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One-loop effective potential in $\mathcal{N}=\frac{1}{2}$ generic chiral superfield model

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Abstract

We obtain the one-loop quantum corrections to the Kählerian and superpotentials in the generic chiral superfield model on the nonanticommutative superspace. Unlike all previous works, we use a method which does not require to rewrite a star-product of superfields in terms of ordinary products. In the Kählerian potential sector the one-loop contributions are analogous to ones in the undeformed theory while in the chiral potential sector the quantum corrections contain a deformation parameter.

Low-energy dynamics of open strings coupled to a constant self-dual graviphoton field strength $\mathcal{C}_{mn}$ can be efficiently described in the terms of field theories with deformed supersymmetry [1]. Remarkable class of such theories is formulated in nonanticommutative (NAC) $\mathcal{N}=\frac{1}{2}$ superspace [2]. Nonanticommutativity means that the multiplication of superfields is described in terms of so-called star-product, which is a fermionic version of the Moyal product. This allows us to work out the details of nonanticommutative supersymmetric field theory using a calculus on conventional $\mathcal{N}=1$ superspace. The component actions of such theories contain the additional couplings with a parameter $\mathcal{C}_{mn}$ what leads to reduction of supersymmetry. Study of the various aspects of supersymmetric theories on deformed superspaces has been carried out in a number of recent papers (see, e.g., [3–5] for $D=4$ models, [6] for $D=2$ models and [7] for extended supersymmetric models in harmonic superspace).

One of the most interesting problems in nonanticommutative supersymmetric theories is a structure of divergences and renormalizability. It is has been demonstrated that $\mathcal{N}=\frac{1}{2}$ supersymmetric Wess–Zumino [3,4] and Yang–Mills [5] models are renormalizable. To be more precise: at the one-loop level renormalizability is lost literally but it can be restored not only at one loop but to all orders of perturbation theory after adding to the classical action the specific terms depending on deformation parameter [3,4]. Divergences are only logarithmic so the induced supersymmetric breaking is soft. From the field-theoretical point of view this property looks rather mysterious because such models contain the higher-dimensional operators and the naive power-counting arguments formally show that the theories are not renormalizable at any order of perturbation theory. However since all such models are consistent only in the Euclidean space, where dotted and undotted spinors are unrelated, the new vertices appearing with the deformation parameter are not accompanied by their conjugates and that provides renormalizability.

Nonlinear sigma-models (bosonic or supersymmetric) attract much attention due to the possibilities of an effective description of infra-red physics. Generic chiral superfield model, which is a generalization of the supersymmetric sigma-model, plays an important role because its relation with the superstring theory as a subsector of an effective theory of $\mathcal{N}=1$ supersymmetric string vacua [8]. Action of this model is written in terms of the Kählerian effective potential $K(\Phi,\bar{\Phi})$ and chiral $W(\Phi)$ and antichiral $\bar{W}(\bar{\Phi})$ superpotentials. Such a model of chiral superfields can be treated as an effective theory, suitable for description of phenomena at

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energies much less than some fundamental scale. Besides, in some cases, a vacuum structure of supersymmetric gauge theories is defined in terms of nonperturbatively induced superpotentials $W(\Phi)$ and $\bar{W}(\bar{\Phi})$.

In ordinary $D=4$ supersymmetric theories there exists the nonrenormalization theorem [9] which states that quantum corrections to the (anti)holomorphic superpotential are very much constrained and sometimes remains nonrenormalizable, whereas the Kähler potential in general gets quantum corrections. Full one-loop corrections to the Kähler potential have been computed, both in the Wess–Zumino model (see, e.g., [10]), in most general renormalizable models [11] and in general supersymmetric sigma-models [12,13].

The component structure of the generic $\mathcal{N}=\frac{1}{2}$ supersymmetric chiral model defined in the two-dimensional [6] and in the four-dimensional [14,15] nonanticommutative superspaces has been investigated in details recently. It has been shown that these theories are described in a closed form by enough simple deformations of the Kähler potential and (anti)holomorphic superpotential. Geometrically, this deformation can be interpreted as a fuzziness in the target space controlled by the vacuum expectation value of the auxiliary field. However the quantum properties of such models have never been studied. In particular the problem of the renormalizability and the problem constructing of the effective action have not been addressed so far.

In this Letter we study the quantum aspects of generic chiral superfield model in NAC superspace. We compute the divergent and leading finite one-loop corrections to the Kähler potential and superpotential using a superfield loop expansion (see, e.g., [16,17]) and the approximation of slowly variating field. Calculation techniques is formulated by a way which preserves the local star-product structure of classical action on the all stages of quantum analysis. The divergence structure of the model is clarified. We show that besides the divergences preserving $\mathcal{N}=1$ supersymmetry there is a new divergent structure explicitly containing the nonanticommutativity parameter.

The classical action for the generic chiral superfield model

$$S=\int d^8z\,K_\star(\Phi,\bar{\Phi})+\int d^6z\,W_\star(\Phi)+\int d^6\bar{z}\,\bar{W}_\star(\bar{\Phi}), \tag{1}$$

up to two derivatives is encoded in three functions of the chiral multiplet: the Kähler potential $K_\star$ that is only required to be a real function and (anti)chiral superpotentials $\bar{W}_\star$, $W_\star$ that are required to be (anti)holomorphic. The Kähler potential and superpotentials are arbitrary functions of chiral $\Phi(y,\theta)$ and antichiral $\bar{\Phi}(y,\theta,\bar{\theta})$ superfields. The subscript $\star$ implies that all functions are understood as expansions in the power series with $\star$-product. The expansions of the Kähler potential and the superpotential are defined as follows

$$K_\star=\sum_{n,\bar{n}=0}^{\infty}K_{n\bar{n}}\frac{1}{n!\bar{n}!}\underbrace{\Phi\star\cdots\star\Phi}_{n}\star\underbrace{\bar{\Phi}\star\cdots\star\bar{\Phi}}_{\bar{n}}\Bigg|_s,\qquad W_\star=\sum_{n=0}W_n\frac{1}{n!}\underbrace{\Phi\star\cdots\star\Phi}_{n}, \tag{2}$$

and analogously for another superpotential $\bar{W}_\star$. Further we will use notation and definition given in papers [2,16,17].

To calculate the one-loop correction we imply a background-quantum splitting of the (anti)chiral fields $\Phi\to\Phi+\phi$ and $\bar{\Phi}\to\bar{\Phi}+\bar{\phi}$. In contrast to our previous paper [4] we do not reduce the star-products of superfields to their ordinary products. Instead of that, we find the explicit operator $\hat{\mathcal{H}}_\star$ which is the second variation of the classical action (1). This operator is formulated completely in terms of $\star$-product and defines the spectrum of quantum fluctuations on a given background. It leads to the following form of the one-loop effective action[1]

$$\Gamma^{(1)}=\mathrm{Tr}\,\ln_\star\hat{\mathcal{H}}_\star. \tag{3}$$

The operator $\hat{\mathcal{H}}_\star$ is a natural generalization of the superspace type operators for the case of a deformed superspace.[2] Analogously to Ref. [18] we call a functional of D_A and $\Phi,\bar{\Phi}$ a star-local polynomial functional if it includes an integral over superspace of a finite sum of monomials so that every monomial is given in terms of star-products of a finite number of D_A and $\Phi,\bar{\Phi}$ taken in the same superspace point. We will see that there are two type of contributions to the effective action. Leading contributions are star-local. However there can be some kind of nonlocal contributions, which are similar to the contributions from nonplanar diagrams. But for the effective potential calculation in the approximation of slowly varying background fields such nonlocal contributions not enter into the game since they lead to higher derivative operators. We mainly focus on the approximation of the constant background

$$D_{\alpha,(\alpha\dot{\alpha})}\Phi=0,\qquad \bar{D}_{\dot{\alpha},(\alpha\dot{\alpha})}\bar{\Phi}=0, \tag{4}$$

where $\bar{D}_{\dot{\alpha}}=\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$, $D_\alpha=\frac{\partial}{\partial\theta^\alpha}+i\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^{\alpha\dot{\alpha}}}$.

[1] Note that the functional integral in the Euclidean space is defined as $\int\mathcal{D}\Phi\,\mathrm{e}^{-S[\Phi]}$ and therefore the one-loop effective action for a complex superfields is defined as $\Gamma=\mathrm{Tr}\,\ln S''[\Phi]$.

[2] The subscript $\star$ at the logarithm ln needs for consistency with the definition of the Green function $G=-\frac{1}{H_\star}\star\delta(z-z')=\int_0^\infty ds\,\mathrm{e}_\star^{sH_\star}\star\delta(z-z')$. Then an arbitrary variation $\delta H_\star$ of the operator $H_\star$ in formal definition $\Gamma^{(1)}$ gives a correct expression $\delta\Gamma^{(1)}=\mathrm{Tr}\,H_\star^{-1}\star\delta H_\star=\int_0^\infty ds\,\mathrm{Tr}\,\mathrm{e}_\star^{sH_\star}\star\delta H_\star=\int_0^\infty\frac{ds}{s}\,\mathrm{Tr}\,(\delta\mathrm{e}_\star^{sH_\star})=\delta\,\mathrm{Tr}\,\ln_\star H_\star$.

Firstly we emphasize the basic properties of $\star$-product used in the further calculation. Because for the mixed powers in the power expansion of the Kähler potential we have $\Phi \star \bar{\Phi} \neq \bar{\Phi} \star \Phi$, we consider the products of superfields to be always fully symmetrized [15]

$$\Phi^1 \star \cdots \star \Phi^n \star \bar{\Phi}^1 \star \cdots \star \bar{\Phi}^m \big|_s = \frac{1}{n!m!}\big(\Phi^1 \star \cdots \star \Phi^n \star \bar{\Phi}^1 \star \cdots \star \bar{\Phi}^m + \text{Perm.}\big). \tag{5}$$

Then, using the cyclic property $\int \Phi_1 \star \Phi_2 \star \cdots \star \Phi_n = \int \Phi_n \star \Phi_1 \star \cdots \star \Phi_{n-1}$ and rules

$$\frac{\delta \Phi(z)}{\delta \Phi(z')} = -\frac{1}{4}\bar{D}^2 \delta^8(z-z'), \qquad \frac{\delta \bar{\Phi}(z)}{\delta \bar{\Phi}(z')} = -\frac{1}{4}D^2 \delta^8(z-z'),$$

where $D^2 = D^\alpha D_\alpha$, $\bar{D}^2 = \bar{D}^{\dot\alpha}\bar{D}_{\dot\alpha}$, one obtains the equations of motion for the model under consideration

$$-\frac{1}{4}\bar{D}^2 K_1 + W_1 = 0, \qquad -\frac{1}{4}D^2 K_{\bar{1}} + \bar{W}_{\bar{1}} = 0, \tag{6}$$

where $K_1 = \frac{\partial K_\star(\Phi,\bar{\Phi})}{\partial \Phi}$, etc.

According to (3) we have to calculate a second functional derivatives of the classical action (1). Part of the action dependent on the Kähler potential leads to three types of the derivatives: mixed derivative $\delta^2 S/\delta\Phi\delta\bar{\Phi}$ and two nonmixed derivatives $\delta^2 S/\delta\Phi^2$, $\delta^2 S/\delta\bar{\Phi}^2$. These derivatives contain both right and left star multiplications. It is easy to check that nonmixed second functional derivatives are equal to zero. Really, the power expansion of the Kähler potential is defined by (2) where all powers are given according to the rule (5). Then second nonmixed derivative leads to the expression

$$\frac{\delta^2}{\delta\Phi(z')\delta\Phi(z)} \int d^8 z\, K_\star = \sum_{n\bar{n}} K_{n\bar{n}} \frac{1}{(n-1)!\bar{n}!} \underbrace{\left(-\frac{1}{4}\bar{D}^2\right)\Phi \star \cdots \star \left(-\frac{1}{4}\bar{D}^2\right)\delta^8(z-z') \star \cdots \star \Phi}_{n-2} \star \bar{\Phi} \star \cdots \star \bar{\Phi}\bigg|_s = 0, \tag{7}$$

where we also suppose a sum over all possible permutations of Φ and $(-\frac{1}{4}\bar{D}^2)\delta^8(z-z')$ according to the definitions (2), (5). The same property is true for another nonmixed variation $\frac{\delta^2}{\delta\bar{\Phi}(z)\delta\bar{\Phi}(z')}\int d^8 z\, K_\star = 0$. The mixed derivative has a quite different structure

$$\begin{aligned}\frac{\delta^2}{\delta\bar{\Phi}(z')\delta\Phi(z)} \int d^8 z\, K_\star &= \sum_{n\bar{n}} K_{n\bar{n}} \frac{1}{(n-1)!\bar{n}!} \left(-\frac{1}{4}\bar{D}^2\right) \underbrace{\Phi \star \cdots \star \Phi}_{n-1} \star \underbrace{\bar{\Phi} \star \cdots \star \left(-\frac{1}{4}D^2\right)\delta^8(z-z') \star \cdots \star \bar{\Phi}}_{\bar{n}-1}\bigg|_s \\ &= K_{1\bar{1}} \star \frac{1}{16}\bar{D}^2 D^2 \delta^8(z-z').\end{aligned} \tag{8}$$

This is a consequence of the following property for the star-product [14]

$$\begin{aligned}&f_1(\theta) \star \cdots \star f_n(\theta) \star \delta(\theta-\theta') \star g_1(\theta) \star \cdots \star g_m(\theta) \\ &\quad = \int d^2\pi\, f_1(\theta + \mathcal{C}\pi) \star \cdots \star f_n(\theta+\mathcal{C}\pi) \star g_1(\theta - \mathcal{C}\pi) \star \cdots \star g_m(\theta - \mathcal{C}\pi) \mathrm{e}^{(\theta-\theta')\pi}.\end{aligned} \tag{9}$$

Therefore using the approximation (4) ($f(\theta+\mathcal{C}\pi) = f(\theta) + \mathcal{C}\pi Df(\theta) + \cdots = f(\theta)$) we obtain

$$f_1(\theta) \star \cdots \star f_n(\theta) \star \delta(\theta-\theta') \star g_1(\theta) \star \cdots \star g_m(\theta) = f_1(\theta) \star \cdots \star f_n(\theta) \star g_1(\theta) \star \cdots \star g_m(\theta) \star \delta(\theta-\theta'). \tag{10}$$

The results (7), (8) for the second functional derivatives and the totally symmetrical form of the expansion (2) lead to simplification of the calculation procedure.

For the parts of the action dependent on the chiral superpotential we have

$$\frac{\delta^2}{\delta\Phi(z')\delta\Phi(z)} \int d^6 z\, W_\star = \sum_n W_n \frac{1}{(n-1)!} \underbrace{\Phi \star \cdots \star \left(-\frac{1}{4}\bar{D}^2\right)\delta^8(z-z') \star \cdots \star \Phi}_{n-2}\bigg|_s = W_2 \star \left(-\frac{1}{4}\bar{D}^2\right)\delta^8(z-z'), \tag{11}$$

and analogously for antichiral superpotential.

The one-loop correction to the effective potential is written as follows

$$\Gamma^{(1)} = \mathrm{Tr}\, \ln_\star \hat{\mathcal{H}}_\star = \int d^8 z\, \ln_\star \hat{\mathcal{H}}_\star \star \delta^8(z-z')\bigg|_{z=z'}, \tag{12}$$

where the operator of second functional derivatives has, according to (7), (8) and (11), the form

$$\hat{\mathcal{H}}_\star = \begin{pmatrix} K_{1\bar{1}}\frac{1}{16}D^2\bar{D}^2 & \bar{W}_{\bar{2}}(-\frac{1}{4}D^2) \\ W_2(-\frac{1}{4}\bar{D}^2) & K_{\bar{1}1}\frac{1}{16}\bar{D}^2 D^2 \end{pmatrix}. \tag{13}$$

It should be especially noted that because we deal with nonanticommutative superspace, all functions are understood as power expansions containing the star-products of superfields. In principle, one can extract for nonanticommutative theories both star-local and star nonlocal contributions but we focus only on the star-local approximation.

Using matrix operator (13) we calculate the leading (Kähler potential) and next-to-leading (chiral potential) contributions. It means that

$$\Gamma^{(1)} = \Gamma^{(1)}_K + \Gamma^{(1)}_W. \tag{14}$$

To find $\Gamma^{(1)}_K$ it is sufficient to consider in the operator (13) the constant background superfields $\bar{W}_{\bar{2}}$ and W_2. For getting the $\Gamma^{(1)}_W$ we have to treat these superfields as slowly varying and take into account the leading terms in their spinor derivatives. It is obviously that the contribution to the antichiral potential $\Gamma^{(1)}_{\bar{W}}$ will be equal to zero (see, e.g., consideration in [4]).

Let us begin with $\Gamma^{(1)}_K$. In this case one can note that the diagonal and off-diagonal blocks of the matrix operator $\hat{\mathcal{H}}_\star$ are commute between each other and, therefore, the logarithm of the matrix can be splitted off into two parts. This fact allows us to rewrite such a contribution in the effective action as

$$\Gamma^{(1)}_K = \mathrm{Tr}\ \ln_\star \begin{pmatrix} K_{1\bar{1}} \frac{1}{16} D^2 \bar{D}^2 & 0 \\ 0 & K_{\bar{1}1} \frac{1}{16} \bar{D}^2 D^2 \end{pmatrix} + \mathrm{Tr}\ \ln_\star \left(1 + \begin{pmatrix} 0 & \frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} (-\frac{D^2}{4\Box}) \\ \frac{1}{K_{\bar{1}1}} \star W_2 (-\frac{\bar{D}^2}{4\Box}) & 0 \end{pmatrix}\right). \tag{15}$$

After calculations the matrix trace and using the projector property we obtain

$$\Gamma^{(1)}_K = \mathrm{Tr}\ \ln_\star (K_{1\bar{1}}) \frac{1}{16} \frac{D^2 \bar{D}^2}{\Box} + \frac{1}{2} \mathrm{Tr}\ \ln_\star \left(1 - \frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{\bar{1}1}} \star W_2 \frac{1}{\Box}\right) \frac{1}{16} \frac{D^2 \bar{D}^2}{\Box} + \mathrm{c.c.} \tag{16}$$

Next point is to analyze the structure of divergence and renormalization properties. The first term in the dimensional regularization scheme is equal to zero. The second term (along with the complex conjugated) gives us

$$\begin{aligned}\Gamma^{(1)}_K &= \mu^{4-d} \int \frac{d^d p}{(2\pi)^d p^2} \ln_\star \left(1 + \frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{\bar{1}1}} \star W_2 \frac{1}{p^2}\right) \\ &= \frac{1}{(4\pi)^2} \left(4\pi\mu^2\right)^{2-d/2} \frac{d/2}{\Gamma(d/2+1)} \int dp^2 \left(p^2\right)^{d/2-2} \ln_\star \left(1 + \frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{\bar{1}1}} \star W_2 \frac{1}{p^2}\right),\end{aligned} \tag{17}$$

where μ is a regularization parameter and d is the space–time dimension. Putting $d = 4 - \epsilon$ we have

$$\Gamma^{(1)}_K = \frac{1}{2(4\pi)^2} \int d^8 z \frac{1}{K_{1\bar{1}}} \star \bar{W}_2 \star \frac{1}{K_{\bar{1}1}} \star W_2 \star \left(\frac{1}{4\pi\mu^2} \frac{1}{K_{\bar{1}1}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{1\bar{1}}} \star W_2\right)^{-\epsilon/2} \Gamma\left(\frac{\epsilon}{2}\right). \tag{18}$$

Using the scheme of minimal subtractions one gets the divergent

$$\Gamma^{(1)}_{K\,\mathrm{div}} = \frac{1}{16\pi^2 \epsilon} \int d^8 z \frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{\bar{1}1}} \star W_2, \tag{19}$$

and finite part

$$\Gamma^{(1)}_{K\,\mathrm{fin}} = -\frac{1}{32\pi^2} \int d^8 z \frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{\bar{1}1}} \star W_2 \star \left(\ln_\star \left(\frac{1}{K_{1\bar{1}}} \star \bar{W}_{\bar{2}} \star \frac{1}{K_{1\bar{1}}} \star W_2 \frac{1}{\mu^2}\right) + \gamma\right), \tag{20}$$

of $\Gamma^{(1)}_K$. Here γ is the Euler constant. When the deformation parameter $\mathcal{C}_{mn} = 0$, the obtained results coincides with ones for undeformed theory [10,11,13]. We point out that in the Kählerian effective potential sector the whole dependence on nonanticommutativity is stipulated only by star-product.

Let us analyze now a structure of the next-to-leading contribution to the effective action $\Gamma^{(1)}_W$. For this purpose we take up a simple form for $K_{1\bar{1}} = 1 + \mathcal{O}(\bar{\Phi})$ and $\bar{W}_{\bar{2}} = \bar{m} + \mathcal{O}(\bar{\Phi})$. The arguments for such a choice of approximation are quite natural [9,19]: contribution to the effective superpotential cannot dependent on the coefficients of the antichiral superpotential. Indeed we can promote each of these coefficients to an antichiral superfield field, whose vev then gives the coupling constants. Holomorphy tells us that these fields cannot appear in an integral over chiral superspace. Since we are interested in computing the effective chiral superpotential, we can consider the antichiral superpotential to be simplest what leads to $\bar{W}_{\bar{2}} = \bar{m}$.

According to the procedure described in [4], the quantity $\Gamma^{(1)}_W$ can be found from (13) in the following form

$$\Gamma^{(1)}_W = \frac{1}{2} \mathrm{Tr}\ \ln_\star \left(1 + \begin{pmatrix} -\bar{m} \frac{D^2 \bar{D}^2}{16\Box^2} W_2 & 0 \\ -\frac{\bar{D}^2}{4\Box} W_2 & 0 \end{pmatrix}\right) = \frac{1}{2} \mathrm{Tr}\ \frac{D^2 \bar{D}^2}{16\Box} \ln_\star \left(1 - \frac{\bar{m}}{\Box} W_2\right). \tag{21}$$

Further consideration is done analogously to one presented in [4]. After writing the integral over whole superspace as $\int d^8z = \int d^6z\,\bar{D}^2$ we obtain the expression for the one-loop correction to the chiral superpotential in the form

$$\Gamma^{(1)}_W = \frac{1}{2}\int d^6z\,\ln_\star\left(1 - \frac{\bar{m}}{\Box}W_2\right) \star \left(-\frac{1}{4}\bar{D}^2\right)\delta^8(z-z')\bigg|_{z=z'}. \tag{22}$$

Analogously to [16,19] such a form can be also obtained after integrating out the antichiral field from the action

$$S(\Phi,\bar{\Phi}) = \int d^8z\,\bar{\Phi}\Phi + \int d^6z\,W_\star(\Phi) + \int d^6\bar{z}\,\frac{\bar{m}}{2}\bar{\Phi}^2. \tag{23}$$

For this purpose ones rewrite the linear and quadratic over $\bar{\Phi}$ part of the above action as follows

$$\int d^8z\left(\frac{\bar{m}}{2}\left(\bar{\Phi} + \frac{1}{\bar{m}}D^2\Phi\right)\frac{1}{\Box}\bar{D}^2\left(\bar{\Phi} + \frac{1}{\bar{m}}D^2\Phi\right) - \frac{1}{2\bar{m}}D^2\Phi\frac{1}{\Box}\bar{D}^2D^2\Phi\right). \tag{24}$$

Now the antichiral superfield can be integrated out in the functional integral. Replacing in the last term $\int d^2\bar{\theta}$ with $\bar{D}^2$ we obtain the action

$$S(\Phi) = \int d^6z\left(-\frac{1}{2\bar{m}}\Phi\Box\Phi + W_\star(\Phi)\right). \tag{25}$$

This action leads to the one-loop effective action in the form (22).

Note that in the undeformed model the expression (22) is equal to zero as it should be in accordance with [9]. In the case under consideration the nonzero result is stipulated by nonanticommutativity. After enough simple calculations within dimensional regularization one gets

$$\Gamma^{(1)}_W = \frac{\bar{m}^2}{64\pi^2}\int d^6z\,W_2 \star W_2 \star \left(\frac{\bar{m}W_2}{4\pi\mu^2}\right)^{-\epsilon/2}\Gamma\left(\frac{\epsilon}{2}\right) \star \delta^2(\theta-\theta')\bigg|_{\theta=\theta'}. \tag{26}$$

To clarify a structure of this result we transform the expression (26) in the form without $\star$-product and keep only the leading term in deformation parameter. One can obtain

$$\Gamma^{(1)}_W = -\frac{\bar{m}^2}{64\pi^2}\int d^6z\,\frac{1}{2}C^2 W_2 Q^2 W_2\left(\frac{\bar{m}W_2}{4\pi\mu^2}\right)^{-\epsilon/2}\Gamma\left(\frac{\epsilon}{2}\right). \tag{27}$$

This expression has the divergent part in the form

$$\Gamma^{(1)}_{W\,\mathrm{div}} = -\frac{\bar{m}^2}{64\pi^2\epsilon}C^2\int d^6z\,W_2 Q^2 W_2, \tag{28}$$

and a finite part. We point out that the divergent part (28) explicitly contains the deformation parameter which cannot be absorbed into $\star$-product.

In the finite part we will try to restore the $\star$-product under the integral over chiral subspace. It leads to

$$\Gamma^{(1)}_{W\,\mathrm{fin}} = \frac{\bar{m}^2}{128\pi^2}C^2\int d^6z\,W_2 Q^2 W_2 \star \ln_\star\left(\frac{\bar{m}W_2}{\mu^2}\right), \tag{29}$$

where $Q = i\partial/\partial\theta$. It is clear that in the expression (29) the deformation parameter cannot be absorbed into initial $\star$-product. We already pointed out the analogous situation in the expression (28).

The relations (28) and (29) define the one-loop chiral effective potential in the model under consideration. In the case of Wess–Zumino nonanticommutative model, the divergent term $\Gamma^{(1)}_{W\,\mathrm{div}}$ takes the form $\Phi Q^2\Phi$ which was recently found (see, e.g., [3,4]).

To conclude, we have presented calculations of the one-loop effective potential for the nonanticommutative generic chiral superfield model. We used the approximation of slowly variating superfields and developed the method which allows to carry out the calculations procedure without explicit rewriting the $\star$-product in terms of ordinary products. The divergence structure of the model is analyzed, we show that besides the divergences analogous to ones for undeformed model, there is a new divergent structure containing the nonanticommutativity parameter and destructing the star-product structure of the model on a quantum level. As a result, the divergent and finite one-loop Kählerian and chiral effective potentials are found in the explicit forms.

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