12} and θ_{23} and the finite mass squared differences Δ_{12} and Δ_{23}, where $\Delta_{ij} = m_i^2 - m_j^2$. Within the framework of three generations, there are two more parameters θ_{13} and δ to be determined. About θ_{13}, only upper bound is obtained from CHOOZ experiment [3] and the information on the CP phase δ is not obtained at all. In order to determine these parameters, several long baseline experiments using artificial neutrino beam will be planned [4], and it is important to study the effect when the neutrino pass through the matter [5]. The main physics goal in these experiments is to measure the value of δ. In this Letter, we study the CP dependence of oscillation probability for all channels in arbitrary matter profile.

Before giving our results, let us review the works on CP violation in three neutrino oscillation. At first we introduce the CP-odd asymmetry $\Delta P_{\alpha\beta}^{\mathrm{CP}} = P(\nu_\alpha \to \nu_\beta) - P(\bar\nu_\alpha \to \bar\nu_\beta)$. In disappearance channel, $\Delta P_{\alpha\alpha}^{\mathrm{CP}}$ is exactly

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equal to 0 in vacuum and independent of δ. However, $\Delta P_{\alpha\alpha}^{\mathrm{CP}}$ is not always equal to 0 in matter. This is due to the genuine CP violation and/or fake CP violation from matter effects. In the case of $\alpha = e$, Kuo and Pantaleone [6] have shown that $P(\nu_e \to \nu_e)$ does not depend on δ in the context of solar neutrino problem. Therefore, $\Delta P_{ee}^{\mathrm{CP}} \neq 0$ arises from matter effects.[1] However, in the case of $\alpha = \mu$, $P(\nu_\mu \to \nu_\mu)$ has the CP-odd term in asymmetric matter profile as pointed out by Minakata and Watanabe [7]. We investigate the CP dependence in more detail in this Letter.

Let us consider the appearance channels. As the CP-odd asymmetry is proportional to $\sin\delta$ in vacuum, $\Delta P_{\alpha\beta}^{\mathrm{CP}} \neq 0$ means that the discovery of CP violation. However, the situation completely changes when the matter effects are taken into account. Namely, $\Delta P_{\alpha\beta}^{\mathrm{CP}} \neq 0$ does not always mean the existence of CP violation [8], because the fake CP violation due to matter effects exists [13–15]. Here, it is difficult to separate genuine CP violation due to δ from fake CP violation. One of the methods to solve these problems is to take into account mass hierarchy approximation $|\Delta_{21}| \ll |\Delta_{32}|$. Actually, some approximate formulae are given by Arafune et al. at low energy region [9] and by Cervera et al. [10] and Freund [11] at high energy region.

Next, we introduce the T-odd asymmetry $\Delta P_{\alpha\beta}^{T} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$. Krastev and Petcov [12] have shown that $\Delta P_{\alpha\beta}^{T}$ is proportional to $\sin\delta$ exactly in constant matter. Recently, Naumov [17], Harrison and Scott [18] have derived the simple identity on the Jarlskog factor J [16] as $\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}\tilde{J} = \Delta_{12}\Delta_{23}\Delta_{31}J$, where quantities with tilde represent those in matter. We can simply understand that $\Delta P_{\alpha\beta}^{T}$ is proportional to $\sin\delta$ from this identity. We have studied the matter enhancement of $\tilde{J}$ [19] taking advantage of this identity. Furthermore, Parke and Weiler have investigated the matter enhancement of the $\Delta P_{e\mu}^{T}$ [20].

There are some works on the deviation from constant matter. In long baseline experiments, we need to estimate the validity of constant density approximation because the earth matter density largely changes along to the path of neutrino. The matter profile of the earth is approximately expressed by Preliminary Reference Earth Model (PREM) [21]. Minakata and Nunokawa [22] give the oscillation probability using mass hierarchy and adiabatic approximations. For the distance less than $L = 3000$ km, the matter density fluctuation is small and the constant density approximation is valid. On the other hand, it has been shown that the fluctuation of the density cannot be ignored for the distance greater than $L = 7000$ km [23–26]. Furthermore, the constant density approximation is not valid in the case that the matter density profile is different from PREM and the asymmetric part exists. It is pointed out that $\Delta P_{\alpha\beta}^{T}$ has the term proportional to $\cos\delta$ in arbitrary matter [27,28]. We investigate this feature in more detail in this Letter.

In previous Letter, we have proposed the new method applicable to constant matter. This method is to estimate the product of effective Maki–Nakagawa–Sakata matrix elements [29] $\widetilde{U}_{\alpha i}\widetilde{U}_{\beta i}^{*}$ without directly calculating $\widetilde{U}_{\alpha i}$. We have shown that the oscillation probability $P(\nu_e \to \nu_\mu)$ is written in the linear combination[2] of $\cos\delta$ and $\sin\delta$ exactly [31]

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B_{e\mu}\sin\delta + C_{e\mu}, \tag{1}$$

in constant matter. In other channels, for example,

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta. \tag{2}$$

It is found that the probability is quadratic polynomial of $\cos\delta$, $\sin\delta$, and the CP dependence was equal to in vacuum [32].

In this Letter, we give the exact CP dependence of oscillation probability for all channels in arbitrary matter profile. For the purpose, we decompose the Hamiltonian H in the form $H = (O_{23}\Gamma_\delta)H'(O_{23}\Gamma_\delta)^\dagger$ using 2–3

[1] Minakata and Watanabe have shown that $P(\nu_e \to \nu_e)$ slightly depends on δ if we take into account the loop correction even in the standard model. In this Letter, we do not consider the loop correction as these effects are safely neglected [7].

[2] It is not so easy to obtain our result from the effective mixing and effective CP phase given by Zaglauer and Schwarzer [30].

rotation matrix O_{23} and CP phase matrix $\Gamma_\delta = \mathrm{diag}(1, 1, e^{i\delta})$. This decomposition plays a key role in our Letter. As a result, we obtain the probability for $\nu_e \to \nu_\mu$ as

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B_{e\mu}\sin\delta + C_{e\mu}. \tag{3}$$

This has the same form with respect to δ as in Eq. (1) in constant matter. On the other hand, for $\nu_\mu \to \nu_\tau$, we show that the probability is given by

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta + E_{\mu\tau}\sin 2\delta. \tag{4}$$

Comparing Eq. (4) with (2), the probability in arbitrary matter profile has the term proportional to $\sin 2\delta$ which does not exist in the probability for constant matter. Furthermore, in the case of symmetric matter profile, we show that this additional term disappears, namely $E_{\mu\tau} = 0$, and the probability reduces to the same form as in constant matter.

2. CP dependence in arbitrary matter profile

In this section, we study the exact CP dependence of neutrino oscillation probability in arbitrary matter profile. The Schrödinger equation for neutrino is

$$i\frac{\partial \nu}{\partial t} = H\nu, \tag{5}$$

where H is the Hamiltonian in matter and ν is flavor eigenstate $\nu = (\nu_e, \nu_\mu, \nu_\tau)^T$. We introduce the MNS matrix which relates the flavor eigenstate ν_α to the mass eigenstate ν_i. The MNS matrix U in the standard parametrization is represented as

$$U = O_{23}\Gamma_\delta O_{13}\Gamma_\delta^\dagger O_{12}, \tag{6}$$

where $\Gamma_\delta = \mathrm{diag}(1, 1, e^{i\delta})$ and

$$O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \tag{7}$$

using the abbreviation $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. O_{13} and O_{12} represent 1–3 and 1–2 rotation matrix like O_{23}, respectively. By using this relation (6), we can rewrite the H as

$$H = \frac{1}{2E}\left[U\,\mathrm{diag}(0, \Delta_{21}, \Delta_{31})U^\dagger + \mathrm{diag}\big(a(t), 0, 0\big)\right] \tag{8}$$

$$= \frac{1}{2E}O_{23}\Gamma_\delta\left[O_{13}O_{12}\,\mathrm{diag}(0, \Delta_{21}, \Delta_{31})O_{12}^T O_{13}^T + \mathrm{diag}\big(a(t), 0, 0\big)\right]\Gamma_\delta^\dagger O_{23}^T, \tag{9}$$

where $a(t)$ is matter potential defined by $a(t) = 2\sqrt{2}G_F N(t)_e E$, and G_F, $N(t)_e$, E are, respectively, Fermi constant, electron number density and neutrino energy. This Eq. (9) means that the Hamiltonian can be decomposed into two parts. One is 1–2 and 1–3 mixing part which contain matter effects. The other is 2–3 mixing and CP phase δ part which does not contain matter effects. It is noted that this decomposition is guaranteed by the relation

$$O_{23}\Gamma_\delta\,\mathrm{diag}\big(a(t), 0, 0\big)(O_{23}\Gamma_\delta)^\dagger = \mathrm{diag}\big(a(t), 0, 0\big). \tag{10}$$

We can separate CP phase δ from matter effects by taking advantage of this decomposition (9). Changing ν to ν' as

$$\nu' = (O_{23}\Gamma_\delta)^\dagger\nu, \tag{11}$$

the Schrödinger equation (5) is rewritten as

$$i\frac{\partial \nu'}{\partial t} = H'\nu', \tag{12}$$

where

$$H' = \frac{1}{2E}\left[O_{13}O_{12}\,\mathrm{diag}(0, \Delta_{12}, \Delta_{13})(O_{13}O_{12})^T + \mathrm{diag}\big(a(t), 0, 0\big)\right]. \tag{13}$$

We emphasise that the reduced Hamiltonian H' does not contain the 2–3 mixing and CP phase and is real symmetric. For anti-neutrino, we obtain the similar results by the replacements $\delta \to -\delta$ and in $a(t) \to -a(t)$.

Next, we introduce the time evolution operator $S(t)$ and $S'(t)$ which is defined by the solution of the Schrödinger equation

$$\nu(t) = S(t)\nu(0), \qquad \nu'(t) = S'(t)\nu'(0). \tag{14}$$

The relation between $S(t)$ and $S'(t)$ is determined by the transformation (11) and is given by

$$S(t) = (O_{23}\Gamma_\delta)S'(t)(O_{23}\Gamma_\delta)^\dagger. \tag{15}$$

By taking the component of (15), the relation between the time evolution operators for each flavour is given by

$$S_{ee} = S'_{ee}, \tag{16}$$

$$S_{\mu e} = S'_{\mu e}c_{23} + S'_{\tau e}s_{23}e^{i\delta}, \tag{17}$$

$$S_{\tau e} = -S'_{\mu e}s_{23} + S'_{\tau e}c_{23}e^{i\delta}, \tag{18}$$

$$S_{\mu\mu} = S'_{\mu\mu}c_{23}^2 + S'_{\mu\tau}c_{23}s_{23}e^{-i\delta} + S'_{\tau\mu}c_{23}s_{23}e^{i\delta} + S'_{\tau\tau}s_{23}^2, \tag{19}$$

$$S_{\tau\mu} = -S'_{\mu\mu}c_{23}s_{23} - S'_{\mu\tau}s_{23}^2e^{-i\delta} + S'_{\tau\mu}c_{23}^2e^{i\delta} + S'_{\tau\tau}c_{23}s_{23}, \tag{20}$$

$$S_{\tau\tau} = S'_{\mu\mu}s_{23}^2 - S'_{\mu\tau}c_{23}s_{23}e^{-i\delta} - S'_{\tau\mu}c_{23}s_{23}e^{i\delta} + S'_{\tau\tau}c_{23}^2. \tag{21}$$

Here, $S_{\alpha\beta}$ represents the transition amplitude for $\nu_\beta \to \nu_\alpha$. $S_{e\mu}$, $S_{e\tau}$ and $S_{\mu\tau}$ are obtained from $S_{\mu e}$, $S_{\tau e}$ and $S_{\tau\mu}$, respectively, by the replacements $S'_{\alpha\beta} \to S'_{\beta\alpha}$, $\delta \to -\delta$. Substituting (16)–(21) into the relation

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}|^2, \tag{22}$$

the oscillation probabilities in arbitrary matter profile are given by

$$P(\nu_e \to \nu_e) = C_{ee}, \tag{23}$$

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B_{e\mu}\sin\delta + C_{e\mu}, \tag{24}$$

$$P(\nu_e \to \nu_\tau) = A_{e\tau}\cos\delta + B_{e\tau}\sin\delta + C_{e\tau}, \tag{25}$$

$$P(\nu_\mu \to \nu_\mu) = A_{\mu\mu}\cos\delta + B_{\mu\mu}\sin\delta + C_{\mu\mu} + D_{\mu\mu}\cos 2\delta + E_{\mu\mu}\sin 2\delta, \tag{26}$$

$$P(\nu_\tau \to \nu_\tau) = A_{\tau\tau}\cos\delta + B_{\tau\tau}\sin\delta + C_{\tau\tau} + D_{\tau\tau}\cos 2\delta + E_{\tau\tau}\sin 2\delta, \tag{27}$$

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta + E_{\mu\tau}\sin 2\delta, \tag{28}$$

and the other probabilities $P(\nu_\mu \to \nu_e)$, $P(\nu_\tau \to \nu_e)$ and $P(\nu_\tau \to \nu_\mu)$ are obtained by the replacements $S'_{\alpha\beta} \to S'_{\beta\alpha}$ and $\delta \to -\delta$ in $P(\nu_e \to \nu_\mu)$, $P(\nu_e \to \nu_\tau)$ and $P(\nu_\mu \to \nu_\tau)$, respectively. Here all coefficients $A_{e\mu}, \ldots, E_{\mu\tau}$ are constructed from the mixing angle θ_{23} and S' including matter effects. See Appendix A for detail. The oscillation probabilities for "anti-neutrino" are also obtained by the replacements $\delta \to -\delta$ and $a(t) \to -a(t)$.

For these Eqs. (23)–(28), we emphasize the following three points. First, the survival probability $P(\nu_e \to \nu_e)$ in Eq. (23) is completely independent of CP phase δ. This coincides with the result by Kuo and Pantaleone [6]. Second, the transition probabilities $P(\nu_e \to \nu_\mu)$ and $P(\nu_e \to \nu_\tau)$ in Eqs. (24) and (25) are linear polynomials of

$\sin\delta$ and $\cos\delta$. These features coincide with the results in constant matter [31,32]. Third, $P(\nu_\mu \to \nu_\tau)$, $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$ in Eqs. (26)–(28) are at most quadratic polynomials of $\sin\delta$ and $\cos\delta$.

We also comment the CP trajectory introduced by Minakata and Nunokawa. This is an orbit in the bi-probability space when δ changes from 0 to 2π [33,34]. Eq. (23) shows that CP trajectory is exactly elliptic even in arbitrary matter profile. In addition, we point out that the dependence of θ_{23} for the oscillation probabilities is completely understood from Eqs. (23)–(27). See Appendix A for detail.

It is also noted that there are two features in asymmetric matter profile. First, the terms proportional to $\sin\delta$ and $\sin 2\delta$ are appeared in $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$. The term proportional to $\sin 2\delta$ are appeared in $P(\nu_\mu \to \nu_\tau)$ and $P(\nu_\tau \to \nu_\mu)$. These terms do not exist in constant matter [31,32]. Second, $\Delta P^T_{\alpha\beta}$ is not proportional to $\sin\delta$ in asymmetric matter as in constant matter [12]. In the next section, we describe these features in more detail.

3. CP dependence in symmetric matter profile

In this section, we study the CP dependence of $P(\nu_\alpha \to \nu_\beta)$ in symmetric matter profile as special case of the previous section. In the case of symmetric matter along neutrino path, the time evolution operator S' becomes symmetric matrix

$$S'_{\alpha\beta} = S'_{\beta\alpha}, \tag{29}$$

for flavour indices [28,35]. As results, the relations between the coefficients of $P(\nu_\alpha \to \nu_e)$ Eqs. (23)–(25) and $P(\nu_e \to \nu_\alpha)$ are given by

$$A_{\mu e} = A_{e\mu}, \qquad B_{\mu e} = -B_{e\mu}, \qquad C_{\mu e} = C_{e\mu}, \tag{30}$$

$$A_{\tau e} = A_{e\tau}, \qquad B_{\tau e} = -B_{e\tau}, \qquad C_{\tau e} = C_{e\tau}. \tag{31}$$

See appendix Appendix B for detail calculation. The probability $P(\nu_e \to \nu_\alpha)$ have the same form with respect to δ as Eqs. (23)–(25) in arbitrary matter profile.

On the other hand, applying the condition (29) to the probability (26)–(28) we obtain the remarkable relations

$$B_{\mu\mu} = B_{\tau\tau} = E_{\mu\mu} = E_{\tau\tau} = E_{\mu\tau} = E_{\tau\mu} = 0, \tag{32}$$

where the detailed calculation is given in Appendix B. Using these relations, the oscillation probabilities (26)–(28) have more simple form such as

$$P(\nu_\mu \to \nu_\mu) = A_{\mu\mu}\cos\delta + C_{\mu\mu} + D_{\mu\mu}\cos 2\delta, \tag{33}$$

$$P(\nu_\tau \to \nu_\tau) = A_{\tau\tau}\cos\delta + C_{\tau\tau} + D_{\tau\tau}\cos 2\delta, \tag{34}$$

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta, \tag{35}$$

and $P(\nu_\tau \to \nu_\mu)$ is simply obtained by replacements $A_{\alpha\beta}$, $B_{\alpha\beta}$, $C_{\alpha\beta}$ and $D_{\alpha\beta}$. Then, the coefficients of $P(\nu_\tau \to \nu_\mu)$ are given by

$$A_{\mu\tau} = A_{\tau\mu}, \qquad B_{\mu\tau} = -B_{\tau\mu}, \qquad C_{\mu\tau} = C_{\tau\mu}, \qquad D_{\mu\tau} = D_{\tau\mu}, \tag{36}$$

where we use the condition (29) for Eqs. (36) or the unitarity for last equation. Here, the point is that the term proportional to $\sin 2\delta$ is dropped in Eq. (35) comparing with Eq. (28). The other point is that the terms proportional to $\sin\delta$ and $\sin 2\delta$ do not exist in $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$.

As the results, the CP dependence of the oscillation probability for each channel in symmetric matter reduces to the same form as in constant matter [32]. This is the generalization of the result in our previous Letter.

Finally, we study the T-odd asymmetry $\Delta P^T_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$. From the unitarity relation, we easily obtain

$$\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}, \tag{37}$$

and in symmetric matter profile we obtain

$$\Delta P^T_{e\mu} = (A_{e\mu} - A_{\mu e})\cos\delta + (B_{e\mu} - B_{\mu e})\sin\delta + (C_{e\mu} - C_{\mu e}) \tag{38}$$

$$= 2B_{e\mu}\sin\delta \tag{39}$$

$$= -4c_{23}s_{23}\,\mathrm{Im}\big[S'^*_{\mu e}S'_{\tau e}\big]\sin\delta, \tag{40}$$

where we use the relations (30) in Eq. (39). In constant matter, Krastev and Petcov [12] have pointed out that $\Delta P^T_{\alpha\beta}$ is proportional to $\sin\delta$. Our result is applicable to the symmetric matter profile, which corresponds to the generalization of their result, even if the oscillation is non-adiabatic.

Let us turn the case of arbitrary matter profile. $\Delta P^T_{\alpha\beta}$ in asymmetric matter is not proportional to $\sin\delta$ because the time evolution operator S' is not symmetric, namely $S'_{\alpha\beta} \neq S'_{\beta\alpha}$. More concretely speaking, the coefficients are not symmetric for flavor indices $A_{\alpha\beta} \neq A_{\beta\alpha}$, $B_{\alpha\beta} \neq -B_{\beta\alpha}$, $C_{\alpha\beta} \neq C_{\beta\alpha}$. We clarify the exact CP dependence of $\Delta P^T_{\alpha\beta}$ in asymmetric matter although this fact is suggested using approximation [27,28].

4. Summary

We summarize the results obtained in this Letter. We have studied the CP dependence of the oscillation probability $P(\nu_\alpha \to \nu_\beta)$ both in arbitrary and in symmetric matter profile.

(i) In arbitrary matter profile, we have found that $P(\nu_\alpha \to \nu_\beta)$ is at most quadratic polynomial of $\sin\delta$ and $\cos\delta$. The CP dependences of the probabilities can be written as

$$P(\nu_e \to \nu_e) = C_{ee}, \tag{41}$$

$$P(\nu_\alpha \to \nu_\beta) = A_{\alpha\beta}\cos\delta + B_{\alpha\beta}\sin\delta + C_{\alpha\beta}, \quad \text{for } (\alpha\beta) = (e\mu), (e\tau), (\mu e), (\tau e), \tag{42}$$

$$P(\nu_\alpha \to \nu_\beta) = A_{\alpha\beta}\cos\delta + B_{\alpha\beta}\sin\delta + C_{\alpha\beta} + D_{\alpha\beta}\cos 2\delta + E_{\alpha\beta}\sin 2\delta,$$
$$\text{for } (\alpha\beta) = (\mu\mu), (\mu\tau), (\tau\mu), (\tau\tau). \tag{43}$$

(ii) In symmetric matter profile, we have shown that the oscillation probabilities $P(\nu_e \to \nu_x)$ have the same form as in arbitrary matter such as

$$P(\nu_e \to \nu_e) = C_{ee}, \tag{44}$$

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B_{e\mu}\sin\delta + C_{e\mu}, \tag{45}$$

$$P(\nu_e \to \nu_\tau) = A_{e\tau}\cos\delta + B_{e\tau}\sin\delta + C_{e\tau}. \tag{46}$$

Furthermore, we have shown that the CP dependences of other probabilities are written in the form as

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta, \tag{47}$$

$$P(\nu_\mu \to \nu_\mu) = A_{\mu\mu}\cos\delta + C_{\mu\mu} + D_{\mu\mu}\cos 2\delta, \tag{48}$$

$$P(\nu_\tau \to \nu_\tau) = A_{\tau\tau}\cos\delta + C_{\tau\tau} + D_{\tau\tau}\cos 2\delta. \tag{49}$$

It is remarkable that the oscillation probability for each channel in symmetric matter reduces to the same form as in constant matter.

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Appendix A. Coefficients in arbitrary matter profile

In this appendix, we give exact CP and 2–3 mixing dependences of the oscillation probabilities in arbitrary matter profile. The probability for each channel is given by

$$P(\nu_e \to \nu_e) = C_{ee} = \left|S'_{ee}\right|^2, \tag{A.1}$$

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B_{e\mu}\sin\delta + C_{e\mu}, \tag{A.2}$$

$$A_{e\mu} = 2\,\mathrm{Re}\left[S'^*_{\mu e}S'_{\tau e}\right]c_{23}s_{23}, \tag{A.3}$$

$$B_{e\mu} = -2\,\mathrm{Im}\left[S'^*_{\mu e}S'_{\tau e}\right]c_{23}s_{23}, \tag{A.4}$$

$$C_{e\mu} = \left|S'_{\mu e}\right|^2 c_{23}^2 + \left|S'_{\tau e}\right|^2 s_{23}^2, \tag{A.5}$$

$$P(\nu_e \to \nu_\tau) = A_{e\tau}\cos\delta + B_{e\tau}\sin\delta + C_{e\tau}, \tag{A.6}$$

$$A_{e\tau} = -2\,\mathrm{Re}\left[S'^*_{\mu e}S'_{\tau e}\right]c_{23}s_{23}, \tag{A.7}$$

$$B_{e\tau} = 2\,\mathrm{Im}\left[S'^*_{\mu e}S'_{\tau e}\right]c_{23}s_{23}, \tag{A.8}$$

$$C_{e\tau} = \left|S'_{\mu e}\right|^2 s_{23}^2 + \left|S'_{\tau e}\right|^2 c_{23}^2, \tag{A.9}$$

$$P(\nu_\mu \to \nu_\mu) = A_{\mu\mu}\cos\delta + B_{\mu\mu}\sin\delta + C_{\mu\mu} + D_{\mu\mu}\cos 2\delta + E_{\mu\mu}\sin 2\delta, \tag{A.10}$$

$$A_{\mu\mu} = 2\,\mathrm{Re}\left[\left(S'_{\mu\mu}c_{23}^2 + S'_{\tau\tau}s_{23}^2\right)^*\left(S'_{\tau\mu} + S'_{\mu\tau}\right)\right]c_{23}s_{23}, \tag{A.11}$$

$$B_{\mu\mu} = -2\,\mathrm{Im}\left[\left(S'_{\mu\mu}c_{23}^2 + S'_{\tau\tau}s_{23}^2\right)^*\left(S'_{\tau\mu} - S'_{\mu\tau}\right)\right]c_{23}s_{23}, \tag{A.12}$$

$$C_{\mu\mu} = \left|S'_{\mu\mu}\right|^2 c_{23}^4 + \left(\left|S'_{\mu\tau}\right|^2 + \left|S'_{\tau\mu}\right|^2\right)c_{23}^2 s_{23}^2 + \left|S'_{\tau\tau}\right|^2 s_{23}^4 + 2\,\mathrm{Re}\left[S'^*_{\mu\mu}S'_{\tau\tau}\right]c_{23}^2 s_{23}^2, \tag{A.13}$$

$$D_{\mu\mu} = 2\,\mathrm{Re}\left[S'^*_{\tau\mu}S'_{\mu\tau}\right]c_{23}^2 s_{23}^2, \tag{A.14}$$

$$E_{\mu\mu} = 2\,\mathrm{Im}\left[S'^*_{\tau\mu}S'_{\mu\tau}\right]c_{23}^2 s_{23}^2, \tag{A.15}$$

$$P(\nu_\tau \to \nu_\tau) = A_{\tau\tau}\cos\delta + B_{\tau\tau}\sin\delta + C_{\tau\tau} + D_{\tau\tau}\cos 2\delta + E_{\tau\tau}\sin 2\delta, \tag{A.16}$$

$$A_{\tau\tau} = -2\,\mathrm{Re}\left[\left(S'_{\mu\mu}s_{23}^2 + S'_{\tau\tau}c_{23}^2\right)^*\left(S'_{\tau\mu} + S'_{\mu\tau}\right)\right]c_{23}s_{23}, \tag{A.17}$$

$$B_{\tau\tau} = 2\,\mathrm{Im}\left[\left(S'_{\mu\mu}s_{23}^2 + S'_{\tau\tau}c_{23}^2\right)^*\left(S'_{\tau\mu} - S'_{\mu\tau}\right)\right]c_{23}s_{23}, \tag{A.18}$$

$$C_{\tau\tau} = \left|S'_{\mu\mu}\right|^2 s_{23}^4 + \left(\left|S'_{\mu\tau}\right|^2 + \left|S'_{\tau\mu}\right|^2\right)c_{23}^2 s_{23}^2 + \left|S'_{\tau\tau}\right|^2 c_{23}^4 + 2\,\mathrm{Re}\left[S'^*_{\mu\mu}S'_{\tau\tau}\right]c_{23}^2 s_{23}^2, \tag{A.19}$$

$$D_{\tau\tau} = 2\,\mathrm{Re}\left[S'^*_{\tau\mu}S'_{\mu\tau}\right]c_{23}^2 s_{23}^2, \tag{A.20}$$

$$E_{\tau\tau} = 2\,\mathrm{Im}\left[S'^*_{\tau\mu}S'_{\mu\tau}\right]c_{23}^2 s_{23}^2, \tag{A.21}$$

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B_{\mu\tau}\sin\delta + C_{\mu\tau} + D_{\mu\tau}\cos 2\delta + E_{\mu\tau}\sin 2\delta, \tag{A.22}$$

$$A_{\mu\tau} = -2\,\mathrm{Re}\left[\left(S'_{\mu\mu} - S'_{\tau\tau}\right)^*\left(S'_{\tau\mu}c_{23}^2 - S'_{\mu\tau}s_{23}^2\right)\right]c_{23}s_{23}, \tag{A.23}$$

$$B_{\mu\tau} = 2\,\mathrm{Im}\left[\left(S'_{\mu\mu} - S'_{\tau\tau}\right)^*\left(S'_{\tau\mu}c_{23}^2 + S'_{\mu\tau}s_{23}^2\right)\right]c_{23}s_{23}, \tag{A.24}$$

$$C_{\mu\tau} = \left(\left|S'_{\mu\mu}\right|^2 + \left|S'_{\tau\tau}\right|^2\right)c_{23}^2 s_{23}^2 + \left|S'_{\mu\tau}\right|^2 s_{23}^4 + \left|S'_{\tau\mu}\right|^2 c_{23}^4 - 2\,\mathrm{Re}\left[S'^*_{\mu\mu}S'_{\tau\tau}\right]c_{23}^2 s_{23}^2, \tag{A.25}$$

$$D_{\mu\tau} = -2\,\mathrm{Re}\left[S'^*_{\tau\mu}S'_{\mu\tau}\right]c_{23}^2 s_{23}^2, \tag{A.26}$$

$$E_{\mu\tau} = -2\,\mathrm{Im}\big[S'^{*}_{\tau\mu} S'_{\mu\tau}\big] c_{23}^2 s_{23}^2, \tag{A.27}$$

and the probabilities for the other channels $P(\nu_\mu \to \nu_e)$, $P(\nu_\tau \to \nu_e)$ and $P(\nu_\tau \to \nu_\mu)$ are obtained by the replacements $S'_{\alpha\beta} \to S'_{\beta\alpha}$ and $\delta \to -\delta$ in $P(\nu_e \to \nu_\mu)$, $P(\nu_e \to \nu_\tau)$ and $P(\nu_\mu \to \nu_\tau)$, respectively. From these expressions, we can see that matter effects is renormalized in $S'_{\alpha\beta}$, which does not contain CP phase δ. Note that matter effects and CP effects are completely separated in the oscillation probability. The mixing angle θ_{23} is also separated from matter effects and all of the oscillation probabilities are quartet polynomials of c_{23} and s_{23}.

Appendix B. Coefficients in symmetric matter profile

In this appendix, we give the relations of the coefficients of the probability in symmetric matter. We use the condition

$$S'_{\alpha\beta} = S'_{\beta\alpha}, \tag{B.1}$$

in symmetric matter profile. First, we calculate the relations between the coefficients of T-conjugate probabilities. From the oscillation probabilities (A.2) and (A.22) in Appendix A and the symmetry of S' (B.1), we obtain

$$A_{e\mu} - A_{\mu e} = 4c_{23}s_{23}\,\mathrm{Re}\big[S'^{*}_{\mu e} S'_{\tau e} - S'^{*}_{e\mu} S'_{e\tau}\big] = 0, \tag{B.2}$$

$$B_{e\mu} + B_{\mu e} = -4c_{23}s_{23}\,\mathrm{Im}\big[S'^{*}_{\mu e} S'_{\tau e} - S'^{*}_{e\mu} S'_{e\tau}\big] = 0, \tag{B.3}$$

$$C_{e\mu} - C_{\mu e} = \big(\big|S'_{\mu e}\big|^2 - \big|S'_{e\mu}\big|^2\big)c_{23}^2 + \big(\big|S'_{\tau e}\big|^2 - \big|S'_{e\tau}\big|^2\big)s_{23}^2 = 0, \tag{B.4}$$

$$A_{\mu\tau} - A_{\tau\mu} = 4\,\mathrm{Re}\big[\big(S'_{\mu\mu} - S'_{\tau\tau}\big)^*\big(S'_{\tau\mu} - S'_{\mu\tau}\big)\big] = 0, \tag{B.5}$$

$$B_{\mu\tau} + B_{\tau\mu} = -4\,\mathrm{Im}\big[\big(S'_{\mu\mu} - S'_{\tau\tau}\big)^*\big(S'_{\tau\mu} - S'_{\mu\tau}\big)\big] = 0, \tag{B.6}$$

$$C_{\mu\tau} - C_{\tau\mu} = \big(\big|S'_{\mu\tau}\big|^2 - \big|S'_{\tau\mu}\big|^2\big)\big(s_{23}^4 + c_{23}^4\big) = 0, \tag{B.7}$$

$$D_{\mu\tau} - D_{\tau\mu} = -4\big(\big|S'_{\mu\tau}\big|^2 - \big|S'_{\tau\mu}\big|^2\big)c_{23}^2 s_{23}^2 = 0. \tag{B.8}$$

The relations between the coefficients for $\nu_e \leftrightarrow \nu_\tau$ are obtained in the same way.

Second, we calculate the coefficients of $\sin\delta$ and $\sin 2\delta$ in $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$ and the coefficients of $\sin 2\delta$ in $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$. From the concrete expression of oscillation probabilities (A.10), (A.16) and (A.22) in Appendix A, we obtain

$$B_{\mu\mu} = -2c_{23}s_{23}\,\mathrm{Im}\big[\big(S'_{\mu\mu}c_{23}^2 + S'_{\tau\tau}s_{23}^2\big)^*\big(S'_{\tau\mu} - S'_{\mu\tau}\big)\big] = 0, \tag{B.9}$$

$$E_{\mu\mu} = 2\,\mathrm{Im}\big[S'^{*}_{\tau\mu} S'_{\mu\tau}\big]c_{23}^2 s_{23}^2 = 2\,\mathrm{Im}\big[\big|S'_{\mu\tau}\big|^2\big]c_{23}^2 s_{23}^2 = 0, \tag{B.10}$$

$$B_{\tau\tau} = 2c_{23}s_{23}\,\mathrm{Im}\big[\big(S'_{\mu\mu}s_{23}^2 + S'_{\tau\tau}c_{23}^2\big)^*\big(S'_{\tau\mu} - S'_{\mu\tau}\big)\big] = 0, \tag{B.11}$$

$$E_{\tau\tau} = 2\,\mathrm{Im}\big[S'^{*}_{\tau\mu} S'_{\mu\tau}\big]c_{23}^2 s_{23}^2 = 2\,\mathrm{Im}\big[\big|S'_{\mu\tau}\big|^2\big]c_{23}^2 s_{23}^2 = 0, \tag{B.12}$$

$$E_{\mu\tau} = -2\,\mathrm{Im}\big[S'^{*}_{\tau\mu} S'_{\mu\tau}\big]c_{23}^2 s_{23}^2 = -2\,\mathrm{Im}\big[\big|S'_{\mu\tau}\big|^2\big]c_{23}^2 s_{23}^2 = 0, \tag{B.13}$$

$$E_{\tau\mu} = 2\,\mathrm{Im}\big[S'^{*}_{\mu\tau} S'_{\tau\mu}\big]c_{23}^2 s_{23}^2 = 2\,\mathrm{Im}\big[\big|S'_{\tau\mu}\big|^2\big]c_{23}^2 s_{23}^2 = 0, \tag{B.14}$$

from the condition (B.1).

References

[1] Homestake Collaboration, B.T. Cleveland, et al., Astrophys. J. 496 (1998) 505;

SAGE Collaboration, J.N. Abdurashitov, et al., Phys. Rev. C 60 (1999) 055801;
GALLEX Collaboration, W. Hampel, et al., Phys. Lett. B 447 (1999) 127;
SuperKamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 82 (1999) 1810;
SuperKamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 82 (1999) 2430;
SNO Collaboration, Phys. Rev. Lett. 89 (2002) 011301;
SNO Collaboration, Phys. Rev. Lett. 89 (2002) 011302.

[2] IMB Collaboration, R. Becker-Szendy, et al., Phys. Rev. D 46 (1992) 3720;
SOUDAN2 Collaboration, W.W.M. Allison, et al., Phys. Lett. B 391 (1997) 491;
SOUDAN2 Collaboration, W.W.M. Allison, et al., Phys. Lett. B 449 (1999) 137;
SuperKamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 82 (1999) 2644;
SuperKamiokande Collaboration, Y. Fukuda, et al., Phys. Lett. B 467 (1999) 185.

[3] CHOOZ Collaboration, M. Apollonio, et al., Phys. Lett. B 466 (1999) 415.

[4] S. Geer, Phys. Rev. D 57 (1998) 6989;
S. Geer, Phys. Rev. D 59 (1999) 039903, Erratum;
OPERA Collaboration K. Kodama, et al., CERN/SPSC 99-20, SPSC/M635, LNGS-LOI 19/99 (1999);
ICARUS, NOE Collaborations, F. Arneodo et al., INFN/AE-99-17, CERN/SPSC 99-25, SPSC/P314 (1999);
MINOS Collaboration, P. Adamson, et al., NuMI-L-337 (1998);
M. Aoki, et al., hep-ph/0112338.

[5] S.P. Mikheev, A.Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913;
L. Wolfenstein, Phys. Rev. D 17 (1978) 2369.

[6] T.K. Kuo, J. Pantaleone, Phys. Lett. B 198 (1987) 406.

[7] H. Minakata, S. Watanabe, Phys. Lett. B 468 (1999) 256.

[8] P. Langacker, S.T. Petcov, G. Steigman, S. Toshev, Nucl. Phys. B 282 (1987) 589.

[9] J. Arafune, J. Sato, Phys. Rev. D 55 (1997) 1653;
J. Arafune, M. Koike, J. Sato, Phys. Rev. D 56 (1997) 3093;
J. Arafune, M. Koike, J. Sato, Phys. Rev. D 60 (1999) 119905, Erratum.

[10] A. Cervera, A. Donini, M.B. Gavela, J.J. Gomez Cadenas, P. Hernandez, O. Mena, S. Rigolin, Nucl. Phys. B 579 (2000) 17;
A. Cervera, A. Donini, M.B. Gavela, J.J. Gomez Cadenas, P. Hernandez, O. Mena, S. Rigolin, Nucl. Phys. B 593 (2001) 731, Erratum.

[11] M. Freund, Phys. Rev. D 64 (2001) 053003.

[12] P.I. Krastev, S.T. Petcov, Phys. Lett. B 205 (1988) 84.

[13] J. Sato, Nucl. Instrum. Methods A 472 (2000) 434.

[14] P. Lipari, Phys. Rev. D 64 (2001).

[15] O. Yasuda, Acta Phys. Pol. B 30 (1999) 3089.

[16] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039.

[17] V.A. Naumov, Int. J. Mod. Phys. D 1 (1992) 379.

[18] P.F. Harrison, W.G. Scott, Phys. Lett. B 476 (2000) 349.

[19] H. Yokomakura, K. Kimura, A. Takamura, Phys. Lett. B 496 (2000) 175.

[20] S.J. Parke, T.J. Weiler, Phys. Lett. B 501 (2001) 106.

[21] F.D. Stacey, Physics of the Earth, 2nd edn., Wiley, 1977.

[22] H. Minakata, H. Nunokawa, Phys. Rev. D 57 (1998) 4403.

[23] M. Koike, J. Sato, Mod. Phys. Lett. A 14 (1999) 1297.

[24] T. Ota, J. Sato, Phys. Rev. D 63 (2001) 093004.

[25] T. Miura, T. Shindou, E. Takasugi, M. Yoshimura, Phys. Rev. D 64 (2001) 073017.

[26] J. Pinney, O. Yasuda, Phys. Rev. D 64 (2001) 093008.

[27] T. Miura, E. Takasugi, Y. Kuno, M. Yoshimura, Phys. Rev. D 64 (2001) 013002.

[28] E.Kh. Akhmedov, P. Huber, M. Lindner, T. Ohlsson, Nucl. Phys. B 608 (2001) 394.

[29] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[30] H.W. Zaglauer, K.H. Schwarzer, Z. Phys. C 40 (1988) 273.

[31] K. Kimura, A. Takamura, H. Yokomakura, Phys. Lett. B 537 (2002) 86.

[32] K. Kimura, A. Takamura, H. Yokomakura, hep-ph/0205295.

[33] H. Minakata, H. Nunokawa, S. Parke, Phys. Lett. B 537 (2002) 249.

[34] H. Minakata, H. Nunokawa, JHEP 0110 (2001) 001.

[35] P.M. Fishbane, P. Kaus, Phys. Lett. B 495 (2000) 369.