The 2\textsuperscript{nd} International Conference on Complexity Science & Information Engineering

Fuzzy economic order quantity model with imperfect items, shortages and inspection errors

Jicheng Liu*, Hui Zheng

North China Electric Power University, No.2 Beinong Rd., Huilongguan, Beijing, 102206, China

Abstract

This paper studies a fuzzy EOQ engineering problem with imperfect quality and shortages. An inspector may make two types of errors while screening the received lot and an imperfect screening process (Raouf et al., 1983) is adopted. The fraction of defectives in the ordered lot is assumed to be a fuzzy number. A fuzzy EOQ model is formulated to describe this inventory engineering problem and optimal solutions are obtained. The effect of fuzziness of fraction of defectives on optimal solutions is illustrated by a numerical example.

© 2011 Published by Elsevier Ltd. Selection and peer-review under responsibility of Desheng Dash Wu.

Keywords: EOQ; Fuzzy set; Imperfect quality; Shortages; Misclassification errors

1. Introduction

In the traditional economic order quantity models, the basic assumption is that 100\% of ordered items are perfect. However, the lot sizes ordered may contain some defective products due to deficient maintenance, weak production control and so on. Then the screening process is adopted to identify the imperfect items. Also, there may be some inspection errors in the imperfect screening process. Recently, many researchers investigate the inventory problems with defective products.

Rosenblatt and Lee (1986) developed an EPQ model with imperfect quality, where the elapsed time from in-control state to out-control state is assumed to be a random variable. Lee and Rosenblatt (1987) investigated an EOQ model with fixed defective rate and inspection and obtained an inspection policy and the optimal order size. Salameh and Jaber (2000) proposed an EPQ model with random defective rate and perfect screening process. A joint lot sizing and inspection strategy is derived. Hayek and Salameh (2001) studied the economic production quantity problem with defective items and rework time. The percentage of defectives is a random variable with a uniform distribution. Chiu (2003) generalized the Hayek and Salameh (2001)’s model by assuming only a part of the imperfect items are reworked to become perfect.

The underlying assumption in the above models is that the screening process is perfect and error-free. In real situation, the human error is unavoidable while screening. Raouf et al. (1983) presented one of the first inspection policies for the human error in inspection process. Duffuaa and Khan (2002) extended Raouf et al. (1983)’s model by incorporating a number of misclassifications. Duffuaa and Khan (2005)

* Corresponding author. Tel.: +86-13601030970; fax: 51963567.
E-mail address: amey2002as@163.com.
studied the optimal inspection policy under different kinds of misclassifications. Khan, Jaber and Bonney (2011) generalized the Salameh and Jaber (2000)'s model by considering the imperfect screening process and adopting the approach in Raouf et al. (1983) to depict the misclassifications.

Recently, it is more reasonable to describe some parameters as fuzzy variables for the unreliability and scarcity of historical data. Chang (2004) studied an EOQ model with fuzzy defective rate and fuzzy demand. Li and Zhang (2008) dealt with the order inventory problem with shortages and imperfect items. The annual demand and cost parameters are assumed to be trapezoidal fuzzy numbers. Vijayan and Kumaran (2009) examined the EOT model with fuzzy arrival rate and fuzzy cost components.

This paper modified the Khan, Jaber and Bonney (2011)'s model by considering shortages and fuzzy fraction of defectives.

2. Notations

\( D \) number of units demanded per year; \( y \) order quantity; \( w \) maximum backorder level allowed; \( c \) unit variable cost; \( k \) fixed ordering cost; \( s \) unit selling price of a nondefective item; \( v \) unit selling price of a defective item; \( x \) screening rate; \( d \) unit screening cost; \( h \) unit holding cost; \( T \) cycle length; \( m_1 \) proportion of nondefective items are classified to be defective; \( m_2 \) proportion of defective items are classified to be nondefective; \( \hat{p} \) percentage of defective items in \( y \), which is a triangular fuzzy number \( \hat{p} = (p - \Delta_1, p, p + \Delta_2) \); \( t_1 \) time to build up a backorder level of \( w \) units; \( t_2 \) time to eliminate the backorder level of \( w \) units; \( t_3 \) time to screen \( y \) units ordered per cycle; \( T \) cycle length; \( B_1 \) number of items that are classified as defective in one cycle; \( B_2 \) number of defective items that are returned from the market in one cycle; \( C_a \) cost of accepting a defective item; \( C_r \) cost of rejecting a nondefective item.

3. Mathematical model

Consider a lot size of \( y \) being replenished instantaneously at the beginning of each cycle. It is assumed that each lot contains a fuzzy proportion \( \hat{p} \) of defective items. Each lot received is screened by an inspector with a screening rate \( x \) and fixed misclassification rate, which means that a proportion \( m_1 \) of nondefective items are classified to be defective and a proportion \( m_2 \) of defective items are classified to be nondefective. The items that are returned from the market are sold at a discounted price at the end of each cycle. The behavior of the inventory level is illustrated in Fig.1.

![Fig.1. Behaviour of the inventory level over time.](image-url)
In a cycle, considering the demand is met from perfect items, the cycle length can be calculated as:

$$ T = \frac{(1 - \hat{p})y(1 - m_i)}{D} $$

(1)

The expected cycle length is:

$$ E[T] = \frac{(1 - E[\hat{p}])y(1 - m_i)}{D} $$

(2)

Referring to Fig.1, we obtain:

$$ t_1 = \frac{w}{D} $$

(3)

$$ t_2 = \frac{w}{(1 - p)x - D} $$

(4)

$$ t_3 = \frac{y}{x} $$

(5)

$$ t_3 - t_2 = \frac{z_2 - z_1 - B_x}{D} $$

(6)

According to equation (4), we have:

$$ z_2 = y - \frac{(1 - p)xw}{(1 - p)x - D} $$

(7)

$$ z_1 $$ is obtained from equations (1) and (2) as follows:

$$ z_1 = yA - w $$

(8)

Where

$$ A = 1 - \frac{D}{x} - (p + m_i) + p(m_i + m_z) $$

According to the above description of the model, the number of items that are classified as defective in one cycle and the number of defective items that are returned from the market in one cycle is given by:

$$ B_x = y(1 - p)m_i + yp(1 - m_z) $$

$$ B_x = ypm_z $$

Thus, the revenue from salvaging defective items and the perfect items are given as:

$$ R_1 = vy(B_1 + B_2) = vy(1 - p)m_i + vyp $$

$$ R_2 = sy(1 - p)(1 - m_i) + sym_z $$

Therefore, the total revenue is:

$$ R = R_1 + R_2 = vy(1 - p)m_i + vyp + sy(1 - p)(1 - m_i) + sym_z $$

(9)
Considering the total cost each cycle is consisted of procurement cost, screening cost, holding cost and shortage cost, the total profit each cycle is

\[
TP = sy(1 - p)(1 - m_i) + sym_p + vy(1 - p)m_i + vyv - k - cy - dy - C_r(1 - p)y_m - C_a pym
\]

\[
= \frac{h}{2}[(\frac{2}{x} - \frac{D}{x^2} + \frac{A^2 + p^2 m^2}{D} \frac{pm}{x})y^2 - yw(\frac{2 - 2p - 2m_i + 2pm_i + pm^2 - 1}{D} - \frac{1}{x})] - \frac{w^2 x(1 - p)(h + \pi)}{2D((1 - p)x - D)}
\]

(10)

The expected total profit each cycle is

\[
E[TP] = sy(1 - E[p])(1 - m_i) + sym_p E[p] + vy(1 - E[p])m_i + vyE[p] - k - cy - dy
\]

\[
- C_r(1 - E[p])y_m - C_a ym_p E[p] - \frac{h}{2}[(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2] + E[p^2] m^2}{D} - \frac{E[p] m^2}{x})y^2
\]

\[
- yw(\frac{2 - 2E[p] - 2m_i + 2E[p] m_i + E[p] m^2}{D} - \frac{1}{x})] - \frac{w^2 x(1 - E[p])(h + \pi)}{2D((1 - E[p])x - D)}
\]

(11)

Referring to equations (2) and (11), the expected annual profit can be written as

\[
E[TPU] = sD + \frac{sDm_p E[p]}{E_3} + \frac{vDm_p E[p]}{E_3} + \frac{vDE[p]}{E_3}
\]

\[
= D\left[\frac{k}{y} + c + d + C_r(1 - E[p])m_i + C_a m_p E[p] + \frac{hE_y}{2}\right]
\]

\[
- \frac{hw}{2E_3} - \frac{w^2 x(h + \pi)}{2yE_1}
\]

(12)

where

\[
E_1 = [(1 - E[p])x - D](1 - m_i) , E_2 = \frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2] + E[p^2] m^2}{D} - \frac{E[p] m^2}{x}
\]

\[
E_3 = (1 - E[p])(1 - m_i) , E_4 = 2 - 2m_i + (2m_i - 2 + m^2)E[p] - \frac{D}{x}
\]

\[
E[A^2] = (1 - \frac{D}{x} - m_i)^2 + (m_i + m^2 - 1)^2 E[p^2] + 2(m_i + m^2 - 1)(1 - \frac{D}{x} - m_i)E[p]
\]

Proposition 1. If \(\bar{p}\) is a triangular fuzzy number, that is \(\bar{p} = (p - \Delta_1, p, p + \Delta_2)\), then

\[
E[\bar{p}^2] = p^2 + \frac{1}{4}(\Delta_1^2 + \Delta_2^2)
\]

(13)

\[
E[\bar{p}^2] = p^2 + \frac{\Delta_1^2 + \Delta_2^2}{6} + \frac{p}{2}(\Delta_2 - \Delta_1)
\]

(14)

Proof: The membership function \(\mu_{\bar{p}}(x)\) and the credibility distribution \(\Phi(x)\) are given by
Then, we have

\[
E[\bar{p}] = \int_0^{p-\Delta_1} [1-0]dx + \int_{p-\Delta_1}^{p} \left[1 - \frac{L(x)}{2}\right]dx + \int_{p}^{p+\Delta_2} \frac{R(x)}{2} dx + \int_{p+\Delta_2}^{+\infty} (1-1)dx = p + \frac{1}{4}(\Delta_1 - \Delta_2)
\]

It follows that

\[
E[\bar{p}^2] = \int_0^{+\infty} C_r(p^2 \geq r)dr - \int_0^{0} C_r(p^2 \leq r)dr = \int_0^{+\infty} C_r(p \geq \sqrt{r})dr
\]

Setting \( x = \sqrt{r} \) yields

\[
E[\bar{p}^2] = \int_0^{+\infty} 2x(1-C_r(p \leq x))dx = p^2 + \frac{\Delta_1^2 + \Delta_2^2}{6} + \frac{p}{2}(\Delta_2 - \Delta_1)
\]

The proof is completed.

**Proposition 2.**

1) \( E[TPU] \) is strictly concave.

2) The optimal order quantity and backorder quantity are

\[
y^* = \frac{8Dk(h + \pi)E_i}{4Dx(h + \pi)E_1E_2 - h^2E_1E_4^2}
\]

(15)

\[
w^* = \frac{hE_4}{2x(h + \pi)E_3}y^*
\]

(16)

**Proof:** The Hessian matrix (H) is as follows:

\[
H = \begin{bmatrix}
\frac{\partial^2 E[TPU]}{\partial y^2} & \frac{\partial^2 E[TPU]}{\partial y \partial w} \\
\frac{\partial^2 E[TPU]}{\partial w \partial y} & \frac{\partial^2 E[TPU]}{\partial w^2}
\end{bmatrix}
\]

Taking partial derivatives of the equation (12), we have

\[
\frac{\partial^2 E[TPU]}{\partial y^2} = -\frac{2Dk}{y^2E_1} - \frac{w^2x(h + \pi)}{y^2E_1} < 0, \quad \frac{\partial^2 E[TPU]}{\partial w^2} = -\frac{x(h + \pi)}{yE_1} < 0, \quad \frac{\partial^2 E[TPU]}{\partial y \partial w} = \frac{wx(h + \pi)}{2y^2E_1}
\]

Thus, the Hessian matrix (H) is negative definite. The expected annual profit \( E[TPU] \) is strictly concave.

2) Setting \( \frac{\partial E[TPU]}{\partial w} = 0 \) and \( \frac{\partial E[TPU]}{\partial y} = 0 \) yields
\[ w = \frac{hE_1E_4}{2x(h+\pi)E_3}, y = \sqrt{\frac{8Dk(h+\pi)E_3}{4Dx(h+\pi)E_1E_2-h^2E_4E_5}}. \]

The proof is completed.

**Inference 1.** If \( p = m_1 = m_2 = 0 \) and \( x \to \infty \), we have \( E_1 \to \infty, \ E_2 = \frac{1}{D}, \ E_3 = 1, \ E_4 = 2, \) then

\[ w_\ast = \frac{h}{h+\pi}y_\ast, \ y_\ast = \sqrt{\frac{2Dk(h+\pi)}{h\pi}} \]

which are the same equations as those given by classical EOQ model with shortages.

### 4. Numerical analysis

Consider an inventory system with imperfect process and inspection errors. The proportion of the defective items contained in each lot is given by expert based on their experience as “about 0.02, not more than 0.03, not less than 0.015”, that is \( \bar{p} = (0.015, 0.020, 0.030), \ \Delta_1 = 0.005, \Delta_2 = 0.010 \).

Most of the following data in the system is taken from the Salameh and Jaber (2000)’s model.

\( D = 50000 \) units/year, \( c = \$25/\text{unit}, \ k = \$100/\text{cycle}, \ s = \$50/\text{cycle,} \ v = \$20/\text{unit,} \ x = 175200 \) units/year, \( d = \$0.5/\text{unit}, \ h = \$5/\text{unit,} \ \pi = \$6/\text{unit,} \ m_1 = 0.02, \ m_2 = 0.02, \ C_a = \$500/\text{unit,} \ C_r = \$100/\text{unit.} \)

From equations (15), (16) and (12), the optimal order quantity and the optimal backorder quantity are \( y_\ast = 1656.30, \ w_\ast = 435.58 \), the annual profit is \( E[TPU] = 1095000 \). The effects of the fuzziness of fraction of defectives on the optimal solutions are analyzed and shown in table 1.

<table>
<thead>
<tr>
<th>( \Delta_1 )</th>
<th>( \Delta_2 )</th>
<th>( y_\ast )</th>
<th>( w_\ast )</th>
<th>( E[TPU] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.010</td>
<td>1657.10</td>
<td>435.09</td>
<td>1094200</td>
</tr>
<tr>
<td>0.005</td>
<td>0.010</td>
<td>1656.90</td>
<td>435.22</td>
<td>1094400</td>
</tr>
<tr>
<td>0.005</td>
<td>0.010</td>
<td>1656.70</td>
<td>435.34</td>
<td>1094600</td>
</tr>
<tr>
<td>0.005</td>
<td>0.010</td>
<td>1656.50</td>
<td>435.46</td>
<td>1094800</td>
</tr>
<tr>
<td>0.005</td>
<td>0.010</td>
<td>1656.30</td>
<td>435.58</td>
<td>1095000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.012</td>
<td>1656.70</td>
<td>435.34</td>
<td>1094600</td>
</tr>
<tr>
<td>0.005</td>
<td>0.014</td>
<td>1657.10</td>
<td>435.09</td>
<td>1094200</td>
</tr>
<tr>
<td>0.005</td>
<td>0.016</td>
<td>1657.50</td>
<td>434.84</td>
<td>1093800</td>
</tr>
<tr>
<td>0.005</td>
<td>0.018</td>
<td>1657.90</td>
<td>434.59</td>
<td>1093400</td>
</tr>
<tr>
<td>0.005</td>
<td>0.020</td>
<td>1658.30</td>
<td>434.34</td>
<td>1093000</td>
</tr>
</tbody>
</table>

From Table 1, as the value of \( \Delta_1 \) increases or the value of \( \Delta_2 \) decreases, that is the proportion of the defective items decreases, the optimal order quantity decreases, while both the optimal backorder quantity and expected annual profit increase. The relationship between the fuzziness of fraction of defectives and the optimal solutions are also illustrated in Fig.1, Fig.2 and Fig.3.

In the numerical example, the shortages allowed causes increase in the optimal order quantity as compared to that in Salameh and Jaber (2000)’s model, and the inspection errors leads to a huge drop in
the profit than that in Salameh and Jaber (2000)’s model. Therefore, it is critical for the corporation to reduce the errors and avoid the shortages to make profit maximized.

5. Conclusion

This paper extends the classical EOQ engineering model by incorporating shortages backordered and imperfect quality contained in each ordered lot in a fuzzy environment. The screening process contains two types of errors, one is that a defective item may be classified to be nondefective, while the other is that a nondefective item may be classified to be defective. The fraction of defectives is assumed to be a triangular fuzzy number. The paper suggests that the increase of fraction of defectives causes the decrease of the expected annual profit. Comparing with the Salameh and Jaber (2000), the optimal order size is larger because of the shortages allowed, while the annual profit is much smaller which signifies the impact of inspection errors. Thus, the inspection errors and fraction of defectives should be eliminated as much as possible in order to achieve the maximum of profit.
5. Conclusion

This paper extends the classical EOQ engineering model by incorporating shortages backordered and imperfect quality contained in each ordered lot in a fuzzy environment. The screening process contains two types of errors, one is that a defective item may be classified to be nondefective, while the other is that a nondefective item may be classified to be defective. The fraction of defectives is assumed to be a triangular fuzzy number. The paper suggests that the increase of fraction of defectives causes the decrease of the expected annual profit. Comparing with the Salameh and Jaber (2000), the optimal order size is larger because of the shortages allowed, while the annual profit is much smaller which signifies the impact of inspection errors. Thus, the inspection errors and fraction of defectives should be eliminated as much as possible in order to achieve the maximum of profit.

References


