Physics Letters B 685 (2010) 120–127

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Non-gaussianity from the tris

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ARTICLE INFO

Article history:
Received 22 October 2009
Received in revised form 19 January 2010
Accepted 25 January 2010
Available online 28 January 2010
Editor: S. Dodelson

ABSTRACT

We use the δN formalism to study the trispectrum T_ζ of the primordial curvature perturbation ζ when the latter is generated by vector field perturbations, considering the tree-level and one-loop contributions. The order of magnitude of the level of non-gaussianity in the trispectrum, $\tau_{\rm NL}$, is calculated in this scenario and related to the order of magnitude of the level of non-gaussianity in the bispectrum, $f_{\rm NL}$, and the level of statistical anisotropy in the power spectrum, g_ζ. Such consistency relations will put under test this scenario against future observations. Comparison with the expected observational bound on $\tau_{\rm NL}$ from WMAP, for generic inflationary models, is done.

1. Introduction

Non-gaussianity in the primordial curvature perturbation ζ is one of the subjects of more interest in modern cosmology, because the non-gaussianity parameters $f_{\rm NL}$ and $\tau_{\rm NL}$ together with the spectrum amplitude A_ζ and spectral index n_ζ allow us to discriminate between the different models proposed for the origin of the large-scale structure (see for example Refs. [1–4]). In most of these cosmological models it is assumed that the n-point correlators of ζ are translationally and rotationally invariants. However, since violations of the translational (rotational) invariance (i.e. violations of the statistical homogeneity (isotropy)) seem to be present in the data [5–10] ([11–15]), many researchers have started to build theoretical models that include those violations, which could be due to the presence of vector field perturbations [16–34], spinor field perturbations [35–37], or p-form perturbations [38–42], due to anisotropic expansion [22,29,35,40,43–48] or due to an inhomogeneous background [16,30,49].

Violation of the statistical isotropy is implemented via modifications of the usual definitions of the statistical descriptors [16,49,50] of the primordial curvature perturbation ζ. For example, to parametrize the statistical anisotropy under the assumption of statistical homogeneity, the power spectrum $P'(\mathbf{k})$ must include an isotropic piece $P(k)$ and an anisotropic piece proportional to the former and exhibiting explicitly the appearance of a preferred direction [50]:

$$P'(\mathbf{k}) = P(k)\big(1 + g_\zeta(\hat{\mathbf{d}}\cdot\hat{\mathbf{k}})^2 + \cdots\big). \tag{1}$$

In the previous expression g_ζ is a dimensionless parameter, $\hat{\mathbf{k}}$ is the unitary wave-vector, and $\hat{\mathbf{d}}$ is the unitary vector along the preferred direction. Some recent papers [11–15] claim for the presence of statistical anisotropy in the five-year data from the NASA's WMAP satellite [51]. In particular, if considering just the quadrupolar term of Eq. (1):

$$P_\zeta(\mathbf{k}) = P_\zeta^{\rm iso}(k)\big(1 + g_\zeta(\hat{\mathbf{d}}\cdot\hat{\mathbf{k}})^2\big), \tag{2}$$

Ref. [11] gives $g_\zeta \simeq 0.290 \pm 0.031$ which rules out statistical isotropy at more than 9σ. Nevertheless, the preferred direction lies near the plane of the solar system, which makes the authors of Ref. [11] believe that this effect could be due to an unresolved systematic error (among other possible systematic errors which have not been demonstrated either to be the source of this statistical anisotropy nor to be completely uncorrelated [11]). Even if the result found in Ref. [11] turns out to be due to a systematic error, some forecasted constraints on g_ζ show that the statistical anisotropy subject is worth studying [52]: $|g_\zeta| \lesssim 0.1$ for the NASA's WMAP satellite [51] if there is no detection, and $|g_\zeta| \lesssim 0.02$ for the ESA's PLANCK satellite [53] if there is no detection.

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0370-2693
doi:10.1016/j.physletb.2010.01.060

Recent works show that the particular presence of vector fields in the inflationary dynamics may generate sizeable levels of non-gaussianity described by $f_{\rm NL}$ [54–56] and $\tau_{\rm NL}$ [57]. As shown in Ref. [56], including vector fields allows us to get consistency relations between the order of magnitude of the non-gaussianity parameter $f_{\rm NL}$ and the amount of statistical anisotropy in the spectrum g_ζ. The above studies may transform the violation of the statistical isotropy in a decisive tool to discriminate among some of the most usual cosmological models.

In this Letter we use the δN formalism to calculate the tree-level and one-loop contributions to the trispectrum T_ζ of ζ including vector and scalar field perturbations. We then calculate the order of magnitude of the level of non-gaussianity in T_ζ including the one-loop contributions and write down formulas that relate the order of magnitude of $\tau_{\rm NL}$ with the amount of statistical anisotropy in the spectrum, g_ζ, and the order of magnitude of the level of non-gaussianity in the bispectrum, $f_{\rm NL}$. Finally, comparison with the expected observational bound from WMAP is done.

2. Trispectrum from vector field perturbations

The δN formalism [58–62] extended to include the possible statistical anisotropy in the primordial curvature perturbation ζ originated from vector field perturbations [30], provides a powerful tool to calculate ζ and its statistical descriptors. Assuming an inflationary dynamics dominated by just one scalar field ϕ and one vector field $\mathbf{A}$, ζ is expressed as [30][1]

$$\zeta(\mathbf{x}) \equiv \delta N\big(\phi(\mathbf{x}), A_i(\mathbf{x}), t\big) = N_\phi \delta\phi + N_A^i \delta A_i + \frac{1}{2} N_{\phi\phi}(\delta\phi)^2 + N_{\phi A}^i \delta\phi \delta A_i + \frac{1}{2} N_{AA}^{ij} \delta A_i \delta A_j, \tag{3}$$

where

$$N_\phi \equiv \frac{\partial N}{\partial\phi}, \qquad N_A^i \equiv \frac{\partial N}{\partial A_i}, \qquad N_{\phi\phi} \equiv \frac{\partial^2 N}{\partial\phi^2}, \qquad N_{AA}^{ij} \equiv \frac{\partial^2 N}{\partial A_i \partial A_j}, \qquad N_{\phi A}^i \equiv \frac{\partial^2 N}{\partial\phi \partial A_i}, \tag{4}$$

and i denotes the spatial indices running from 1 to 3. Now, we define the power spectrum P_ζ and the trispectrum T_ζ for the primordial curvature perturbation, through the Fourier modes of ζ, as:

$$\big\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\big\rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta(\mathbf{k}) \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(\mathbf{k}), \tag{5}$$

$$\begin{aligned}\big\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\big\rangle &\equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \\ &\equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \frac{(2\pi^2)^3}{k_1^3 k_2^3 |\mathbf{k}_2 + \mathbf{k}_3|^3} \mathcal{T}_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4).\end{aligned} \tag{6}$$

As shown in Ref. [30], the tree-level contribution to the spectrum has the form of Eq. (2). This is simply obtained by using Eqs. (3) and (5). Assuming again only tree-level contributions, the level of non-gaussianity $f_{\rm NL}$ in the bispectrum B_ζ was calculated in Ref. [54]. The same calculation was performed in Ref. [56] but this time including also one-loop contributions and considering them to be dominant over the tree-level terms. In both works, P_ζ and $f_{\rm NL}$ were shown to exhibit anisotropic contributions coming from the vector field perturbation. In this Letter we show that it is possible to obtain an analogous expression for the level of non-gaussianity $\tau_{\rm NL}$ in the trispectrum T_ζ. To do it, we first need to calculate the expressions for $\mathcal{P}_\zeta$ and $\mathcal{T}_\zeta$, defined in Eqs. (5) and (6). Considering contributions up to one-loop order, we find[2]:

$$\mathcal{P}_\zeta^{\rm tree}(\mathbf{k}) = N_\phi^2 \mathcal{P}_{\delta\phi}(k) + N_A^i N_A^j \mathcal{T}_{ij}(\mathbf{k}) = N_\phi^2 \mathcal{P}_{\delta\phi}(k) + N_A^2 \mathcal{P}_+(k) + (\mathbf{N}_A \cdot \hat{\mathbf{k}})^2 \mathcal{P}_+(k)(r_{\rm long} - 1), \tag{7}$$

$$\mathcal{P}_\zeta^{\text{1-loop}}(\mathbf{k}) = \int \frac{d^3p\, k^3}{4\pi |\mathbf{k}+\mathbf{p}|^3 p^3} \bigg[\frac{1}{2} N_{\phi\phi}^2 \mathcal{P}_{\delta\phi}\big(|\mathbf{k}+\mathbf{p}|\big) \mathcal{P}_{\delta\phi}(p) + N_{\phi A}^i N_{\phi A}^j \mathcal{P}_{\delta\phi}\big(|\mathbf{k}+\mathbf{p}|\big) \mathcal{T}_{ij}(\mathbf{p}) + \frac{1}{2} N_{AA}^{ij} N_{AA}^{kl} \mathcal{T}_{ik}(\mathbf{k}+\mathbf{p}) \mathcal{T}_{jl}(\mathbf{p})\bigg], \tag{8}$$

$$\begin{aligned}\mathcal{T}_\zeta^{\rm tree}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = {} & N_\phi^2 N_{\phi\phi}^2 \big[\mathcal{P}_{\delta\phi}(k_2) \mathcal{P}_{\delta\phi}(k_4) \mathcal{P}_{\delta\phi}\big(|\mathbf{k}_1 + \mathbf{k}_2|\big) + 11\ \text{perm.}\big] \\ & + N_A^i N_A^j N_{AA}^{kl} N_{AA}^{mn} \big[\mathcal{T}_{ik}(\mathbf{k}_2) \mathcal{T}_{jm}(\mathbf{k}_4) \mathcal{T}_{ln}(\mathbf{k}_1 + \mathbf{k}_2) + 11\ \text{perm.}\big] \\ & + N_\phi^2 N_{A\phi}^i N_{A\phi}^j \big[\mathcal{P}_{\delta\phi}(k_2) \mathcal{P}_{\delta\phi}(k_4) \mathcal{T}_{ij}(\mathbf{k}_1 + \mathbf{k}_2) + 11\ \text{perm.}\big] \\ & + N_A^i N_A^j N_{A\phi}^k N_{A\phi}^l \big[\mathcal{T}_{ik}(\mathbf{k}_2) \mathcal{T}_{jl}(\mathbf{k}_4) \mathcal{P}_{\delta\phi}\big(|\mathbf{k}_1 + \mathbf{k}_2|\big) + 11\ \text{perm.}\big] \\ & + N_\phi N_{\phi\phi} N_A^i N_{A\phi}^j \big[\mathcal{P}_{\delta\phi}(k_2) \mathcal{T}_{ij}(\mathbf{k}_4) \mathcal{P}_{\delta\phi}\big(|\mathbf{k}_1 + \mathbf{k}_2|\big) + 23\ \text{perm.}\big] \\ & + N_\phi N_A^i N_{A\phi}^j N_{AA}^{kl} \big[\mathcal{P}_{\delta\phi}(k_2) \mathcal{T}_{ik}(\mathbf{k}_4) \mathcal{T}_{jl}(\mathbf{k}_1 + \mathbf{k}_2) + 23\ \text{perm.}\big],\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{T}_{\zeta_A}^{\text{1-loop}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = {} & N_{AA}^{ij} N_{AA}^{kl} N_{AA}^{mn} N_{AA}^{op} \int \frac{d^3p\, k_1^3 k_3^3 |\mathbf{k}_3 + \mathbf{k}_4|^3}{4\pi p^3 |\mathbf{k}_1 - \mathbf{p}|^3 |\mathbf{k}_3 + \mathbf{p}|^3 |\mathbf{k}_3 + \mathbf{k}_4 + \mathbf{p}|^3} \\ & \times \mathcal{T}_{im}(\mathbf{p}) \mathcal{T}_{jk}(\mathbf{k}_1 - \mathbf{p}) \mathcal{T}_{np}(\mathbf{k}_3 + \mathbf{p}) \mathcal{T}_{lo}(\mathbf{k}_3 + \mathbf{k}_4 + \mathbf{p}),\end{aligned} \tag{10}$$

where

$$\mathcal{T}_{ij}(\mathbf{k}) \equiv T_{ij}^{\rm even}(\mathbf{k}) \mathcal{P}_+(k) + i T_{ij}^{\rm odd}(\mathbf{k}) \mathcal{P}_-(k) + T_{ij}^{\rm long}(\mathbf{k}) \mathcal{P}_{\rm long}(k), \tag{11}$$

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[1] This expression corrects Eq. (3.14) of Ref. [30], and Eq. (3) of Ref. [54], where a factor 2 in the fourth term of the expansion is missing.

[2] Eq. (8) corrects a mistake in Eq. (4.12) of Ref. [30] where the infinitesimal volume element d^3p was incorrectly expressed in terms of dp.

and

$$T_{ij}^{\text{even}}(\mathbf{k}) \equiv \delta_{ij} - \hat{k}_i\hat{k}_j, \qquad T_{ij}^{\text{odd}}(\mathbf{k}) \equiv \epsilon_{ijk}\hat{k}_k, \qquad T_{ij}^{\text{long}}(\mathbf{k}) \equiv \hat{k}_i\hat{k}_j. \tag{12}$$

Eq. (7) was written in the form of Eq. (2) with $\hat{\mathbf{d}} = \hat{\mathbf{N}}_A$, $\mathbf{N}_A$ being a vector with magnitude $N_A \equiv \sqrt{N_A^i N_A^i}$, and $r_{\text{long}} \equiv \mathcal{P}_{\text{long}}/\mathcal{P}_+$, where $\mathcal{P}_{\text{long}}$ is the power spectrum of the longitudinal component, and $\mathcal{P}_+$ and $\mathcal{P}_-$ are the parity conserving and violating power spectra defined by

$$\mathcal{P}_\pm \equiv \frac{1}{2}(\mathcal{P}_R \pm \mathcal{P}_L), \tag{13}$$

with $\mathcal{P}_R$ and $\mathcal{P}_L$ denoting the power spectra of the transverse components with right-handed and left-handed polarizations [30]. Eq. (10) only includes terms coming from vector field perturbations; this is because the complete expression (including the scalar and the mixed terms) is too large and in the current paper we are assuming that the contributions to T_ζ coming *only* from vector fields dominate over all the other contributions.

3. Vector field contributions to the statistical descriptors

When statistical anisotropy is assumed, there is an important restriction from observation: one related to the amount of statistical anisotropy present in the spectrum, which is given by the parameter g_ζ in Eq. (2). Recent studies of the data coming from the WMAP experiment, set an upper bound over g_ζ: $g_\zeta \lesssim 0.383$ [11]. The latter observational constraint is fully satisfied when we assume that the contributions coming from vector fields in Eqs. (7) and (8) are smaller than those coming from scalar fields. That means that the first term in Eq. (7) dominates over all the other terms, even those coming from one-loop contributions.

In our study we will assume that the terms coming only from the vector field dominate over those coming from the mixed terms and from the scalar fields only, except for the case of the tree-level spectrum, where we will assume that the scalar term is the dominant one.[3] Of course, for an actual realization of this scenario, we need to show that such constraints are fully satisfied. From the above assumptions it follows that:

$$\mathcal{P}_\zeta{}^{\text{tree}}(\mathbf{k}) = \mathcal{P}_{\zeta}{}_{\phi}^{\text{tree}}(k) + \mathcal{P}_{\zeta}{}_{A}^{\text{tree}}(\mathbf{k}), \tag{14}$$

$$\mathcal{P}_\zeta^{\text{1-loop}}(\mathbf{k}) = \mathcal{P}_{\zeta}{}_{A}^{\text{1-loop}}(\mathbf{k}), \tag{15}$$

$$\mathcal{T}_\zeta^{\text{tree}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \mathcal{T}_{\zeta}{}_{A}^{\text{tree}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4), \tag{16}$$

$$\mathcal{T}_\zeta^{\text{1-loop}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \mathcal{T}_{\zeta}{}_{A}^{\text{1-loop}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4), \tag{17}$$

where the subscripts ζ_ϕ and ζ_A mean scalar field or vector field contributions to ζ. The above expressions lead us to two different possibilities that let us study and probably get a high level of non-gaussianity:

- Vector field spectrum ($\mathcal{P}_{\zeta_A}$) and trispectrum ($\mathcal{T}_{\zeta_A}$) dominated by the tree-level terms.
- Vector field spectrum ($\mathcal{P}_{\zeta_A}$) and trispectrum ($\mathcal{T}_{\zeta_A}$) dominated by the one-loop contributions.

Other possibilities are not viable because it is impossible to satisfy simultaneously that the vector field spectrum ($\mathcal{P}_{\zeta_A}$) is dominated by the tree-level terms and the trispectrum ($\mathcal{T}_{\zeta_A}$) is dominated by the one-loop contributions, or the vector field spectrum ($\mathcal{P}_{\zeta_A}$) is dominated by the one-loop contributions and the trispectrum ($\mathcal{T}_{\zeta_A}$) is dominated by the tree-level terms.[4] This is perhaps related to the fact that we have taken into account only one vector field. Such a conclusion may be relaxed if we take into account more than one vector field, as analogously happens in the scalar multi-field case [63,64].

In order to study the above possibilities, we need to estimate the integrals coming from loop contributions. From Eqs. (8), (10), (15), and (17) the integrals to solve are:

$$\mathcal{P}_\zeta{}^{\text{1-loop}}(\mathbf{k}) = \frac{1}{2}N_{AA}^{ij}N_{AA}^{kl}\int \frac{d^3p\,k^3}{4\pi\,p^3|\mathbf{k}+\mathbf{p}|^3}\mathcal{T}_{ik}(\mathbf{k}+\mathbf{p})\mathcal{T}_{jl}(\mathbf{p}), \tag{18}$$

$$\begin{aligned}\mathcal{T}_{\zeta_A}^{\text{1-loop}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = {} & N_{AA}^{ij}N_{AA}^{kl}N_{AA}^{mn}N_{AA}^{op}\int \frac{d^3p\,k_1^3k_3^3|\mathbf{k}_3+\mathbf{k}_4|^3}{4\pi\,p^3|\mathbf{k}_1-\mathbf{p}|^3|\mathbf{k}_3+\mathbf{p}|^3|\mathbf{k}_3+\mathbf{k}_4+\mathbf{p}|^3} \\ & \times \mathcal{T}_{im}(\mathbf{p})\mathcal{T}_{jk}(\mathbf{k}_1-\mathbf{p})\mathcal{T}_{np}(\mathbf{k}_3+\mathbf{p})\mathcal{T}_{lo}(\mathbf{k}_3+\mathbf{k}_4+\mathbf{p}).\end{aligned} \tag{19}$$

The above integrals cannot be done analytically, but they can be estimated using the same technique shown in Appendix A; it is found that the integrals are proportional to $\ln(kL)$ (where L is the box size) if the spectrum is scale invariant. Following it, we find from Eqs. (18) and (19):

$$\mathcal{P}_{\zeta_A}^{\text{1-loop}}(\mathbf{k}) = \frac{1}{2}N_{AA}^{ij}N_{AA}^{kl}(2\mathcal{P}_+ + \mathcal{P}_{\text{long}})\delta_{ik}\mathcal{T}_{jl}(\mathbf{k})\ln(kL), \tag{20}$$

$$\mathcal{T}_{\zeta_A}^{\text{1-loop}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = N_{AA}^{ij}N_{AA}^{kl}N_{AA}^{mn}N_{AA}^{op}\ln(kL)(2\mathcal{P}_+ + \mathcal{P}_{\text{long}})\delta_{im}\big[\mathcal{T}_{jk}(\mathbf{k}_1)\mathcal{T}_{np}(\mathbf{k}_3)\mathcal{T}_{lo}(\mathbf{k}_4+\mathbf{k}_3)\big]. \tag{21}$$

[3] The power spectrum P_ζ must be dominated by the tree-level terms. Otherwise there would be too much scale dependence in conflict with the current observational limit on n_ζ.

[4] See the relevant discussion regarding the vector field bispectrum $\mathcal{B}_{\zeta_A}$ in Ref. [56].

Observations are available within the observable universe and, except for the low multipoles of the CMB, all observations probe scales $k \gg H_0$. To handle them, one should choose the box size as $L = H_0^{-1}$ [65]. A smaller choice would throw away some of the data while a bigger choice would make the spatial averages unobservable. Low multipoles ℓ of the CMB anisotropy explore scales of order H_0^{-1}/ℓ not very much smaller than H_0^{-1}. To handle them one has to take L bigger than H_0^{-1}. For most purposes, one should use a box, such that $\ln(LH_0)$ is just a few (i.e. not exponentially large) [66–68]. When comparing the loop contribution with observation one should normally set $L = H_0^{-1}$, except for the low CMB multipoles where one should choose $L \gg H_0^{-1}$ with $\ln(kL) \sim 1$. With the choice $L = H_0^{-1}$, $\ln(kL) \sim 5$ for the scales explored by the CMB multipoles with $\ell \sim 100$, while $\ln(kL) \sim 10$ for the scales explored by galaxy surveys. Since we are interested in giving orders of magnitude and simple mathematical expressions, we will set $\ln(kL) \sim 1$ without loss of generality.

4. Calculation of the non-gaussianity parameter τ_{NL}

The non-gaussianity parameter τ_{NL} is defined by [69]:

$$\tau_{NL} = \frac{2\mathcal{T}_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{[\mathcal{P}_\zeta(\mathbf{k}_1)\mathcal{P}_\zeta(\mathbf{k}_2)\mathcal{P}_\zeta(\mathbf{k}_1 + \mathbf{k}_4) + 23 \text{ perm.}]}. \tag{22}$$

Remember that the isotropic contribution in Eq. (2) is always dominant compared to the anisotropic one so that we may write in the above expression only the isotropic part of the spectrum ${\mathcal{P}_\zeta}^{\text{iso}}(k)$:

$$\tau_{NL} = \frac{2\mathcal{T}_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{[{\mathcal{P}_\zeta}^{\text{iso}}(k_1){\mathcal{P}_\zeta}^{\text{iso}}(k_2){\mathcal{P}_\zeta}^{\text{iso}}(|\mathbf{k}_1 + \mathbf{k}_4|) + 23 \text{ perm.}]}. \tag{23}$$

Using the above expression, we will estimate the possible amount of non-gaussianity generated by the anisotropic part of the primordial curvature perturbation, taking into account different possibilities and assuming that the non-gaussianity is produced solely by vector field perturbations.

4.1. Vector field spectrum ($\mathcal{P}_{\zeta_A}$) and trispectrum ($\mathcal{T}_{\zeta_A}$) dominated by the tree-level terms

In this first case, we assume that the trispectrum is dominated by vector field perturbations and that the higher order terms in the δN expansion in Eq. (3) involving the vector field are sub-dominant against the first-order term: $N_A^i \delta A_i \gg N_{AA}^{ij} \delta A_i \delta A_j$. The latter implies that both the spectrum and the trispectrum are dominated by the tree-level terms, i.e. ${\mathcal{P}_\zeta}_A^{\text{tree}} \gg {\mathcal{P}_\zeta}_A^{\text{1-loop}}$ and ${\mathcal{T}_\zeta}_A^{\text{tree}} \gg {\mathcal{T}_\zeta}_A^{\text{1-loop}}$. Thus, we have from Eq. (23):

$$\tau_{NL} = \frac{2{\mathcal{T}_\zeta}_A^{\text{tree}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{[{\mathcal{P}_\zeta}^{\text{iso}}(k_1){\mathcal{P}_\zeta}^{\text{iso}}(k_2){\mathcal{P}_\zeta}^{\text{iso}}(|\mathbf{k}_1 + \mathbf{k}_4|) + 23 \text{ perm.}]}, \tag{24}$$

which, in view of Eqs. (9) and (16), looks like:

$$\tau_{NL} \simeq \frac{2N_A^i N_A^j N_{AA}^{kl} N_{AA}^{mn}[\mathcal{T}_{ik}(\mathbf{k}_2)\mathcal{T}_{jm}(\mathbf{k}_4)\mathcal{T}_{ln}(\mathbf{k}_1 + \mathbf{k}_2) + 11 \text{ perm.}]}{[{\mathcal{P}_\zeta}^{\text{iso}}(k_1){\mathcal{P}_\zeta}^{\text{iso}}(k_2){\mathcal{P}_\zeta}^{\text{iso}}(|\mathbf{k}_1 + \mathbf{k}_4|) + 23 \text{ perm.}]}. \tag{25}$$

We will just consider here the order of magnitude of τ_{NL}. Therefore, we will ignore the specific $\mathbf{k}$ dependence of $\mathcal{T}_{ij}$. Instead, we will assume that $\mathcal{P}_{\text{long}}$, $\mathcal{P}_+$, and $\mathcal{P}_-$ are all of the same order of magnitude, which is a good approximation for some specific actions (see for instance Ref. [30]), and take advantage of the fact that the spectrum is almost scale invariant [70]. Thus, after getting rid of all the $\mathbf{k}$ dependences, the order of magnitude of τ_{NL} looks like:

$$\tau_{NL} \simeq \frac{\mathcal{P}_A^3 N_A^2 N_{AA}^2}{({\mathcal{P}_\zeta}^{\text{iso}})^3}, \tag{26}$$

where $\mathcal{P}_A = 2\mathcal{P}_+ + \mathcal{P}_{\text{long}}$. Employing our assumption that $N_A \delta A > N_{AA} \delta A^2$, and since the root mean squared value for the vector field perturbation δA is $\sqrt{\mathcal{P}_A}$, the contribution of the vector field to ζ is given by $\zeta_A \sim \sqrt{\mathcal{P}_{\zeta_A}} \sim N_A \sqrt{\mathcal{P}_A}$. An upper bound for τ_{NL} is therefore given by:

$$\tau_{NL} \lesssim \frac{{\mathcal{P}_\zeta}_A^2}{({\mathcal{P}_\zeta}^{\text{iso}})^3}. \tag{27}$$

Since the order of magnitude of g_ζ is $\mathcal{P}_{\zeta_A}/{\mathcal{P}_\zeta}^{\text{iso}}$, under the assumptions made above we get:

$$\tau_{NL} \lesssim 8 \times 10^6 \left(\frac{g_\zeta}{0.1}\right)^2, \tag{28}$$

where $({\mathcal{P}_\zeta}^{\text{iso}})^{1/2} \simeq 5 \times 10^{-5}$ [70] has been used. Eq. (28) gives an upper bound for the level of non-gaussianity τ_{NL} in terms of the level of statistical anisotropy in the power spectrum g_ζ when the former is generated by the anisotropic contribution to the curvature perturbation. Comparing with the expected observational limit on τ_{NL} coming from future WMAP data releases, $\tau_{NL} \sim 2 \times 10^4$ [71],[5] we

[5] The trispectrum in this scenario might be either of the local, equilateral, or orthogonal type. We are not interested in this Letter on the shape of the non-gaussianity but on its order of magnitude. Being that the case, comparing with the expected bound on the *local* τ_{NL} [71] makes no sensible difference under the assumption that the expected bounds on the equilateral and orthogonal τ_{NL} are of the same order of magnitude, as analogously happens in the f_{NL} case for single-field inflation [72].

conclude that in this scenario a large level of non-gaussianity in the trispectrum T_ζ of ζ is possible, leaving some room for ruling out this scenario if the current expected observational limit is overtaken.

As an example of this scenario, we apply the previous results to a specific model, e.g. the vector curvaton model [17–19], where the N-derivatives are [54]:

$$N_A = \frac{2}{3A} r, \tag{29}$$

$$N_{AA} = \frac{2}{A^2} r, \tag{30}$$

where $A \equiv |\mathbf{A}|$ is the value of vector field just before the vector curvaton field decays and the parameter r is the ratio between the energy density of the vector curvaton field and the total energy density of the Universe just before the vector curvaton decay.

First, we check if the conditions under which the vector field spectrum and trispectrum are always dominated by the tree-level terms are fully satisfied. From Eqs. (7), (9), (20) and (21) our constraint leads to:

$$\mathcal{P}_A N_A^2 \gg \mathcal{P}_A^2 N_{AA}^2, \tag{31}$$

$$\mathcal{P}_A^3 N_A^2 N_{AA}^2 \gg \mathcal{P}_A^4 N_{AA}^4, \tag{32}$$

which mean that the if the vector field spectrum is dominated by the tree-level terms so is the vector field trispectrum. An analogous situation happens when the vector field spectrum is dominated by the one-loop terms: the vector field trispectrum is also dominated by this kind of terms. As a result, it is impossible that simultaneously the vector field spectrum is dominated by the tree-level (one-loop) terms and the vector field trispectrum is dominated by the one-loop (tree-level) terms. Following Eq. (31), we get:

$$\mathcal{P}_A \ll \left(\frac{N_A}{N_{AA}}\right)^2, \tag{33}$$

which, in view of $\zeta_A \sim \sqrt{\mathcal{P}_{\zeta_A}} \sim N_A \sqrt{\mathcal{P}_A}$ and Eqs. (29) and (30), reduces to:

$$r \gg 2.25 \times 10^{-4} g_\zeta^{1/2}. \tag{34}$$

This lower bound on the r parameter has to be considered when building a realistic particle physics model of the vector curvaton scenario.

Second, looking at Eq. (26), we obtain the level of non-gaussianity $\tau_{\rm NL}$ for this scenario:

$$\tau_{\rm NL} \simeq \frac{2 \times 10^{-2}}{r^2} \left(\frac{g_\zeta}{0.1}\right)^3. \tag{35}$$

This is a consistency relation between $\tau_{\rm NL}$, g_ζ, and r which will help when confronting the specific vector curvaton realization against observation. Indeed, a similar consistency relation between $f_{\rm NL}$ and g_ζ was derived for this scenario in Ref. [56]:

$$f_{\rm NL} \simeq \frac{4.5 \times 10^{-2}}{r} \left(\frac{g_\zeta}{0.1}\right)^2. \tag{36}$$

Thus, in the framework of the vector curvaton scenario, the levels of non-gaussianity $f_{\rm NL}$ and $\tau_{\rm NL}$ are related to each other via the r parameter in this way:

$$\tau_{\rm NL} \simeq \frac{2.1}{r^{1/2}} f_{\rm NL}^{3/2}, \tag{37}$$

in contrast to the standard result

$$\tau_{\rm NL} = \frac{36}{25} f_{\rm NL}^2, \tag{38}$$

for the scalar field case (including the scalar curvaton scenario) found in Ref. [73].

4.2. Vector field spectrum ($\mathcal{P}_{\zeta_A}$) and trispectrum ($\mathcal{T}_{\zeta_A}$) dominated by the one-loop contributions

From Eqs. (21) and (23) we get

$$\tau_{\rm NL} \simeq \frac{N_{AA}^{ij} N_{AA}^{kl} N_{AA}^{mn} N_{AA}^{op} \ln(kL)(2\mathcal{P}_+ + \mathcal{P}_{\rm long})\delta_{im}[\mathcal{T}_{jk}(\mathbf{k}_1)\mathcal{T}_{np}(\mathbf{k}_3)\mathcal{T}_{lo}(|\mathbf{k}_4 + \mathbf{k}_3|)]}{[\mathcal{P}_\zeta{}^{\rm iso}(k_1)\mathcal{P}_\zeta{}^{\rm iso}(k_2)\mathcal{P}_\zeta{}^{\rm iso}(|\mathbf{k}_1 + \mathbf{k}_4|) + 23 \text{ perm.}]}. \tag{39}$$

Assuming again that $\mathcal{P}_{\rm long}$, $\mathcal{P}_+$, and $\mathcal{P}_-$ are all of the same order of magnitude, and that the spectrum is scale invariant, we end up with:

$$\tau_{\rm NL} \simeq \frac{\mathcal{P}_A^4 N_{AA}^4}{(\mathcal{P}_\zeta{}^{\rm iso})^3}. \tag{40}$$

Performing a similar analysis as done in the previous subsection, but this time taking into account that the vector field spectrum is dominated by the one-loop contribution and therefore $\zeta_A \sim \sqrt{\mathcal{P}_{\zeta_A}} \sim N_{AA}\mathcal{P}_A$, we arrive at:

$$\tau_{\rm NL} \sim \frac{\mathcal{P}_{\zeta_A}^2}{(\mathcal{P}_\zeta{}^{\rm iso})^3} \sim 8 \times 10^6 \left(\frac{g_\zeta}{0.1}\right)^2. \tag{41}$$

The above result gives a relation between the non-gaussianity parameter $\tau_{\rm NL}$ and the level of statistical anisotropy in the power spectrum g_ζ.

Now, we call a similar result that we found for the non-gaussianity parameter $f_{\rm NL}$ in Ref. [56], that is:

$$f_{\rm NL} \sim 10^3 \left(\frac{g_\zeta}{0.1}\right)^{3/2}. \tag{42}$$

By combining Eqs. (41) and (42) we get:

$$\tau_{\rm NL} \sim 8 \times 10^2 f_{\rm NL}^{4/3}, \tag{43}$$

which gives a consistency relation between the non-gaussianity parameters $f_{\rm NL}$ and $\tau_{\rm NL}$ for this particular scenario. The consistency relations in Eqs. (41), (42), and (43) will put under test this scenario against future observations. In particular, the consistency relation in Eq. (43) differs significantly from those obtained when ζ is generated only by scalar fields (see e.g. Eq. (38) and Ref. [73]).

Again when we apply our result to the vector curvaton scenario, we get from Eqs. (7), (9), (20), (21), (29) and (30):

$$r < 2.25 \times 10^{-4} g_\zeta^{1/2}, \tag{44}$$

which is an upper bound on the r parameter that must be considered when building a realistic particle physics model of the vector curvaton scenario.

5. Conclusions

We have studied in this Letter the order of magnitude of the level of non-gaussianity $\tau_{\rm NL}$ in the trispectrum T_ζ when statistical anisotropy is generated by the presence of one vector field. We have shown that it is possible to get an upper bound on the order of magnitude of $\tau_{\rm NL}$ if we assume that the tree-level contributions dominate over all higher order terms in both the vector field spectrum ($\mathcal{P}_{\zeta_A}$) and the trispectrum ($\mathcal{T}_{\zeta_A}$); this bound is given in Eq. (28). We also show that it is possible to get a high level of non-gaussianity $\tau_{\rm NL}$, easily exceeding the expected observational bound from WMAP, if we assume that the one-loop contributions dominate over the tree-level terms in both the vector field spectrum ($\mathcal{P}_{\zeta_A}$) and the trispectrum ($\mathcal{T}_{\zeta_A}$). $\tau_{\rm NL}$ is given in this case by Eq. (41), where we may see that there is a consistency relation between the order of magnitude of $\tau_{\rm NL}$ and the amount of statistical anisotropy in the spectrum g_ζ. Two other consistency relations are given by Eqs. (42) and (43), this time relating the order of magnitude of the non-gaussianity parameter $f_{\rm NL}$ in the bispectrum B_ζ with the amount of statistical anisotropy g_ζ and the order of magnitude of the level of non-gaussianity $\tau_{\rm NL}$ in the trispectrum T_ζ. Such consistency relations let us fix two of the three parameters by knowing about the other one, i.e. if the non-gaussianity in the bispectrum (or trispectrum) is detected and our scenario is appropriate, the amount of statistical anisotropy in the power spectrum and the order of magnitude of the non-gaussianity parameter $\tau_{\rm NL}$ (or $f_{\rm NL}$) must have specific values, which are given by Eqs. (42) (or (41)) and (43). A similar conclusion is reached if the statistical anisotropy in the power spectrum is detected before the non-gaussianity in the bispectrum or the trispectrum is.

Appendix A. One-loop integral for P_ζ

We sketch in this appendix the mathematical procedure to estimate the integrals in Eqs. (8) and (10). We only work one integral since the other ones are estimated in a similar way.

The one-loop contribution to the spectrum is:

$$\mathcal{P}_\zeta^{\text{1-loop}}(\mathbf{k}) = \int \frac{d^3p\,k^3}{4\pi|\mathbf{k}+\mathbf{p}|^3p^3}\left[\frac{1}{2}N_{\phi\phi}^2\mathcal{P}_{\delta\phi}\big(|\mathbf{k}+\mathbf{p}|\big)\mathcal{P}_{\delta\phi}(p) + N_{\phi A}^iN_{\phi A}^j\mathcal{P}_{\delta\phi}\big(|\mathbf{k}+\mathbf{p}|\big)\mathcal{T}_{ij}(\mathbf{p}) + \frac{1}{2}N_{AA}^{ij}N_{AA}^{kl}\mathcal{T}_{ik}(\mathbf{k}+\mathbf{p})\mathcal{T}_{jl}(\mathbf{p})\right]. \tag{45}$$

As we can see, the total contribution to $\mathcal{P}_\zeta^{\text{1-loop}}$ corresponds to three integrals, each one having two singularities: one in $\mathbf{p}=0$ and the other one in $\mathbf{p}=-\mathbf{k}$. If the fields spectra are scale invariant, the first integral may be written as:

$$\mathcal{P}_\zeta^{\text{1-loop(a)}}(\mathbf{k}) = \frac{1}{8\pi}\mathcal{P}_{\delta\phi}^2N_{\phi\phi}^2\int\frac{d^3p\,k^3}{4\pi|\mathbf{k}+\mathbf{p}|^3p^3}, \tag{46}$$

so the actual integral to estimate is:

$$I = \int_{L^{-1}} \frac{d^3p\,k^3}{|\mathbf{k}+\mathbf{p}|^3p^3}. \tag{47}$$

This integral is logarithmically divergent at the zeros in the denominator, but there is a cutoff at $k = L^{-1}$. The subscript L^{-1} indicates that the integrand is set equal to zero in a sphere of radius L^{-1} around each singularity, and the discussion makes sense only for $L^{-1} \ll k \ll k_{max}$. If we consider the infrared divergences, that means $\mathbf{p} \ll \mathbf{k}$, we may write:

$$I = \int_{L^{-1}}^{k} \frac{d^3p}{p^3} \sim 4\pi\ln(kL). \tag{48}$$

To calculate the contribution coming from the other singularity we can make the substitution $\mathbf{q} = \mathbf{k} + \mathbf{p}$. After evaluating this latter integral, we find that the contribution is again $4\pi \ln(kL)$. The integral in Eq. (47) may be finally estimated by adding the contributions of the two singularities, that means:

$$I = \int \frac{d^3 p\, k^3}{|\mathbf{k}+\mathbf{p}|^3 p^3} = 8\pi \ln(kL). \tag{49}$$

More details to evaluate these integrals may be found in Refs. [67,69,74].

The technique to evaluate this kind of integrals when considering vector fields is the same, although the procedure is algebraically more tedious. Nevertheless, one can finally arrive to the same conclusion. A more detailed discussion about this issue will be found in a forthcoming publication [75].

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