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# On the consistency of Lorentz invariance violation in QED induced by fermions in constant axial-vector background

J. Alfaro<sup>a</sup>, A.A. Andrianov<sup>b,c,\*</sup>, M. Cambiaso<sup>a</sup>, P. Giacconi<sup>c</sup>, R. Soldati<sup>c</sup>

<sup>a</sup> *Facultad de Física, Pontificia Universidad Católica de Chile, Macul, Santiago de Chile, Casilla 306, Chile*

<sup>b</sup> *V.A. Fock Department of Theoretical Physics, St. Petersburg State University, 198504 Sankt-Petersburg, Russia*

<sup>c</sup> *Dipartimento di Fisica, Università di Bologna & INFN, Sezione di Bologna, 40126 Bologna, Italia*

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## Abstract

We show for the first time that the induced parity-even Lorentz invariance violation can be unambiguously calculated in the physically justified and minimally broken, dimensional regularization scheme, suitably tailored for a spontaneous Lorentz symmetry breaking in a field theory model. The quantization of the Lorentz invariance violating quantum electrodynamics is critically examined and shown to be consistent either for a light-like cosmic anisotropy axial-vector or for a time-like one, when in the presence of a bare photon mass.

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## 1. Introduction

The Lorentz and CPT invariance violation (LIV) in quantum electrodynamics [1,2] has not yet been seen [3] but, in principle, it might arise in several ways reviewed in [4,5]. In particular, spontaneous symmetry breaking [6] may cause LIV after condensation of massless axion-like fields [7,8] and/or of certain vector fields [9] (maybe, of gravitational origin [10]), as well as short-distance space–time asymmetries may come from string [11] and quantum gravity effects [12,13] and non-commutative structure of the space–time [14]. Whereas the empirical parameterization of LIV does not represent a tedious and difficult task [2,3], the consistent unraveling of its dynamical origin is far more subtle and involved.

If LIV occurs spontaneously in QED, due to some vector-like condensate, then the related low-energy effective action is actually dominated by the classical gauge invariant Maxwell–

Chern–Simons modified electrodynamics [1], with a Chern–Simons (CS) fixed vector  $\eta_\mu$ . In addition, the low-energy effective action has to be indeed supplemented by a Lorentz-invariant bare photon mass  $\mu_\gamma$  and take into account the contribution from the one-loop radiative corrections [15–17] induced by the fermion sector, in which some constant axial-vector  $b_\mu = \langle B_\mu \rangle = \langle \partial_\mu \theta \rangle$  does appear. The latter one might represent the vacuum expectation value of a vector field  $B_\mu(x)$ , such as some torsion field of a cosmological nature, or of a gradient of some axion field or quintessence field  $\theta(x)$ , or anything else. Whatever it is, it turns out to be responsible of the Lorentz invariance violation and, correspondingly, this model will be henceforth referred to as Lorentz invariant violating quantum electrodynamics (LIVQED).

If LIV manifests itself as a fundamental phenomenon in the large-scale universe, it is quite plausible that LIV is induced universally by different species of fermion fields coupled to the very same axial-vector  $b^\mu$ , albeit with different magnitudes depending upon flavors. Then both LIV vectors become [15,16] collinear, i.e.,  $\eta^\mu = \zeta b^\mu$ . Meantime it has been found [7,18–20] that a consistent quantization of photons just requires the CS

\* Corresponding author.

E-mail addresses: [andrianov@bo.infn.it](mailto:andrianov@bo.infn.it), [alexandr.andrianov@bo.infn.it](mailto:alexandr.andrianov@bo.infn.it) (A.A. Andrianov).

vector to be space-like, whereas for the consistency of the spinor free field theory a space-like axial-vector  $b^\mu$  is generally not allowed but for the pure space-like case which, however, is essentially ruled out by the experimental data [21].

In our Letter the quantization of the Lorentz invariance violating quantum electrodynamics is critically examined and shown to be consistent either for a light-like cosmic anisotropy axial-vector or for a time-like one, when in the presence of a bare photon mass. To this purpose, we show for the first time that the induced parity-even Lorentz invariance violation can be unambiguously calculated in the physically justified and minimally broken, dimensional regularization scheme, suitably tailored for a spontaneous Lorentz symmetry breaking in a field theory model.

Two main paths will be focused, along which the incompatibility in the LIV quantization of the photon and fermion fields can be actually removed. A first road towards the quantum intrinsic consistency of LIVQED is opened by the light-like vector  $b^\mu$ . As well, the cure can be owed to a radiatively generated mass for the photon. The relevant LIVQED Lagrange density in the photon sector reads

$$\begin{aligned} \mathcal{L}_\gamma = & -\frac{1}{4} \left( 1 + \xi \frac{b^2}{m_e^2} \right) F^{\nu\lambda} F_{\nu\lambda} + \xi \frac{b_\nu b^\rho}{2m_e^2} F^{\nu\lambda} F_{\rho\lambda} \\ & - \frac{1}{2} \zeta b_\nu A_\lambda \tilde{F}^{\nu\lambda} + \frac{1}{2} m_\gamma^2 A_\nu A^\nu + B \partial_\nu A^\nu, \end{aligned} \quad (1)$$

with the photon mass assembling both a bare and an induced one,  $m_\gamma^2 = \mu_\gamma^2 + \delta m_\gamma^2$ . As the gauge invariance is broken by the photon mass, we have suitably introduced the Stückelberg’s auxiliary field  $B(x)$ , together with the usual dual field tensor  $\tilde{F}^{\nu\lambda} \equiv (1/2)\epsilon^{\nu\lambda\rho\sigma} F_{\rho\sigma}$  and the electron mass  $m_e$ .

As it will be shown in next section, the induced photon mass squared  $\delta m_\gamma^2$  is of order  $\alpha b_\nu b^\nu \equiv \alpha b^2$ ,  $\alpha = e^2/4\pi$  being the fine structure constant. The electric charge  $e$  is included also into the induced dimensionless constants  $\xi \sim \alpha$  and  $\zeta \sim \alpha$  and the related LIV vertices are estimated to be indeed very small [1,3]—see the discussion and concluding remarks in the last section.

## 2. Induced coupling constants

Let us now evaluate the constants  $\xi$  and  $\zeta$  as well as the photon mass  $m_\gamma$  when they are induced by fermions in an axial background of a cosmological origin. To this aim, one has to compute the one-loop induced effective action from the classical spinor Lagrange density for a given fermion species  $f$  of electric charge  $eq_f$ : namely,

$$\mathcal{L}_f = \bar{\psi}_f (i \not{\partial} - m_f - b_f^\mu \gamma_\mu \gamma_5 + eq_f \not{A}) \psi_f, \quad (2)$$

which leads to the Feynman’s propagators in the four-dimensional momentum space,

$$\begin{aligned} S_f(p) = & i(\gamma^\nu p_\nu + m_f + b_f^\nu \gamma_\nu \gamma_5) \\ & \times \frac{p^2 + b_f^2 - m_f^2 + 2(p \cdot b_f + m_f b_f^\lambda \gamma_\lambda) \gamma_5}{(p^2 + b_f^2 - m_f^2 + i\varepsilon)^2 - 4[(p \cdot b_f)^2 - m_f^2 b_f^2]}. \end{aligned} \quad (3)$$

The one-loop vacuum polarization tensor is formally determined to be

$$\begin{aligned} \Pi_2^{\nu\sigma}(k; b_f, m_f) = & -ie^2 q_f^2 \int \frac{d^4 p}{(2\pi)^4} \text{tr} \{ \gamma^\nu S_f(p) \gamma^\sigma S_f(p-k) \}. \end{aligned} \quad (4)$$

The above formal expression for  $\Pi_2^{\nu\sigma}$  exhibits, by power counting, ultraviolet divergencies. In the presence of LIV due to the background axial-vectors  $b_f^\mu$ , the physical cutoff in fermion momenta does emerge as a result of fermion–antifermion pair creation at very high energies. To this concern, it has been proved [16] that the calculations with such a physical cutoff do actually give the same results of a Lorentz invariance violating dimensional regularization scheme (LIVDRS)

$$\int \frac{d^4 p}{(2\pi)^4} \longrightarrow \mu^{4-2\omega} \int \frac{d^{2\omega} p}{(2\pi)^{2\omega}}$$

suitably tailored in order to strictly preserve the residual Lorentz symmetry. This LIVDRS coincides with the conventional one, with ’t Hooft–Veltman–Breitenlohner–Maison algebraic rules for gamma-matrices, when it is applied to the integrand (4) with fermion propagators (3) having the spinor matrices in the numerator. The general structure of the regularized polarization tensor turns out to be

$$\text{reg} \Pi_2^{\nu\sigma} = \text{reg} \Pi_{2,\text{even}}^{\nu\sigma} + \text{reg} \Pi_{2,\text{odd}}^{\nu\sigma}. \quad (5)$$

The regular odd part has been unambiguously evaluated in [16] with the help of LIVDRS and for small  $|b^\mu| \ll m_e$  reads

$$\text{reg} \Pi_{2,\text{odd}}^{\nu\sigma} = \left( \frac{\alpha}{\pi} \right) 2i \epsilon^{\nu\sigma}{}_{\rho\lambda} k^\lambda \sum_f q_f^2 b_f^\rho, \quad (6)$$

so that we can eventually identify

$$\zeta b^\mu = 2 \left( \frac{\alpha}{\pi} \right) \sum_f q_f^2 b_f^\mu. \quad (7)$$

In the case of a universal Lorentz symmetry breaking, that means the very same axial-vector  $b^\mu$  for all fermion species, one can derive

$$\zeta = 2q^2 \left( \frac{\alpha}{\pi} \right), \quad (8)$$

where  $\sum_f q_f^2 \equiv q^2$  is a sum over the normalized fermion electric charges,  $q^2 = 3 \cdot 3 \cdot (1/9 + 4/9) + 3 = 8$  being the result for three generations of quarks and leptons in the standard model.

With the help of the LIVDRS [16], the even part of the vacuum polarization tensor can be also found unambiguously. In this work we focus our attention on the LIV deviations of free photons on mass shell  $k^2 \sim 0$ . The latter ones are expected to be really small  $\Delta k^2 \ll m_e^2$  and therefore it makes sense to retain only leading orders in  $k^2$  and  $b^\mu$ . Correspondingly this part of the polarization tensor takes the form,

$$\begin{aligned} \text{reg} \Pi_{2,\text{even}}^{\nu\sigma} = & (k^2 g^{\nu\sigma} - k^\nu k^\sigma) \Pi_{\text{div}} \\ & + \frac{2\alpha}{3\pi} \sum_f q_f^2 (b_f^2 g^{\nu\sigma} - m_f^{-2} S_f^{\nu\sigma}), \end{aligned} \quad (9)$$

in which we have set

$$S_f^{\nu\sigma} \equiv g^{\nu\sigma} [(b_f \cdot k)^2 - b_f^2 k^2] - (b_f \cdot k)(b_f^\nu k^\sigma + b_f^\sigma k^\nu) + k^2 b_f^\nu b_f^\sigma + b_f^2 k^\nu k^\sigma. \quad (10)$$

Here the first term  $\Pi_{\text{div}}$  is logarithmically divergent and does renormalize the electric charges in a conventional way, whereas the second term is finite and involves the sum over the charged fermions of the standard model. Thus, for a universal  $b^\mu$  the induced constants in the Lagrangian density are eventually determined as follows:

$$\delta m_\gamma^2 = -\frac{2\alpha}{3\pi} \sum_f q_f^2 b_f^2 \longrightarrow -\frac{16\alpha}{3\pi} b^2 \quad \text{for SM}, \quad (11)$$

$$\xi = \frac{2\alpha}{3\pi} \sum_f q_f^2 \left(\frac{m_e}{m_f}\right)^2 \simeq \frac{2\alpha}{3\pi}, \quad (12)$$

as the electron is the lightest charged particle.

### 3. LIV dispersion law

Let us now analyze the modified Maxwell's equations

$$\left(1 + \xi \frac{b^2}{m_e^2}\right) \partial_\lambda F^{\lambda\nu} - \frac{\xi}{m_e^2} (b^\rho b_\lambda \partial_\rho F^{\lambda\nu} - b^\nu b_\lambda \partial_\rho F^{\lambda\rho}) + m_\gamma^2 A^\nu - \zeta b_\lambda \tilde{F}^{\nu\lambda} = \partial^\nu B, \quad (13)$$

$$\partial_\nu A^\nu = 0. \quad (14)$$

After contraction of Eq. (13) with  $\partial_\nu$  we find

$$\partial^2 B(x) = 0, \quad (15)$$

whence it follows that the auxiliary field is a decoupled massless scalar field, which is not involved in dynamics.

After using Eq. (14) one can rewrite the field equations in terms of the gauge potential, i.e.,

$$\left(1 + \xi \frac{b^2}{m_e^2}\right) \partial^2 A^\nu - (\xi/m_e^2) [(b \cdot \partial)^2 A^\nu - \partial^\nu (b \cdot \partial)(b \cdot A) + b^\nu \partial^2 (b \cdot A)] + m_\gamma^2 A^\nu - \zeta \epsilon^{\nu\lambda\rho\sigma} b_\lambda \partial_\rho A_\sigma = 0. \quad (16)$$

After contraction of Eq. (16) with  $b^\nu$  we get

$$(\partial^2 + m_\gamma^2)(b \cdot A) = 0, \quad (17)$$

for the special component  $b \cdot A$  of the vector potential. Thus, for this polarization, we actually find the ordinary dispersion law of a real massive scalar field, whereas the two further components with polarizations orthogonal to both  $k_\nu$  and  $b_\nu$  are affected by the fermion induced LIV radiative corrections.

Going to the momentum representation, the equations of motion take the form

$$K^{\nu\sigma} A_\sigma(k) = 0, \quad k^\sigma A_\sigma(k) = 0, \quad (18)$$

where we have set

$$K^{\nu\sigma} \equiv (k^2 - m_\gamma^2) g^{\nu\sigma} - k^\nu k^\sigma - \xi (D/m_e^2) e^{\nu\sigma} + i\zeta \epsilon^{\nu\lambda\rho\sigma} b_\lambda k_\rho. \quad (19)$$

In order to pick out the two independent field degrees of freedom, we have introduced the quantity

$$D \equiv (b \cdot k)^2 - b^2 k^2 \quad (20)$$

and the projector onto the two-dimensional hyperplane orthogonal to  $b_\nu$  and  $k_\nu$ ,

$$e^{\nu\sigma} \equiv g^{\nu\sigma} - \frac{b \cdot k}{D} (b^\nu k^\sigma + b^\sigma k^\nu) + \frac{k^2}{D} b^\nu b^\sigma + \frac{b^2}{D} k^\nu k^\sigma. \quad (21)$$

One can always select two real orthonormal four-vectors corresponding to the linear polarizations in such a way that

$$e_{\nu\sigma} = -\sum_{a=1,2} e_\nu^{(a)} e_\sigma^{(a)}, \quad g^{\nu\sigma} e_\nu^{(a)} e_\sigma^{(b)} = -\delta^{ab}. \quad (22)$$

It is also convenient to define another couple of four-vectors, in order to describe the left- and right-handed polarizations: in our case, those generalize the circular polarizations of the conventional QED. To this aim, let us first define

$$\epsilon^{\nu\sigma} \equiv D^{-1/2} \epsilon^{\nu\lambda\rho\sigma} b_\lambda k_\rho. \quad (23)$$

Notice that we can always choose  $e_\lambda^{(a)}$  to satisfy

$$\epsilon^{\nu\sigma} e_\sigma^{(1)} = e^{(2)\nu}, \quad \epsilon^{\nu\sigma} e_\sigma^{(2)} = -e^{(1)\nu}. \quad (24)$$

Let us now construct the two orthogonal projectors

$$P_{\nu\sigma}^{(\pm)} \equiv \frac{1}{2} (e_{\nu\sigma} \pm i \epsilon_{\nu\sigma}) \quad (25)$$

and set, e.g.,

$$\epsilon_\nu^{(L)} \equiv \frac{1}{2} (e_\nu^{(1)} + i e_\nu^{(2)}) = P_{\nu\sigma}^{(+)} e^{(1)\sigma}, \quad (26)$$

$$\epsilon_\nu^{(R)} \equiv \frac{1}{2} (e_\nu^{(1)} - i e_\nu^{(2)}) = P_{\nu\sigma}^{(-)} e^{(1)\sigma}. \quad (27)$$

We remind that, actually, the left- and right-handed (or chiral) polarizations only approximately [16] correspond to the circular ones of Maxwell QED. In the presence of the CS kinetic term, the field strengths of electromagnetic waves are typically not orthogonal to the wave vectors.

Once the physical meaning of polarizations has been suitably focused, one can readily find the expression of the dispersion relations for the doubly transversal photon modes,

$$\left\{ k^2 - \frac{\xi}{m_e^2} [(b \cdot k)^2 - b^2 k^2] - m_\gamma^2 \right\}^2 - \zeta^2 [(b \cdot k)^2 - b^2 k^2] = 0. \quad (28)$$

Evidently real solutions exist only iff

$$D = (b \cdot k)^2 - b^2 k^2 \geq 0,$$

consistently with our previous notations. Notice that on the photon mass shell, deviations off the light-cone are of order  $|b_\nu|^2$ . As a consequence, the on-shell momentum dependence of the polarization tensor (5) is dominated by the lowest order  $k^2 = 0$ , whereas the higher orders in  $k^2$  do represent simultaneously higher orders in  $b_\nu$ , which are neglected in the present analysis.

#### 4. LIVQED consistency

It is worthwhile to recall that a time-like axial-vector  $b^\mu$  is required for a consistent fermion quantization [16,19,20]. Nonetheless, one has to take into account that, on the one hand, in the lack of a bare photon mass and/or a bare CS vector of different direction a time-like vector  $b^\mu$  just leads to a tachyonic massive photon [18,19] and instability of the photodynamics, that means imaginary energies for the soft photons. On the other hand, a space-like vector  $b^\mu$  causes problems for fermion quantization [16,20] and *a fortiori* for the very meaning of the radiative corrections. There are two ways to avoid this obstruction.

(A) If we adopt classical photons in Lorentz invariant QED to be massless  $\mu_\gamma = 0$  then, in such a situation, a fully induced LIV appears to be flawless only for *light-like* axial-vectors  $b^\mu$ . In particular, for a light-like universal axial-vector  $b^\mu = (|\vec{b}|, \vec{b})$ , we find the dispersion relations for the LIV 1-particle states of a fermion species  $f$  that read

$$p_+^0 + |\vec{b}| = \pm \sqrt{(\vec{p} + \vec{b})^2 + m_f^2}, \tag{29}$$

$$p_-^0 - |\vec{b}| = \pm \sqrt{(\vec{p} - \vec{b})^2 + m_f^2}. \tag{30}$$

Now, it turns out that the requirement  $p_\pm^2 > 0$  for the LIV free 1-particle spinor physical states just drives to the high momenta cut-off  $|\vec{p}| \leq m_e^2/4|\vec{b}|$  which is well compatible with the LIVDRS treatment of fermion loops.

Then one can use the induced values of the LIV parameters (7), (12) in the case  $b^2 = 0$  and the dispersion law for photons (28) is reduced to

$$k^2 - (\xi/m_e^2)(b \cdot k)^2 \pm \zeta b \cdot k = 0. \tag{31}$$

For photon momenta  $|\vec{k}| \gg |\vec{b}|$  one approximately finds the relationship for positive energies (frequencies)

$$k_0 \simeq |\vec{k}|(1 + \delta c_\theta) \mp \zeta |\vec{b}| \sin^2 \theta/2, \\ \cos \theta \equiv \frac{\vec{b} \cdot \vec{k}}{|\vec{b}||\vec{k}|}, \quad \delta c_\theta \equiv \frac{2\xi}{m_e^2} |\vec{b}|^2 \sin^4 \theta/2, \tag{32}$$

and a similar expression for negative energies (frequencies). One can see clearly that LIV entails an increase  $\delta c_\theta$  of the light velocity, which makes it different from its decrement generated by quantum gravity in the leading order [12,13]. Both the variation in the light velocity and the birefringence effect [1] caused by a phase shift between left- and right-polarized photons—alternate signs in (32)—depend upon the direction of the wave vector  $\vec{k}$ . Both effects do vanish in the direction collinear with  $\vec{b}$ . Thus the compilation of the UHECR data in search for deviations of the speed of light must take into account this possible anisotropy of photon spectra.

This is also true for the compilation of the data on polarization plane rotation for radio waves from remote galaxies. The earlier search for this effect [1,22,23] led to the very stringent upper bound on values of  $|\vec{b}| < 10^{-31}$  eV.

However, in addition to the previous remark on the photon spectrum anisotropy, we would like to give more arguments in favor of a less narrow room for the possibility of LIV and CPT breaking in the universe. Indeed one must also take into account the apparent time variation of an anisotropic CS vector, when its origin derives from the v.e.v. of a parity-odd quintessence field [24] very weakly coupled to photons. That v.e.v. may well depend on time and obtain a tiny but sizeable value in the later epoch of the universe evolution [25], just like the cosmological constant [26] might get. As well a non-vanishing CS vector may be induced also by the non-vanishing v.e.v. of a dark matter component if its coupling to gravity is CPT odd. Eventually it means that, for large distances corresponding to earlier epochs in the universe, one may not at all experience this kind of LIV and CPT breaking. Conversely, in a later time such a CS term may gradually rise up. Then, the earlier radio sources—galaxies and quasars with larger Hubble parameters—may not give any observable signal of birefringence, whereas the individual evidences from a nearest radio source may be of a better confidence. So far we cannot firmly predict on what is an actual age of such CPT odd effects and therefore, to be conservative, one has to rely upon the lab experiments and meantime pay attention to the data from quasars of the nearest universe. Thus one may certainly trust to the estimations [3] performed in the laboratory and the nearest universe observations. So far the most conservative value of the LIV parameter from [3,21] arises from hydrogen maser experiments: namely,  $|\vec{b}_e| < 10^{-18}$  eV for electrons.

(B) Another way to implement the LIV, solely by fermion coupling to an axial-vector background, is to start with the Maxwell’s photodynamics supplemented by a bare and *Lorentz invariant* photon mass  $\mu_\gamma$ , so that

$$m_\gamma^2 = -\frac{2\alpha}{3\pi} \sum_f q_f^2 b_f^2 + \mu_\gamma^2. \tag{33}$$

Then, for a genuine time-like  $b^\mu = (\sum_f q_f b_f^0, 0, 0, 0)$  one finds from Eq. (28) and the definition (6) the following dispersion laws: namely,

$$k_0^2 = \left(1 + \frac{\xi b_0^2}{m_e^2}\right) \left(|\vec{k}| \pm \frac{1}{2} \zeta b_0\right)^2 + m_\gamma^2 \\ - b_0^2 \left\{ \frac{1}{4} \zeta^2 + O(b_0 |\vec{k}|/m_e^2) \right\}. \tag{34}$$

Hence, if  $m_\gamma \geq \zeta b_0/2 = 8\alpha b_0/\pi$ , then the photon energy keeps real for any wave vector  $\vec{k}$  and LIVQED happens to be consistent. Meantime the longitudinal photon polarization exhibits the entire mass  $m_\gamma$ . Then the present day very stringent experimental bound on the photon mass [27],  $m_\gamma < 6 \times 10^{-17}$  eV, does produce the limit  $b_0 < 3 \times 10^{-15}$  eV.

To conclude we would like to make few more comments on estimates for the LIV vector components.

- (1) There are no better bounds on  $b_\mu$  coming from the UHECR data on the speed of light for photons. This is because the increase of the speed of light depends quadratically on components of  $b_\mu$ . Thus, for example, the data cited in [2, 3] do imply less severe bounds on  $|\vec{b}|$  or  $b_0$  than those ones above mentioned.
- (2) For the LIVQED examined in the present Letter, the typical bounds on LIV and CPT breaking parameters in the context of quantum gravity phenomenology are not good enough to compete with the laboratory estimations. They are, in fact, of a similar order of magnitude as other LIV effects in the high energy astrophysics.
- (3) An interesting bound on deviations of the speed of light is given in [28] where, in the spirit of quantum gravity phenomenology, space–time fluctuations are addressed to produce modifications of the speed of light and, as well, of the photon dispersion relations exhibiting helicity dependent effects. Using an interferometric technique, the authors of Ref. [28] were able to estimate  $\Delta c < 10^{-32}$ . However this estimation does not imply a better bound for a LIV vector  $b_\mu$ , as it actually gives  $b_0 < 10^{-12}$  eV, which is certainly in agreement with the more stringent bounds discussed above.

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