Calculation of Wall Temperature for Aircraft Exhaust System with Considering Gas Radiation Heat Transfer

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Abstract

The coupled numerical simulation of flow field, solid temperature field, species concentration field and gas radiation transfer/energy field based on statistical narrow-band correlated-$\kappa$ (SNBCK) model, is employed to accurately predict aerothermodynamic characteristic of aircraft exhaust system. A series of methods to increase computational efficiency and descend computational resources make it possible to finish the calculation in PC. The parameters of narrow-band model are evaluated by HITEMP line-by-line database. Three examples have proved the accuracy of using these methods to solve flow heat transfer coupled problem and radiation transfer/energy equation, which are the calculation of temperature distribution of water-cooling nozzle in rocket engine, the calculation of carbon dioxide absorbptivity at 4.3 micron band, and the gas radiation heat transfer evaluation of the cylindrical furnace. Finally, the inner flaps temperature distribution of ejecting nozzle with floating outer flaps is computed, under high-altitude, high-speed and afterburning conditions. Two completely different air-inlet schemes of ejecting channel almost achieve the same effect in cooling inner flaps.

Keywords: numerical simulation; gas radiation; exhausting nozzles; coupled model; statistical narrow-band correlated-$\kappa$ model

1. Introduction

Many factors affect the solid wall temperature distribution in aircraft exhaust system. Besides gas convective heat transfer and solid wall heat conduction, radiation heat transfer between solid wall and high-temperature gas (a typical kind of selective radiation absorption/emission medium on after-burning conditions) cannot be neglected. These different heat transfer modes coexist during the working process of aircraft exhaust system, affecting and restricting each other. Therefore, only by developing a set of computational fluid dynamics (CFD) programs to conduct coupled simulation for the physical phenomenon described above, can we accurately predict the temperature distribution of various components in aircraft exhaust system.

There are various models describing the characteristics of gas absorption and radiation. Line-by-line model[1] is accurate, but it is only used to do verified computations for other models because of its huge computation burden. Wide-band model[2] is often adopted for gas radiation heat transfer, computing dozens of spectral bands. Statistical narrow-band (SNB) model[1], which needs to compute 200-2,000 spectral bands, is often used to calculate gas infrared radiation signal. The parameters of narrow-band model are evaluated by HITEMP line-by-line database[3-4]. In most domestic or foreign references of the recent years, radiation transmission/heat transfer is calculated on condition that distribution of other physical quantities has been known[5-6], coupled model is hardly used. Commercial CFD software usually adopts weighted sum of gray gases model (WSGGM)[7] with lower accuracy to calculate gas radiation. The reason lies in two aspects: first, the computation burden of using wide/narrow-band model with high accuracy is heavy; second, the traditional wide/narrow-band model could just provide gas transmittance within certain optical depth, not the gas absorption coefficient needed for finite volume method (FVM)[1]. This problem can be solved by wide/narrow-band correlated-$\kappa$ model at present[8-9].

The article is organized as follows. In Section 2, to accurately predict nozzle aerothermodynamic characteristics, FVM coupled solution is employed for N-S equations, heat conduction equations, species transport equations and gas radiation transfer/energy equations based on statistical narrow-band correlated-$\kappa$ (SNBCK)
model. Then, to guarantee that the computation can be performed by a common 32 bites PC, in Section 3, a series of algorithms is used in this program to optimize calculation efficiency and computer resource needed. In Section 4, three cases have respectively verified the accuracy of using the above-mentioned methods to calculate flow heat transfer coupled problems and radiation transfer/heat exchange problems. In Section 5, under high-altitude, high-speed afterburning conditions, the inner flaps temperature distribution of ejecting nozzle with floating outer flaps is computed.

2. Mathematical Models

2.1. Model of flow heat transfer coupled problem

In this part, finite volume method and second-order Roe’s scheme\(^1\) is employed to discretize Eq.(1).

\[
\frac{\partial}{\partial t} \int_V \rho Q dV + \oint_S (\mathbf{F} - \mathbf{F}_w) dS = \oint_S \mathbf{G} dV
\]  

(1)

Eq.(1) is a 3D generalized N-S equation\(^1\) in conservation form, where \(V\) and \(S\) are volume and surface area of grid cell respectively, \(\mathbf{Q}\) and \(\mathbf{F}\) are conserved vector and convective-diffusive flux respectively, \(\mathbf{F}_w = \begin{bmatrix} 0 & 0 & 0 & q_w + q_{nw} \\ 0 & 0 & 0 & q_w \\ 0 & 0 & 0 & q_{ng} \end{bmatrix}\), \(q_w\) and \(q_{nw}\) is wall heat flux, \(\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & q_{ng} \end{bmatrix}^T\), \(q_{ng}\) is gas radiation heat flux.

The turbulence model employs renormalization group (RNG) \(k-e\) two-equation model and standard wall function\(^1\).

The discrete form of 3D solid heat conduction equations is as follows:

\[
a_{w,j,k} T_{i-1,j,k} + a_{e,j,k} T_{i,j-1,k} + a_{s,j,k} T_{i,j,k} + a_{d,j,k} T_{i,j,k+1} = 0
\]  

(2)

At present, the third boundary condition is employed to treat flow field/solid temperature field coupled problems in most domestic or foreign references. The reason is that directly transferring interface temperature and heat flux will cause computational instability or even divergence\(^1-4\).

In Fig.1, \(T_{i-1}, T_{i}, T_{i+1}\) are unknown temperatures of solid grid cell. \(T_0\) and \(T_{n+1}\) are temperatures of fluid in near wall grid cell. The matrix form for 1D flow heat transfer coupled problem is

\[
\begin{bmatrix}
\epsilon_0 + a & -a & \\
-a & 2a & -a & \\
\vdots & & \ddots & \\
-a & 2a & -a & \epsilon_0 + a
\end{bmatrix}
\begin{bmatrix}
T_0 \\
T_1 \\
\vdots \\
T_n \\
T_{n+1}
\end{bmatrix}
= \begin{bmatrix}
b_0 \\
0 \\
\vdots \\
0 \\
b_{n+1}
\end{bmatrix}
\]  

(3)

where \(\epsilon = 1/(1/h + \Delta x/2\lambda)\), \(b_0 = \epsilon_0 T_0\), \(b_{n+1} = \epsilon_{n+1} T_{n+1}\), \(\lambda = \lambda / \Delta x\), \(\lambda\) is thermal conductivity, \(h\) the convective heat transfer coefficient between fluid and solid wall, and \(q_w = \epsilon_q (T_i - T_0)\).

Fig.1 Sketch map of computation meshes of 1D flow heat transfer coupled problem.

2.2. Radiation transfer and energy equation

Eq.(4) is gas radiation transfer equation discretized by FVM\(^1\).

\[
\int_{\Omega^r} \int_S \left( \frac{k}{\pi} E_{b,\nu} - k_{v,\nu} I_{v,\nu} \right) dV d\Omega = \int_{\Omega^r} \int_S \left[ \sum_{m} k_{m,\nu} I_{v,\nu} d\Omega \right] dV
\]  

(4)

where \(I\) is radience, \(k\) the gas absorption coefficient, \(\mathbf{n}\) the external normal unit vector of grid cell surface, \(s\) the radiation direction, \(\Omega\) the radiation direction spatial angle, \(E_b\) the black body emittance, and subscript \(\nu\) the radiation wave number.

Gas and solid wall radiation heat flux are obtained by solving radiation energy equations\(^1\), which are coupled with Eq.(1) as energy source terms.

\[
\dot{q}_{lw} = \int_{0}^{\infty} \left( -4k_{v,\nu} E_{b,\nu} + \sum_{m} k_{m,\nu} I_{v,\nu} d\Omega \right) d\nu
\]  

(5)

where \(\epsilon\) is radiation emissivity.

2.3. Model of gas radiation

HITEMP database is an upgrading version of HITRAN (the parameter database of high resolution atmosphere absorption lines), which is edited by geo-physics laboratory of American Air Force\(^1\). According to the parameters of gas spectral lines from HITEMP database, \(\bar{b}_g\) (the average gas absorption coefficient of band for SNB model) and \(\bar{b}_s\) (the average interval of spectral lines) can be obtained through Young numerical average method. Details could be found in Refs.[3]-[4]. \(\bar{b}_g\) (the average semi-width of fuel gas spectral lines) is evaluated by using Eq.(6)\(^1\).
$$\bar{F}_{CO_2} = \frac{p}{p_{\text{ref}}} \left( T_{\text{ref}} \right)^{0.7} \left( 0.058 + 0.012 x_{CO_2} + 0.042 x_{H_2O} \right)$$

$$\bar{F}_{H_2O} = \frac{p}{p_{\text{ref}}} \left[ 0.462 x_{H_2O} T_{\text{ref}} + (T_{\text{ref}})^{0.4} \right]$$

$$T_{\text{ref}} = 296 \text{ K}, p_{\text{ref}} = 101325 \text{ Pa}$$

(6)

The basis of correlated-k model is that for any radiation quantity $\phi$, which is solely dependent on $k$, integration over $\nu$ can be replaced by integration over $k$.

$$\frac{1}{\nu} \int \phi(k_i) d\nu = \int \phi(g^{-1}) dg$$

(7)

where $g(k)$ is the cumulative distribution function about $k$. The absorption rate for Malkmus line strength distribution SNB model can be calculated by\(^9\)

$$\alpha = 1.0 - \exp \left[ -2 \frac{\bar{F}_{c}}{\bar{F}} \left( \sqrt{1 + \bar{F}_{c} p x L \bar{F}_{\text{ref}} d_{\nu}} - \bar{d}_{\nu} - 1 \right) \right]$$

(8)

By performing Laplace transform on Eq.(8), the expression of $g(k)$ can be obtained\(^9\). Eq.(7) is evaluated using four points Gauss-Legendre integral\(^{16}\). Absorption coefficient of gas mixture under the same band is the sum of each species absorption coefficient. These two methods of approximate treatment have little effect on calculation accuracy\(^{9,17}\).

3. Calculation Methods

Multi-grid accelerating lower-upper symmetric Gauss-Seidel (LUSGS) implicit time marching method is employed in the solution of Eq.(1).

Transport equation of two-equation turbulence model is only performed on fine grid and Harten flux modification total variation diminishing (TVD) explicit algorithm ("stiffless" for short) is used to overcome the stiffness\(^{18}\).

Bi-conjugate gradient stabilized method (BI-CGSTAB)\(^{19}\) algorithm with precondition matrix is applied to the solution of solid heat conduction equation. This algorithm, based on Galerkin principle, has high efficiency, low memory occupation and favorable computational stability\(^{20}\).

BI-CGSTAB algorithm with LUSGS precondition is adopted to solve Eq.(4). Then, radiation heat flux of gas and solid wall is obtained from Eq.(5).

3.1. Solution of ill-conditioned matrix in flow heat transfer coupled problem

The matrix form of Eq.(2) is

$$AT = B$$

(9)

For the high efficiency/stability of BI-CGSTAB algorithm, pretreatment on matrix $A$ must be done to improve its condition number.

$$M^{-1}_{L} AT = M^{-1}_{L} B \quad \Leftrightarrow \quad AT = B$$

$$AM^{-1}_{R} MB = T \quad \Leftrightarrow \quad AT = B$$

(10)

where $M^{-1}_{L}$ and $M^{-1}_{R}$ are left and right pretreatment matrices respectively. In the process of using BI-CGSTAB algorithm to solve flow heat transfer coupled problem, when the solid region is quite thin and heat conductive coefficient is very high, $a$ will be much higher than $h$, thereby $\varepsilon \approx h << a$ in Eq.(3) (The calculation accuracy is optimal when near wall grid’s $y’ \approx 30\(^{21}\)$ and $\varepsilon = 0.001a$ order). The left side in Eq.(3) is a typical ill-conditioned matrix with high condition number\(^{16}\). The commonly used preconditioned matrices are LUSGS pretreatment and incomplete lower-upper decomposition (ILU(0)) pretreatment\(^{22}\), both of which are based on approximate LU factorization of matrix. In reality, neither of them has the capability to overcome the ill-conditioned matrix.

$$M_{L} = \left(D + Z\right)^{-1} \left(D + Y\right)^{-1} \left(D + X\right)^{-1}$$

which is based on Brian alternating direction implicit (BADI) algorithm\(^{23}\), is the left preconditioned matrix used in this article. In it, coefficient matrix $X$ consists of $a_{i,j,k}$ and $a_{i,j,k}$; $Y$ consists of $a_{i,j,k}$ and $a_{i,j,k}$; $Z$ consists of $a_{i,j,k}$ and $a_{i,j,k}$; Diagonal matrix $D$ consists of $a_{i,j,k}$; $A = D + X + Y + Z; (D + X)^{-1}$; $(D + Y)^{-1}$ and $(D + Z)^{-1}$ are three tridiagonal matrixes. The product of them and the vector can be solved through chasing method\(^{18}\).

This preconditioned matrix can effectively reduce condition number in $A$, because ADI algorithm solves Eq.(10) accurately in the same direction. That means ADI algorithm can transform matrix $A$ into unit matrix $E$ in 1D problem.

3.2. Treatment of symmetry plane in radiation transfer equation

The calculation program in the current article treats radiation transfer symmetry plane as a mirror surface (follow mirror-reflection law). Symmetry plane and non-blackbody solid wall (diffuse reflection) are both boundary conditions in solving Eq.(4). Their incident radiance from a certain spatial angle $\Omega^{*}$ is obtained by iterations.

4. Verification Cases

The program is compiled on Microsoft VC++ 6.0 and operated on Intel Core2 Duo E8200 CPU (the master frequency is 2.66 GHz).

The computational reliability and computed-mesh independence, using the program for the simulations of flow and temperature fields, have been validated\(^{11}\).
4.1. Calculation of wall temperature distribution for water-cooling nozzle in rocket engine

As shown in Fig.2, the designed nozzle pressure ratio (NPR) of the axisymmetric nozzle \(^{[24]}\) is about 17, calculation NPR is 5.1, and inlet total temperature is 843.33 K. The outer wall of nozzle is cooled by running water, whose temperature distribution along the Z-axis is regarded as known parameter provided by test (see Fig.3). The comparison between prediction about temperature distribution of inner wall of nozzle and experiment measurement is satisfactory (see Fig.4).

![Fig.2 Mach number and temperature distribution of water cooling nozzle.](image)

![Fig.3 Temperature distribution of outer wall of nozzle.](image)

4.2. Numerical simulation of radiation heat transfer characteristics of cylindrical furnace wall

The furnace is a closed cylinder of 6 m in length and 2 m in diameter. The furnace wall is a black-surface whose temperature is 500 K. Temperature distribution of gray gas in furnace \(^{[25]}\) can be seen in Fig.5. Computational grids are very rough in this case and resolution of spatial angle, employed in solving radiation transport equations, only reaches 10×10. However, comparing the calculated results of furnace wall radiation heat flux with the theoretical data \(^{[25]}\), the error is acceptable (see Fig.6), which also shows that the treatment of the radiation symmetric plane is accurate and effective.

![Fig.4 Comparison between computation result and experimental data for temperature distribution of inner wall of nozzle.](image)

![Fig.5 Temperature distributions of furnace gas and radiation computation meshes.](image)

![Fig.6 Comparison of furnace wall radiation heat transfer between calculated results and theoretical ones with different gas absorption coefficients \(k\).](image)
4.3. Calculation of gas spectral absorption coefficient with SNBCK model

Databases of the average gas absorption coefficient \( \kappa_\nu (T) \) (atm\(^{-1}\)·cm\(^{-1}\)), 1 atm = 101.325 kPa and the average interval of spectral lines \( \Delta \nu = f(T) \) (cm\(^{-1}\)) for each band of SNB model are calculated by using methods in Section 2.3. The results of H\(_2\)O (vapor) listed in Table 1 are completely consistent with those in Ref.[4], while comparison between the results of CO\(_2\) in Table 2 and those in Ref.[3] shows that there is some discrepancy at high temperature (1 500 K).

### Table 1 SNB model parameters for H\(_2\)O 2.7 micron band

<table>
<thead>
<tr>
<th>( \nu/\text{cm}^{-1} )</th>
<th>( T = 500 \text{ K} )</th>
<th>( T = 1 000 \text{ K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \kappa_\nu )</td>
<td>( 1/\Delta \nu )</td>
</tr>
<tr>
<td>3 300</td>
<td>0.010 6</td>
<td>2.257 0</td>
</tr>
<tr>
<td>3 350</td>
<td>0.018 7</td>
<td>2.468 9</td>
</tr>
<tr>
<td>3 400</td>
<td>0.034 7</td>
<td>2.397 0</td>
</tr>
<tr>
<td>3 450</td>
<td>0.059 9</td>
<td>2.006 3</td>
</tr>
<tr>
<td>3 500</td>
<td>0.189 8</td>
<td>1.744 0</td>
</tr>
<tr>
<td>3 550</td>
<td>0.318 5</td>
<td>1.369 2</td>
</tr>
<tr>
<td>3 600</td>
<td>0.479 6</td>
<td>0.826 6</td>
</tr>
<tr>
<td>3 650</td>
<td>0.385 9</td>
<td>1.036 6</td>
</tr>
<tr>
<td>3 700</td>
<td>0.671 6</td>
<td>1.394 1</td>
</tr>
<tr>
<td>3 750</td>
<td>0.475 6</td>
<td>0.920 0</td>
</tr>
<tr>
<td>3 800</td>
<td>0.386 2</td>
<td>0.685 7</td>
</tr>
<tr>
<td>3 850</td>
<td>0.239 8</td>
<td>0.892 4</td>
</tr>
<tr>
<td>3 900</td>
<td>0.352 4</td>
<td>1.363 2</td>
</tr>
<tr>
<td>3 950</td>
<td>0.075 2</td>
<td>1.363 2</td>
</tr>
</tbody>
</table>

### Table 2 SNB model parameters for CO\(_2\) 2.7 micron band

<table>
<thead>
<tr>
<th>( \nu/\text{cm}^{-1} )</th>
<th>( T = 1 000 \text{ K} )</th>
<th>( T = 1 500 \text{ K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \kappa_\nu )</td>
<td>( 1/\Delta \nu )</td>
</tr>
<tr>
<td>3 450</td>
<td>0.077 5</td>
<td>115.783 2</td>
</tr>
<tr>
<td>3 500</td>
<td>0.179 9</td>
<td>74.052 5</td>
</tr>
<tr>
<td>3 550</td>
<td>0.259 3</td>
<td>51.401 3</td>
</tr>
<tr>
<td>3 600</td>
<td>0.239 8</td>
<td>58.863 5</td>
</tr>
<tr>
<td>3 650</td>
<td>0.362 3</td>
<td>57.636 4</td>
</tr>
<tr>
<td>3 700</td>
<td>0.438 5</td>
<td>32.790 8</td>
</tr>
</tbody>
</table>

Fig.7 shows comparison of calculated data using SNB model, FVM-SNBCK model and Ludwing experimental data\(^{26}\) for CO\(_2\) absorption coefficient at 4.3 micron band. The calculation conditions are: \( p = 15 900 \text{ Pa} \), \( T = 1 200 \text{ K} \), stroke length \( L = 15.05 \text{ cm} \); concentration of CO\(_2\) is 100%, and 1D computational grid number of FVM is 10.

It can be found that the first two results match each other very well. The discrepancy from experimental data is due to wider calculation band and approximation solution of the average semi-width of spectral lines.

5. Calculation Results and Analysis

The main nozzle of an ejecting nozzle is an axisymmetric convergent-divergent one. Environmental air flows into ejecting channel through an annular gap, which is the first air-inlet scheme. Outer flaps of nozzle float in the position where the moment, generated by air pressure and friction acting on hinges of flap, reaches balance. The circumferential angle covered by computational region is 72°. The total computation grid number is about 610 000 as shown in Fig.8 (black grid is the solid zone). Geometric parameters and computational boundary conditions of nozzle are shown in Table 3 and Table 4, respectively.

The computational process contains two phases. Considering the little influence of gas radiation heat transfer on the whole flow field and the huge amount of calculation in the solution of gas radiation transfer equations, coupled solution only for flow field, solid temperature field and gas species concentration field is performed in the first phase. Coarse grids modification V cycle is employed to solve Eq.(1). It iterates 10 steps on fine grids, while iterates 15 steps on coarse grids (adjacent 8 fine grids merge together) during each cycle period. The solutions for turbulence model trans-
Table 3 Geometric parameters of ejecting nozzle

<table>
<thead>
<tr>
<th>Component</th>
<th>Geometric parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-main nozzle</td>
<td>Throat diameter/mm</td>
<td>349.0</td>
</tr>
<tr>
<td></td>
<td>Wall thickness/mm</td>
<td>0.8-3.2</td>
</tr>
<tr>
<td></td>
<td>Cycle of wall thickness in circumferential direction(°)</td>
<td>24</td>
</tr>
<tr>
<td>2-heat shield</td>
<td>Height/mm</td>
<td>3.6-7.2</td>
</tr>
<tr>
<td></td>
<td>Cycle of height in circumferential direction(°)</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Wall thickness/mm</td>
<td>0.8</td>
</tr>
<tr>
<td>3-outer flaps</td>
<td>Mechanical limit of convergent angle(°)</td>
<td>0-14.5</td>
</tr>
<tr>
<td></td>
<td>Annular gap width/mm</td>
<td>80.0</td>
</tr>
</tbody>
</table>

Table 4 Computational boundary conditions of ejecting nozzle

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Boundary parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-pressure inlet of main nozzle</td>
<td>Total pressure/Pa</td>
<td>159 000</td>
</tr>
<tr>
<td></td>
<td>Total temperature/K</td>
<td>2 000</td>
</tr>
<tr>
<td></td>
<td>N₂ (mass concentration; below is the same)</td>
<td>73.38%</td>
</tr>
<tr>
<td></td>
<td>O₂</td>
<td>7.43%</td>
</tr>
<tr>
<td></td>
<td>H₂O</td>
<td>5.57%</td>
</tr>
<tr>
<td></td>
<td>CO₂</td>
<td>13.62%</td>
</tr>
<tr>
<td>Radiation transmittance</td>
<td>Total pressure/Pa</td>
<td>159 000</td>
</tr>
<tr>
<td></td>
<td>Total temperature/K</td>
<td>450</td>
</tr>
<tr>
<td>5-pressure inlet of heat shield</td>
<td>N₂</td>
<td>77%</td>
</tr>
<tr>
<td></td>
<td>O₂</td>
<td>23%</td>
</tr>
<tr>
<td></td>
<td>H₂O</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>CO₂</td>
<td>0%</td>
</tr>
<tr>
<td>Radiation transmittance</td>
<td>Static pressure/Pa</td>
<td>26 500</td>
</tr>
<tr>
<td></td>
<td>Static temperature/K</td>
<td>223.3</td>
</tr>
<tr>
<td></td>
<td>M₁</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>N₂</td>
<td>77%</td>
</tr>
<tr>
<td></td>
<td>O₂</td>
<td>23%</td>
</tr>
<tr>
<td></td>
<td>H₂O</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>CO₂</td>
<td>0%</td>
</tr>
<tr>
<td>Radiation transmittance</td>
<td>Radiation emissivity</td>
<td>0.75</td>
</tr>
</tbody>
</table>

In this section, gas specific heat $c_p$, adiabatic compression coefficient $c_p/c_v$, prandtl number $Pr$, are all set as the functions of gas temperature and constituent. Details can be found in Ref.[27]. Fig.9 shows the Mach number distribution of ejecting nozzle inner-out flow.

Fig.9  Mach number distribution of ejecting nozzle inner-out flow.

Fig.10 shows the gas radiation heat transfer distribution in boundary layer of inner-wall of main nozzle. The nearer to wall the gas is, the higher the temperature becomes, and the more energy radiated to the surrounding low temperature gas/wall will be.

Fig.10  Gas radiation heat transfer distribution in boundary layer of inner-wall of main nozzle.

Calculated wall temperature distribution in main nozzle (see Fig.11) has shown that, when considering solid radiation, the convergent section temperature of the main nozzle is much higher than that when neglecting radiation, which is absolutely caused by solid-wall radiation heat transfer of high temperature heat shield. When considering gas and solid radiation, the calculated wall temperature near the main nozzle outlet is lower than that when just considering solid radiation, because gas in nozzle outlet core region has a high speed and low temperature, which make its absorptive radiation from solid wall higher than the radiation emission ability of itself (although gas temperature in boundary layer is higher than solid wall temperature, it has no effective influence on the latter because of too small value of optical thickness and too low absorption/emission rate). In the same way, with low velocity and high temperature/pressure, gas near the nozzle throat has stronger radiation emission ability than solid wall, thereby the calculated temperature of the main species concentration field, an iterative solution for gas radiation transfer equations is implemented on the coarsest grids. And then, $\dot{q}_r$ and $\dot{q}_{sw}$, obtained by solving radiation energy equations on the coarsest grids, are transported to the finest grids through bilinear interpolation as an energy source term of N-S equations.
nozzle near throat is higher than that just considering solid radiation.

Fig.11 Calculated wall temperature distribution of main nozzle.

Fig.12 shows the computational residual convergence history and time ratio of occupying CPU of each term. Memory usage for the whole calculation is 1.2 GB. The solution for gas radiation transfer equations accounts for more than 60% of the whole computation time (see Table 5).

The residual curve fluctuation in the first phase is due to the fact that computed grids change dynamically with the position of floating outer flaps. Computation is regarded as convergence when the moment of outer flaps reaches a sufficiently small value. The reason in the second phase is that radiation heat transfer can be got once after every 750-step iteration in the solution of other physical field. Computation reaches convergence when the fluctuation peak value of residual curve is small enough.

Fig.13 shows the influence of different solution algorithms for flow field on the whole computation efficiency (just considering solid radiation), when the location of floating outer flaps is known (the divergent angle is 1.78°). Results show good agreement with Ref.[18]. The multi-grid method and the “stiffless” algorithm which can overcome stiffness of two-equation turbulence model, manage to double the computational efficiency respectively.

Fig.14 shows the iteration convergence information in the solution of solid temperature field for the first time, based on the boundary conditions passed from flow field. It can be found that using BI-CGSTAB algorithm to solve the ill-conditioned matrix in Section 3.1, the converging velocity of BADI pretreatment is fourfold as fast as that of LUSGS pretreatment (see Fig.14(a)). When the material nozzle is replaced by aluminum alloy with better heat conduction effect (only suitable for cold working condition), as the matrix morbibity increases, the difference of convergence velocity between the two enlarges to ten times (see Fig.14(b)).

The second air-inlet scheme of ejecting channel is to bleed air with total pressure of 77 825 Pa (80% of outflow total pressure) and total temperature of 323.8 K from aircraft inlet. Comparison of ejecting channel flow field between the two schemes is shown in Fig.15. It can be found that in the second scheme, the airflow

<table>
<thead>
<tr>
<th>Table 5 CPU time ratio of each term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational term</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Flow field</td>
</tr>
<tr>
<td>Solid temperature field</td>
</tr>
<tr>
<td>Species concentration field</td>
</tr>
<tr>
<td>Radiation transfer/energy field</td>
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Fig.12 Computation residual convergence history of flow field and outer flaps’ moment of ejecting nozzle.
is downstream in most regions, and the convergence angle of outer flaps also decreases to 0° under high pressure in ejecting channel; while in the first scheme, the airflow is counter current in most regions.

By using the second air-inlet scheme of ejecting channel, the calculated wall temperature distribution of the main nozzle is shown in Fig.16. Downstream flow in ejecting channel does not generate better cooling effect on wall surface of the main nozzle. The reason is that, NPR of ejecting nozzle in calculation condition is much higher than the designed NPR of main nozzle. In the first scheme, the inlet pressure of ejecting channel is very low. The high back pressure generated by high-pressure jet expanding at the main nozzle outlet makes airflow in ejecting channel flow back at quite a high velocity; therefore, good convective cooling effect on outer wall of the main nozzle is obtained.

Fig.16  Main nozzle wall temperature distribution of ejecting nozzle (Scheme 2).

6. Conclusions

A procedure has been presented for verification of computational efficiency improvement by using two algorithms: the “stiffless” algorithm to overcome the stiffness of two-equation turbulence model, and BL-CGSTAB algorithm with BADI pretreatment to overcome the ill-conditioned matrix appearing in solution of solid wall temperature field. They are combined with the multi-grid accelerating LUSGS implicit-time marching method and the radiation symmetry plane treatment based on mirror-reflection principle. This makes possible the coupled solution on 32 bites PC for N-S equations, solid temperature diffusion equations, species transport equation and gas radiation transfer/energy equations based on SNBCK model.

Three calculated examples, including the calculation of temperature distribution of water-cooling nozzle in rocket engine, the calculation of carbon dioxide absorptivity at 4.3 micron band, and the gas radiation heat transfer evaluation of the cylindrical furnace, have verified that the program presented in this article is capable of doing coupled calculations for flow field, solid temperature field, species concentration field, gas radiation transfer/energy field and getting accurate results.

The difference between calculated temperature distribution of nozzle’s wall reaches the highest value of 70 K when considering solid or both gas and solid radiation. It also indicates the significant application value of the above-mentioned coupled calculation for thermal design and protection in exhaust system.

Compared to the method of bleeding air from air-
craft inlet, the way of opening gaps on ejecting channel outside can reach an equal or even better cooling effect on the main nozzle’s wall of ejecting nozzle with floating outer flaps. The latter can also reduce aircraft drag coefficient and configuration weight obviously.

References


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