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Disformal inflation

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Abstract

We show how short inflation naturally arises in a non-minimal gravity theory with a scalar field without any potential terms. This field drives inflation solely by its derivatives, which couple to the matter only through the combination $\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi$. The theory is free of instabilities around the usual Minkowski vacuum. Inflation lasts as long as $\dot{\phi}^2 > m^4$, and terminates gracefully once the scalar field kinetic energy drops below m^4. The total number of e-folds is given by the initial inflaton energy $\dot{\phi}_0^2$ as $\mathcal{N} \simeq \frac{1}{3}\ln(\frac{\dot{\phi}_0}{m^2})$. The field ϕ can neither efficiently reheat the universe nor produce the primordial density fluctuations. However this could be remedied by invoking the curvaton mechanism. If inflation starts when $\dot{\phi}_0^2 \sim M_P^4$, and $m \sim m_{\rm EW} \sim$ TeV, the number of e-folds is $\mathcal{N} \sim 25$. Because the scale of inflation is low, this is sufficient to solve the horizon problem if the reheating temperature is $T_{\rm RH} \gtrsim$ MeV. In this instance, the leading order coupling of ϕ to matter via a dimension-8 operator $\frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$ would lead to fermion–antifermion annihilation channels $f\bar{f} \to \phi\phi$ accessible to the LHC, while yielding very weak corrections to the Newtonian potential and to supernova cooling rates, that are completely within experimental limits.

The recurring challenge to our attempts to understand Nature is the origin of hierarchies between the scales we observe. Familiar examples are the hierarchy between the Planck scale $M_P \sim 10^{19}$ GeV and the electroweak scale $m_{\rm EW} \sim$ TeV, $M_P/m_{\rm EW} \sim 10^{16}$, and the hierarchy between the Planck scale and the present horizon scale, $H_0 \sim 10^{-33}$ eV, $M_P/H_0 \sim 10^{61}$. These problems are usually dealt with separately. In the former case, models of particle dynamics such as strong gauge field dynamics [1,2], supersymmetry [3] or large extra dimensions [4,5] are invoked to explain the dichotomy between the Planck and electroweak scales. In the latter case, the leading contender to explain the horizon scale is inflation [6], which posits that the universe has been blown up really large by a period of exponential expansion in the past, and then subsequent expansion generates the rest of the hierarchy between M_P and H_0. If inflation starts near the Planck scale, it should blow up the universe by at least $\mathcal{N}_* \sim 65$ e-folds, or by a factor of at least $e^{\mathcal{N}_*} \sim 10^{28}$. The approximate relation $e^{\mathcal{N}_*} \sim (M_P/m_{\rm EW})^2$ is typically viewed as an accident. In fact, the usual models of inflation predict that the universe has expanded by much more than the current necessary minimum to explain the present horizon scale [7]. This would indicate that there is nothing special about the present horizon scale. We just happen to make our observations now,

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but some other being could have seen a completely different horizon scale at some other time, as the cosmic evolution marches on.

Yet there are indications that we might live at a special moment in the history of the universe. Indeed observations have uncovered the cosmic coincidences: the current cosmological densities of various forms of matter inhabiting our universe, such as dark energy, dark matter, baryons, photons and neutrini are within a few orders of magnitude of each other [8]. Some of the coincidences are presently very mysterious, such as explaining the scale of dark energy from first principles. Other coincidences, such as the near equality of the energy densities of dark matter, baryons and photons, may be understood in particle physics models which contain weakly-interacting particles with masses and couplings set by the electroweak scale $m_{\rm EW}$. Any definitive clue in favor of spatial curvature within a few orders of magnitude of the critical density of the universe would further underscore that we live in a special epoch, requiring that inflation were short. It should have ceased after the necessary minimum of e-folds was achieved, in order to avoid completely flattening the spatial slices. Other clues of short inflation might emerge from observing non-trivial topology of the universe [9], low power in low ℓ CMB multipoles [10], substructure in the CMB [11] or holographic considerations [12]. A natural explanation for such coincidences would be to relate the dynamics which control their evolution, including cosmology, with a particular hierarchy of scales governing microphysics, such as $M_P/m_{\rm EW}$.

Building models of inflation capable of stopping after few tens of e-folds has been especially hard (for some models, see [13–16]). In this Letter, we consider a mechanism where inflation can be very short. The inflaton is a massless singlet pseudoscalar, whose dynamics respects the shift symmetry $\phi \to \phi + \mathcal{C}$ and reflection, $\phi \leftrightarrow -\phi$. Its couplings to the matter sector are introduced via a modification of the gravitational coupling to matter, of the form

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi. \tag{1}$$

Here the metric $g_{\mu\nu}$ is the canonically normalized metric with the kinetic term given by the usual Einstein–Hilbert action, and ϕ is normalized as usual such that it has dimension of mass. The mass scale m is the coupling parameter of the inflaton sector to matter, which couples covariantly to the combination $\bar{g}_{\mu\nu}$. We will discuss the acceptable range of values for it below. Theories with scalars coupled to matter in ways including (1) have been considered by Bekenstein in 1992 [17], who looked for generalizations of Riemannian geometry that do not violate the weak equivalence principle and causality. He found that the extensions of the standard general relativity based on coupling the matter to the combinations of the form he referred to as *disformal* transformation

$$\bar{g}_{\mu\nu} = \mathcal{A}\big(\phi, (\partial\phi)^2\big)g_{\mu\nu} - \frac{\mathcal{B}(\phi, (\partial\phi)^2)}{m^4}\partial_\mu\phi\partial_\nu\phi \tag{2}$$

preserve causality and the weak equivalence principle. In contrast to conformal transformation, the disformal transformation (2) does not preserve the angles between the geodesics of $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. We confine our attention to a specialized form (1), taking $\mathcal{A} = \mathcal{B} = 1$ (constants other than unity can be absorbed away by rescaling M_P and m), in order to enforce the symmetries $\phi \to \phi + \mathcal{C}$, $\phi \leftrightarrow -\phi$ which protect the inflaton from the matter loop corrections. This implies the stability of the slow roll regime under the Standard Model radiative corrections.

With the choice of the mass scale $m \sim m_{\rm EW} \sim$ TeV, the resulting dynamics is equivalent to low scale inflation with $V^{1/4} \sim$ TeV, lasting about 25 e-folds [18]. This is just enough to solve the horizon problem if the reheating is $T_{\rm RH} \gtrsim$ MeV [19]. The reheating and the generation of density perturbations are however involved. The field ϕ which drives inflation cannot efficiently reheat the universe, nor produce the scale-invariant spectrum of perturbations to match the COBE amplitude, because it is too weakly coupled to the Standard Model, and the scale of inflation is so low. The model also does not solve the curvature problem, because it requires the initial curvature of the universe to be small in order not to prevent the onset of the low scale inflation. However these problems are common in low scale inflation. The reheating and the generation of density perturbations may be solved by invoking a curvaton field [20]. We will outline a scenario that could accomplish this. Solving the curvature problem requires additional dynamics, such as a stage of very early inflation [21] or holographic considerations [22].

We define the theory by the action principle $\delta S = 0$, where the action is

$$S = \int d^4x \left\{ \sqrt{g}\left[\frac{M_P^2}{2} R - \frac{1}{2}(\partial\phi)^2 \right] - \sqrt{\bar{g}}\mathcal{L}_M\big(\psi, \partial\psi, \bar{g}^{\mu\nu}\big) \right\}, \tag{3}$$

where $g = \det(-g_{\mu\nu})$, etc. Because of the shift symmetry of ϕ, general covariance and reflection $\phi \leftrightarrow -\phi$ we can treat the matter Lagrangian $\mathcal{L}_M$ as fully quantum, including all the Standard Model loop corrections. The shift symmetry operates as in the case of pseudo-Nambu–Goldstone inflatons [23,24], excluding corrections which are polynomial in ϕ. The reflection $\phi \leftrightarrow -\phi$ precludes the operators of the form $\partial_\mu \phi j^\mu$ where the scalar couples derivatively to some conserved current. Finally, general covariance of the matter sector protects the universality of matter couplings to only $\bar{g}^{\mu\nu}$, which can be seen by rewriting the action (3) in terms of only barred variables and recalling that by matter loops we mean these loop diagrams which involve only matter internal lines. Specifically, the Standard Model corrections do not change the coupling constant $1/m^4$. Varying (3) yields the field equations, which using the shorthand $U_\mu = \frac{1}{m^2}\partial_\mu\phi$ are

$$M_P^2 G^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}g^{\mu\nu}(\partial\phi)^2 + \sqrt{1-U^2}\,\bar{T}^{\mu\nu}, \tag{4}$$

$$\bar{\nabla}_\mu \bar{T}^{\mu\nu} = 0, \tag{5}$$

$$\nabla^2\phi + \frac{1}{m^4}\sqrt{1-U^2}\,\bar{T}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu\phi = 0. \tag{6}$$

Eq. (4) is the modified Einstein's equation, (5) stands for the matter field equations, designating that the matter fields couple to $\bar{g}_{\mu\nu}$, and (6) the inflaton field equation, which includes the matter-inflaton derivative couplings. Raising and lowering of the indices of unbarred tensors is to be done with $(g^{\mu\nu}, g_{\mu\nu})$, and of barred tensors with $(\bar{g}^{\mu\nu}, \bar{g}_{\mu\nu})$. It is straightforward to derive several useful relations between key barred and unbarred quantities; using (1), one finds $\bar{g} = (1 - U^2)g$, $\bar{g}^{\mu\nu} = g^{\mu\nu} + \frac{1}{1-U^2}U^\mu U^\nu$, $\bar{U}^2 = \frac{U^2}{1-U^2}$, $\bar{U}^\mu = \frac{1}{1-U^2}U^\mu$.

Before proceeding we ought to mention that the theories of this form have been considered in the context of the so-called variable speed of light cosmologies [25]. The motivation was to argue that if $\partial\phi \neq 0$, the lightcones of the metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ are different, suggesting that the electromagnetic waves propagate faster than gravity waves, with a speed which varies in space and time. This "superluminal" propagation of light is then supposed to solve the horizon problem without inflation, since it would seem to allow for communication at superhorizon scales. We strongly caution against considering the theory (1), (3) in this way. Namely, because the ϕ-field equation (6) is homogeneous in $\partial\phi$, it admits solutions $\phi = \text{const}$, which are identical to $\phi = 0$ by the shift symmetry. This is the vacuum of the theory. In this vacuum there is no difference in the propagation speed of any excitations in the theory, matter or gravitational. Thus the presence of two different lightcone structures, one for the graviton and another for the matter fields, is an environmental effect, which emerges because the initial state of the universe began with $\partial\phi \neq 0$. This is analogous to the propagation of light in a dielectric, or to the propagation of massless charged particles in an external electric field. An observer who sees that the trajectories of these probes deviate from the null geodesics in the vacuum does not invoke a changing speed of light at a fundamental level to explain this. Instead she notes that the probes interact with the environment, which breaks Poincaré symmetry because $\partial\phi \neq 0$. The breaking is soft, in the sense that as $\partial\phi$ diminishes in the course of the evolution of the universe, the symmetries are restored. This is reminiscent to a spontaneously broken gauge symmetry, where because of the breaking the gauge fields become massive, and their quanta propagate along timelike instead of null geodesics. Because in this case the scalar field gradients $\partial\phi \neq 0$ break Poincaré symmetry instead of the electromagnetic gauge symmetry, the photons remain massless and move along null geodesics, while the gravitons move along timelike geodesics. One ought to interpret the double lightcone structure induced by $\partial\phi \neq 0$ as a signature of the slow-down of gravitons due to their strong interactions with $\partial\phi$, which makes the early universe opaque to them. This helps with the horizon problem not because it allows for superhorizon correlations, but because it arrests the gravitational instability, preventing the growth of inhomogeneities. However, in the frame where the matter fields are canonically normalized this looks precisely like inflation. Hence in what follows we adopt this view and focus on the effective field theory description of inflation.

Let us now establish when the model based on (3)–(6) is meaningful. Consider first the low energy limit. As indicated above, we define the vacuum by setting $\phi = 0$. For simplicity we further assume that in the vacuum $\bar{T}_{\mu\nu} = 0$ and so $g_{\mu\nu} = \eta_{\mu\nu}$, i.e., that the vacuum is the usual Minkowski space. In order to ensure its perturbative stability we must show that it is a minimum energy state, without negative energy excitations and/or runaway modes (i.e., ghosts and tachyons). Constructing the matter sector in the usual way ensures that there are no such degrees of freedom in $\mathcal{L}_M$. The form of the gravitational action in (3) further guarantees that the metric degrees of freedom are safe too. What remains to check is that the scalar ϕ does not produce instabilities. Now, if we consider small perturbations of (6) around the vacuum $\phi = \bar{T}_{\mu\nu} = 0$, $g_{\mu\nu} = \eta_{\mu\nu}$, we see that the scalar ϕ is just a massless canonically normalized scalar field too, without any pathologies. Thus the vacuum is stable.

However there still might be runaway scalar modes around some fixed classical background with $\bar{T}_{\mu\nu} \neq 0$. Even if the vacuum were exactly stable, it would be disastrous if infinitesimally small distributions of matter are not. To check this does not occur we consider the spectral decomposition of ϕ in the presence of a point mass. This will be sufficient since any other distribution of energy–momentum can be obtained by superposition and boosting of such sources. Before looking at the details, however, we note that the dimensionless coefficient controlling the correction is given by $\sim \rho/m^4$, where ρ is the energy density of the distribution. If we smooth the distribution over a whole Hubble volume, this reaches its upper value if the total mass M is of the order of the mass in the observable universe: $M \sim \rho_0/H_0^3 \sim M_P^2/H_0$, which is at most $\sim \frac{M_P^2 H_0^2}{m^4}$. This is smaller than unity as long as $m > 10^{-3}$ eV. In fact, we will see below that m is at least m_{EW}, and so the perturbation is really tiny, with ξ at most 10^{-60}. The parameter ξ approaches unity only in the limit $\rho \to m^4$. From this we expect that the ϕ excitations will not destabilize the background as long as the densities are below m^4. Similar conclusions remain true for localized sources too. To see this explicitly, we expand (6) around a point mass. Picking $\phi = 0$ and $g_{\mu\nu} = \eta_{\mu\nu}$ for the background outside of the mass source, we find the equation for the excitations of ϕ,

$$\partial^2\phi + \frac{1}{m^4} M \delta^{(3)}(\vec{x})\ddot{\phi} = 0, \tag{7}$$

where M is the mass of the source at $\vec{x} = 0$ and $\delta^{(3)}(\vec{x})$ is the Dirac δ-function. Using (7), after simple algebra we can write the matrix propagator equation for $\Delta(\omega, \vec{k}) = i\langle\phi(\omega, \vec{k})\phi(0)\rangle$ in momentum space,

$$\big(\vec{k}^2 - \omega^2(1-\xi)\big)\Delta(\omega, \vec{k}) + \xi\omega^2 \sum_{\vec{q}\neq\vec{k}} \Delta(\omega, \vec{q}) = -i, \tag{8}$$

where $\xi = \frac{M H_0^3}{m^4} = \frac{M H_0}{M_P^2}\frac{M_P^2 H_0^2}{m^4} \ll 1$, and we imagine that the universe is a lattice of size $1/H_0$ with a lattice spacing $1/\Lambda$. We can solve Eq. (8) perturbatively using ξ as the expansion parameter, to find

$$\Delta(\omega, \vec{k}) = \frac{i}{\omega^2(1-\xi) - \vec{k}^2 + i\epsilon} \times \left(1 + \xi \sum_{\vec{q}\neq\vec{k}} \frac{\omega^2}{\omega^2(1-\xi) - \vec{q}^2 + i\epsilon} + \cdots\right). \tag{9}$$

This shows that the full propagator in the presence of a mass source M contains admixtures of all plane wave modes with very slightly shifted frequencies $\omega^2 \to \omega^2(1-\xi)$. However when $\xi \ll 1$ all the poles occur only when $\omega^2 > 0$, and thus there are no runaway, exponentially growing modes. Moreover, the momenta on the lattice are $\vec{p} = H_0\vec{n}$, where $\vec{n} \in \mathsf{Z}^3$, and therefore at the poles $\omega^2(1-\xi) - \vec{q}^2 = \vec{p}^2 - \vec{q}^2 = H_0^2(\vec{n}_p^2 - \vec{n}_q^2)$. Hence $\xi\frac{\omega^2}{\omega^2(1-\xi)-\vec{q}^2+i\epsilon}$ is maximized when $\vec{p}^2 - \vec{q}^2 \simeq |\vec{n}|H_0^2 \simeq H_0\omega$, reaching $\xi\omega/H_0 \leqslant \xi\Lambda/H_0$. Hence as long as the theory is cutoff at a scale $\Lambda \leqslant H_0/\xi \lesssim M_P$ the residues are positive, and so there are no negative energy excitations either. Thus the Minkowski vacuum of the theory (3)–(6) is perturbatively stable.

These conclusions are valid as long as the energy density of the Standard Model matter in $\bar{T}_{\mu\nu}$ does not exceed $\mathcal{O}(m^4)$. As it increases towards m^4, the expansion parameter ξ approaches unity and the perturbative analysis yielding (9) breaks down. This is not necessarily detrimental: it means that the theory based on (3) must be given a proper UV completion.

Thus to ensure the validity of the effective field theory description of ϕ as defined by (3) we should cutoff the matter sector at $\lambda_{\rm SM} \sim m$. Once this is done, the $g_{\mu\nu}, \phi$ sector may remain well-defined all the way up to some high energy scale $\Lambda \sim M_P$ which regulates the gravity-ϕ sector. Such frameworks were discussed in, for example, [26], who suggested that the Standard Model is completed by a TeV-scale little string theory, which couples to gravity that remains weak up to the usual Planck scale. Note that although we imagine that $\partial\phi$ can reach energy scales as high as M_P^2, this does not destabilize the Standard Model sector because it couples to ϕ only through $\bar g^{\mu\nu} = g^{\mu\nu} + \frac{1}{1-(\partial\phi)^2/m^4}\partial^\mu\phi\partial^\nu\phi/m^4$. Therefore, the dependence on the cutoff Λ cancels to the leading order, entering only through terms $\sim m^4/\Lambda^4$, leaving m in full control of the Standard Model as long as we ignore gravity and ϕ loops.

The inclusion of the Standard Model corrections to the $g_{\mu\nu}, \phi$ sector does not destabilize the leading order terms in $g_{\mu\nu}, \phi$ in (3). Because the Standard Model is cutoff at m, and because it only couples to $\bar g_{\mu\nu}$, general covariance implies that the Standard Model corrections are organized as an expansion in the higher-derivative invariants of $\bar g_{\mu\nu}$. The only dimensional scale weighing them is m:

$$\mathcal{L}_{\rm corrections} = \sqrt{\bar g}\left(a_0 m^4 + a_1 m^2 \bar R + a_2 \bar R^2 + a_3 \bar\nabla^2 \bar R + \frac{a_4}{m^2}\bar R^3 + \cdots\right), \tag{10}$$

where the coefficients $a_0, a_1, a_2, \ldots$ are all numbers of order unity. We can now add the leading order terms for $g_{\mu\nu}$ and ϕ from (3), $\sqrt{g}[\frac{M_P^2}{2}R - \frac{1}{2}(\partial\phi)^2]$. The full effective Lagrangian rewritten in terms of the variables $\bar g_{\mu\nu}$ and ϕ becomes symbolically

$$\mathcal{L}_{\rm eff} = \sqrt{\bar g}\left(\frac{1}{\sqrt{1-(\bar\partial\phi)^2/m^4}} \times \left[\frac{M_P^2}{2}\bar R - \frac{M_P^2(\bar\partial\phi)^2}{m^4}\bar R - \frac{1}{2}(\bar\partial\phi)^2\right] + a_0 m^4 + a_1 m^2 \bar R + a_2 \bar R^2 + a_3 \bar\nabla^2 \bar R + \frac{a_4}{m^2}\bar R^3 + \cdots\right), \tag{11}$$

where we have ignored the tensor structure in the terms like $\frac{M_P^2}{m^4}\partial_\mu\phi\partial\phi_\nu \bar R^{\mu\nu}$, choosing to write them instead as $\frac{M_P^2}{m^4}(\bar\partial\phi)^2\bar R$, which is sufficient to analyze their scaling, and relative importance in the effective action with the Standard Model corrections included. When $\partial\phi < m^2$, the corrections are obviously small. In the regime $\partial\phi \sim \Lambda^2$, in the background (27) each derivative contributes a power of $\bar H \simeq m^2/M_P$, and so the expansion becomes a series of the form

$$\mathcal{L}_{\rm eff} = \sqrt{\bar g}\left(m^2\Lambda^2 + m^4 + a_0 m^4 + a_1\frac{m^6}{M_P^2} + a_2\frac{m^8}{M_P^4} + a_3\frac{m^8}{M_P^4} + a_4\frac{m^{10}}{M_P^6} + \cdots\right), \tag{12}$$

where the leading order terms $\sim m^2\Lambda^2$ and $\sim m^4$ come entirely from the classical background, and the corrections affect the background only slightly through the cosmological term $\sim a_0 m^4$, while all other effects from terms $\propto a_k$ remain completely negligible. We stress however that in general the corrections from the $g_{\mu\nu}, \phi$ loops are not under control, and to understand what happens with them one must seek an embedding of the theory (3) into some more fundamental theory with a UV completion which is under control. That task is beyond the scope of the present work. We do see however, that like in natural inflation scenarios [23], that the conditions for slow roll regime are protected from the matter radiative corrections.

The presence of a new degree of freedom ϕ leads to many new processes, some of which could affect the low energy experiments. This yields important observational bounds on m. The strongest arises from collider data. The operator (15) opens up the channel for annihilation of any two standard model fermions into two ϕ's, $f\bar f \to \phi\phi$. The cross-section for this process goes as

$$\sigma_{f\bar f\to\phi\phi} \sim \frac{s^3}{m^8}, \tag{13}$$

where $\sqrt{s}$ is the center-of-mass energy. Taking $\sqrt{s} \sim 100$ GeV and requiring that $\sigma \leqslant 1/m_{\rm EW}^2$ in order for this channel not to be ruled out by present data, we find a bound

$$m \gtrsim m_{\rm EW}. \tag{14}$$

If this bound is saturated, the detection of ϕ's may be within reach of the future colliders such as the LHC.

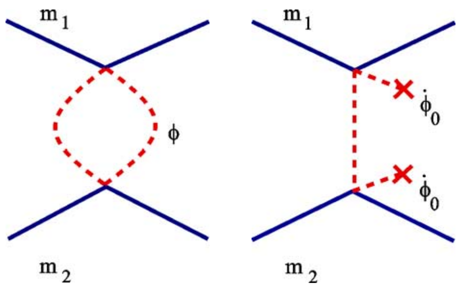

Fig. 1. Feynman diagrams for the force mediated by ϕ.

The massless scalar ϕ mediates a new force, that could be long-range, modifying the Newton's law. Even though (1) preserves weak equivalence principle, the force generated by ϕ should be constrained by solar system tests of gravity just like in the usual Brans–Dicke theory. However in this case the corrections to the Newton's law are very small, and the Solar system tests are easy to pass. This can be seen as follows. We can compute the potential from ϕ exchange using Feynman diagrams. Expanding (3) around the vacuum, we find that the ϕ-matter interaction vertex is given by the dimension-8 operator

$$\mathcal{L}_I = \frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}, \tag{15}$$

where we can drop the bar from $T^{\mu\nu}$ whenever we expand around the vacuum. The leading-order diagrams correcting the Newtonian potential are given in Fig. 1.

The diagram in Fig. 1(a) involves a double ϕ exchange. The loop integral is divergent and so we need to cut it off at some scale Λ. The result is the expansion

$$\Lambda^4 + \Lambda^2\vec{k}^2 + \vec{k}^4\log\vec{k}^2 + \cdots. \tag{16}$$

Both of the cutoff-dependent terms are contact interactions, corresponding to shrinking both, or one of the propagators in the loop to a point, and they should be subtracted away, leaving the $\propto \vec{k}^4\log\vec{k}^2$ term as the physical loop contribution. This yields

$$V_1 \sim \frac{1}{m^8}\frac{m_1 m_2}{r^7} = \frac{1}{M_P^2}\frac{m_1 m_2}{r}\frac{M_P^2}{m^8 r^6}, \tag{17}$$

where the latter parameterization makes the comparison with the experimental data more transparent. Because of the rapid drop of this potential with distance, the strongest bounds will come from the shortest scales that have been probed so far, i.e., from table-top experiments [27]. Thus taking $r \sim 0.1$ mm, we must choose m such that $\frac{M_P^2}{m^8(0.1\ \mathrm{mm})^6} < 1/100$. We find

$$m^8 \gtrsim 10^8\frac{M_P^2}{\mathrm{mm}^6}, \tag{18}$$

or numerically $m > \mathrm{MeV}$. Hence as long as $m > \mathrm{MeV}$, the force which ϕ mediates is very weak, and short-ranged. In fact, if we take $m \sim m_{\mathrm{EW}}$, which as we will see below is the strongest bound on m, the force becomes strong only at distances $r \leqslant 40$ fermi, where the effect would, remarkably, appear as a sudden opening of six new dimensions. This is very similar to the theories with large extra dimensions [4] or CFT effects [28] in cutoff AdS braneworlds [29].

The diagram in Fig. 1(b) involves a single ϕ exchange between two masses m_1, m_2, with two lines ending on the cosmological background $\dot\phi_0$. The potential arising from this diagram is, after cancelling the contact terms, the velocity-dependent contribution to the potential, which arises because the coupling $\frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$ vanishes in the static limit when the mass sources are at rest:

$$V_2 \sim \frac{\dot\phi_0^2}{m^8}\frac{m_1 m_2}{r^3}\vec{v}_1\cdot\vec{v}_2. \tag{19}$$

Because today $\dot\phi_0$ is at most of the order of $\sqrt{\rho_0} \sim M_P H_0$, $\frac{\dot\phi_0^2}{m^8} < \frac{M_P^2 H_0^2}{m^8} \ll \frac{1}{m^2 M_P^2}$. The bound (14) renders the effects of this term ignorable tiny at distances $r > m^{-1}$.

The bounds which one obtains from astrophysics considerations are also consistent with (14). They arise because the coupling of ϕ to the matter degrees of freedom via the dimension-8 operator (15) leads to the ϕ production which could enhance the cooling rates of astrophysical objects. The analysis is similar to the one performed in theories with large extra dimensions [4]. The leading order process that governs the ϕ production is $\mathbf{p} \to \mathbf{p} + \phi + \phi$, given by the diagram in Fig. 2. Here $\mathbf{p}$ is a typical particle in the supernova which is dressed by thermal effects, as denoted by the double-line in Fig. 2. Since it has thermal width it can shake off two ϕ's, decreasing its thermal energy. The "decay rate" governing this process is easy to estimate from (15) simply by dimensional analysis. Since the typical kinetic energy

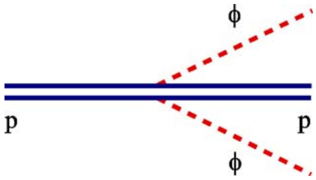

Fig. 2. The process for the $\mathbf{p} \to \mathbf{p} + \phi + \phi$ "decay".

of a particle in a star is $E \sim T$, where T is the temperature, and since the decay rate is proportional to the square of the transition amplitude, and thus to $1/m^8$, we find

$$\Gamma \sim \frac{T^9}{m^8}. \tag{20}$$

In a typical process each ϕ carries off energy $\sim T$, and thus the total energy loss per unit time of a star due to the ϕ emission is $\dot{E}_T \sim -N\frac{T^{10}}{m^8} \sim -\frac{M_S}{m_p}\frac{T^{10}}{m^8}$, where $N \sim M_S/m_p$ is the number of particles in a star of mass M_S. Because stars are typically predominantly made up of hydrogen, m_p is the proton mass. Rather than analyzing all the sources of data, we merely quote the strongest bound which comes from the supernova SN1987a. In order to agree with the observations, the total output of ϕ's cannot exceed the luminosity of about 10^{53} erg/s $\sim 10^{32}$ GeV^2. Since $M_S \sim M_\odot \sim 1.6 \times 10^{57}$ GeV and $T \sim 30$ MeV, requiring $\dot{E}_T \leqslant 10^{32}$ GeV^2 we find

$$m \geqslant 30 \text{ GeV}, \tag{21}$$

which is weaker than (14). Hence because of the bound (14) the supernova cooling is not significantly affected by ϕ emission. We note that similar bounds were also obtained from considering Goldstone boson interactions in braneworlds [30]. Although these theories are different, the bounds are similar because of the Goldstone boson equivalence theorem.

We now turn to the cosmology of the model. Let us restrict to the spatially flat FRW backgrounds for now. Starting with the usual metrics for $g_{\mu\nu}$, the line element defining the graviton–inflaton geometry is

$$ds^2 = -dt^2 + a^2\, d\vec{x}^2. \tag{22}$$

The translational symmetries require $\partial_k \phi = 0$, and hence using (3) we find that the metric in which the Standard Model fields dwell is

$$\begin{aligned} d\bar{s}^2 &= -\left(1 + \frac{\dot{\phi}^2}{m^4}\right) dt^2 + a^2\, d\vec{x}^2 \\ &= -d\bar{t}^2 + a^2(\bar{t})\, d\vec{x}^2, \end{aligned} \tag{23}$$

where $d\bar{t} = dt\sqrt{1 + \dot{\phi}^2/m^4}$. In this case the field equations (4)–(6) reduce to

$$\begin{aligned} &3H^2 = \frac{1}{M_P^2}\left(\rho_\phi + \frac{1}{\sqrt{1-U^2}}\bar{\rho}_{\rm SM}\right), \\ &\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}\left(\rho_\phi + 3p_\phi + \frac{\bar{\rho}_{\rm SM}}{\sqrt{1-U^2}} + 3\sqrt{1-U^2}\,\bar{p}_{\rm SM}\right), \\ &\frac{d\bar{\rho}}{d\bar{t}} + 3\bar{H}(\bar{\rho} + \bar{p}) = 0, \qquad \bar{p}_{\rm SM} = \bar{w}\bar{\rho}_{\rm SM}, \\ &\ddot{\phi} + 3H\dot{\phi} - \frac{\bar{\rho}_{\rm SM}}{m^4(1-U^2)^{3/2}}\left(\ddot{\phi} - 3H\bar{w}\left(1-U^2\right)\dot{\phi}\right) \\ &\quad = 0, \end{aligned} \tag{24}$$

where we are still employing the obvious notations without and with bars to distinguish the quantities built from metrics (22) and (23), and bearing in mind that $U^2 = -\dot{\phi}^2/m^4$ and $\rho_\phi = p_\phi = \dot{\phi}^2$. Here we are approximating the Standard Model influences with a perfect fluid, obeying the equation of state $\bar{p}_{\rm SM} = \bar{w}\bar{\rho}_{\rm SM}$ for some $\bar{w}$. Note that the two next-to-last equations can be immediately integrated to yield $\bar{\rho}_{\rm SM} = \bar{\rho}^0_{\rm SM}(a_0/a)^{3(1+\bar{w})}$, where $\bar{\rho}^0_{\rm SM}$ is the initial value of the Standard Model energy density when the description based on (24) became valid.

Although Eqs. (24) look quite formidable, it is very simple to deduce their qualitative properties. In the regime $m^4 \lesssim \dot{\phi}^2 \lesssim \Lambda^4 \sim M_P^4$, one finds the following inequalities:

$$\begin{aligned} &\frac{\bar{\rho}_{\rm SM}}{\sqrt{1-U^2}} \simeq \frac{m^2}{\dot{\phi}}\bar{\rho}_{\rm SM} \ll m^4 < \rho_\phi, \\ &\sqrt{1-U^2}\,\bar{p}_{\rm SM} \simeq \frac{\dot{\phi}}{m^2}\bar{p}_{\rm SM} \leqslant m^2\dot{\phi} < p_\phi = \rho_\phi, \\ &\frac{\bar{\rho}_{\rm SM}}{m^4(\sqrt{1-U^2})^{1/2}} \simeq \frac{m^2}{\dot{\phi}}\frac{\bar{\rho}_{\rm SM}}{m^4} \ll 1, \\ &\frac{\bar{\rho}_{\rm SM}}{m^4(\sqrt{1-U^2})^{3/2}} \simeq \frac{m^6}{\dot{\phi}^3}\frac{\bar{\rho}_{\rm SM}}{m^4} \ll 1. \end{aligned} \tag{25}$$

Because of these inequalities, all of the Standard Model contributions in (24) are completely subleading to the $\dot\phi$ sources in the regime $m^4 \lesssim \dot\phi^2 \lesssim \Lambda^4 \sim M_P^4$. This in fact is exactly a tell-tale sign of inflation: the matter contributions become irrelevant as the inflationary dynamics sets in. Substituting these inequalities in (24) we find the simple equations

$$3H^2 = \frac{1}{M_P^2}\frac{\dot\phi^2}{2},$$
$$\ddot\phi + 3H\dot\phi = 0, \tag{26}$$

i.e., precisely the equations of a cosmology dominated by a stiff fluid $p_\phi = \rho_\phi = \dot\phi^2/2$. Assuming that initially the universe started with a Planckian curvatures, as in chaotic inflation [7], the solution is

$$a = a_0\left(\frac{t}{t_P}\right)^{1/3},$$
$$\phi = \phi_0 + \sqrt{\frac{2}{3}}M_P \ln\left(\frac{t}{t_P}\right). \tag{27}$$

Even though the geometry of (27) is the same as the holographic cosmology background of [22], the difference is that here the geometry is sourced by a simple scalar field ϕ whereas in the context of holographic cosmology it emerges in response to a black hole gas. Thus the fluctuations around the background would be very different. Now, because $\dot\phi = \sqrt{\frac{2}{3}}M_P/t$, the matter frame metric is, using (23),

$$d\bar s^2 = -\left(1 + \frac{2}{3}\frac{M_P^2}{m^4t^2}\right)dt^2 + a_0^2\left(\frac{t}{t_P}\right)^{2/3} d\vec x^2, \tag{28}$$

or therefore, using $d\bar t = dt\sqrt{1 + \frac{2}{3}\frac{M_P^2}{m^4t^2}} \simeq \sqrt{\frac{2}{3}}\frac{M_P}{m^2}\frac{dt}{t}$,

$$d\bar s^2 = -d\bar t^2 + a_0^2 e^{\frac{2}{3}\frac{m^2}{M_P}\bar t}\, d\vec x^2, \tag{29}$$

i.e., precisely the slow-roll inflation, with an almost constant Hubble parameter,

$$\bar H \simeq \frac{m^2}{\sqrt{6}M_P}. \tag{30}$$

Thus the effective field theory description of ϕ-dominated cosmology is a low scale inflation. An indication of the presence of an inflationary attractor in theories which in the matter frame metric contain similar operators to those present here was noted in [31]. When $m \sim m_{\rm EW} \sim$ TeV, the Hubble scale during inflation is $\bar H \sim \text{mm}^{-1}$, i.e., $V_{\rm eff} \sim \text{TeV}^4$, corresponding to TeV scale inflation as in the examples of [18]. We stress that this mechanism of inflation is different than the so-called k-inflation [32]. This can be readily seen by rewriting the theory (3) completely in terms of the metric $\bar g_{\mu\nu}$ to which the matter couples, and noting that it contains operators $\propto \frac{M_P^2}{m^4}\partial_\mu\phi\partial_\nu\phi\bar R^{\mu\nu}$, which play a key role here and are absent in k-inflation.

The inflationary stage terminates gracefully because as the time goes on, $\dot\phi \sim M_P/t$ decreases. When it reaches m^2, inflation ceases. Indeed, in the regime $\dot\phi^2 < m^4$, because ϕ is completely without a potential, its energy density scales as $1/a^6$, and so the scale factor rapidly changes behavior, scaling as some low power of t after inflation, while $\dot\phi$ is diluted very fast. It is straightforward to determine the duration of the inflationary phase. Using the form of the solution (27) during the inflationary regime, we can rewrite the scale factor as a function of $\dot\phi$: $a = a_0(\dot\phi_0/\dot\phi)^{1/3}$, where $\dot\phi_0 \sim M_P^2$ is the initial value of the inflaton gradient. Thus, the total amount of inflation is given by the number of e-folds

$$\mathcal{N} = \ln\frac{a_{\rm exit}}{a_0} \simeq \frac{1}{3}\ln\frac{\dot\phi_0}{m^2}. \tag{31}$$

Taking the initial condition for the inflaton to correspond to the Planckian energy density, $\rho_0 \simeq \dot\phi_0^2 \sim M_P^4$ [7], and choosing $m \gtrsim m_{\rm EW} \sim$ TeV to saturate (14) we get

$$\mathcal{N} \simeq \frac{2}{3}\ln\frac{M_P}{m} \lesssim 25. \tag{32}$$

This suffices to solve the horizon problem if the reheating temperature after inflation is $\sim$ MeV, because inflation started late, with the initial horizon size $\sim \bar H^{-1} \sim$ mm. In this case the formula linking the number of e-folds needed for the post-inflationary entropy production to the reheating temperature and the scale of inflation [19], $\mathcal{N} \simeq 67 - \ln(M_P/m) - \frac{1}{3}\ln(m/T_{\rm RH})$, gives exactly $\mathcal{N} \simeq 25$ for $m \sim m_{\rm EW} \sim$ TeV and $T_{\rm RH} \sim$ MeV, agreeing with (32). If $m > m_{\rm EW}$, inflation would be shorter, reducing its efficiency for solving the horizon problem.

The processes of reheating and generation of the primordial density fluctuations are somewhat involved. We first discuss the nature of the problems, and then turn to a specific solution based on another light field [20]. Since the inflaton energy density after inflation scales as $1/a^6$, after inflation the cosmological evolution rapidly falls under the control of the matter sector. However, inflaton reheating is very inefficient. The symmetries $\phi \to \phi + \mathcal{C}$, $\phi \leftrightarrow -\phi$ which protect the inflaton from the Standard Model corrections prevent strong inflaton-matter couplings and hamper reheating. This is similar to other non-oscillatory models of inflation, where the inflaton after inflation does not fall into a minimum of a potential [33]. In fact, this model is an extreme non-oscillatory model, because the matter couples only to the metric $\bar{g}_{\mu\nu}$. Thus the only particle production is gravitational, driven by the evolution of the vacuum of the quantum field theory of matter in the $\bar{g}_{\mu\nu}$ background. This means that the reheating temperature is given by [34]

$$T_{\rm RH} \sim \bar{H} \sim \frac{m^2}{M_P}. \tag{33}$$

Requiring $T_{\rm RH} \gtrsim$ MeV in order to have nucleosynthesis, one needs $m \gtrsim 10^5$ TeV, reducing the number of e-folds to ~ 17. This is too few e-folds to accommodate a solution of the horizon problem, but could be useful for other model building purposes.

In the usual potential-driven models of inflation, the inflaton quantum fluctuations on the potential plateau generate density perturbations, which later serve as the seeds for structure formation in the post-inflationary universe [35,36]. The key reason why this mechanism for generation of density perturbations is successful is that during inflation the inflaton fluctuations are imprinted on the background as curvature inhomogeneities which are stretched to scales greater than the apparent horizon, where they freeze out: their amplitude rapidly approaches a constant value, leading to $\delta\rho/\rho \sim H^2/\dot{\phi} \sim$ const. Thus the resulting spectrum is scale-invariant, fixed by inflationary dynamics, and protected from the details of subsequent evolution.

Our analysis shows that this does not happen with the ϕ-field fluctuations in this model, contrary to the claims of [25]. The problem is that in this case the curvature perturbations never freeze out. The reason is quite simple: the canonically normalized fluctuations couple to the background metric $g_{\mu\nu}$, and the background is given by (27), with $a \sim t^{1/3}$. This is a decelerating geometry, singular at $t = 0$ and with the apparent horizon given by $l_H = 3t$, which is spacelike. Since the wavelength of the fluctuations obeys $\lambda = \lambda_0 a/a_0 = \lambda_0 (t/t_0)^{1/3}$, it grows more slowly than the horizon l_H as time goes on. The evolution

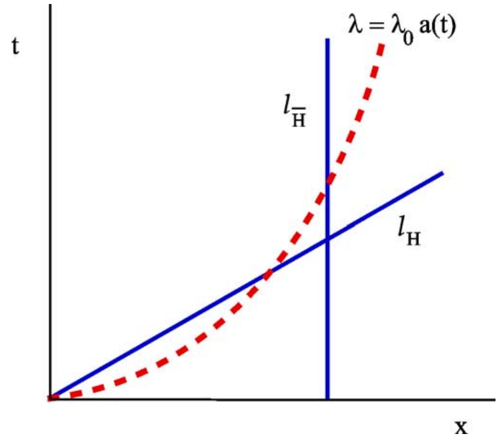

Fig. 3. Apparent horizons $l_H = H^{-1}$, $l_{\bar{H}} = \bar{H}^{-1}$ and the wavelength λ of a typical fluctuation of ϕ as functions of time t.

of the horizons and a characteristic wavelength is given in Fig. 3. So a fluctuation which originates inside the horizon l_H remains inside of it forever. The fact that its wavelength will become greater than the apparent horizon in the matter frame, $l_{\bar{H}} = \bar{H}^{-1}$, is of little dynamical consequence since in this frame the fluctuations do not have canonical kinetic terms. This only serves to set the proper normalization for the momenta, and wavelengths, of the fluctuations after inflation. The perturbations which are generated from the quantum fluctuations of ϕ will therefore continue being redshifted away, and will end up being exponentially small.

To see this we use the gauge-invariant cosmological perturbation theory [35,36]. Although the dynamics of the perturbations will be governed by a linearized theory which will differ significantly from the standard perturbation theory during inflation, because of the couplings in (1), because the theory is generally covariant we can use the formalism of [35] to identify the gauge invariant potentials. In the longitudinal gauge the background + perturbations are defined by

$$\begin{aligned} ds^2 &= a^2(\eta)\big[-\big(1+2\Psi(\eta,\vec{x})\big)\,d\eta^2 \\ &\qquad + \big(1+2\Phi(\eta,\vec{x})\big)\,d\vec{x}^2\big], \\ \phi &= \phi(\eta) + \delta\phi(\eta,\vec{x}). \end{aligned} \tag{34}$$

The conformal time η is related to the usual comoving FRW time t by $dt = a\,d\eta$, which gives $t = t_P(\frac{3\eta}{2t_P})^{3/2}$, and $a(\eta), \phi(\eta)$ are obtained from (27). The potentials Φ, Ψ and the inflaton perturbation $\delta\phi$ are related by momentum conservation as $\Psi = -\Phi$, $\phi'\delta\phi = -2m_P^2(\Phi' + \mathcal{H}\Phi)$, where $\mathcal{H} = a'/a$, and primes denote derivatives with respect to η. One then defines the curvature perturbations as the perturbations on the isodensity spatial slices. In terms of the gauge-invariant potential $\Theta = \Phi - \frac{\mathcal{H}}{\phi'}\delta\phi$ they are

$$\frac{\delta\mathcal{R}_3}{R} = \frac{1}{3a^2H^2}\vec{\nabla}^2\Theta(\eta,\vec{x}). \tag{35}$$

The canonically normalized scalar field corresponding to this perturbation is

$$\varphi = a\delta\phi - \frac{a\phi'}{\mathcal{H}}\Phi = -\mathcal{Z}\Theta. \tag{36}$$

Following a common practice we have defined $\mathcal{Z} = \frac{a\phi'}{\mathcal{H}} = \frac{a\dot{\phi}}{H}$ [36]. Expanding in Fourier modes, and using the definition of the power spectrum $\mathcal{P}(k)\delta^{(3)}(\vec{k}-\vec{q}) = \frac{k^3}{2\pi^2}\langle\Theta_{\vec{k}}(\eta)\Theta^\dagger_{\vec{q}}(\eta)\rangle$ yields

$$\mathcal{P}(k)\delta^3(\vec{k}-\vec{q}) = \frac{k^3}{2\pi^2}\left(\frac{H}{\dot{\phi}}\right)^2\left\langle\frac{\varphi_{\vec{k}}}{a}\frac{\varphi^\dagger_{\vec{q}}}{a}\right\rangle, \tag{37}$$

where $\langle\mathcal{O}\rangle$ stands for the quantum expectation value of the 2-point operator $\mathcal{O}$ in the quantum state of inflation. The curvature perturbation in the gravitational frame is $(\frac{\delta\rho}{\rho}) \sim \mathcal{P}^{1/2}(k)$.

However, we are interested in the curvature perturbation as seen in the matter frame, in terms of the variables adopted to the metric $\bar{g}_{\mu\nu}$. From the relation (1) and the background solution (27) it is straightforward albeit tedious to compute the relation of the curvature perturbations in the gravitational frame Θ and the matter frame $\bar{\Theta}$. Keeping terms up to linear order in perturbations, redefining the conformal time according to $d\bar{\eta} = \sqrt{1+\phi'^2/m^4}\,d\eta$, and then performing an infinitesimal diffeomorphism $d\bar{\eta} \to d\bar{\eta} + \frac{\phi'\delta\phi}{a^2m^4\sqrt{1+\phi'^2/m^4}}$, we find that in the limit $\dot{\phi}^2 \gg m^4$, valid during inflation, they obey the relationship

$$\bar{\Theta} = \Theta + \frac{m^4}{\dot{\phi}^2}\frac{\mathcal{H}}{\phi'}\delta\phi + \frac{1}{3am^2}\frac{d}{d\bar{\eta}}\left(\frac{m^4}{\dot{\phi}^2}\delta\phi\right) + \mathcal{O}\left(\left(\frac{m^4}{\dot{\phi}^2}\right)^2\right). \tag{38}$$

Hence to the leading order $\bar{\Theta} = \Theta$, and so we can simply compute Θ in the gravitational frame, where the scalar and graviton modes have canonical kinetic terms, and carry over the result to the matter frame. The main difference between the frames arises because in the matter frame we should compare the curvature perturbation Θ to the background curvature of the matter frame metric $\bar{g}_{\mu\nu}$. This yields

$$\frac{\delta\bar{\rho}}{\bar{\rho}} \simeq \left(\frac{R}{\bar{R}}\right)^{1/2}\frac{\delta\rho}{\rho} \simeq \left(\frac{H}{\bar{H}}\right)\frac{\delta\rho}{\rho}. \tag{39}$$

We can now estimate the perturbations in the long wavelength limit.

Since (27) implies that $\mathcal{Z}''/\mathcal{Z} = -\frac{1}{4\eta^2}$ to the leading order, one sees that the Fourier modes of φ obey the field equation

$$\varphi''_{\vec{k}} + \left(k^2 + \frac{1}{4\eta^2}\right)\varphi_{\vec{k}} = 0, \tag{40}$$

with the solutions

$$\varphi_{\vec{k}} = \sqrt{\frac{\eta}{t_P}}\big(A_{\vec{k}}J_0(k\eta) + B_{\vec{k}}Y_0(k\eta)\big), \tag{41}$$

where J_0, Y_0 are Bessel functions of index zero. Because inflation progresses as t grows, the long wavelength limit behavior of the modes is encoded in the limit $k\eta \gg 1$. Because in this limit

$$J_0 \to \sqrt{\frac{2}{\pi k\eta}}\cos\left(k\eta - \frac{\pi}{4}\right),$$
$$Y_0 \to \sqrt{\frac{2}{\pi k\eta}}\sin\left(k\eta - \frac{\pi}{4}\right), \tag{42}$$

the mode functions behave as

$$\varphi_{\vec{k}} \to \frac{1}{\sqrt{k}}\big(a_{\vec{k}}e^{-ik\eta} + a^\dagger_{\vec{k}}e^{ik\eta}\big), \tag{43}$$

where we have defined $a_{\vec{k}}, a^\dagger_{\vec{k}}$ from $A_{\vec{k}}, B_{\vec{k}}$ in an obvious way. Therefore in the limit $k\eta \gg 1$ after a simple algebra we get

$$\mathcal{P}(k) \to \frac{k^2}{M_P^2}n(k)\left(\frac{t_P}{t}\right)^{2/3}, \tag{44}$$

and therefore, using (27) and (39) and defining the physical momentum of the fluctuations $p = k/a$,

$$\frac{\delta\bar{\rho}}{\bar{\rho}} \simeq \frac{p}{\bar{H}}n^{1/2}(p)e^{-3\bar{H}\bar{t}}. \tag{45}$$

Here $n(p)$ is the occupation number of modes as a function of their momentum in the initial state of inflation $\langle\ \rangle$. From this formula we see that unless the initial state of ϕ is very precisely fine-tuned such that $n(p) = \alpha/p^2$, the spectrum of fluctuations will not be flat. This is very hard to justify because in the limit $t \to t_P$ the solution is singular, and the fluctuations of ϕ are random, and very large. More importantly, the curvature fluctuations do not freeze out as inflation proceeds. Indeed, if inflation lasts $\mathcal{N} = \bar{H}\bar{t}_{\rm exit} \sim 25$ e-folds, the amplitude of density perturbations in horizon-size modes, which were the first to leave the matter-frame apparent horizon $l_{\bar{H}}$, is diluted by the factor $(e^{25})^3 = e^{75} \sim 10^{32}$ by the time inflation terminates: $\delta\bar{\rho}/\bar{\rho} \sim 10^{-32}$. Thus these fluctuations are much too small when compared to the COBE amplitude $\delta\bar{\rho}/\bar{\rho} \sim 10^{-5}$, and cannot give rise to the observed structure in the universe. However, at least they do not destabilize inflation once it sets in.

A cure to the problems of reheating and generation of density perturbations may be the curvaton mechanism [20]. In this case the curvaton should be a very light field σ, with a mass $\mu \lesssim \bar{H} \sim m^2/M_P$. If it is stuck at a large vev initially, say $\sigma \sim M_P$, it will remain there all the way through inflation. It will give rise to a small cosmological term, $\sim \mu^2 M_P^2 \lesssim m^4$, which however does not significantly affect the background as we have discussed following Eq. (12). Once inflation terminates and $\bar{H}$ starts to decrease, σ will begin to roll towards its minimum. It will have initial energy density $\rho_\chi \sim \mu^2\sigma_0^2 \sim \bar{H}^2 M_P^2 \sim m^4$, and will immediately take over the control of the cosmological evolution from ϕ, scaling like cold dark matter. It could reheat the post-inflationary universe efficiently if it couples to a fermion field ψ in the matter sector with a Yukawa coupling

$$(m_\psi - g\sigma)\bar{\psi}\psi. \tag{46}$$

As the field σ moves towards the minimum, within a time $\sim 1/\mu$ it will scan all possible values. When it reaches $\chi = m_\psi/g$ it will copiously preheat the fermions ψ through the parametric resonance phenomenon [37]. One can give a crude estimate of the number density of fermions n_ψ produced in this way by recalling that the decay rate of χ into two fermions is $\Gamma \sim g^2\mu$, and that the number density of χ's is $n_\chi \simeq \mu\sigma^2$, so that using the continuity equation for the fermion number density, $\frac{1}{a^4}\frac{d}{dt}(a^4 n_\psi) \sim \Gamma n_\chi$ [37]. The fermion production lasts a fraction of $1/\mu$ so that the resulting fermion number density is

$$n_\psi \simeq \mu m_\psi^2. \tag{47}$$

When the field σ settles down in the minimum, the fermion energy density will be $\rho_\psi \sim n_\psi m_\psi \sim \mu m_\psi^3$. The fermions ψ need to quickly decay into the Standard Model particles to complete the reheating process. Taking $m_\psi \sim m$, the reheating temperature is given by

$$T_{\rm RH} \sim \left(\mu m_\psi^3\right)^{1/4} \sim \left(\frac{m}{M_P}\right)^{1/4} m, \tag{48}$$

or $T_{\rm RH} \lesssim 100$ MeV if $m \sim m_{\rm EW} \sim$ TeV, which may be sufficient to have conditions for a successful nucleosynthesis. The proper treatment of non-linear effects may further enhance the reheating efficiency [37]. There may also be other possibilities for curvaton reheating, as discussed in [38].

The same field may also produce the required density fluctuations. During inflation, because the curvaton dwells in the matter frame geometry defined by $\bar{g}_{\mu\nu}$, its fluctuations freeze out just like the fluctuations of any light scalar during inflation. They obey a field equation [20]

$$\frac{d^2\sigma_{\vec{k}}}{d\bar{t}^2} + 3\bar{H}\frac{d\sigma_{\vec{k}}}{d\bar{t}} + \left(\frac{k^2}{a^2(\bar{t})} + \mu^2\right)\sigma_{\vec{k}} = 0, \tag{49}$$

and thus in the limit $k^2/a^2 \ll 1$ they yield $\sigma_{\vec{k}} \to \alpha_{\vec{k}} + \beta_{\vec{k}}/a^3$. The perturbations are Gaussian, and start off as isocurvature perturbations, which however are converted into adiabatic perturbations after inflation [20], giving a nearly scale-invariant spectrum with the amplitude

$$\mathcal{P}^{1/2} \sim r\frac{\bar{H}}{\pi\sigma_*}, \tag{50}$$

where r is the curvaton fraction of the total energy density after inflation, and $\bar{H}$ and σ_* are the values of the Hubble parameter and the curvaton near the end of inflation. In the case of low scale inflation, one must also ensure that the curvaton mass changes rapidly after inflation in order not to spoil nucleosynthesis [39], but such models are possible in principle.

So far we have been ignoring the curvature problem. The model of inflation discussed here does not solve it. This is reminiscent of other models of low scale inflation. None of them are stand-alone solutions of the curvature problem. If inflation begins at a scale $\bar{H} \ll M_P$, something else must have kept the universe from collapsing until it reached the age $\sim \bar{H}^{-1}$, where late inflation can begin. Thus for low scale inflation to start, k/a^2 must be very small. However, since the solution (27) is decelerating when expressed in terms of the metric $g_{\mu\nu}$, the curvature problem here is more severe than in other low scale inflation models. To see this, note that in order to get a sufficiently small curvature today, we must ensure roughly $\mathsf{k}/a_{\rm now}^2 \lesssim H_0^2/100$, and therefore at the end of inflation the curvature term must satisfy $\mathsf{k}/a_{\rm exit}^2 \lesssim mH_0/100$. Hence using the solution (27), we see that since initially $H \lesssim M_P$, we must have $\mathsf{k}/(a_0^2 M_P^2) \lesssim (M_P/m)^{4/3}(mH_0/M_P^2)/100 \sim 10^{-57}$, which is a bit better than the required amount of fine tuning without any inflation. However one still needs to explain the origin of such a small number. Problems with curvature were noticed in [40]. There are several different possibilities for ensuring that k/a^2 is small at the onset of low scale inflation. One possibility is to have an early stage of inflation, driven by some other gravitationally coupled scalar, followed by the low scale inflation [21]. Such models might arise in the context of little string theories at a TeV [26], where the early inflation would take a Planck-scale universe and blow it up to TeV^{-1} size while generating the Planck-electroweak hierarchy. Another possibility might be the holographic cosmology [22]. In that case, the very early universe would start in the most entropically dense state, with a nearly vanishing curvature, which would evolve towards the regime of low density where conventional evolution can take place [22].

In closing, we have shown that a theory of gravity based on a special case of Bekenstein's disformal couplings, where matter couples to a combination $g_{\mu\nu} - \frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi$, gives rise to an epoch of short, low scale inflation. Here ϕ is a pseudoscalar field, which is invariant under $\phi \to \phi + \mathcal{C}$, $\phi \leftrightarrow -\phi$. These symmetries protect the inflaton sector from the Standard Model radiative corrections. The leading order coupling of ϕ to the matter fields is via the universal dimension-8 operator $\frac{1}{m^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$. Because of such couplings, the presence of ϕ is not in violation of any experimental bounds, even though it is massless, and may even lead to new signatures accessible to future collider experiments when $m \sim$ TeV. For that value of the mass m the number of e-folds of inflation is $\mathcal{N} \sim 25$, which is just enough to solve the horizon problem. The fluctuations of ϕ do not give rise to the satisfactory spectrum of density perturbations, and its couplings are too weak for efficient reheating, but both of these problems can be solved by adding other light scalar(s) such as the curvaton [20]. It would be interesting to explore further implications of this mechanism and see if it can arise from some fundamental theory.

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