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# Loop corrections in higher dimensions via deconstruction

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## Abstract

We calculate the one-loop corrections to the Kaluza–Klein gauge boson excitations in the deconstructed version of the 5D QED. Deconstruction provides a renormalizable UV completion of the 5D theory that enables to control the cut-off dependence of 5D theories and study a possible influence of UV physics on IR observables. In particular we calculate the cut-off-dependent non-leading corrections that may be phenomenologically relevant for collider physics. We also discuss the structure of the operators that are relevant for the quantum corrections to the gauge boson masses in 5D and in deconstruction.

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## 1. Introduction

In the past few years, high energy physics ventured to explore phenomenological aspects of space–times involving more than four dimensions. From the hierarchy to the flavor problem, from supersymmetry to electroweak symmetry breaking, from proton stability to the number of the Standard Model generations, from dark matter abundance to neutrino oscillations, many puzzles that jeopardize our 4D understanding of quantum field theory could find a solution when extra dimensions are involved. So one is naturally led to wonder what is so special about extra dimensions? The notion of locality/sequestering is definitively an essential tool in suppressing any dangerous radiative operator. It was then realized [1] that locality in physical extra dimension can be advantageously mimicked by locality in theory space along which 4D gauge symmetry is multi-replicated. At tree-level, by a matching in the IR of the mass spectra and the interaction patterns, a precise correspondence has been established between higher-dimensional theories and 4D deconstructed theories. This correspondence is believed to hold all the way long from the perturbative to the non-perturbative regime [2].

Higher-dimensional gauge theories are non-renormalizable and valid only below certain physical cut-off scale  $\Lambda$ . Calculating quantum corrections in such theories requires a careful choice of a regularization scheme as, in general, there is a clash between the gauge invariance and the need for a cut-off [3]. The question of regularization arises even for those radiative corrections that are expected to be UV finite (i.e., dominated by IR

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physics). Deconstruction can serve as a renormalizable UV completion of higher-dimensional gauge theories and, within such scheme, calculation of quantum correction is totally unambiguous. Moreover, in deconstructed theory, radiative corrections include the effects due to a finite cut-off  $\Lambda$ . Although they are specific for this particular UV completion, they illustrate how the predictions of higher-dimensional theories can be disturbed by UV physics.

Recently, one-loop corrections to the masses of the gauge boson excitations have been calculated [4–6]. In the present Letter we calculate analogous corrections in the renormalizable deconstruction setup and compare the results. We will restrict ourselves to 5D QED compactified on a circle (see [7] for the corresponding setup), the group theory factors associated to the non-Abelian nature of the interactions being identical in the 5D and the 4D computations anyway. In this simple case, it was shown in Refs. [4–6] that the interactions with a single 5D fermion of electric charge  $e_5$  shift all the masses of the 4D KK gauge bosons by an amount

$$\delta m_n^2 = -\frac{\zeta(3)e_0^2}{4\pi^4 R^2}, \quad (1)$$

where  $e_0 = e_5/\sqrt{2\pi R}$  is the 4D gauge coupling and  $R$  is the radius of the compact fifth dimension (the 4D massless photon remains of course massless by gauge invariance). Meanwhile, the massless scalar field corresponding to the component of the 5D gauge field along the compact dimension acquires a mass given by [8]

$$\delta m^2 = -\frac{3\zeta(3)e_0^2}{4\pi^4 R^2}. \quad (2)$$

The phenomenological relevance of one-loop corrections to the 4D gauge boson masses in 5D gauge theories has been stressed in Ref. [4] where it was noticed that, due to the degeneracy of KK spectrum at tree-level, decay channels are controlled by radiative corrections, thus a slight modification in the modification, in particular from UV physics, can affect collider signals [4] as well as the abundance of dark matter [9]. Thus the importance of our computation in the deconstruction regularization where we have a full control on the UV physics. Let us also mention that in models where the Higgs boson is identified as a component of a gauge boson in higher dimensions [10], the radiative corrections we are interested in ultimately control the electroweak symmetry breaking and determine the Higgs mass. Finally, computing the radiative corrections to gauge bosons masses in 4D deconstructed theories is also important for the following reason: in Ref. [11], it was shown that the spectrum of a product of  $\mathcal{N} = 1$  supersymmetric  $SU(N)$  gauge theories broken to the diagonal  $SU(N)$  exhibit a  $\mathcal{N} = 2$  supersymmetry. Even though this extended supersymmetry seems accidental from the 4D point of view, it is actually dictated by the underlying 5D Lorentz invariance of the corresponding higher-dimensional theory. Our computation can be extended to show that the  $\mathcal{N} = 2$  supersymmetry indeed survives at one-loop.

## 2. Framework

### 2.1. Tree-level matching between the 5D and 4D theories

As outlined in the introduction, we restrict ourselves to the case of 5D QED and a massless Dirac fermion of electric charge  $e$ , the fifth dimension being compactified on a circle of radius  $R$ . The deconstructed setup (see also [7]) corresponds to a product of  $N$  copies of  $U(1)$  gauge group<sup>1</sup> linked together by  $N$  scalar fields  $\Phi_p$  of charge  $(e, -e)$  under  $U(1)_p \times U(1)_{p+1}$  (the site indices being periodically identified as  $N + p \sim p$ ). Once the link fields acquire a VEV,  $\langle \Phi_p \rangle = v/\sqrt{2}$ , the product gauge group is broken to the diagonal  $U(1)$  and the gauge boson

<sup>1</sup> For definiteness we take  $N$  to be odd.

spectrum is made of a massless photon:

$$A_\mu^{(0)} = \frac{1}{\sqrt{N}} \sum_{p=1}^N A_{\mu,p}, \quad (3)$$

and a tower of massive excitations doubly degenerated in mass ( $n = 1, \dots, (N-1)/2$ )

$$A_\mu^{(n)} = \sqrt{\frac{2}{N}} \sum_{p=1}^N \cos \frac{2n(p-1)\pi}{N} A_{\mu,p} \quad \text{and} \quad A_\mu^{(-n)} = \sqrt{\frac{2}{N}} \sum_{p=1}^N \sin \frac{2n(p-1)\pi}{N} A_{\mu,p} \quad (4)$$

with mass

$$m_{\pm n} = 2ev \sin \frac{n\pi}{N}. \quad (5)$$

The shift symmetry of the setup, i.e., the fact that the electric charges and VEVs do not depend on the site index, corresponds to the translational symmetry of the fifth dimension compactified on a circle.

The deconstruction setup can be thought of as a discretization of the fifth dimension at points  $y_p = 2p\pi R/N$ ,  $p = 1, \dots, N$ , the 5D gauge field being matched to the 4D degrees of freedom in the following way. The 4D components of the gauge field at the point  $y_p$ ,  $A_\mu(x_\nu, y_p)$ , are identified with the 4D gauge field at the site  $p$ ,  $A_{\mu,p}(x_\nu)$ . The component along the extra dimension of the 5D gauge field,  $A_5(x_\nu, y_p)$ , is matched to the link field  $\Phi_p(x_\nu)$ , as it can be seen in the broken phase of the deconstruction theory. Indeed let us split the link fields as  $\Phi_p = \frac{1}{\sqrt{2}}(v\mathbf{I} + \Sigma_p + iG_p)$ . For a number of sites large enough, a gauge invariant renormalizable scalar potential can depend on the link fields only in the combination  $\Phi_p^* \Phi_p$ . In consequence, the scalar sector of the theory possesses an additional  $U(1)^N$  global symmetry (acting as  $\Phi_p \rightarrow e^{i\alpha_p} \Phi_p$ ), which is completely broken when the links acquire VEVs. This global symmetry pattern results in the presence of  $N$  massless Goldstone bosons,  $N-1$  of which actually being eaten by the massive gauge bosons. The remaining physical Goldstone boson, identified as  $G_{(0)} = (G_1 + \dots + G_N)/\sqrt{N}$ , is precisely what matches the zero mode of  $A_5$ . Finally, the real parts of the link fields,  $\Sigma_p$ , can acquire a mass of the order of the deconstruction scale and thus they do not match any degrees of freedom of the 5D theory below its cut-off  $\Lambda$ .

To reproduce the fermionic KK modes, we need to introduce  $N$  pairs of chiral fermions  $(\psi_p, \chi_p)_{p=1\dots N}$  of charge  $(e, e)$  under  $U(1)_p$ . After the breaking to the diagonal  $U(1)$ , the correct KK spectrum is recovered in the large  $N$  limit at the condition to correctly fine-tune the Yukawa couplings of the fermions as follows [11]:

$$\mathcal{L} = \sum_{p=1}^N (i\bar{\psi}_p \sigma^\mu D_{\mu,p} \psi_p + i\bar{\chi}_p \sigma^\mu D_{\mu,p} \chi_p + \sqrt{2}e\Phi_p \bar{\chi}_p \psi_{p+1} - ev\bar{\chi}_p \psi_p + \text{h.c.}), \quad (6)$$

where  $D_{\mu,p}$  stands for the covariant derivative for the  $U(1)_p$  gauge group,  $D_{\mu,p} = \partial_\mu + ieA_{\mu,p}$ . After symmetry breaking down to the diagonal  $U(1)$ , the fermionic spectrum is made of one massless Dirac fermion and a tower of massive Dirac fermions with the same mass as the gauge boson ones (see [12] for details about the mode decomposition). Note that due to the normalization factor appearing in the massless photon (3), all these fermions carry a charge  $e_0 = e/\sqrt{N}$  under the unbroken  $U(1)$  gauge group.

The comparison of the spectrum and the interactions in both the compactified 5D theory and the deconstructed 4D theory leads to the following identification of the parameters [1]

$$e_0 = \frac{e_5}{\sqrt{2\pi R}} = \frac{e}{\sqrt{N}} \quad \text{and} \quad \frac{1}{R} = \frac{2\pi}{N} ev. \quad (7)$$

The cut-off scale,  $\Lambda$ , of the 5D theory is also related to the 4D parameters by  $\Lambda = ev$ .

### 2.2. Renormalization setup

At the quantum level, the 4D deconstructed theory constitutes a UV completion of the 5D gauge theory, and the framework can be arranged to be renormalizable. Therefore, at an arbitrary level of perturbation theory, all observables are unambiguously determined up to the freedom of adjusting a finite number of counterterms. Note that the form of the counterterms is additionally constrained by the discrete shift symmetry inherited from the 5D translational invariance.

The bare and renormalized quantities are related to one another as follows:

$$A_{\mu,p}^B = Z_A^{1/2} A_{\mu,p}, \quad \Phi_p^B = Z_\Phi^{1/2} \Phi_p, \quad g^B = Z_A^{-3/2} (g + \delta_g), \quad v^B = Z_\Phi^{1/2} (v - \delta_v), \quad (8)$$

where  $Z_A = 1 + \delta_A$ ,  $Z_\Phi = 1 + \delta_\Phi$  are the wave function renormalization of the gauge boson and the link fields.

Let us first discuss the loop corrections to the mass of the massive gauge bosons  $A_\mu^{(n)}$ . Of course, there are no reasons to expect that the loop corrections are finite, nevertheless, since the setup is renormalizable, all divergences can be absorbed into counterterms. From Eq. (8) we find that the allowed counterterms corresponding to gauge boson masses are given by

$$\mathcal{L}_{ct} = \frac{1}{2} \delta_M g^2 v^2 \sum_{p=1}^N (A_{\mu,p} - A_{\mu,p+1})^2, \quad (9)$$

where  $\delta_M$  can be expressed in terms of the wave function and gauge coupling renormalization as  $\delta_M = 2\delta_g/g + \delta_\Phi + \delta_A - 2\delta_v/v$ . Expressing the gauge fields in terms of the mass eigenstates these counterterms become ( $N = 2s + 1$ )

$$\mathcal{L}_{ct} = \frac{1}{2} \delta_M \sum_{n=-s}^s m_n^2 A_\mu^{(n)} A_\mu^{(n)}. \quad (10)$$

By adjusting  $\delta_M$  we can remove any divergence proportional to  $m_n^2$  that may appear in loop calculations of the gauge boson masses. The finite part of  $\delta_M$  depends on the regularization scheme, and therefore the renormalization of an overall scale of the gauge boson masses cannot be unambiguously calculated in deconstruction. On the other hand, any loop corrections to the gauge boson masses that are not proportional to  $m_n^2$  (including a constant,  $n$ -independent, shift) are, in the deconstruction formalism, unambiguous predictions.

Consider now how loop corrections to the mass of 4D massless scalar, i.e., the zero mode of the fifth component of the gauge field  $A_{5,(0)}$ , appear. To this end we need to analyze the possible form of the counterterms containing a mass term for the Goldstone boson  $G_{(0)}$  and which descend from the counterterms involving the link fields  $\Phi_p$ . At the level of dimension  $\leq 4$  operators and assuming  $N > 4$ , we have only the following ‘non-holomorphic’ operators:

$$\mathcal{L}_d = \sum_{p=1}^N \delta_{d1} |\Phi_p|^2 + \sum_{p,q=1}^N \delta_{d2} |\Phi_p|^2 |\Phi_q|^2. \quad (11)$$

As a result of the translational invariance along the discrete lattice direction,  $\delta_{d1}$  is independent of the lattice position  $p$  while  $\delta_{d2}$  can only depend in the lattice distance  $|p - q|$ . These operators can be induced with a divergent coefficient. However, effectively, they do not introduce any incalculable corrections to the mass of  $G_{(0)}$ . Indeed, once the link fields acquire a VEV, the Lagrangian (11) contains both a mass term for the Goldstone boson  $G_{(0)}$  and a tadpole for the real part,  $\Sigma_p$ , of the link fields:

$$\mathcal{L}_d = \delta_T \left( \frac{2v}{N} \sum_p \Sigma_p + G_{(0)}^2 \right) + \dots, \quad (12)$$

where  $\delta_T$  is some function of the coefficients  $\delta$ 's in Eq. (11)—note that the shift symmetry was essential to factorize the  $\delta_T$  dependence in (12). Now adjusting the counterterms in order to remove the tadpoles automatically cancels the mass term for  $G_{(0)}$  as well. However the  $G_{(0)}$  mass can be renormalized by gauge invariant ‘holomorphic’ operators like, e.g.,  $\Phi_1\Phi_2\cdots\Phi_N$ . For  $N > 4$  the holomorphic operators are non-renormalizable and are induced at loop level with a finite, calculable coefficient. We conclude that loop corrections to the  $G_{(0)}$  mass are unambiguously calculable in deconstruction, once we fix the counterterms such that the  $\Sigma_p$  tadpole term is vanishing.

### 3. Diagrammatic computation

#### 3.1. Mass corrections to $A_5$

Let us start with computing the radiative correction to the mass of the Goldstone boson that remains massless at tree-level. Similar calculation, but in a non-renormalizable non-linear sigma model setup, was performed in Ref. [13]. As discussed in the previous section, in the renormalizable formalism the first step is to calculate the diagrams that contribute to the tadpoles of the real part of link scalar fields,  $\Sigma_p$ , in order to determine the mass counterterm  $\delta_T$ , see Eq. (12). Then the mass correction of the physical Goldstone boson,  $G_{(0)}$ , is obtained by calculating the two point function of this Goldstone mode and subtracting the contribution of  $\delta_T$ . The decomposition of the action in terms of the mass eigenstates leads to standard Feynman rules (see for instance [12]) which we can use to compute the two point function. After rather long but trivial manipulations, we obtain

$$\delta m^2 = -4e_0^2 \sum_{k=-(N-1)/2}^{(N-1)/2} \int \frac{d^4 l_E}{(2\pi)^4} \frac{l_E^2 \cos(2k\pi/N) - m_k^2}{(l_E^2 + m_k^2)^2}. \quad (13)$$

First we perform the momentum integration using dimensional regularization (we present at the end of Appendix A a computation of the mass correction where the summation over the KK modes is first performed). Divergent terms cancel for  $N > 2^2$  and for the finite part we get:

$$\delta m^2 = -\frac{4e_0^2}{(4\pi)^2} (2ev)^2 (-S_2(N) + 2S_4(N) + 3\Sigma_2(N) - 4\Sigma_4(N)), \quad (14)$$

where the sums  $S_{2m}$  and  $\Sigma_{2m}$  are defined by ( $N = 2s + 1$ )

$$S_{2m}(N) = \sum_{k=-s}^s \sin^{2m} \frac{k\pi}{N} \quad \text{and} \quad \Sigma_{2m}(N) = \sum_{k=-s}^s \sin^{2m} \frac{k\pi}{N} \log \sin^2 \frac{k\pi}{N}. \quad (15)$$

The sums  $S_{2m}$  are trivially performed (see Appendix A) and quite remarkably the sums  $\Sigma_{2m}$  can also be performed analytically and they are expressed in terms of the digamma function  $\Psi(z) \equiv \frac{\Gamma'(z)}{\Gamma(z)}$  (see Appendix A for details). So the mass correction is finally written as

$$\delta m^2 = -\frac{2e_0^2}{(4\pi)^2} (2ev)^2 (\Psi(1 + 1/N) - \Psi(1 - 2/N) + \Psi(1 - 1/N) - \Psi(1 + 2/N)). \quad (16)$$

<sup>2</sup> For  $2 < N \leq 4$  the behaviour of the two-point function is softer than expected from the discussion renormalizability because of the little-Higgs mechanism [13]. However for  $N > 4$  the mass correction in deconstruction is calculable at any order of perturbation theory irrespectively of the little-Higgs arguments.

By Taylor expanding the digamma function  $\Psi$  around the unity, we easily obtain an  $1/N$  expansion of the mass correction. In particular, using Eq. (A.16), the leading terms in the correction are given by

$$\delta m^2 = -\frac{3e_0^2}{4\pi^2} \left(\frac{2ev}{N}\right)^2 \left(\zeta(3) + \frac{5\zeta(5)}{N^2} + \dots\right). \tag{17}$$

Identifying the parameters of the 5D and 4D theories as in Eq. (7) we can translate this result as

$$\delta m^2 = -\frac{3e_0^2}{4\pi^4 R^2} \left(\zeta(3) + \frac{5\zeta(5)}{(2\pi \Lambda R)^2} + \dots\right). \tag{18}$$

The first term agrees with the mass correction (2) obtained by directly performing the computations in the 5D theory [4–6], while the second represents a correction due to a finite value of the 5D cut-off realized in the deconstruction setup.

### 3.2. Mass corrections to $A_\mu$

Let us now turn to the more involved computation of the corrections to the gauge boson masses. To this end, we need to evaluate the two point function of the tree-level mass eigenstate gauge field  $A_\mu^{(n)}$  which we split into a transverse and a longitudinal part:

$$\mathcal{M}_n = (p_\mu p_\nu - \eta_{\mu\nu} p^2) \Pi_1^n(p^2) + \eta_{\mu\nu} \Pi_2^n(p^2). \tag{19}$$

Then the shift of the mass at the  $k^{\text{th}}$  level is given by

$$\delta m_n^2 = \Pi_2^n - m_n^2 \Pi_1^n. \tag{20}$$

After some algebra, the two form factors  $\Pi_i^n$  are calculated to be ( $N = 2s + 1$ )

$$\Pi_1^n(p^2) = 8e_0^2 \sum_{k=-s}^s \int_0^1 dx F_1^{n,k}(x) \quad \text{and} \quad \Pi_2^n(p^2) = -4e_0^2 \sum_{k=-s}^s \int_0^1 dx F_2^{n,k}(x) \tag{21}$$

with

$$F_1^{n,k}(x) = \int \frac{d^d l_E}{(2\pi)^d} \frac{x(1-x)}{(l_E^2 + xm_k^2 + (1-x)m_{n+k}^2 - x(1-x)p^2)^2}, \tag{22}$$

$$F_2^{n,k}(x) = \int \frac{d^d l_E}{(2\pi)^d} \frac{(1-2/d)l_E^2 + m_k m_{n+k} \cos \frac{k\pi}{N} - x(1-x)p^2}{(l_E^2 + xm_k^2 + (1-x)m_{n+k}^2 - x(1-x)p^2)^2}. \tag{23}$$

In the previous integrals,  $d = 4$  is the dimension of the space–time and it will be promoted to  $d = 4 - \epsilon$  in order to compute the integrals over the momenta using the usual recipes of dimensional regularization. The mass shift is then written as

$$\delta m_n^2 = -\frac{2e_0^2}{(4\pi)^2} \left( -\frac{1}{3} N \left( \frac{2}{\epsilon} - \gamma + \log(4\pi) \right) m_n^2 + \sum_{k=-(N-1)/2}^{(N-1)/2} \int_0^1 dx f^{n,k}(x) \right) \tag{24}$$

with

$$f^{n,k}(x) = (m_{n+k}^2 + m_n^2 - m_k^2 + 2x(m_k^2 - 2m_n^2 - m_{n+k}^2) + 4x^2 m_n^2) \times \log(m_{n+k}^2 - x(p^2 - m_k^2 + m_{n+k}^2) + p^2 x^2). \tag{25}$$

Let us first note that the mass of the massless gauge boson does not get shifted ( $\delta m_0^2 = 0$ ) as a consequence of the unbroken  $U(1)$  gauge symmetry. For the massive gauge bosons ( $n \neq 0$ ), the mass correction is divergent, but, as it should be according to our general analysis of the renormalization setup, the divergence is proportional to tree-level  $m_n^2$  and so it can be absorbed into counterterms. We keep only the finite part in the following formulae and evaluate the mass correction on-shell, for  $p^2 = m_n^2$ . After integration over the Feynman parameter and lengthy trigonometric manipulations and after absorbing the finite terms proportional to  $m_n^2$  into the counterterms, we end up with the expression

$$\begin{aligned} \delta m_n^2 = & -\frac{2e_0^2 e^2 v^2}{3\pi^2} \left( S_2(N) + \left( 1 - \frac{3n\pi}{N} \sin \frac{2n\pi}{N} \right) S_4(N) - 2\frac{n\pi}{N} \cot \frac{n\pi}{N} \left( 1 - 4 \sin^2 \frac{n\pi}{N} \right) S_6(N) \right) \\ & - \frac{e_0^2 e^2 v^2}{\pi^2} (\Sigma_2(N) + 2\Sigma_4(N) - 4\Sigma_6(N)), \end{aligned} \quad (26)$$

where the sums are  $S_{2m}(N)$  and  $\Sigma_{2m}(N)$  have been defined previously, see Eq. (17). Using again the formulae from Appendix A to evaluate these sums, we obtain:

$$\begin{aligned} \delta m_n^2 = & -\frac{e_0^2 e^2 v^2}{8\pi^2} \left( 3\Psi \left( 1 + \frac{1}{N} \right) + 3\Psi \left( 1 - \frac{1}{N} \right) - 4\Psi \left( 1 + \frac{2}{N} \right) - 4\Psi \left( 1 - \frac{2}{N} \right) + \Psi \left( 1 + \frac{3}{N} \right) \right. \\ & \left. + \Psi \left( 1 - \frac{3}{N} \right) \right) \\ & - \frac{e_0^2 e^2 v^2}{24\pi^2} \left( 10N - 9n\pi \cot \frac{n\pi}{N} - n\pi \frac{\cos \frac{3n\pi}{N}}{\sin \frac{n\pi}{N}} \right). \end{aligned} \quad (27)$$

The first term of the sum does not depend on the mass level  $n$  and corresponds to the constant shift of the massive KK levels which, in the 5D setup, was found in Ref. [4]. The second term does depend on  $n$  and it appears here because deconstruction is a regularization that does not preserve 5D Lorentz invariance in UV. These terms however vanish when the continuum limit is taken. Indeed, using the expansion of the digamma function given in Appendix A, the leading terms in  $1/N$  expansion of Eq. (27) read

$$\delta m_n^2 = -\frac{e_0^2}{4\pi^2} \left( \frac{2ev}{N} \right)^2 \left( \zeta(3) - \frac{5\zeta(5)}{N^2} \right) + \frac{11\pi^2 e_0^2}{108} \left( \frac{ev}{N} \right)^2 \frac{n^4}{N} + \dots \quad (28)$$

which, in terms of 5D parameters, translates into

$$\delta m_n^2 = -\frac{e_0^2}{4\pi^4 R^2} \left( \zeta(3) - \frac{5\zeta(5)}{(2\pi R\Lambda)^2} - \frac{11\pi^3 n^4}{216\Lambda R} + \dots \right). \quad (29)$$

In the continuum limit  $\Lambda \rightarrow \infty$  we recover the mass correction (1) obtained by directly performing the computations in the 5D theory [4–6]. But for a finite value of the cut-off the correction depends on the UV completion of the 5D theory. In particular, we can infer that, for a cut-off scale not much higher than the compactification scale, the prediction of the constant shift of the massive levels can be disturbed by UV physics, which may then play an important role for collider experiments.

#### 4. Operator analysis

In Section 2 we signaled that operators responsible for the mass correction to the Goldstone boson  $G_{(0)}$  are of the holomorphic structure  $\Phi_1 \Phi_2 \dots \Phi_N$ . From the 5D point of view such operators correspond to non-local Wilson lines winding around the extra dimension. The renormalizable deconstruction setup offers thus a convenient setting to study loop induced non-local operators in a higher-dimensional theory. Indeed, the one-loop Coleman–Weinberg

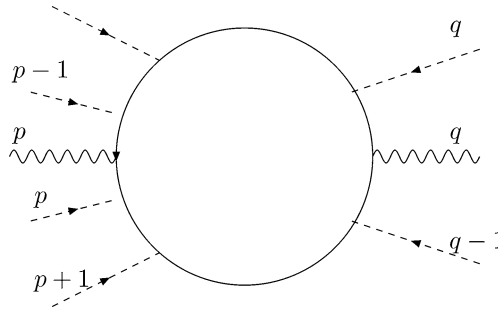


Fig. 1. One-loop diagram contributing to the mass shift of the KK gauge bosons. In the 5D language, an effective non-local operator involving derivative of Wilson lines is generated.

potential for the gauge invariant phase  $\phi \equiv \frac{1}{2iN} \log\left(\frac{\Phi_1 \Phi_2 \dots \Phi_N}{\Phi_1^* \Phi_2^* \dots \Phi_N^*}\right)$  can be expressed [7,13] as

$$V(\phi) = -\frac{e^4 v^4}{\pi^2} \sum_{k=-(N-1)/2}^{(N-1)/2} \sin^4\left(\frac{k\pi}{N} + \frac{\phi}{2}\right) \log \sin^2\left(\frac{k\pi}{N} + \frac{\phi}{2}\right). \tag{30}$$

Using the expressions for the sums  $\Sigma_{2m}(N)$  introduced previously, we can easily find the Taylor expansion around  $\phi = 0$ . In particular we can obtain this way the mass of the Goldstone boson. Indeed at linear order,  $\phi = G_{(0)}/(v\sqrt{N})$ , thus the quadratic term in this expansion of the effective potential (30) is directly related to the one loop mass of  $G_{(0)}$ . We obtain

$$V(\phi) = cst - \frac{e^4 v^4}{\pi^2} (6\Sigma_2(N) - 8\Sigma_4(N) + 7S_2(N) - 8S_4(N)) \left(\frac{\phi}{2}\right)^2 + \dots \tag{31}$$

and using the formulae of Appendix A we end up for the mass of  $G_{(0)}$  with the same expression (16) obtained by a diagrammatic calculation.

Quite analogously, the constant shift of the heavy gauge boson mass levels can be ascribed to holomorphic operators that are interpreted as non-local from the 5D point of view. For instance, the diagram in Fig. 1 induces an operator of the form

$$\mathcal{L} \sim (A_{\mu,p} \Phi_p \dots \Phi_{q-1} A_{\mu,q} \Phi_q \dots \Phi_{p-1}). \tag{32}$$

This operator is invariant only under global transformations of the product group and so it must be a part of some locally invariant operator. Let us define the ‘Wilson-line’ operators,  $W(p, q) \equiv \Phi_p \dots \Phi_{q-1}$ , and their covariant derivatives,  $D_\mu W(p, q) \equiv \partial_\mu W(p, q) + ieA_{\mu,p} W(p, q) - ieW(p, q) A_{\mu,q}$ . Then locally (and shift symmetry) invariant operator which contains that of Eq. (32) is given by

$$\mathcal{L} \sim \sum_{p,q=1, p \neq q}^N D_\mu W(p, q) D_\mu W(q, p) + \text{h.c.} \tag{33}$$

when the links get VEVs such operators yield mass terms for the gauge bosons of the form

$$\mathcal{L} \sim v^2 \sum_{p,q=1}^N (A_{\mu,p} - A_{\mu,q})^2. \tag{34}$$

Inserting the mode decomposition for  $A_{\mu,p}$  we get precisely the constant shift of the massive KK modes

$$\mathcal{L} \sim v^2 \sum_{n \neq 0} A_\mu^{(n)} A_\mu^{(n)}. \tag{35}$$



The 5D non-local operator that corresponds to the deconstructed operator of (33) involves covariant derivatives of the Wilson lines

$$S \sim \frac{1}{R^4} \int d^4x \int_0^{2\pi R} dy_1 dy_2 D^\mu \left( e^{i \int_{y_1}^{y_2} d\tilde{y} g_5 A_5} \right) D_\mu \left( e^{i \int_{y_2}^{y_1} d\tilde{y} g_5 A_5} \right). \quad (36)$$

This operator yields a constant shift of the massive KK gauge bosons of the form (1). However to be able to determine the exact value of the mass shift, one should compute not only the coefficient of the operator (32) but also the coefficients of infinite number of other holomorphic operators, like for instance

$$\mathcal{L} \sim (A_{\mu,p} \Phi_p^k \cdots \Phi_{q-1}^k A_{\mu,q} \Phi_q^k \cdots \Phi_{p-1}^k), \quad (37)$$

and non-holomorphic operators like

$$\mathcal{L} \sim (A_{\mu,p} \Phi_p \cdots \Phi_{q-1} A_{\mu,q} \Phi_q \cdots \Phi_{r-1} |\Phi_r|^{2k} \Phi_r \cdots \Phi_{p-1}). \quad (38)$$

Whether it exists an appropriate choice of variable, like in (30), that allows to sum all those operators is an open question that deserves further scrutiny.

In any case the 4D analysis leads to an identification of non-local operators that are responsible for the mass shift of both  $A_\mu$  and  $A_5$  in five-dimensional gauge theories.

## 5. Conclusions

In this Letter we calculated one-loop corrections to the Kaluza–Klein gauge boson excitations in the deconstructed version of the 5D QED. The deconstructed setup, being a renormalizable UV completion of the 5D theory, is a useful framework for studying quantum corrections. Moreover, it enables to control the cut-off dependence of 5D theories and study a possible influence of UV physics on IR observables. Our results are consistent with those obtained in Refs. [4–6] by direct computations in the 5D theory. We calculate the  $A$ -dependent non-leading corrections and point out that sensitivity of the 5D theory to UV physics may be phenomenologically relevant. We also discuss the structure of operators that are relevant for the quantum corrections to the gauge boson masses in 5D and in deconstruction.

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## Appendix A. Reference formulae

In this appendix we present formulae for various sums appearing in diagrammatic computations and we collect various properties of the digamma function.

The sums,  $S_{2m}$ , involving even powers of sines can be computed using a Chebychev decomposition of  $\sin^{2m} \theta$ :

$$S_{2m}(2s + 1) = \sum_{k=-s}^s \sin^{2m} \left( \frac{k\pi}{2s + 1} \right) = \frac{(2m - 1)!!}{(2m)!!} (2s + 1). \tag{A.1}$$

The sum  $\Sigma_{2m}$  is defined as

$$\Sigma_{2m}(2s + 1) = \sum_{k=-s}^s \sin^{2m} \left( \frac{k\pi}{2s + 1} \right) \log \sin^2 \left( \frac{k\pi}{2s + 1} \right), \tag{A.2}$$

and it can be performed analytically by the use of the Gauss' theorem about the digamma function. For  $0 < p < 2s + 1$  we have

$$\Psi \left( \frac{p}{2s + 1} \right) = -\gamma - \log(4s + 2) - \frac{\pi}{2} \cot \left( \frac{p\pi}{2s + 1} \right) + \sum_{k=1}^s \cos \left( \frac{2pk\pi}{2s + 1} \right) \log \sin^2 \left( \frac{k\pi}{2s + 1} \right). \tag{A.3}$$

Here  $\gamma \sim 0.577 \dots$  is the Euler–Mascheroni constant and  $\Psi(z)$  stands for the digamma function, which is defined as the logarithmic derivative of the Euler gamma function,  $\Gamma(z)$ :

$$\Psi(z) = \Gamma'(z) / \Gamma(z). \tag{A.4}$$

From the Gauss' digamma theorem one can derive the general expressions ( $0 < 2m < N$ ):

$$\Sigma_0(N) = \log N - (N - 1) \log 2, \tag{A.5}$$

$$\Sigma_{2m}(N) = \frac{1}{2^{2m-1}} \left( -\binom{2m}{m} (\gamma + N \log 2) + \sum_{k=1}^m (-1)^k \binom{2m}{m-k} \left( 2\Psi \left( \frac{k}{N} \right) + \pi \cot \left( \frac{k\pi}{N} \right) \right) \right). \tag{A.6}$$

In particular, using the following relations about the digamma function

$$\Psi(z) = \Psi(1 - z) - \pi \cot(\pi z), \tag{A.7}$$

$$\Psi(1 + z) = \Psi(z) + \frac{1}{z}, \tag{A.8}$$

$$\Psi(1) = -\gamma, \tag{A.9}$$

one obtains:

$$\Sigma_2(N) = -\frac{1}{2} \Psi \left( 1 + \frac{1}{N} \right) - \frac{1}{2} \Psi \left( 1 - \frac{1}{N} \right) - (N \log 2 + \gamma) + \frac{N}{2}, \tag{A.10}$$

$$\Sigma_4(N) = -\frac{1}{2} \Psi \left( 1 + \frac{1}{N} \right) - \frac{1}{2} \Psi \left( 1 - \frac{1}{N} \right) + \frac{1}{8} \Psi \left( 1 + \frac{2}{N} \right) + \frac{1}{8} \Psi \left( 1 - \frac{2}{N} \right) - \frac{3}{4} (N \log 2 + \gamma) + \frac{7N}{16}, \tag{A.11}$$

$$\begin{aligned} \Sigma_6(N) = & -\frac{15}{32} \Psi \left( 1 + \frac{1}{N} \right) - \frac{15}{32} \Psi \left( 1 - \frac{1}{N} \right) + \frac{3}{16} \Psi \left( 1 + \frac{2}{N} \right) + \frac{3}{16} \Psi \left( 1 - \frac{2}{N} \right) \\ & - \frac{1}{32} \Psi \left( 1 + \frac{3}{N} \right) - \frac{1}{32} \Psi \left( 1 - \frac{3}{N} \right) - \frac{5}{8} (N \log 2 + \gamma) + \frac{37N}{96}. \end{aligned} \tag{A.12}$$

In order to find the  $1/N$  expansion of these results we introduce the  $n$ th polygamma function,  $\Psi^{(n)}(z)$ , which is defined as the  $(n - 1)$ th derivative of the  $\Psi(z)$  function. From the series representation of the  $\Gamma$  function, the polygamma function can be related to the Hurwitz  $\zeta$  function defined by  $\zeta(s, a) = \sum_{k=0}^{\infty} (k + a)^{-s}$  (the prime meaning that the possible value of  $k$  such that  $k + a = 0$  is omitted in the sum)

$$\Psi^{(n)}(z) = (-1)^{n+1} n! \zeta(n + 1, z). \tag{A.13}$$

In particular, we get that

$$\Psi^{(2)}(1) = -2!\zeta(3), \quad (\text{A.14})$$

$$\Psi^{(4)}(1) = -4!\zeta(5), \quad (\text{A.15})$$

where  $\zeta(s) = \sum_{k=1}^{\infty} k^{-s}$  is the usual Riemann  $\zeta$  function. We thus find

$$\frac{1}{2} \left( \Psi \left( 1 + \frac{a}{N} \right) + \Psi \left( 1 - \frac{a}{N} \right) \right) = -\gamma - \frac{\zeta(3)a^2}{N^2} - \frac{\zeta(5)a^4}{N^4} + \dots \quad (\text{A.16})$$

Let us finally mention that we can alternatively compute the mass correction (16) to  $A_5$  by first performing the summation over the KK mode in Eq. (13) and then performing the momentum integration. To this end, the following sum is needed

$$\sum_{k=-(N-1)/2}^{(N-1)/2} \frac{1}{\sinh^2 x + \sin^2 k\pi/N} = \frac{2N \operatorname{coth} Nx}{\sinh 2x}. \quad (\text{A.17})$$

This relation can be proved by a pole decomposition of the right-hand side. And the resulting momentum integration reduces to

$$\int_0^{\infty} dx \frac{\sinh^3(x/2) \cosh(x/2)}{\sinh^2(Nx/2)} = \frac{1}{2N^2} \left( \Psi \left( \frac{N+1}{N} \right) - \Psi \left( \frac{N-2}{N} \right) + \Psi \left( \frac{N-1}{N} \right) - \Psi \left( \frac{N+2}{N} \right) \right). \quad (\text{A.18})$$

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