Changing anisotropy of $G_0$ in Hostun sand during drained monotonic and cyclic loading

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Received 4 July 2014; received in revised form 2 July 2015; accepted 25 July 2015
Available online 26 September 2015

Abstract

The inherent and stress-induced anisotropy of saturated Hostun sand has been examined by means of shear wave velocity measurements under drained monotonic and cyclic triaxial conditions. Tests were carried out on samples prepared by moist tamping for a wide range of initial densities in a fully instrumented stress-path triaxial apparatus equipped with bender element instrumentation in three directions. Data have been normalised to account for the variations in void ratio and mean effective stress. The evolution of shear stiffness under anisotropic stress conditions has been explored by performing both constant mean effective stress tests and constant radial stress tests in compression and extension. Some samples were subsequently subjected to multiple drained deviatoric loading cycles, and the evolution of shear stiffness was examined. The tests showed that the initial inherent anisotropy was small, namely, lower than that previously observed in dry pluviated samples of the same sand in a cubical cell apparatus. Changing stress levels in compression or extension resulted in stress-induced anisotropy even when there were no changes in the values for $p'$, indicating that the widely-used empirical expressions for $G_0$ under isotropic stress states are not strictly applicable under anisotropic stress conditions. The comparison between stress-induced anisotropy in compression and in extension is discussed, and the evolution of shear stiffness during drained cyclic loading is examined. It is concluded that the changes in inherent anisotropy during these tests were small, but that new constitutive models should account for such changes in state with appropriate normalization.

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Keywords: Sandy soil; Stress ratio; Anisotropy; Laboratory tests

1. Introduction

The elastic properties of soils are of special importance in the design of foundations subjected to static and dynamic loading, since they are used in the prediction of deformations under working loads. It is well known that the elastic properties depend on both the stress state and the soil density. Soils are non-linear in nature, and thus, constitutive models are needed to simulate the non-linear response of soils. Many models include a small-strain region in which the response is assumed to be elastic, and the elastic stiffness is frequently normalised to account for the variation in soil state with depth.

An empirical expression for elastic shear modulus $G_0$, that accounts for the variation in mean effective stress $p'$ and void ratio at the isotropic stress states, was developed by early researchers (Hardin and Richart, 1963; Hardin and Drnevich, 1972), and then
expression for 1985; Hardin and Blandford, 1989; Lo Presti, 1995). The principal effective stress levels (Roesler, 1979; Stokoe et al.,

\[ p_{\text{ref}} = \frac{1}{3} \left( \frac{\sigma_{\text{ref}}}{\rho \left( V_s^2 - V_{\text{ref}}^2 \right)} \right) \]

where \( f(e) \) is the void ratio function describing the dependence of \( G_0 \) on the void ratio and typically obtained by empirical regression, \( p_{\text{ref}} \) is the reference pressure taken here as being equal to 100 kPa and used in order to have non-dimensional empirical coefficients, \( \sigma_{\text{ref}} \) is the principal effective stress in the direction of shear wave propagation, \( \sigma'_{a} \) is the principal effective stress in the direction of shear wave polarization, \( A \) is the value of stiffness at a reference state with stress equal to \( p_{\text{ref}} \) and \( f(e) \) equal to 1.0, \( n \) is the empirical coefficient that accounts for the effect of the principal effective stress \( \sigma'_{a} \) and \( m \) is the empirical coefficient that accounts for the effect of the principal effective stress \( \sigma'_{b} \).

It has been reported that coefficients \( n \) and \( m \) are both equal to 0.25 (Stokoe et al., 1985; Hardin and Blandford, 1989; Bellotti et al., 1996); and therefore, Eq. (1) is frequently used with these values. Roesler (1979) obtained a value \( n < m \), but the sum was equal to 0.50. On the other hand, Iwasaki and Tatsuoka (1977) found that coefficients \( n \) and \( m \) differed from each other depending on the type of soil. In the present authors’ experience, it is difficult to reliably determine separate values for \( m \) and \( n \) and, in practice, it is common to use Eq. (1) to express the dependence of \( G_0 \) with \( p' \) and to ignore the effect of any variations with anisotropic stress.

The research described herein investigates the degree of inherent anisotropy of small strain stiffness \( G_0 \) in reconstituted samples of Hostun sand prepared by a moist tamping method. The stiffness levels are compared with similar measurements in samples of the same material prepared by dry pluviation and tested in a cubical cell apparatus (Sadek, 2006). The variation in stiffness with density and the isotropic stress level are examined, and a suitable void ratio function is obtained. Stress-induced anisotropy is explored for samples sheared at constant \( p' \) and under constant radial stress \( \sigma'_{b} \). Finally, the evolution of elastic stiffness, in tests that are cycled under drained conditions, is presented.

2. Test apparatus and procedure

Tests were conducted on Hostun RF sand which is a granular siliceous material with an angular to sub-angular grain shape, \( D_{50}=0.35 \text{ mm, } \epsilon_{\text{min}}=0.656, \) and \( \epsilon_{\text{max}}=1.00 \) (Doanh and Ibraim, 2000; Alvarado, 2000). This material
has been widely used in the past for experimental research and constitutive modelling (Sadek, 2006; Doanh and Ibraim, 2000; Gajo and Muir Wood, 1999). Some physical properties of Hostun RF sand are given in Table 1 along with the critical state parameters. More information on other properties of Hostun RF sand can be found in Flavigny et al. (1990). Basic details of the test program described here are given in Table 2.

The testing was carried out using a hydraulic Bishop–Wesley triaxial apparatus (Bishop and Wesley, 1975). The sample dimensions were 75 mm in diameter and 150 mm in height, with porous stones of a similar diameter. Cell, ram, and back pressure was controlled by stepper-motor driven air pressure regulators, which were connected to the cell via the air-water interfaces. Local strain levels were measured by miniature LVDTs, model D5/200WRA/1316 from RDP Electronics (Cuccovillo and Coop, 1997); however, data obtained with this instrumentation is not presented in this paper. Other instrumentation consisted of an internal load cell, an external LDT, and an Imperial College type of volume gauge. TRIAX software (Toll, 1993; Nash, 2002) was used to control the stress levels during testing.

Three pairs of bender elements were used in all the tests (Pennington et al., 1997). They consisted of vertical benders, fixed at the end caps of the triaxial cell and protruding through the porous disc into the sample by 4 mm, and pairs of lateral bender elements, protruding 3 mm into the sample at mid-height. The lateral bender elements were inserted through pre-cut slots in the membrane that were covered by a small piece of aluminium foil. The benders were mounted in small metal pots that were held in place by a rubber grommet that was glued to the membrane and then sealed with latex. Then, where required, the internal LVDTs were mounted on the specimen. A fixed epoxy connection was used between the load cell and the top cap of the sample for all the tests.

The acquisition system for the bender element readings is schematically shown in Fig. 1. A function generator creates the desired form shape, frequency, and peak-to-peak voltage of the input wave fed to the transmitter bender and used to trigger the

Table 1
Physical properties and critical state parameters of Hostun RF sand.

<table>
<thead>
<tr>
<th>Type of sand</th>
<th>Uniformly graded medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest mineralogical component</td>
<td>SiO₂ &gt; 98%</td>
</tr>
<tr>
<td>Specific gravity (G)</td>
<td>2.646</td>
</tr>
<tr>
<td>Degree of roundness of particle</td>
<td>Angular to subangular</td>
</tr>
<tr>
<td>Maximum void ratio (eₘₐₓ)</td>
<td>1.000</td>
</tr>
<tr>
<td>Minimum void ratio (eₘᵢₙ)</td>
<td>0.655</td>
</tr>
<tr>
<td>Uniformity coefficient Cu</td>
<td>0.17</td>
</tr>
<tr>
<td>Coefficient of curvature Cc</td>
<td>0.25</td>
</tr>
<tr>
<td>D₅₀: mm</td>
<td>0.35</td>
</tr>
<tr>
<td>Critical state angle φₜₜ</td>
<td>32</td>
</tr>
<tr>
<td>Slope of CSL in a v-log(p) plane λ</td>
<td>0.031</td>
</tr>
<tr>
<td>Intercept of CSL at p = 1 kPa in a v-log(p) plane</td>
<td>2.650</td>
</tr>
</tbody>
</table>

Table 2
Testing program.

<table>
<thead>
<tr>
<th>Tests</th>
<th>e₀</th>
<th>η</th>
<th>Δq (kPa)</th>
<th>N</th>
<th>Test stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC01</td>
<td>0.920</td>
<td>0.50</td>
<td>–</td>
<td>–</td>
<td>Stage 1: Isotropic consolidation from 50 to 500 kPa.</td>
</tr>
<tr>
<td>IC02</td>
<td>0.856</td>
<td>0.80</td>
<td>–</td>
<td>–</td>
<td>Stage 2: Unloading to p' = 100 kPa.</td>
</tr>
<tr>
<td>IC03</td>
<td>0.768</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>Stage 3: Shear to η₀ at constant radial stress</td>
</tr>
<tr>
<td>IC04</td>
<td>0.730</td>
<td>1.40</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>AC01</td>
<td>0.961</td>
<td>0.50</td>
<td>30</td>
<td>4550</td>
<td>Stage 1: Isotropic consolidation to</td>
</tr>
<tr>
<td>AC02</td>
<td>0.726</td>
<td>0.50</td>
<td>30</td>
<td>5000</td>
<td>p' = 100 kPa.</td>
</tr>
<tr>
<td>AC03</td>
<td>0.968</td>
<td>1.00</td>
<td>15</td>
<td>3800</td>
<td>Stage 2: Shear to η at constant mean effective stress.</td>
</tr>
<tr>
<td>AC04</td>
<td>0.727</td>
<td>1.00</td>
<td>15</td>
<td>5200</td>
<td>Stage 3: Drained cyclic loading at 1 cycle per 5 min, amplitude Δq, and number of cycles N.</td>
</tr>
<tr>
<td>AC05</td>
<td>0.731</td>
<td>1.20</td>
<td>33</td>
<td>5400</td>
<td></td>
</tr>
<tr>
<td>AC06</td>
<td>0.920</td>
<td>1.30</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>AC07</td>
<td>0.730</td>
<td>1.50</td>
<td>22</td>
<td>4500</td>
<td></td>
</tr>
<tr>
<td>AC08</td>
<td>0.713</td>
<td>—</td>
<td>0.30</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>AC09</td>
<td>0.982</td>
<td>—</td>
<td>0.49</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>AC10</td>
<td>0.719</td>
<td>—</td>
<td>0.72</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>AC11</td>
<td>0.949</td>
<td>0.10</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>AC12</td>
<td>0.696</td>
<td>—</td>
<td>0.10</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 1. Control system for automatic bender element readings.](image-url)
signal acquisition. The three pairs of transmitter and receiver benders were connected to a switching unit that allowed changing from one pair of benders to another by using the digital output from an 8255 I/O card in the PC. Cables carrying the transmitted and received signals were connected to a two channel PICO ADC-216A/D converter that was used to obtain and to store the data before it was downloaded to the PC.

Stress-path control software was used to initiate the acquisition of bender data at predetermined stages of the test. Measurements of three shear wave velocities (Fig. 2) were obtained: \( V_{s(hh)} \), \( V_{s(hv)} \), and \( V_{s(vh)} \), where the first subscript denotes the direction of the shear wave propagation and the second subscript represents the direction of the shear wave polarization. The three velocities, \( V_s \), were calculated from the estimated wave travel time and with the tip-to-tip distance between the transmitter bender and receiver bender \( d \). Elastic shear modulus \( G_0 \) was then computed as the product of bulk density \( \rho \) and the square of the shear wave velocity.

The wave travel time was obtained by visual identification of the shear wave arrival from the peak of the transmitted signal to the peak of the received signal (peak-to-peak method). This method was compared with the cross-correlation method (Viggiani and Atkinson, 1995) which measures the similarity of two waveforms as a function of a time-lag applied to one of them. In this method, the arrival time is found from the time shift between the transmitted and the received signals required to maximise their dot-product (Viggiani and Atkinson, 1995). Both methods yielded similar results in these cylindrical samples (Escribano, 2014).

Samples of Hostun sand were prepared by moist tamping under compaction (Ladd, 1978) to a range in initial densities above and below the critical state line on a void ratio \(-\log(p')\) plane. After sample preparation, the specimen was held under a suction of 25 kPa and its dimensions were measured. After the application of a cell pressure of 25 kPa and the simultaneous removal of the internal suction, samples were fully saturated. This was achieved through the initial circulation of carbon dioxide for 30 minutes, the circulation of de-aired water through the sample until there was a flow equal to twice the total volume of the sand, and finally an automatic increase in back pressure together with cell pressure at a rate of 50 kPa per hour. The saturation process resulted in a Skempton B value between 0.95 and 0.98 using back pressure between 200 and 400 kPa.

Samples were isotropically consolidated and anisotropically consolidated, as indicated in Table 2, by raising the effective stress at a rate of 60 kPa per hour. IC tests were loaded in stages up to a maximum confining pressure of 500 kPa and then unloaded to 100 kPa before increasing the axial stress to a chosen stress ratio of \( \eta = q/p' \) with the radial stress held constant. Here, \( q \) is the deviator stress equal to the difference between vertical principal effective stress \( \sigma_v'' \) and horizontal principal effective stress \( \sigma_h'' \). It can take positive values in compression and negative values in extension. \( p' \) is the mean effective stress determined as the average value of the principal effective stress acting in a triaxial sample \( (p' = (\sigma_{v'} + 2\sigma_{h'})/3 \) in triaxial compression). Finally, the sample was sheared to failure at an axial strain rate of 5% per hour. In the AC tests (Table 2), samples were initially isotropically consolidated to a mean effective stress of \( p' = 100 \) kPa, and then anisotropically consolidated in compression or extension following a constant \( p' \) path under controlled stress, as indicated in Table 2. The stress was then held constant for a maximum of 2 h, to allow some creep deformation, before bender element readings were taken.

In some tests, cyclic loading was then applied by varying the ram pressure sinusoidally at a frequency of 1 cycle per 5 min to ensure well-defined cycles and homogeneous pore water pressure inside the sample under fully drained conditions. In the initial tests, a mid-height pore pressure transducer was used to ensure that excess pore pressure was minimal. Table 2 shows the different peak-to-peak cyclic amplitudes \( \Delta q \) that were applied. Loading symmetry and stress amplitude were maintained at constant levels by software through automatic control of the pressure with periodic checking and correction of the stress levels if deemed necessary. In this series of tests, a maximum of 5400 cycles were applied with measurements of shear wave velocity being undertaken at regular intervals.

Table 2 summarizes the testing program and gives the basic details of the tests conducted. All tests had bender element readings in three directions. Initial void ratio \( e_v \) was corrected for changes in volume during saturation and during initial isotropic consolidation. Table 2 also indicates the stress ratio at the end of consolidation, \( \eta = q/p' \), and the number of loading cycles \( N \) for the corresponding AC tests. Fig. 3 shows the stress path followed by each set of tests and the stages described in Table 2.

3. Inherent and stress-induced anisotropy

It is useful to distinguish the inherent anisotropy due to fabric effects from the stress-induced anisotropy. In order to
evaluate the inherent anisotropy of the soil, bender element readings were firstly made in the isotropically consolidated IC samples (Table 2) under confining pressure between 50 kPa and 500 kPa. The normalization of \( G_0 \) was undertaken by evaluating its dependence with the void ratio and \( p' \). The data were initially analysed in terms of void-ratio dependence through a regression analysis. A power function was obtained (expression (2)) with an average exponent of \(-1.10\), fairly similar to the values obtained by Lo Presti et al. (1997) of \(-1.30\) for Toyoura sand and Quiou sand, and by Fioravante (2000) of \(-0.80\) for Ticino sands.

\[
f(e) = e^{-1.10}
\]

Fig. 4 shows the influence of \( p' \) on \( G_0 \) in a double logarithmic plot, with \( G_0 \) normalized by the void ratio function (expression (2)). By plotting all the normalized data from the IC tests and fitting the power trendlines, it may be seen that the data are consistent with the empirical Eq. (1), which in this case has been expressed in terms of \( p' \). From the equations for the trendlines, an average exponent of \( n+m = 0.45 \) was obtained for normalised stiffness \( G_{0(hv)}/f(e) \) with vertically propagated waves, and a value of \( 0.48 \) was obtained for both normalised stiffness \( G_{0(hv)}/f(e) \) and \( G_{0(hv)}/f(e) \) with horizontally propagated waves. When stiffness is normalised to take account of the variation in both \( e \) and \( p' \), reference stiffness \( A \), in Eq. (1), ranges from 85 to 96 MPa when reference pressure \( p_{ref} \) is taken as 100 kPa. These values are shown in Table 3 together with the average of the normalised data from the measurements at an isotropic confining pressure of 100 kPa in the different groups of tests. It may be seen that the average measured stiffness varies by a few percent depending on which data set are used. In the discussion that follows, the normalised stiffness is based on the average values measured in all the tests and these values are used to calculate the stiffness using Eq. (1). As a check, all the measurements for \( G_0 \) made under an isotropic confining pressure of 100 kPa are plotted against the void ratio in Fig. 5 together with the relationships calculated using Eq. (1) with the void ratio function given by expression (2). It can be seen that although there is some scatter in the data, the trendlines fit the data well.

4. Inherent anisotropy

Through an evaluation of the shear wave velocities and the corresponding elastic shear stiffness, the degree of inherent anisotropy of Hostun sand, due to the fabric effects produced by the sample preparation, has been quantified. Similar ways to explore the inherent anisotropy have been proposed in the past (Jamiolkowski et al., 1995; Bellotti et al., 1996; Pennington et al., 1997). For these cylindrical samples that were reconstituted by moist tamping, it may be expected that samples exhibit cross-anisotropy with a vertical axis of symmetry. Fig. 6 presents shear wave velocities \( V_{sh(v)} \) and \( V_{sh(h)} \) measured under isotropic stress plotted against measurements of \( V_{sh(h)} \) at the same state (IC tests, Table 2). It can be observed that the degree of inherent anisotropy is small, with average ratios of \( V_{sh(h)}/V_{sh(v)} = 1.02 \) and \( V_{sh(b)}/V_{sh(h)} = 1.09 \). The ratio \( V_{sh(b)}/V_{sh(h)} \) is believed to be the most reliable indication of inherent anisotropy since both measurements are made using the same horizontal travel path. The horizontally polarised shear waves apparently travel only slightly faster than the vertically polarized shear waves, although these differences are a little more pronounced when comparing shear wave velocities \( V_{sh(h)} \) and \( V_{sh(b)} \).

Previous research on the degree of inherent anisotropy of sands has been performed with the triaxial apparatus by Sunyer (2007) for Hostun sand, Kuwano (1999) for Ham River sand, and Fioravante (2000) for Ticino sand. In these previous tests, the height to diameter ratio of the samples was equal to 2, the sands all had an angular to subangular shape, and all the tests were undertaken on dry pluviated samples. The preceding authors obtained ratios of \( V_{sh(b)}/V_{sh(h)} \) between 1.04 and 1.05. Shear wave
velocity measurements on samples of sand prepared by dry pluviation with a height to diameter ratio of 1 were made by Sadek (2006) who tested Hostun sand in a cubical cell, Belloti et al. (1996) who tested Ticino sand in a calibration chamber, and Wang et al. (2008) who tested Toyoura subangular sand in a true triaxial apparatus, after preparing them by dry tamping and then fully saturating them before testing. Ishibashi et al. (1991) observed differences of over 30% in some in-situ testing in sands (Hight et al., 1997). Jardine, 2002; Sadek, 2006), and similar differences have also been observed in in-situ testing in sands (Hight et al., 1997). Nash et al. (1999) observed differences of over 30% in some tests in the triaxial cell for reconstituted Gault clay, and these differences disappeared when the specimens were removed from the triaxial cell and were tested on a laboratory bench. Ishibashi et al. (1991) observed that $G_{hh}$ was lower than $G_{vb}$.

The previous comparison of ratios $V_{s(hh)}/V_{s(hv)}$ and $G_{0(hh)}/G_{0(hv)}$ suggests that the sample preparation method has little influence on the fabric anisotropy and that the moist tamping method results in lower fabric anisotropy than dry pluviation. It is also seen that the geometry of the sample has some influence, as cylindrical samples with a height to diameter ratio of 2 yield lower ratios compared to samples with a geometry ratio of 1. The differences between samples with different geometries could perhaps be an effect of the restrained boundary conditions imposed on a triaxial sample. The results from this research and those given in the literature suggest that particle shape may also have some influence on the degree of inherent anisotropy, as rounded particles seem to produce a fabric with stiffness in the horizontal plane that is lower than in the vertical plane.

Theoretically, the two shear wave velocities, $V_{s(hh)}$ and $V_{s(hv)}$, should be the same in an homogeneous and cross-anisotropic elastic material. However, previous research works have found differences between the $V_{s(hh)}$ and $V_{s(hv)}$ measured in the laboratory samples (Nash et al., 1999; Kuwano and Jardine, 2002; Sadek, 2006), and similar differences have also been observed in in-situ testing in sands (Hight et al., 1997). Nash et al. (1999) observed differences of over 30% in some tests in the triaxial cell for reconstituted Gault clay, and these differences disappeared when the specimens were removed from the triaxial cell and were tested on a laboratory bench. Nash et al. (1999) attributed these differences to the end restraint affecting the measurements of $V_{s(hh)}$. Kuwano (1999) and Kuwano and Jardine (2002) attributed the differences between $V_{s(hh)}$ and $V_{s(hv)}$ to the particulate nature of the

Table 3
Normalized stiffness (MPa) at isotropic stress of 100 kPa and inherent anisotropy ratios.

<table>
<thead>
<tr>
<th>From trendlines:</th>
<th>$G_{0(hh)}/f(e)$</th>
<th>$G_{0(hv)}/f(e)$</th>
<th>$G_{0(hh)}/f(e)$</th>
<th>$G_{0(hh)}/G_{0(hv)}$</th>
<th>$G_{0(hh)}/G_{0(hv)}$</th>
<th>$G_{0(hh)}/G_{0(hv)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC tests</td>
<td>77.6</td>
<td>91.4</td>
<td>99.0</td>
<td>1.28</td>
<td>1.08</td>
<td>0.85</td>
</tr>
<tr>
<td>AC tests (12)</td>
<td>85.5</td>
<td>92.0</td>
<td>94.7</td>
<td>1.11</td>
<td>1.03</td>
<td>0.93</td>
</tr>
<tr>
<td>All tests (16)</td>
<td>84.8</td>
<td>92.4</td>
<td>95.6</td>
<td>1.13</td>
<td>1.04</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Fig. 5. $G_0$ measurements at isotropic stress of 100 kPa against void ratio.

Fig. 6. Comparison between horizontal shear wave velocity $V_{s(hh)}$ and vertical shear wave velocities $V_{s(hv)}$ and $V_{s(hv)}$ for $p' = 50–500$ kPa and $e = 0.730–0.920$ (IC tests, Table 2).
material and particle shape. Sadek (2006) obtained a difference of 3% for cubical cell samples of Hostun sand with flexible boundaries and isotropic geometry. On the other hand, Hight et al. (1997) compared in situ seismic measurements for three different materials: a marine sand, London Clay, and a clayey silty fine sand. For all three materials, Hight et al. (1997) obtained higher values for $V_s(hv)$ than $V_s(vh)$, with a ratio $V_s(hv)/V_s(vh)$ between 1.05 and 1.11, differences that could perhaps be accounted for by the layering of the naturally deposited materials.

In this research, an average ratio of $V_s(hv)/V_s(vh)=1.07$ was obtained in the Hostun sand which is close to the values of 1.11 and 1.10 obtained by Sunyer (2007) and Kuwano (1999), respectively. A much lower value of 1.03 was obtained by Sadek (2006). The nature of these differences is not entirely understood, although evidence from laboratory tests and in situ testing seems to show that a non-homogeneous material will invariably present differences between $V_s(hv)$ and $V_s(vh)$. The particle shapes of Hostun sand and Ham River sand, tested by Kuwano (1999), are both angular to subangular, and may well generate a non-homogeneous fabric independent of the sample preparation method used. Yet, it is interesting to compare tests conducted in the cubical cell with those performed in the triaxial apparatus, suggesting that the larger differences observed in the triaxial tests may arise more from geometric and boundary effects than from departures from elasticity.

In conclusion, it can be said that the horizontal plane of reconstituted samples of Hostun sand is slightly stiffer than the vertical plane. Even though sample preparation by air pluviation and by moist tamping result in different fabrics, the degree of inherent stiffness anisotropy is similar. The slightly lower degree of inherent anisotropy found in this research, compared to the results from the literature, is thought to be the result of the different sample preparation methods as well as the geometry of the samples.

5. Stress-induced changes in stiffness

In order to study the stress-induced anisotropy of Hostun sand, shear wave velocity measurements were undertaken in samples consolidated firstly under isotropic stress and then further consolidated to an anisotropic stress state reached by either a stress path maintaining mean effective stress $p'$, constant at 100 kPa (AC tests), or with axial stress increased with radial stress $s'_r$, held constant at 100 kPa (IC tests). Details of the stress conditions are given in Table 2 and Fig. 3. It is appropriate to interpret the data using Eq. (1) taking into account the variation in stress in the plane of shear wave propagation $\sigma'_a$ and polarization $\sigma'_b$. The normalised stiffness data are plotted on double logarithmic plots against the product of effective stress $\sigma'_a \cdot \sigma'_b$ in Fig. 7. In interpreting the plots, it is helpful to bear in mind how the product of $\sigma'_a$ and $\sigma'_b$ varies with $\eta$ along both a constant $p'$ stress path (Fig. 8a) and a constant $\sigma'_r$ path (Fig. 8b). It may be seen that with mean stress $p'$ maintained constant, while the product $\sigma'_a \cdot \sigma'_b$ varies with the stress ratio, the product $\sigma'_a \cdot \sigma'_b$ remains approximately...
and m path. (a) Constant s in extension. With the cell pressure held constant, the product constant in compression, while decreasing slightly in extension. With the cell pressure held constant, the product $\sigma'_h \cdot \sigma'_h$ remains constant, while both the square of $p'$ and the product $\sigma'_h \cdot \sigma'_v$ vary with the stress ratio.

![Fig. 8](image1.png)

Fig. 8. Changes in the product of principal effective stress, $\sigma'_a$ and $\sigma'_b$, with stress ratio for (a) constant mean stress path and (b) constant radial stress path. (a) Constant $p'$ (b) Constant radial stress.

Similarly, tests along a stress path at constant $\sigma'_h$ (IC tests shown in Fig. 7b and d) result in little change in $G_{hhhh}/f(e)$, since product $\sigma'_h \cdot \sigma'_h$ does not change (Fig. 8b), but the stiffness values $G_{vhvh}/f(e)$ and $G_{vhvh}/f(e)$ in the vertical plane vary as a result of the increasing vertical stress, as expected from the variation in the product $\sigma'_v \cdot \sigma'_h$. At high stress ratios, they drop below the relationships obtained from Eq. (1) probably as a result of fabric changes at large stress ratios.

To clarify the changes in elastic stiffness as the stress ratio is varied, the stiffness under anisotropic stress has been normalised by the initial values under a confining pressure of 100 kPa, as shown in Fig. 9a and b, where the values for $G_{ah}/G_{hh}$ are plotted against $\eta$ for the three sets of measurements for the two sets of data (IC and AC tests). Here, $G_{0a}$ denotes the normalised elastic stiffness $G_{ah}/f(e)$, measured under an anisotropic stress state, and $G_{hh}$ denotes its initial value, measured under an isotropic stress of 100 kPa. For tests along the constant $p'$ path (AC tests Fig. 9a), the changes in void ratio were very small, and although they were taken into account in the normalisation, the f(e) terms have been omitted from the expressions for simplicity. It is observed that the ratio $G_{0hh}/G_{0hh}$ increases in extension and decreases in the principal effective stress values remains nearly constant (Fig. 8a).

![Fig. 9](image2.png)

Fig. 9. Ratio of normalized $G_{0h}/G_{hh}$ for AC tests and IC tests against average stress ratio. Solid line: $G_{0h}/G_{0h}$ from Eq. (1); Dotted line: $G_{0h}/G_{0h}$ and $G_{0h}$ from Eq. (1). (a) AC tests at constant $p'$ (b) IC tests at constant radial stress.

In this case, the values for exponents $n$ and $m$ in Eq. (1) will be similar. Therefore, for a constant $p'$ condition, the values for $G_{0vh}/f(e)$ and $G_{0vh}/f(e)$ do not change significantly, as shown in Fig. 7c, since the product of

$\frac{\eta}{\alpha_0 \cdot \beta_0}$ terms have been omitted from the expressions for simplicity. It is observed that the ratio $G_{0h}/G_{0h}$ increases in extension and decreases in

$\eta = \frac{\alpha_0 \cdot \beta_0}{(\sigma'_v \cdot \sigma'_v)}$.

$G_{0h}/G_{0h}$ is less spread out, which re
compression, while the ratios in the vertical plane, \( G_{G(hh)A}/G_{G(hh)A} \) and \( G_{G(hv)A}/G_{G(hv)A} \), decrease in extension and stay approximately constant in compression. These changes occur despite maintaining \( p' \) at a constant value and are consistent with Eq. (1) when the variation in stress values \( \sigma'_h \) and \( \sigma'_v \) are taken into account. When stress ratio \( \eta \) is increased above the critical state stress ratio in compression \( M_{cs} \), it is observed that the values for \( G_{G(hh)A}/G_{G(hh)A} \) continue to decrease. The ratios \( G_{G(hh)A}/G_{G(hh)A} \) and \( G_{G(hv)A}/G_{G(hv)A} \) also decrease at high stress ratios; this is perhaps associated with the dilation of the sand that produces important changes in fabric and occurs despite the normalisation of the void ratio. For tests along a stress path at constant \( \sigma'_h \) (IC tests shown in Fig. 9b), once again, the variations up to a stress ratio of 0.5 were much as predicted from Eq. (1). However, at high stress ratios, the values decreased, as observed in the AC tests. Previous researchers (Nakamura et al., 1999; Kuwano, 1999) have also observed that the elastic stiffness decreases with the onset of dilation. While the empirical Eq. (1) does imply a decrease in elastic stiffness if the void ratio is increasing, the void ratio normalisation is not sufficient to explain the changes.

6. Stress-induced changes in anisotropy

Under isotropic confining stress, the ratios for stiffness define the inherent (or fabric) anisotropy of the soil, as discussed above. As the stress changes, the stiffness is altered by different amounts. The anisotropy ratio is defined here as the ratio between \( G_{G(hh)} \) and the values for \( G_{G(hh)} \) and \( G_{G(hv)} \). Under isotropic stress states, the inherent anisotropy gives slightly higher values for elastic stiffness in the horizontal plane than in the vertical plane, as discussed previously (Table 3). For the AC tests, sheared at constant \( p' \) in compression (positive stress ratio \( \eta \)) and extension (negative stress ratio \( \eta \)), the changing anisotropy ratios \( G_{G(hh)}/G_{G(hh)} \) and \( G_{G(hh)}/G_{G(hv)} \) are shown in Fig. 10a. All these AC tests were firstly isotropically consolidated under \( p' \) of 100 kPa. As the soil was compressed, the anisotropy ratios decreased as the vertical effective stress increased and the horizontal stress decreased. The reverse happened in extension and the anisotropy ratios increased. Similar results were found from the IC tests compressed at constant radial effective stress \( \sigma'_h \), as shown in Fig. 10b. These observations are broadly consistent with the trends predicted by Eq. (1) taking into account the variation in the product of the stress values for \( \sigma'_h \) and \( \sigma'_v \) with \( \eta \), as shown in Fig. 8. When stress ratio \( \eta \) exceeds the critical state stress ratio in compression \( M_{cs} \), the decrease in anisotropy ratios is interrupted and the ratios increase again slightly – a consequence of the changes in dilation and fabric.

The ratios \( G_{G(hh)/G_{G(hv)}} \), for the two sets of tests are plotted in Fig. 10. As expected, this ratio remains approximately constant for all the tests, although it is generally less than 1.0, as noted above. The difference from 1.0 is probably due to the geometric and boundary effects in the triaxial samples, as well as the possible effects of layering.

7. Elastic stiffness during cyclic loading

The degree of inherent anisotropy reflects the fabric of the soil, but the degree of anisotropy is also strongly influenced by the anisotropic stress in the soil. Drained cyclic loading will generate changes in a soil fabric as a result of a reorientation of grains and a changing arrangement of force chains (interparticle forces). This leads to the accumulation of strain and changes in void ratio that are expected to result in changes in small strain stiffness.

The evolution of \( G_0 \) during drained cyclic loading was investigated by performing long-term low amplitude drained cyclic triaxial tests and measuring the three shear wave velocities at regular intervals during the tests with bender elements. Fig. 11 shows the evolution of \( G_0 \) during drained cyclic loading for tests on dense and loose samples. The value for \( G_0 \) is normalized with \( f(e) \) and divided by the initial value for \( G_0 \) before cyclic loading. It is seen that the variations in \( G_{G(hv)/G_{G(hv)}} \) were almost negligible. The values for \( G_{G(hv)/G_{G(hv)}} \) appear to be more affected by cyclic loading; however, the variations are small and do not follow any specific pattern. Similar findings were reported by Tatsuoka et al. (1979) and Lo Presti et al. (1993), based on resonant column tests on sand, and by Wichmann (2005) who undertook drained cyclic triaxial tests on sand. These results indicate that even through the soil experiences plastic deformation, and therefore, changes in fabric, the values for \( G_0 \) are not affected.

This contrasts with the secant modulus, which increases during cyclic loading as the soil contracts, but decreases above \( M_{ct} \) as the soil dilates. Under increasing amplitude of cyclic loading, \( G_0 \) is generally found to remain approximately
constant when normalised to take account of the changes in void ratio and stress, in contrast to the decrease in secant stiffness widely observed (e.g., Hardin and Drnevich, 1972). However, from observations of the changes in anisotropy under static loading to high stress ratios reported in Fig. 10, it may be inferred that \( G_0 \) would change in soils that are similarly highly stressed during cyclic loading due to major changes in fabric and particle rearrangement.

8. Conclusions

Measurements of shear wave velocities in reconstituted triaxial samples of Hostun sand, prepared by moist tamping with under-compaction, have been undertaken using three pairs of bender elements. This has permitted the evaluation of \( G_{0(hv)}, G_{0(hv)}, \) and \( G_{0(hv)} \), and has enabled the anisotropy of the sand samples to be quantified.

From the results at isotropic stress states, empirical correlations were obtained to characterise the elastic shear stiffness in the vertical and horizontal directions, and their dependence on the void ratio and mean effective stress \( p' \). A comparison of the shear wave velocities in three directions revealed that the inherent anisotropy is rather small in these samples prepared by moist tamping. Comparisons were made with previous measurements in Hostun sand samples prepared by pluviation and tested in the triaxial apparatus and in a cubical cell, and the ratio \( V_{s(hv)}/V_{s(hv)} \) varied from 1.02 (present research) to 1.10 (cubical cell). Slightly larger differences were observed in the triaxial tests when comparisons were made with \( V_{s(hv)} \), but these had not been found in the cubical cell tests. These differences may arise more from geometric, boundary, and fabric effects in the triaxial samples than departures from elasticity.

In tests carried out following a stress path at constant \( p' \), it was found that there were significant changes in \( G_0 \) and that the development of stress-induced anisotropy varied from 0.7 in compression to 1.5 in extension. When the data were interpreted using the product of the stress values in the plane of shear wave polarisation (Eq. (1)), the normalised stiffness was broadly consistent with that found from measurements at isotropic stress states. These results indicate that the normalisation of stiffness by a function of mean effective stress \( p' \) alone, as is common in current constitutive models for granular soils, does not capture the significant effects of stress-induced anisotropy. It is concluded that new constitutive models should account for changes in the stress state with appropriate normalisation.

During drained cyclic loading, the elastic stiffness did not change significantly nor was it affected by the associated fabric changes. These findings suggest that even though a granular soil without pronounced fabric is non-linear at larger strain levels, the elastic properties are recoverable and can be thought of as constant.

Acknowledgements

This research was carried out in the Geomechanics Laboratory at the University of Bristol as part of the PhD research by the first author. Financial support from Conicyt Scholarship in Chile is gratefully acknowledged.

References


