Suitable cost functions for signalized arterials and freeways, in the user equilibrium assignment problem

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Abstract

The aim of this paper is to design and prove new link travel time functions (TTF), which are combinations of known delay functions. The proposed TTF are useful for modeling two types of arc: light signalized arcs and limited access arcs. These TTF are used for formulating and solving the User Equilibrium Assignment (UEA) problem by means the Frank-Wolfe algorithm, obtaining more accurate results. The process for solving the UEA problem was improved because usually it is solved for only one travel time function or by flow-speed curves, or without considering overflow periods. An application on a Mexico City sub-network is presented.

1. Introduction

Currently, planning and evaluation of transport projects are based on performance indicators which refer to travel time on the arcs and routes, and traffic volume (Afandizadeh [1]). Some indicators are related to level of congestion, travel costs and pollutant emissions. The estimation of flow through the user equilibrium traffic assignment (UEA) has been used in macroscopic studies, where travel time functions on arcs are flow dependent. These functions must incorporate the parameters and variables which generate a good representation of the delay, in terms of physical and operational components for each arc, in order to obtain the best prediction of travel costs,
traffic flows and emissions. Some researchers have tested the sensitivity of link travel time, flow parameters, traffic signal (for example, the green interval and cycle length) and degree of saturation (Roess [2]; Qi et al., [3]).

The UEA problem has been extensively studied. The basic problem is to find the link flows given the origin-destination trip rates, the network, and the link performance functions (Sheffi [4]). Recently, Bar-Gera [5] presented a review of several algorithms for solving this problem; the algorithms seek efficiency, speed and equitable distribution of paths flow. However, a large amount of research and UEA’s commercial software use just one travel time function, the same for all the arcs of the network. Many works have used the UEA problem with the BPR function (function proposed by U.S. Bureau of Public Roads), due to its simplicity and mathematical properties (Kuang et al., [6] and others). Some software use travel time functions which are time dependent but sub-estimate the delay on arcs, because they do not consider overflow periods (PTV AG [7]).

Yang et al. [8] presented a study on impedance functions for UTN, for both continuous and discontinuous ways. Those functions are different to the presented in this paper. Furthermore, although they proved their functions on a traffic assignment model, did not analytically work on the UEA problem. Other researchers recognize the need of real link travel time functions and the trouble for obtaining them. Gastaldi and Rossi [9] estimate link travel time by using field-data taken from signaling devices and vehicle detection, through experiments with a simulator driver; those functions were compared with travel time information estimated by loop detectors.

This paper proposes the use of the UEA with travel time functions which came from the traffic flow theory. These functions can describe the delays on limited access roads (freeway) and light signalized arterials; they are dependent on traffic flow and congestion time, and on other physical and operating characteristics of arcs. The UEA problem is solved by the Frank-Wolfe algorithm (F-W), which under convexity conditions ensures a unique solution. The obtained link flow is very similar to the current flow; the proposed travel time functions are better than other functions for representing the performance of an urban transportation network (UTN).

The rest of the paper is formed as follows. First, the UEA problem is described; then the objective function is reformulated to incorporate the variability of travel time functions, according the operational characteristics of arcs. Then, some flow dependent delay functions, which describe the link delay components for limited access arcs and light signalized arcs, are presented. Subsequently, the F-W algorithm is described, and applied to a case study, using the proposed functions and other functions, in order to compare results. Then, results area analyzed. Finally conclusions and references are presented.

2. User equilibrium assignment problem

The UEA problem is to estimate the flows on each arc, such that each user minimizes its costs according to traffic conditions. It is assumed that travelers have perfect information about the conditions of all possible routes and want to minimize their travel times or costs. In equilibrium solution, all used routes offer the same travel time and no traveler will improve travel time by unilaterally changing his/her route (Wardrop cited by Sheffi [4]).

The UEA problem was mathematically formulated by Beckmann as a convex problem with an objective function for fixed and variable demand, subject to linear constraints and non-negativity variables (Sheffi [4]). Later, Jorgensen proposed an optimization formulation with network equilibrium conditions, fixed demand and where cost functions are separable arc (Boyce et al., [10]).

Let be a directed network \( G = (N, A) \) (N: set of nodes and A: set of arcs) and \( C \subseteq N \times N \) a set of costs or travel time between origin-destination (O-D) pairs. For each \( l^{mn} \in K \) \( (l \) is a path between origin \( m \) and destination \( n \), \( f^{mn}_l \) is the flow on the path), a flow rate \( q_l \) is assigned to the network for each O-D pair. The UEA problem is to find the flow in the arc or path that satisfies the user equilibrium criterion (UE), which is achieved when all O-D pairs have been properly assigned to all routes (Sheffi [4]). This flow pattern can be obtained using the mathematical program (1) to (4), known as the Beckmann’s transformation, formulated for separable arcs (flow
on arc \( a \) only depends on travel time on arc \( a \), non on other arc flow) (Seffi [4]). We introduce a superscript \( \zeta \) on the link travel time function, which indicate the type of link.

**Nomenclature**

\( G = (N, A) \) directed network; \( N \) node set; \( A \) arc set

\( R \) set of origin nodes, \( R \subseteq N \); \( S \) set of destination nodes, \( S \subseteq N \); \( K \) set of route connecting O-D pair \( r-s \)

\( x_a \) flow on arc \( a \) or arriving rate at light signalized arc approach

\( t_a^\zeta (\cdot) \) travel time function on arc \( a \)

\( q_l \) trips on the route \( l_{mn} \) connecting pair \( m-n \) and \( l_{mn} \in K \)

\( t_a^{0,\zeta} = t_a^\zeta (0) \) travel time on arc \( a \) with \( x_a = 0 \), of type \( \zeta \); \( t_a^0 = L_a/v_f \)

\( L_a \) length of arc \( a \); \( v_f \) : maximum speed

\( u_{rs}^n \) minimum travel time route between O-D pair \( r-s \) at the \( n \)th iteration

\( d_a^1 \) uniform delay component of flow entering on arc \( a \), s/veh

\( d_a^2 \) overflow delay component of flow entering on arc \( a \), s/veh

\( Q_a \) capacity of intersection approach \( Q_a = S_a \left( g_a/C_a \right) \) or capacity of arc \( a \), veh/h

\( C_a \) cycle length, s; \( g_a \) : green time, s

\( S_a \) saturation flow on arc approach, veh/h-green

\( X_a \) volume to capacity ratio, \( v_a/Q_a \), or degree of saturation \( x_a/Q_a \)

\( T_f \) analysis period or overflow time

\( A_a \) Akcelik’s parameter for arc \( a \)

\( \alpha_a, \beta_a \) BPR function’s parameters

\( u_{rs}^n \) minimum travel time route connecting pair \( r-s \) in the \( n \)th iteration of the F-W algorithm

\( \delta^l_{a^l, \cdot} \) \( \begin{cases} 1 & \text{if link } a \text{ of type } \zeta \text{ is on route } l \\ 0 & \text{otherwise} \end{cases} \)

\( \zeta \) define the type of arc \( \begin{cases} \text{LA if the arc is limited access type} \\ \text{LS if the arc is light signalized type} \end{cases} \)

\[
\min z(x) = \sum_a \int_0^{x_a} t_a^\zeta (w) dw
\]

Subject to:

\[
\sum_l f^l_k = q_l, \quad \forall l
\]

\[
f^l_k \geq 0, \quad \forall l, k
\]

\[
x_a = \sum_k f^l_k \delta^l_{a_k}, \quad \forall a \in A
\]
Flow on an arc is a nonnegative vector \( f = (x_a)_{a \in A} \) describing the rate of traffic on each arc. The link delay functions or travel time \( t_a^\zeta (\cdot) \) of the arc \( a \) type \( \zeta \), expresses the time required to traverse arc \( a \) with flow \( x_a \) per hour. This cost function is non-decreasing and nonnegative (Sheffi [4]). The objective function \( z(x) \) is the sum of the integrals of the travel time functions of arcs, \( t_a^\zeta (\cdot) \).

In section 4 are presented new link travel time functions (TTF), which are combinations of known delay functions. The proposed TTF are useful for modeling two types of arc: light signalized arcs and limited access arcs. These TTF are used for formulating and solving the User Equilibrium Assignment (UEA) problem by means the Frank-Wolfe algorithm. This algorithm solves the UEA problem when objective function (1) is strictly convex.

3. Time travel function for arterials and urban freeways, and delays dependent on traffic flow and overflow times

Any main urban road network may include light signaled arterial corridors and limited access roads. In a macroscopic analysis of urban traffic patterns, travel time function corresponds to the arc operation type and should include three types of delay: travel time under the actual speed, light signal delay and overflow delay. Travel time under the actual speed can be expressed as the sum of free flow travel time and the flow dependent delay on the arc. The light signal delay is composed of uniform delay or fixed delay, and overflow delay (Roess et al. [2]). The components of travel time function are the following (Roess et al. [2]):

- Free flow travel time, \( t_a^0 \), is travel time at limit speed or length-maximum speed ratio.
- Control delay is defined for light signal and STOP device. This includes time-in-queue delay plus the acceleration-deceleration delay component.
- Fixed delay or uniform delay \( d_a \) is defined for arrival and departure of vehicles on green signal, without accruing no-delay.
- Random delay is the additional delay ranging above and beyond uniform delay on short intervals time.
- Overflow delay, \( d_a^2 \), is the additional delay that occurs when the capacity of individual phase or series of phases is lower than the demand.

This paper adopts overflow delay for both, light-signalized arterial and limited-access road, which are dependent on vehicular volume and congestion time (assuming that delay is lower than one hour). It is considered the adjustment factor for including the impact of platoon on uniform delay. Many current models combine random and overflow delays into a single function referred as overflow delay (Roess et al. [2]).

Analytical models could use these three delay component for each arc type, and a deterministic approach based on mean values, such as (5):

\[
t_a^\zeta (x) = t_a^0 + d_a^1 + d_a^2
\]

We reviewed some travel time functions used in traffic flow theory (appropriate for the UEA problem), which include the delays of (5): Webster’s delay, Akcelik’s function, HCM’s 2000 control delay and BPR’s function.

3.1. Webster’s uniform delay

This function was proposed by Webster in 1958 and is the average of the vehicles that both arrive in red phase and depart in green phase signal; it is expressed in [s/veh], such as (6) (Roess et al. [2]):
During the oversaturation period, the uniform delay is constant because \( v/c \) ratio takes the maximum value of 1.00. Then, Webster proposed the overflow delay model (UD) \( d_{a}^{2} \) when \( x_{a}/Q_{a} \geq 1 \), such as (7) (Roess et al. [2]):

\[
UD_{Webster} = 0.5C_{a} \left[ 1 - \frac{g_{a}}{C_{a}} \right] + \frac{T_{f}}{2} \left[ X_{a} - 1 \right]
\]  

(7)

3.2. Akcelik’s travel time function

The function proposed by Akcelik [11; 12] was developed for the analysis of signalized intersections, to estimate the flow-dependent delay for both limited access arcs and arcs controlled by traffic lights, under oversaturation condition \( x_{a}/Q_{a} \geq 1 \) and non-saturated condition \( x_{a}/Q_{a} < 1 \), such as (8):

\[
t_{a}(X_{a})_{Akcelik} = \frac{1}{4}T_{f} \left[ (X_{a} - 1) + \sqrt{(X_{a} - 1)^{2} + \frac{8A_{k}}{Q_{a}T_{f}}}X_{a} \right], \text{[s/km]}
\]  

(8)

3.3. BPR’s travel time function

The BPR travel time function was proposed by U.S. Bureau of Public Roads (1964). This has been used in the cost function of the UEA problem, for macroscopic analysis of UTN, because of its good mathematical performance: continuous, differentiable and increasing. This includes free flow travel time and other term which are increasing according flow to capacity ratio \( x_{a}/Q_{a} \), such as (9) (Sheffi [4]):

\[
t_{a}(x_{a})_{BPR} = t_{a}^{0}\left[ 1 + \alpha_{a}\left( \frac{x_{a}}{Q_{a}} \right)^{\beta_{a}} \right]
\]  

(9)

3.4. HCM’s 2000 control delay

The control delay of the HCM 2000 is applied for light signalized intersection approach, such as (10) (Roess et al. [2]):

\[
t_{a}(X_{a}) = d_{a}^{1}PF + d_{a}^{2} + d_{a}^{3}
\]  

(10)

Where \( d_{a}^{1} \) is Webster’s uniform delay on light signalized intersection approach of the arc \( a \) (6), and \( d_{a}^{2} \) is overflow delay component, such as (11):

\[
d_{a}^{2} = 900T_{f} \left[ (X_{a} - 1) + \sqrt{(X_{a} - 1)^{2} + \frac{8kIX_{a}}{Q_{a}T_{f}}} \right]
\]  

(11)
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PF: progression adjustment factor.
k: incremental delay factor for actuated controller setting; 0.50 for all pre-timed controllers.
l: upstream filtering/metering adjustment factor; 1.00 for all individual intersection analyses.

3
ad : delay due to pre-existing queue [s/veh].

HCM’s 2000 delay model and Akcelik’s delay model could be used for formulating piecewise continue cost functions for the UEA model.

4. Webster’s and Akcelik’s delay models on the user equilibrium assignment problem

We designed a new link travel time function by means of the combination of two of the above described delay functions, and then formulate the objective cost function of the UEA problem (equation 1), for a UTN which contains light signalized arterials and limited access roads. In order to facilitate the computer program, we include a multiplicative 1-0 variable, for differentiate if the delay is for limited access road or light signalized arterial. The designed travel time function is appropriated for both types of arc, and it is convex, non-decreasing, non-negative and piecewise continues, as such (12):

\[ t^*_a(x) = t^0_a + \phi d^1_a + d^2_a, \text{Akcelik} \]

Then, substituting (6) and (8) in (12), results (13):

\[ t_a(X_a) = t^0_a + \phi \left( \frac{C_a}{2} \right) + \left( \frac{1 - \frac{g_a}{C_a}}{1 - \frac{g_a}{C_a}} \right) + \frac{1}{4} T_f \left( X_a - 1 \right) + \frac{8A}{Q_a T_f} X_a \]

\[ \forall X_a \in \left( 0 < X_a < 1 \right) \cup (x \geq 1) \]

Where:

\[ \phi = 1 \quad \text{if arc is light signalized type (LS)} \]
\[ \phi = 0 \quad \text{if arc is limited access type (LA)} \]

Then (13) is replaced in (1), and results (15):

\[ z(x) = \sum_a \int_0^{t^*_a} \left[ t^0_a + \phi \left( \frac{C_a}{2} \right) + \left( \frac{1 - \frac{g_a}{C_a}}{1 - \frac{g_a}{C_a}} \right) + \frac{1}{4} T_f \left( X_a - 1 \right) + \frac{8A}{Q_a T_f} X_a \right] dw \]

\[ \forall X_a \in \left[ 0 < X_a < 1 \right] \cup (x \geq 1) \]

The objective function (1), z(x), is the sum of integrals of the \( t^*_a(X_a) \) for each link type. The objective function z(x) in (15) is convex, non-decreasing, non-negative, and piecewise continue, and its solution is unique. Equation (15) was used, instead of equation (1), in the UEA problem. The F-W algorithm was used for solving a real case, which is described below (Process 2). Some constants were required for the dimensional analysis. The procedure is described below:

Initial step: Perform all-or nothing assignment with \( t^0_a = t_a(0), \forall a \). This yields \{ x^1_a \}. Set counter n=1.

Step 1: Update \( t^n_a = t_a(x^n_a), \forall a \) and calculate \( z(x) \) such as (16), (17) and (18)
\[ z(x) = \sum \left\{ t_a^0 x_a^n - \phi \left( \frac{C_a}{7200} \left[ 1 - \left( \frac{g_a}{C_a} \right)^2 \right] \ln 1 - \left( \frac{x_a^n g_a}{Q_a C_a} \right) + \phi B_1 \right) \right\}, \quad \forall X_a \in \left[ 0 < X_a < 1 \right] \]  

(16)

\[ z(x) = \sum \left\{ t_a^0 x_a^n + \phi \left( \frac{C_a}{7200} \left[ 1 - \left( \frac{g_a}{C_a} \right)^2 \right] x_a^n + B_1 \right) \right\}, \quad \forall X_a \in \left[ X_a > 1 \right] \]  

(17)

Where:

\[ B_1 = \begin{cases} r_a \left( \frac{x_a^n - 1}{Q_a} \right)^2 - r_s + r_a \left( \frac{x_a^n}{Q_a} - s_a \right) + a_s^2 + r_s a_s^2 \ln \left( 1 - \frac{x_a^n}{Q_a} - s_a + a_s^2 + \left( \frac{x_a^n}{Q_a} - s_a \right) \right) \end{cases} \]

\[ + r_s s_a - r_s a_s^2 \ln \left( 1 - s_a \right) \]

\[ r_a = \frac{T_f Q_a}{8} ; \quad s_a = \left[ 1 - \left( \frac{4A_a}{Q_a T_f} \right) \right] ; \quad a_s^2 = \left[ \frac{4A_a}{Q_a T_f} \left( 2 - \frac{4A_a}{Q_a T_f} \right) \right] \]  

(18)

Step 2: Direction finding. Perform all-or-nothing assignment with base on \{ t_a^0 \}. This yields a set of auxiliary flows \{ y_a^n \}.

Paso 3: Line search \( \lambda \) that solves (19):

\[ \min_{0 \leq \lambda \leq 1} \sum_a \int_0^{\lambda a + \lambda (y_a^n - x_a^n)} t_a(\omega) d\omega \]  

(19)

Then, found \( \lambda \) that minimize \( z(x_a^n + \lambda (y_a^n - x_a^n)) \), by \( dz[x_a^n + \lambda (y_a^n - x_a^n)]/d\lambda \), formulated as a continuous piecewise function, within the following ranges \( 0 < \left( \frac{x_a^n + \lambda (y_a^n - x_a^n)}{Q_a} \right) < 1 \cup \left( \frac{x_a^n + \lambda (y_a^n - x_a^n)}{Q_a} \right) \geq 1 \). The problem (19) was solved in (20) y (21) and (18):

\[ \sum_a (y_a^n - x_a^n) \left\{ t_a^0 + \phi \left( \frac{C_a}{7200} \left[ 1 - \left( \frac{g_a}{C_a} \right)^2 \right] \right) + B_2 \right\}, \quad \forall \left( \frac{x_a^n + \lambda (y_a^n - x_a^n)}{Q_a} \right) < 1 \]  

(20)

\[ \sum_a (y_a^n - x_a^n) \left\{ t_a^0 + \phi \left( \frac{C_a}{7200} \left[ 1 - \left( \frac{g_a}{C_a} \right)^2 \right] \right) + B_2 \right\}, \quad \forall \left( \frac{x_a^n + \lambda (y_a^n - x_a^n)}{Q_a} \right) \geq 1 \]  

(21)

Where, \( B_2 = \left\{ \left( \frac{x_a^n + \lambda (y_a^n - x_a^n)}{Q_a} \right) - 2 \right\} + \frac{2 t_a}{Q_a} \left\{ \left( \frac{x_a^n + \lambda (y_a^n - x_a^n)}{Q_a} \right)^2 + a_s^2 \right\} \]

Then, \( \lambda \) is founded in (20) y (21), by the bisection algorithm.
Step 4: Move, set 
\[ x_a^{n+1} = x_a^n + \lambda_a(y_a^n - x_a^n), \quad \forall a \]  
(22)

Step 5: Convergence test. Review convergence by means (23). If a convergence criterion is met, stop and the current solution \( x_a^{n+1} \) is the set of equilibrium link flows; otherwise, set \( n=n+1 \) and go to step 1.

\[ \sum_{rs} \frac{|u_{rs}^n - u_{rs}^{n-1}|}{u_{rs}^n} \leq k, \quad \text{for example } k = 0.001, \]  
(23)

We also propose another piecewise continuous travel time function, \( t^*_a(x) \), and use it in the case study (Process 1). This function combines Webster’s fixed and overflow delays, for light signalized arcs, and BPR function for limited access arcs, as such (24):

\[ t_a = t_a^0 + \phi \left( \frac{C_a}{2} \right) \left[ \frac{(1 - g_a/C_a)^2}{1 - \min(1, x_a)*g_a/C_a} \right] + \phi \frac{T_f}{2} [X_a - 1](1 - \phi)\tau_a^0 \left( \alpha_a \left( \frac{x_a}{Q_a} \right)^\beta_a \right) \]

\[ \forall X_a \in [0 < X_a < 1) \cup (x \geq 1)] \quad \text{and} \quad T_f < 4 \left[ t_a^0 + \left( \frac{C_a}{2} \right) \left( 1 - \left( \frac{g_a}{C_a} \right) \right) \right] \]  
(24)

5. Case study

The UEA mathematical program was formulated for three different network loading processes and proved on a sub-network of Mexico City consisting of two corridors connecting one pair O-D, a light signalized arterial and an urban freeway, by means the F-W algorithm. The difference for type of arc is in the summands of delay that make the equation (5) and making up the UEA problem objective function \( z(x) \). We define the difference by the following processes:

Process 1. The UEA mathematical program was formulated with \( t(x_a) = t_lf + d_{a1}^{LS-Webster} + d_{a2}^{2-LS-Webster} + d_{a2}^{2-LA-BPR} \), equation (24). The analytical expressions of the five steps of the FW algorithm were solved but are omitted due to lack of space.

Process 2. The UEA mathematical program was formulated with \( t(x_a) = t_lf + d_{a1}^{LS-Webster} + d_{a2}^{2-LS-Webster} + d_{a2}^{2-LA-Akcelik-SC} + d_{a2}^{2-LA-Akcelik} \), equations (12) to (14). The analytical expressions of the five steps of the FW algorithm were solved, equations (16) to (23).

Process 3. The UEA mathematical program was formulated with \( t(x_a) = f_{BPR} \) for all arcs and the solution is well known.

The application sub-network has 32 different arcs and 32 nodes. The assigned demand was 11,500 trips/h, and the overflow time (Tf) was 15 minutes.

Results, shown in Table 1, indicate that flow-dependent functions are increasing piecewise continuous and appropriate for the UEA problem. The F-W algorithm found equilibrium for the three processes. Parameters of all of functions were calibrated. The more realistic estimated times were obtained by means process 2. Process 3 was performed in order to make comparisons.

The scope of this test was to compare the performance of the travel time functions in the F-W algorithm. The lower cost in the network was obtained by using the unique BPR function, since this function ignore the overflow fixed delay due to traffic lights. The better performance (closer to actual flow) was obtained by using the proposed combined Webster-Akcelik function (process 2).

The case study analysis includes the sensibility test of the congestion time variable (T), for the travel time on light signalized arcs and limited access arcs, which is not usually included in the flow estimation and route choice
of traffic load processes from commercial software. This allows obtain best furcating without need of simulation of delays on lanes or temporal congestion propagation.

Table 1: Summary of results of the Mexico City Case Study

<table>
<thead>
<tr>
<th>Travel time function</th>
<th>( z(x_a) ) (h)</th>
<th>Route flow on light signalized arterial (veh/h)</th>
<th>Route flow on freeway (veh/h)</th>
<th>Route time on light signalized arterial (h)</th>
<th>Route time on freeway (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: ( t(x_a) = t_{ff} + d_{a1}^{LS-Webster} + d_{a2}^{LS-Webster} + d_{a2}^{LA-BPR} )</td>
<td>1648</td>
<td>277</td>
<td>11223</td>
<td>0.205278</td>
<td>0.205275</td>
</tr>
<tr>
<td>2: ( t(x_a) = t_{ff} + d_{a1}^{LS-Webster} + d_{a2}^{LS-Akcelik} + d_{a2}^{LA-Akcelik} )</td>
<td>2697</td>
<td>4624</td>
<td>6876</td>
<td>0.544863</td>
<td>0.543624</td>
</tr>
<tr>
<td>3: ( t(x_a) = f_{BPR} )</td>
<td>1491</td>
<td>3428</td>
<td>8072</td>
<td>0.151746</td>
<td>0.151729</td>
</tr>
</tbody>
</table>

6. Conclusion

This paper designed and tested two new link travel time functions (TTF), which are combinations of well known delay functions. These new functions are useful for macroscopic traffic analysis on urban transportation networks, where two types of arcs exist: light signalized arcs and limited access arcs. These TTF were used for formulating and solving the User Equilibrium Assignment (UEA) problem by means the Frank-Wolfe algorithm (F-W).

The proposed functions are dependent on the type of arc, by means a 1-0 variable (it stands for light signalized arc and controlled access arc).

The combined Akcelik-Webster function (process 2) resulted very accurate for modeling a network with light signalized arcs and limited access arcs. This functions is composed as follows: i) free flow travel time plus overflow Akcelik’s delay, which describe overflow delay dependent on both time and vehicular flow, for limited access arcs; and ii) free flow travel time plus uniform Webster delay and overflow Akcelik’s delay, which describe overflow delay dependent on both time and vehicular flow, for lights signalized arcs.

This composed function presents suitable characteristics to be used for solving the UEA problem, by means the F-W algorithm, and produce a more accurate estimation of travel time and flows.

Hence, this paper introduced better functions and improved the applying of the UEA process on urban networks, where usually the problem is solved for only one travel time function (the same for all types of arc) or by flow-speed curves, or overflow periods are not considered producing a sub-estimation of the delay on arcs. The application was performed on a large sub-network of Mexico City.

Other authors have recommended travel time functions and have used the UAE process (Dowling [13] and others), but they consider separated conditions, one for freeways and another one for arteries, using flow-speed curves. These curves cannot estimate travel time in temporary overflow condition.

In addition, this work is very different from previous ones, as it uses combinations of travel time functions to formulate the UEA problem, by a delay model suitable for both light signalized arcs and limited access arcs.

The methodology used in this work includes the formulation of a mathematical programming problem (UEA) with a convex objective function and linear constraint, which is solved by using an algorithm that finds minimal critical point. We analytically solved the mathematical program discreetly through a C# program. The parameters
of the travel time functions were incorporated into the model as attributes of the arc (length, capacity, green split, operative characteristics, etc), providing a flexible, accurate, efficient and transferable model. The obtained results from the case study (on a real network) shown that the algorithm had good performance with these combined cost functions (functions have good mathematical properties). The objective cost functions and others functions in the algorithm F-W were piecewise defined and their limit parameters were specified. This represents a contribution to the development on transportation planning software.

References


