Characteristics of structural loess strength and preliminary framework for joint strength formula

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Abstract: The strength of structural loess consists of the shear strength and tensile strength. In this study, the stress path, the failure envelope of principal stress (f_K line), and the strength failure envelope of structurally intact loess and remolded loess were analyzed through three kinds of tests: the tensile strength test, the uniaxial compressive strength test, and the conventional triaxial shear strength test. Then, in order to describe the tensile strength and shear strength of structural loess comprehensively and reasonably, a joint strength formula for structural loess was established. This formula comprehensively considers tensile and shear properties. Studies have shown that the tensile strength exhibits a decreasing trend with increasing water content. When the water content is constant, the tensile strength of the structurally intact soil is greater than that of remolded soil. In the studies, no loss of the originally cured cohesion in the structurally intact soil samples was observed, given that the soil samples did not experience loading disturbance during the uniaxial compressive strength test, meaning there is a high initial structural strength. The results of the conventional triaxial shear strength test show that the water content is correlated with the strength of the structural loess. When the water content is low, the structural properties are strong, and when the water content is high, the structural properties are weak, which means that the water content and the ambient pressure have significant effects on the stress-strain relationship of structural loess. The established joint strength formula of structural loess effectively avoids overestimating the role of soil tensile strength in the traditional theory of Mohr-Coulomb strength.

Key words: structurally intact loess; remolded loess; tensile strength; shear strength; stress path; failure envelope of principal stress (f_K line); strength failure envelope; joint strength formula

1 Introduction

The strength characteristics of structural loess, a type of soil material, are presented in terms of shear properties and tensile properties. In loess plateau regions, numerous towering steep slopes of structural loess can be found, with some slopes reaching as high as 10 m or more. The tensile strength of structural loess is an important factor, and a high tensile strength can keep structural loess slopes upright and maintain long-term stability. Therefore, the
mechanical properties of unsaturated structural loess should be studied in order to analyze the relationship between its structural properties and strength properties (i.e., shear properties and tensile properties). Moreover, establishing a reasonable joint strength formula for structural loess is equally important.

In studying soil tensile properties, direct and indirect tensile test methods are mainly used to assess soil tensile strength. Lü et al. (2013) developed a set of tensile strength apparatuses to test the tensile strength of swell-shrink soil. Zhang et al. (2010) developed a new type of horizontal tension apparatus for soil tests. Lu et al. (2007) employed a simple direct tensile strength testing apparatus to analyze the tensile strength for unsaturated sand. Ibarra et al. (2005) presented a method of measuring the tensile strength for unsaturated sandy loam soil. Zhou et al. (2012) carried out an axial tensile test of chlorine saline soil specimens to understand the tensile strength behavior of chlorine saline soil. Consoli et al. (2010) carried out tensile strength tests and uniaxial compressive strength tests to analyze the tensile strength and compressive strength of artificially cemented sand. Wang et al. (2007) investigated the tensile strength of clay through the uniaxial tension test on cylindrical compacted specimens.

A direct tensile test is advantageous because of its simple operation. Moreover, it provides clear stress states of soil deformation and failure processes. However, the equipment required for direct tensile tests exerts a number of adverse impacts on test operations, thus leading to frequent errors. In addition, determining the tensile strength of soil is difficult when direct tensile tests are employed in the case that the central axis between the soil sample and loading system is shifted during the tensile test operation. An indirect tensile test also shows the stress of a soil sample, albeit the tensile stress state is not as clear as the result of a direct tensile test because the tensile strength in indirect tensile test is obtained indirectly by the bending moment. Its advantages include a simple test process, easy-to-control operation, and, often, a more reliable tensile strength result.

As for research on the shear strength of structural loess, Shao et al. (2008) studied the relationship between the soil shear strength and the soil structural property, and proposed that the cohesion ($c$) presents hyperbolic changes along with structural parameters, while the friction angle ($\phi$) basically remains unchanged. Hu et al. (2013) conducted analyses on the relationship between the initial compaction degree, the initial water content, and the strength of the compacted Malan loess in the Lüliang region. Ma et al. (2013) analyzed the strength characteristics of unsaturated structural loess. The abovementioned studies positively promoted theoretical research on the strength and structural parameters of structural soil. However, they did not consider the comprehensive tensile and shear properties and their results were not assessed by the tensile test and corresponding theoretical strength formula.

In general, for unsaturated structural loess, the cohesion measured in tests typically reaches tens or even hundreds of kPa, and, thus, the role of the tensile strength of structural loess cannot be ignored. Therefore, in order to consider the need of practical geotechnical
engineering or the requirement to improve loess mechanical systems, researchers should pay more attention to the tensile strength of structural soil. Moreover, the tensile strength and shear strength of structural loess are equally important. Thus, research on the reasonable evaluation of tensile properties and shear properties of structural loess is urgently required.

In this study, the stress path, the strength failure envelope, and the failure envelope of principal stress (\( K_i \) line) of the structurally intact loess and remolded loess were analyzed through three kinds of tests: the tensile strength test, the uniaxial compressive strength test, and the conventional triaxial shear strength test. To achieve a more reasonable description of the tensile property and the shear property of structural loess, a concept of structural loess joint strength that comprehensively considers tensile properties and shear properties is proposed in this paper.

2 Strength tests of structural loess

The experimental soil samples in this study were collected from the Bailuyuan region, in Xi’an City, Shaanxi Province, at a soil sampling depth of approximately 3.5 to 4.5 m. The soil samples obviously contained strong structural loess. Data determined through a conventional indoor test indicated that the natural water content of the samples was 15.23\%, the natural density was 1.78 g/cm\(^3\), and the dry density was 1.54 g/cm\(^3\).

To establish the formula for the joint strength of structural loess with the tensile strength, the uniaxial compressive strength, and the triaxial shear strength, corresponding tests were performed on soil samples.

The uniaxial compressive strength test and the conventional triaxial shear strength test are currently well developed, and they offer stable test results. At present, existing devices for soil tensile strength tests cannot determine the tensile strength of a structural soil body under conditions of high water content.

2.1 Tensile strength test and analyses

To study the tensile strength property of unsaturated structural soil, we developed a tensile strength testing device based on the concepts of devices for soil beam bending testing. The device can be applied to structurally intact soil and remolded soil under various humidity conditions. It features electric control and automatic data collection. Our device received a patent grant (Li et al. 2012). It mainly includes an operating system of the vacuum pressure chamber, a loading system, and a measurement system. The specific structure of the testing device is shown in Fig. 1.

The bending test of the soil beam involves applying a concentrated force to the supporting beam until cracking damage occurs in the middle of soil beam. During the test, the tensile strength (\( \sigma_t \)) for the destruction of the beam is calculated with the following formula:

\[
\sigma_t = \frac{3PL}{2bh^2}
\]
where $P$ refers to the applied maximal external force, $L$ is the effective length of the soil beam, $b$ the width of the cross-section of the soil beam, and $h$ the height of the cross-section of the soil beam.

The tensile strengths of the structurally intact soil and remolded soil beams with different water contents measured via tensile strength tests are shown in Fig. 2.

Fig. 1 Developed device for soil tensile strength test

Fig. 2 Relation between water content $w$ and tensile strength $\sigma_t$

Fig. 2 shows that the tensile strength decreases as the water content increases for both structurally intact and remolded soils. With a constant water content, the tensile strength of the structurally intact soil is greater than that of the remolded soil. When the water content increases, the difference between the tensile strength of the structurally intact soil and the remolded soil decreases gradually. By analyzing this phenomenon and the resulting differences, their intrinsic mechanism can be explained from two aspects. First, much more soluble salts in the soil skeleton particles dissolve in water with the increase in water content, thus gradually weakening the cohesion among particles, and reducing the structural strength of structural loess. Second, the originally cured cohesion, i.e., the original structural property, of structural soil is almost completely destroyed in the remolded soil, thus causing the tensile strength to decrease to a degree that is significantly less than the tensile strength of the structurally intact loess.

2.2 Uniaxial compressive strength test and analyses

From axial stress-strain curves (Fig. 3) established through the uniaxial compressive strength test of structurally intact loess and remolded loess with water contents of 5%, 10%, and 15%, we can see that both the structurally intact and remolded soils have obvious peak values in curves of the relation between axial stress ($\sigma$) and axial strain ($\varepsilon_t$). After the stress peak appears, the cured cohesion among certain particles is damaged and microcracks occur. As axial deformation increases, the microcracks in the soil skeleton continue enlarging. Although the shape of the stress-strain curve of the structurally intact soil sample is similar to that of the remolded soil sample during the uniaxial compressive strength test, two differences
are still observed. First, the peak strength of the structurally intact soil is significantly greater than that of the remolded soil. Second, taking the axial strain as the reference, the peak strength of the remolded soil appears earlier than that of the structurally intact soil, indicating that the cohesive strength of the remolded soil is weaker than that of the structurally intact soil. The phenomenon indicates that no loss of the originally cured cohesion in the structurally intact soil samples is observed, given that the soil samples did not experience loading disturbance during the uniaxial compressive strength test, meaning there is a high initial structural strength.

![Fig. 3 Uniaxial compressive stress-strain curves for structurally intact soil and remolded soil with different water contents](image)

### 2.3 Conventional triaxial shear strength test and analyses

Fig. 4 and Fig. 5 show the variation of the generalized shear stress ($\sigma_1 - \sigma_3$) against the axial strain ($\varepsilon_1$) developed through the conventional triaxial shear strength testing of the structurally intact loess and the remolded loess with constant water contents of 5%, 10%, and 15%, in which $\sigma_1$ is major principal stress and $\sigma_3$ is minor principal stress. When the water content is low, the stress-strain curves for the structurally intact soil and the remolded soil exhibit a softening characteristic, with a peak that often occurs in the middle of stress-strain curves. When the water content is high, damage to the microstructure of the structurally intact soil and the remolded soil occurs, as a result of the ambient pressure during consolidation. Consequently, the generalized shear stress increases against the axial strain, and there is no peak occurring in the middle of stress-strain curves. Under the same amount of ambient pressure, the peak value of the stress-strain curve with lower water content is greater than the maximum of the curve with higher water content. These relationships show that the water content is correlated with the strength of structural loess. When the water content is low, the structural property is strong. When the water content is high, the structural property is weak. With the same water content, the peak value of the stress-strain curve under a lower amount of ambient pressure is smaller than that of the curve under a higher amount of ambient pressure. In sum, the water content and the ambient pressure have significant effects on the stress-strain relationship of structural loess.
3 Formula for determining joint strength of structural loess

3.1 Joint strength of structural soil

In the following section, the viewpoint and the approach to establish the joint strength are discussed according to data from three kinds of strength tests.

The shear strength of structural soil is generally believed to conform to the Mohr-Coulomb strength theory, as shown in Fig. 6, where $\sigma$ is normal stress, and $\tau$ is shear stress. However when the tensile strength of structural soil is larger, the failure envelope will no longer be a straight line. On the $\sigma$-$\tau$ plane, with $\sigma_3 = -\sigma_1$, stress circle $\circled{1}$ demonstrates the stress state of tensile failure, stress circle $\circled{2}$ demonstrates that neither tensile failure nor Mohr-Coulomb shear failure occurs, and stress circle $\circled{3}$ demonstrates the stress state of the shear failure according to the Mohr-Coulomb strength theory.

For soil samples with the same dry density and water content, tensile strength, compressive strength, and shear strength can be determined through the tensile strength test, the uniaxial compressive strength test, and the conventional triaxial shear strength test, respectively. Then, the failure envelope of principal stress ($K_i$ line) can be defined by connecting failure points on the stress path of the three tests. The stress path in the $\sigma$-$\tau$ coordinates and the $K_i$ line obtained from the tests are shown in Fig. 7, wherein $\tau = (\sigma_1 - \sigma_3)/2$ and $\sigma = (\sigma_1 + \sigma_3)/2$. 

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**Fig. 4** Triaxial shear stress-strain curves of structurally intact soil with different water contents

**Fig. 5** Triaxial shear stress-strain curves of remolded soil with different water contents
The segment $OA$ indicates the stress path in the uniaxial compressive strength test, on which point $A$ represents the uniaxial compressive strength failure state. Segments $C_1D_1$, $C_2D_2$, and $C_3D_3$ respectively represent the stress paths in the conventional triaxial shear strength test under the ambient pressures of $C_1$, $C_2$, and $C_3$, on which points $D_1$, $D_2$, and $D_3$ refer to the failure state under different ambient pressures. The oblique straight line $ED_3$ is the $K$ line for the Mohr-Coulomb strength, while the curve $TSD_3$ is the $K$ line for the joint strength. Segment $OS$ represents the stress path in the tensile strength test, on which point $S$ represents the strength failure state under uniaxial tension. Segment $OT$ represents the three-directional tensile stress path, on which point $T$ represents the strength failure state under three-directional tension. Given that the strength test under three-directional tension is currently difficult to conduct in experimental studies, point $T$ is difficult to determine. However, considering that the three-directional tensile strength must be less than the uniaxial tensile strength, the upper limit of the three-directional tensile strength variation is the test value of the uniaxial tensile strength. Furthermore, considering that the strength failure envelope is convex, the lower limit for the three-directional tensile strength variation is $22\sqrt{2}/2$ of the test value of the uniaxial tensile strength, as inferred from the geometric relations in Fig. 7. Based on these findings, a mathematical formula can be obtained through fitting of the experimental data. Thus, a strength formula that can reflect both tensile and shear properties of structural soil is established.

### 3.2 Stress path and $K_f$ line for joint strength

In this section, based on data from three kinds of strength tests with different water contents (Section 2), test results of the structurally intact soil and the remolded soil with a water content of 15% are taken as an example (Fig. 8 to Fig. 9) to establish the joint strength formula for structural loess.

The failure stress circles of the remolded soil under the three experimental conditions are shown in Fig. 8(a). The values of the cohesion ($c$) and the friction angle ($\varphi$) are 100 kPa and 27.2°, respectively. The stress circles of different line types in Fig. 8(a) demonstrate the failure states in the tensile strength test, the uniaxial compressive strength test and the conventional triaxial shear strength test, respectively. The joint strength envelope is formed by the smooth
connection of failure points under three experimental conditions. The stress path corresponding to each stress circle and the $K_f$ lines for the remolded soil are presented in Fig. 8(b).

![Fig. 8 Stress paths and strength characteristic lines for remolded soil with water content of 15%](image)

Based on the analysis, an intuitive conclusion is drawn as follows: In Fig. 8 (b), if the extended segment ($\sigma < 0$) of the $K_f$ line for the Mohr-Coulomb strength is adopted to describe the tensile properties of structural soil, then the tensile strength will be significantly exaggerated; if the tensile strength is completely neglected, the treatment of tensile strength will be too conservative.

The failure stress circles of the structurally intact soil under three experimental conditions are shown in Fig. 9(a), and the stress paths and the $K_f$ lines of the structurally intact soil are shown in Fig. 9(b), where the values of the cohesion ($c$) and the friction angle ($\phi$) are 168 kPa and 25.39°, respectively.

![Fig. 9 Stress paths and strength characteristic lines for structurally intact soil with water content of 15%](image)

By comparing the strength envelopes of the structurally intact soil with those of the remolded soil, the uniaxial compressive strength failure point of the remolded soil is found to fall on the Mohr-Coulomb strength envelope, while the uniaxial compressive strength failure point of the structurally intact soil is beyond the Mohr-Coulomb strength envelope. Analysis provides the reasons for these findings. The straight Mohr-Coulomb strength envelope is
typically the failure envelope determined by the conventional triaxial shear strength test, without reflecting initial structural properties of structural soil because damages to the microstructure of structural loess result from shear action after loading consolidation. The remolded soil has a smaller initial structural strength mainly because of secondary structures in which soil particles’ reconstruction resulted from the compaction during the remolding process. Thus, the uniaxial compressive strength failure point of the remolded soil falls on the Mohr-Coulomb strength envelope. While the structurally intact soil samples are free from the disturbance of consolidation loading during the uniaxial compressive strength test, in this case, the initial structural property becomes more evident, and the uniaxial compressive strength failure point of the structurally intact soil is beyond the Mohr-Coulomb strength envelope. Based on the preceding discussion, the initial structural properties of structural soil are worthy of further research.

3.3 Deduction of formula joint strength

To establish a joint strength formula not only judging tensile failure, but also determining shear failure, we changed the straight portion of the $K_t$ line for the Mohr-Coulomb strength with $\bar{\sigma} < 0$ into a smooth curve by fitting the experimental data.

To fit a hyperbolic curve of the joint strength envelope on the $\sigma$-$\tau$ plane, we assume that the horizontal axis intercept of the curve represents the tensile strength $\sigma_t$, and that the Mohr-Coulomb strength envelope is regarded as the asymptote. Then, the following formula is derived using coordinate translation and arithmetic derivatives:

$$\tau^2 = (c + \sigma \tan \varphi)^2 - (c - \sigma_t \tan \varphi)^2 \quad (2)$$

When we transform Eq. (2) into $\sigma$-$\tau$ space through coordinate translation, Eq. (2) can be rewritten as:

$$\tau^2 = \sin^2 \varphi \left[ (\sigma + c \cot \varphi)^2 - (c \cot \varphi - 0.9\sigma_t)^2 \right] \quad (3)$$

The horizontal axis intercept of the $K_t$ line for the joint strength was considered to be 0.9 $\sigma_t$ because the intercept represents the tensile strength under three-directional tension in $\sigma$-$\tau$ space. Although this three-directional tension stress state is difficult to realize in actual operations, the value must be less than the uniaxial tensile strength. Therefore, the three-directional tensile strength could be obtained by reducing the value of the uniaxial tensile strength. The reduction coefficient was set at 0.9 based on Yao et al. (2004). With three known soil strength parameters, i.e., $c$, $\varphi$, and $\sigma_t$, the $K_t$ line, considering both tensile failure and shear failure, can be determined by Eq. (3). The $K_t$ lines for the joint strength of the remolded soil and the structurally intact soil with a water content of 15% are shown in Fig. 10. The horizontal axis intercept of the $K_t$ line for the joint strength of the structurally intact soil is greater than that of the remolded soil.
If the initial structural property of the structurally intact soil needs to be considered, the tensile strength and initial structural strength should be considered in the joint strength.

According to the experimental analysis in this study, the tensile strength failure point and the uniaxial compressive strength failure point are connected with a smooth curve. Then, the curve is gradually extended upward, parallel to the $K_t$ line for the Mohr-Coulomb strength. The $K_t$ line for the joint strength that is achieved in this manner can reasonably reflect the effect of the initial structural strength of the structurally intact soil.

To fit a hyperbolic curve of the joint strength envelope considering the initial structural strength on the $\sigma-\tau$ plane, we assume that the horizontal axis intercept of the curve represents the uniaxial tensile strength $\sigma_t$, and that the fitted curve runs through the uniaxial compressive strength failure point and is parallel to the Mohr-Coulomb strength envelope. Eq. (4) is thus derived as follows:

$$\tau^2 = \left[ \frac{q_u}{2} \left( \frac{1 - \sin \varphi}{\cos \varphi} \right) + \sigma \tan \varphi \right]^2 - \left[ \frac{q_u}{2} \left( \frac{1 - \sin \varphi}{\cos \varphi} \right) - \sigma_t \tan \varphi \right]^2$$

where $q_u$ is the uniaxial compressive strength.

When we transform Eq. (4) into $\bar{\sigma}-\bar{\tau}$ space through coordinate translation and process it with the three-directional tensile strength as stated before, Eq. (4) can be rewritten as:

$$\bar{\tau}^2 = \sin^2 \varphi \left[ \bar{\sigma} + \frac{q_u}{2} \left( \frac{1 - \sin \varphi}{\cos \varphi} \right) \cot \varphi \right]^2 - \left[ \frac{q_u}{2} \left( \frac{1 - \sin \varphi}{\cos \varphi} \right) \cot \varphi - 0.9\sigma_t \right]^2$$

The $K_t$ line for the joint strength considering the initial structural property of the structurally intact soil with a water content of 15% is shown in Fig. 11.

According to Eqs. (4) and (5), the difference in joint strength between the remolded soil and the structurally intact soil exhibits a decreasing trend as the structural loess property decreases, because the tensile strength of structural soil decreases with the increasing water content.

Therefore, in order to consider the initial structural property of the structurally intact soil, the upper limit of the $K_t$ line for the joint strength shall be determined by Eq. (5). The results are of significance to future study on the initial structural property of structural loess.
4 Conclusions

Shear properties and tensile properties are two aspects of strength characteristics of structural loess. In this study, in order to achieve a more reasonable description of tensile properties and shear properties, a preliminary joint strength formula for structural loess has been established, and the following conclusions are drawn:

(1) A device for tensile strength tests of unsaturated and saturated structural soils has been developed. The tensile strength exhibits a decreasing trend with the increasing water content, for both structurally intact soil and remolded soil, and the tensile strength of the structurally intact soil is greater than that of the remolded soil.

(2) During the uniaxial compressive strength test, no loss of the originally cured cohesion in the structurally intact soil samples can be observed, given that the soil samples do not experience loading disturbance, meaning that there is a high initial structural strength.

(3) The results of the conventional triaxial shear strength test show that the water content is correlated with the strength of the structural loess. When the water content is low, the structural properties are strong, and when the water content is high, the structural properties are weak, which means that the water content and the ambient pressure have significant effects on the stress-strain relationship of structural loess.

(4) By comparing uniaxial compressive strength properties of the structurally intact soil with those of the remolded soil, the uniaxial compressive strength failure point of the remolded soil is determined to fall on the Mohr-Coulomb strength envelope, whereas the uniaxial compressive strength failure point of the structurally intact soil falls beyond the Mohr-Coulomb strength envelope. This phenomenon is interesting because it reflects the initial structural strength of the structurally intact soil before loading disturbance.

(5) A joint strength formula further reflecting the initial structural strength of structural soil was developed. The difference in joint strength between the remolded soil and the structurally intact soil exhibits a decreasing trend with increasing water content and weaker structural loess properties.

The joint strength formula effectively avoids overestimating the tensile strength of structural soil in the Mohr-Coulomb strength theory, thus providing a more realistic strength theory to reasonably evaluate the tensile property and the initial structural property of structural soil.
References


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