Fractal Coding Using Gradient Direction Based Tag Matrix And Score Value

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Abstract

Reducing encoding complexity and improving the tradeoff between quality of image and compression ratio are active areas of research in the field of fractal image compression. In this paper two techniques are proposed to reduce the encoding complexity and to improve the PSNR of the reconstructed image. The first method is the prediction of affine transformation, based on the gradient direction of domain and range block using a Tag Matrix and the second involves calculation of score value based on the maximum value of intensity difference of range and domain block. An important merits of this new method is that encoding time is highly reduced and PSNR is also improved than the existing fractal coding techniques. This technique will be very applicable in situation where we have limited storage space and bandwidth but have to store and transmit enormous amount of data.

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1. Introduction

Fractal is defined as an object having fractional dimension and self similarity. Self similarity in an image means either a part of the image can be expressed as a geometric transform of whole image or a smaller part of an image can be represented as the geometric transform of any other bigger part in the image. The idea of fractal geometry is put forward by the mathematician Benoid Mandelbrot. All natural images have fractal geometry. So to represent a natural object in compact form, the technique of fractal compression is most suitable. But this technique has a major drawback of unsymmetrical nature of encoding-decoding phase that prevent it from accepting it as an international standard. In this paper two techniques are proposed to improve the performance of fractal image compression. The concept of fractal image compression is introduced by the scientist Michael Barnsley. The mathematical concept of Fractal image compression is based on the principle of Collage Theorem. At present there are a lot of research work on reducing the encoding complexity of fractal coding techniques.
Exhaustive search for a best matching domain block is the major drawback of BFIC (Basic Fractal Image Compression)\textsuperscript{6}. Some algorithms such as Fisher’s 3 classes method and APCC-based block, are based on domain classification, so that searching is restricted in specific classes but the PSNR value is found to be decreased in these methods\textsuperscript{7,8}. Some researchers adopted the method of Particle Swarm Optimization in fractal encoding phase, which speed up the encoder efficiency with a little decay in image quality, since this is also an area-restricted searching method\textsuperscript{9}. Some fast-fractal coding methods were proposed based on a selected part of the domain pool to improve the efficiency of search phase, for example, searching the matching domain block from the adjacent domain block of the current range block\textsuperscript{10,11}. A lot of hybrid fractal encoding works were done by exploiting the multi-resolution capability of Discrete Wavelet Transform and self similarity of fractal images\textsuperscript{12,13}. Some research works are based on improvement of search speed to make encoding and decoding phase more symmetric by using tree structured search method\textsuperscript{14,15} quad tree partitioning of range block\textsuperscript{16,17,18} or parallel search\textsuperscript{19,20}. A few works were based on affine transformation prediction to reduce the cost of encoding process\textsuperscript{21}. But this research paper proposes a new gradient based geometrical transformation prediction algorithm which shows a better performance than the previous existing methods. Fractal image Compression is based on the principle of Collage Theorem.

2. Proposed Method

This is a hybrid encoding technique, where multi-resolution property of wavelet and self similarity property of fractal is blended. This new prediction algorithm reduces the encoding time and improves the PSNR of reconstructed image based on gradient direction and maximum value of intensity difference of domain and range block. Affine transformations are predicted using a Tag Matrix to reduce the encoding complexity. Here the self similarity between the sub bands in each level is exploited to predict the affine transformation of each range block defined in each sub-band. The coefficients of the lowest coarser sub-band are encoded directly, i.e. without fractal encoding. Sub bands at higher scale are divided into higher entropy block and chosen as domain block and the adjacent lower scale sub band is divided in to lower entropy block of same size and considered as range block. The affine transformation of range block is predicted using the proposed Tagging algorithm. In this proposed method a six level wavelet decomposition (Haar wavelet) is used. Parameters of the Iterated Function system is converted in to binary stream with an appropriate bit allocation to each parameter.

2.1. Theory of Tagging Algorithm

In Tagging phase a gradient matrix\textsuperscript{2} is to be defined and chosen. Gradient matrix is a matrix having gradient in all direction. Gradient direction of range block and domain blocks are determined using this reference matrix (gradient matrix).

\textit{Definition of gradient}: In an image, gradient is defined as the directional change in image intensity or color.

\begin{equation}
\text{Gradient of an image } f(x,y), \nabla f(x,y) = (f(x,y), f(x,y)) \\
i.e. \nabla f(x,y) = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j
\end{equation}

Gradient direction gives the direction of maximal change in the intensity.

2.2. Explanation of Tag Generation Technique

The direction of maximum intensity variation in an image can be determined by gradient operator. So in the succeeding step a tag is generated for the domain block and range block by comparing with reference matrix and it gives the relative affine transformation with respect to the reference block. This is stored as tag1 and tag 2 respectively for domain and range block respectively. Tag1 is that relative geometric transformation of the matrix F
(shown below) which shows best matching with range block. Tag2 is that relative geometric transformation of the matrix F which shows best matching with domain block. Also a score value, which gives the information of maximum value of intensity difference, is also calculated for each domain and range block and this score value is also tagged. During the search among the best matching domain blocks skip those blocks whose score difference is above some threshold.

Table 1. Tag Matrix to find the most suitable affine transformation.

<table>
<thead>
<tr>
<th>Tag1 Tag2</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>6</td>
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<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>6</td>
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<td>8</td>
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<td>3</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>1</td>
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<td>6</td>
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<td>1</td>
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<tr>
<td>7</td>
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<td>5</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

The best matching contractive affine transformation is determined using the Tag Matrix, which is given in Table 1. This table gives the result of composition of two geometric transformations. An example of matrix having gradient in all direction (gradient matrix), which is used in the proposed method is,

\[
F = \begin{bmatrix}
1 & \frac{2}{4}
\end{bmatrix}
\]  

(2)

After creating the tag for each range and domain block, next process is to predict the best matching affine transformation for each range block using the Tag Matrix. For Example: If the Tag value of the range block is 3 and Tag value of the present domain block is 5, the best affine transformation is predicted from the table as 6.

2.3. Proposed Algorithm

2.3.1. Encoding Phase

Step 1: Load the image

Step 2: Perform 6 level wavelet decomposition (cA1-cA6, cH1-cH6, cV1-cV6, cD1-cD6) using Haar wavelet
Step 3: Encode $cA^6$ directly using a lossless binary transformation

Step 4: Select $cH_d, cV_d, cD_d$ as domains and $cH(d-1), cV(d-1), cD(d-1)$ as range where $d=6, 5, 4, \ldots 2$.

Step 5: Divide $cH_d, cV(d-1), cD(d-1)$ into block of sizes,

$$B = 2^{(d_{\text{max}} - d)}$$

where $d_{\text{max}} = 6$ and $d = 6, 5, \ldots 2$

Step 6: Find a best suitting affine transformation using gradient based affine transformation prediction (Tagging algorithm) for $d=6, 5, \ldots 2$.

Step 7: Encode the fractal codes for each scale

2.3.1.1. Tagging algorithm

Step 1: Initialize a reference block (gradient matrix) with gradient in all direction and store it in 8 arrays with 8 affine transformations.

Step 2: Load the image either range and domain block.

Step 3: Decimate each block into the size of reference block (2x2).

Step 4: Match each block with 8 arrays and store the transformation as a tag for the same block.

Step 5: Find score of each block using the equation

$$\text{Score}(R | D) = \text{abs}(\max(\max(R | D)) - \min(\min(R | D)))$$

and store it along with the tag.

Step 6: During the search among the best matching domain blocks skip those blocks whose score difference is above some threshold.

Step 7: For a block in the range if affine transformation is tag1 and for range it is tag2 then the affine transformation is predicted using Table 1.

Step 8: Using the predicted affine transformation find whether it is the best matching block or not.

Step 9: Repeat step 8 until all range blocks are compared and store the affine transformation, offset, scale and domain location of the best matched block.

Step 10: Repeat step 7-9 for each range blocks.

2.3.2. Decoding Phase

Step 1: Consider an empty matrix of size 512x512

Step 2: Do a 6 level wavelet decomposition.
Step 3: \( c_{Hd}, c_{Vd}, c_{Dd} \) as domains and \( c_{H(d-1)}, c_{V(d-1)}, c_{D(d-1)} \) as ranges apply iterated function system to predict the range from the fractal code.

Step 4: Go to step 2 while no of iteration is not completed.

3. Experimental Results and Analysis

Experiments are simulated using MatLab12 intel core2duo 1.2 ghz processor. In this vector form of hybrid compression approach Haar wavelet is used for wavelet decomposition. This proposed gradient based affine transformation prediction method using a Tagging algorithm is performed on four images and the parameters, PSNR, Encoding time, No of computation are compared keeping compression ratio a constant value.

<table>
<thead>
<tr>
<th>Images</th>
<th>BFIC</th>
<th>WFC</th>
<th>PROPOSED METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No of mse computation</td>
<td>PSNR</td>
<td>Encoding time</td>
</tr>
<tr>
<td></td>
<td>Pepper</td>
<td>11383080</td>
<td>34.77</td>
</tr>
<tr>
<td></td>
<td>Lena</td>
<td>13743680</td>
<td>33.86</td>
</tr>
<tr>
<td></td>
<td>cameraman</td>
<td>11951121</td>
<td>30.50</td>
</tr>
<tr>
<td></td>
<td>Barbara</td>
<td>13579120</td>
<td>31.82</td>
</tr>
</tbody>
</table>

Table 2: Simulation results for encoding time (sec), PSNR (db) and No of mse calculations
The results of Proposed Gradient based prediction method is compared with that of Basic Fractal Image Compression (BFIC) and Hybrid Wavelet – Fractal Image compression (WFC) techniques. This vector form of image representation guarantees a very good value for the encoding time when comparing with other existing fractal coding techniques (like BFIC and WFC) and provides a comparatively good trade-off between compression ratio and quality of image when compares with the recent technique of wavelet-fractal compression (WFC). Also the result shows that no of mse computation required is less that that of the recent techniques of fractal compression. So this method will be applicable in situation where we have only a low band width communication channel or small storage space and to deal with a large amount of data size.

The main drawback of the existing fractal coding techniques are the unsymmetrical nature of encoding - decoding process. The most important feature of this work is very low value of encoding time than that of the existing fractal coding techniques. This ensures a very good symmetry between the encoding-decoding phases. From the table IV, it is observed that there is a performance improvement of 92.26% in encoding time, 98.27% in no of mse calculations in the proposed method over BFIC and 82.44% in encoding time, 88.89% in no mse calculations in the proposed method over the recent technique of WFC. PSNR of the proposed method is also improved by 2.3% over the recent technique of WFC.

Highly reduced value of encoding time and comparatively high value of PSNR of the reconstructed image makes this new technique superior to other existing techniques.

4. Conclusion

Experimental results infers that fractal coding using gradient direction based Tag matrix method and score value gives superior performance. These methods have good performance in terms of encoding time over both wavelet-fractal and existing fractal coding techniques. There is a good trade-off between compression ratio and the quality of reconstructed image that was lacking in recent technique of wavelet-fractal encoding.
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