



Can one phase induce all CP violations including leptogenesis?

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Abstract

In the framework of a SUSY $SO(10)$ model a phase is generated spontaneously for the $B - L$ breaking VEV. Fitting this phase to the observed CP-violating K, B decays all other CP breaking effects are uniquely predicted. In particular, the amount of leptogenesis can be explicitly calculated and found to be in the right range and sign for the BAU.

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CP violation is directly observed only in the decays of the K and B mesons. The present experimental results [1] are consistent at the moment with the standard model (SM). I.e., CP breaking is induced by a phase in the Cabibbo, Kobayashi, Maskawa (CKM) mixing matrix of the quarks.

Extensions of the SM using right-handed (RH) neutrinos, that account for the neutrino oscillations, involve in general phases which allow for CP violation in the leptonic sector also. This CP breaking is difficult to observe but may be detected as soon as neutrino factories are available. The observation of neutrino-less double beta decays may be also an indication for Majorana phases in the neutrino sector [2].

Spontaneous generation of baryon asymmetry in the universe (BAU) needs CP violation [3]. It is clear now that it requires also extension of the SM, while baryon asymmetry in the universe (BAU) à la Fukugita and Yanagida [4] due to leptogenesis [5] is the most popular and promising theory for the BAU.

Where is the CP breaking coming from?

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CP breaking can be induced via phases in the Yukawa coupling, in the interactions of the LH and RH gauge bosons and in the VEVs. Phases in the spontaneously generated VEVs lead naturally to violation of CP. This spontaneous breaking can also help to solve the strong CP problem [7,8].

The spontaneous violation of CP was already suggested long ago by Lee [9]. In the framework of $SO(10)$ GUT spontaneous breaking was first discussed by Harvey, Reiss and Ramond [10]. Recently, Bento and Branco [11] added to the SM a heavy Higgs scalar with a $B - L$ violating VEV to generate spontaneous CP violation.

In general, the known CP violation in the hadronic sector is not related to the leptonic one. Even the CP breaking needed for leptogenesis is usually independent of that in the leptonic sector. Hence, CP violation in the leptonic sector is in general not predictable. Predictability can be gained only in terms of a specific model. There are quite a few models relating CP violation in the neutrino sector to leptogenesis [12] but no conventional SUSY GUT which connects the leptogenesis to the observed violation in the K and B decays is presently known.

I would like to suggest in this Letter that the one and only origin for CP violation is a spontaneous breaking at high energies. A phase in the $B - L$ breaking VEV can induce all manifestations of CP violation. This phase can be fixed by the observed breaking in the K and B decays and the other CP violations are then predicted. In particular, we will show explicitly that within a SUSY $SO(10)$ model the amount of leptogenesis is exactly that needed to have the right BAU.

Let me first show how a phase can be spontaneously generated in the $SU(5)$ singlet component of a scalar $\mathbf{126}$ representation of $SO(10)$. It was already pointed out by Harvey, Ramond and Reiss [10] that there is a natural way to break CP spontaneously at high energies. This is due to the fact that $(\mathbf{126})^4$ is $SO(10)$ invariant. $\Phi_{\mathbf{126}}$ is the Higgs representation used to break down $B - L$. Its $SU(5)$ singlet component gives also masses to the heavy RH neutrinos. The corresponding large VEV induces also small VEVs in the components of $\Phi_{\mathbf{126}}$ that transform like $\mathbf{2}_L$ under the SM [13] which play a role in the light fermion mass matrices.

Assume that all the parameters in the $SO(10)$ invariant Lagrangian are real. If the three fermionic families are in $\Psi_{\mathbf{16}}$'s, only $\Phi_{\mathbf{10}}$, $\Phi_{\mathbf{126}}$ and $\Phi_{\mathbf{120}}$ can contribute to the mass terms:

$$\mathbf{16} \times \mathbf{16} = (\mathbf{10} \oplus \mathbf{126})_S \oplus (\mathbf{120})_{AS}. \quad (1)$$

Suppose we have chosen global symmetries that dictate a (super-)potential of the form¹ [11]

$$V(\lambda_1, \lambda_2, \dots) = V_0 + [\dots + \lambda_1 (\Phi_{\mathbf{10}})_S^2] [(\Phi_{\mathbf{126}})_S^2 + (\Phi_{\mathbf{126}})_{\mathbf{2}_S}^2] + \lambda_2 [(\Phi_{\mathbf{126}})_S^4 + (\Phi_{\mathbf{126}})_{\mathbf{2}_S}^4] \quad (2)$$

and that those are the only phase dependent terms after the spontaneous breaking.² If the $SU(5)$ singlet component of $\Phi_{\mathbf{126}}$ and $\Phi_{\mathbf{126}}$ acquire a VEV as well as the right component of $\Phi_{\mathbf{10}}$:

$$\langle \Phi_{\mathbf{10}} \rangle = \frac{v}{\sqrt{2}}, \quad \langle \Phi_{\mathbf{126}} \rangle = \frac{\gamma}{\sqrt{2}} e^{i\alpha}. \quad (3)$$

The phase dependent part of the potential can be then written as

$$V(v, \gamma, \alpha) = A \cos(2\alpha) + B \cos(4\alpha). \quad (4)$$

For B positive and $|A| > 4B$ the absolute minimum of the potential is obtained with

$$\alpha = \frac{1}{2} \arccos\left(\frac{A}{4B}\right). \quad (5)$$

This spontaneous generation of a phase in the large VEV γ , will generate also phases in the induced small VEVs which give mass to the light fermions. Those will lead to CP violation in the quark and lepton sectors. The

¹ Note that $\mathbf{10}$ is a real representation.

² For a detailed discussion of possible scalar potentials see Ref. [10]. The $[(\Phi_{\mathbf{126}})_S^4 + (\Phi_{\mathbf{126}})_{\mathbf{2}_S}^4]$ part serves also to break the continuous global symmetries avoiding massless Nambu–Goldstone bosons.

value of the spontaneously generated phase α depends on arbitrary parameters in the Higgs potential. Its actual value can be however fixed by the requirement that the phases of the induced light VEVs will give the observed CP violation in the K, B decays. All other manifestations of CP violation will then be explicitly given. In particular the amount of leptogenesis is then predicted in models where M_ν^{Dirac} is known.

Let me now explicitly calculate the amount of leptogenesis in a SUSY $SO(10)$ model where a phase is generated spontaneously in the $B - L$ breaking VEV. The model was developed in a series of papers [14,15]. It was originally aimed to find explicitly the mixing angles which are hidden in the SM, like RH rotations. Those allow to calculate explicitly, e.g., the proton decay branching ratios as well as all mass matrices and in particular the Dirac neutrino mass matrix and the RH neutrino mass matrix which are needed for the calculation of the leptogenesis. We will use here the mass matrices given in Ref. [15]. This is a renormalizable SUSY $SO(10)$ model, i.e., $B - L$ is broken via $\Phi_{\overline{126}} + \Phi_{126}$ while $\Phi_{\overline{126}}$ gives mass to the RH neutrinos (without using non-renormalizable contributions). The origin of CP breaking in the model is a phase in the $SU(5)$ singlet component of one $\Phi_{\overline{126}}$. A global horizontal symmetry $U(1)_F$ dictates the asymmetric Fritzsch texture [16] for the fermionic mass matrices and the possible VEVs in the different Higgs representations. By fitting the free parameters to the observed masses and CKM matrix a set of non-linear equations is obtained. These equations have five solutions which obey all the restrictions, i.e., five sets of explicit mass matrices. The Dirac neutrino mass matrices have the texture:

$$M_\nu^{\text{Dirac}} = \begin{pmatrix} 0 & A & 0 \\ B & 0 & C \\ 0 & D & E \end{pmatrix}. \tag{6}$$

They are given explicitly in Table 1.

The RH neutrino mass matrices have the following form in our model:

$$M_{\nu R} = e^{i\alpha} \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & -b \end{pmatrix} M_R. \tag{7}$$

Where the real $a, b > 0$. The corresponding eigenmasses are given in Table 2.

What is leptogenesis?

Out of equilibrium CP-violating decays of RH neutrinos, N_i , produce excess of the lepton number $\delta L \neq 0$. This will induce baryon asymmetry through $B + L$ conserving sphaleron processes [4–6].

Table 1
The Dirac neutrino mass matrices for the five solutions (for $\tan \beta = 10$) in GeV

Solution GeV	1	2	3	4	5
$\text{Re}(M_\nu^{\text{Dirac}})_{12}$	17.486	26.953	-41.320	-41.320	-28.274
$\text{Im}(M_\nu^{\text{Dirac}})_{12}$	0.0394	0.0607	0.0929	-0.0929	-0.06356
$\text{Re}(M_\nu^{\text{Dirac}})_{21}$	17.654	27.120	-41.218	-41.218	-28.172
$\text{Im}(M_\nu^{\text{Dirac}})_{21}$	0.0394	0.0607	0.0929	-0.0929	-0.06356
$(M_\nu^{\text{Dirac}})_{23}$	-113.425	-142.425	116.073	82.073	102.073
$(M_\nu^{\text{Dirac}})_{32}$	-14.700	14.302	10.695	44.695	24.695
$\text{Re}(M_\nu^{\text{Dirac}})_{33}$	-127.913	-176.670	146.103	146.103	78.715
$\text{Im}(M_\nu^{\text{Dirac}})_{33}$	-0.3152	-0.4249	0.2788	0.2788	0.1271

Table 2
The masses of the RH neutrinos for the five solutions in 10^{13} GeV

Solution 10^{13} GeV	1	2	3	4	5
$M_1 = M_2$	5.2	9.1	16	18	12
M_3	8×5.2	7×9.1	3×16	3×18	2×12

The amount of CP violation in these decays is:

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow L_i + \Phi) - \Gamma(N_i^\dagger \rightarrow L_i^\dagger + \Phi^\dagger)}{\Gamma(N_i \rightarrow L_i + \Phi) + \Gamma(N_i^\dagger \rightarrow L_i^\dagger + \Phi^\dagger)}.$$

Knowing the details of CP violation in the leptonic sector as well as the RH mixing angles,³ one is able to calculate explicitly the BAU via leptogenesis. This is the main test of the model.

Let us denote the Dirac neutrino mass matrix M_v^{Dirac} in the basis where M_{ν_R} is real diagonal with positive eigenvalues: M_D . In this basis ϵ_i can be expressed as follows

$$\epsilon_i = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j \neq 1} \text{Im}[(M_D^\dagger M_D)_{ij}^2] f(M_j^2/M_i^2),$$

where

$$f(x) = \sqrt{x} \left[\ln \left(1 + \frac{1}{x} \right) + \frac{2}{x-1} \right]$$

and $v = 174 \times \sin \beta$ GeV.⁴

M_{ν_R} is given in Eq. (7) and its eigenmasses in Table 2.

It is diagonalized by a matrix U

$$U^T M_{\nu_R} U = \text{diag}(M_1, M_2, M_3) = M_3 \text{diag} \left(\frac{M_1}{M_3}, \frac{M_2}{M_3}, 1 \right),$$

$$U = O P, \quad \text{where } P = e^{-(i/2)\alpha} \text{diag}(i, 1, i)$$

and

$$O = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In this basis, in terms of Eq. (6)

$$M_D^\dagger M_D = \begin{pmatrix} 1/2(|A|^2 + |B|^2 + |D|^2) & i/2(|A|^2 - |B|^2 + |D|^2) & 1/\sqrt{2}(B^\dagger C - D^\dagger E) \\ -i/2(|A|^2 - |B|^2 + |D|^2) & 1/2(|A|^2 + |B|^2 + |D|^2) & i/\sqrt{2}(B^\dagger C + D^\dagger E) \\ 1/\sqrt{2}(BC^\dagger - DE^\dagger) & -i/\sqrt{2}(BC^\dagger + DE^\dagger) & |C|^2 + |E|^2 \end{pmatrix}.$$

This gives the following general results

$$\begin{aligned} \text{Im}((M_D^\dagger M_D)_{12} (M_D^\dagger M_D)_{12}) &= \text{Im}((M_D^\dagger M_D)_{21} (M_D^\dagger M_D)_{21}) = 0, \\ (M_D^\dagger M_D)_{11} &= (M_D^\dagger M_D)_{22}. \end{aligned} \quad (8)$$

Due to the degeneracy of N_1, N_2 , the decay of both contributes to ϵ_i . However, Eq. (8) avoids the possible singularity in $f(x)$. Hence,

$$\epsilon_L = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} (\text{Im}[(M_D^\dagger M_D)_{13}^2] + \text{Im}[(M_D^\dagger M_D)_{23}^2]) f(M_3^2/M_1^2).$$

The BAU is given then (in the minimal supersymmetric SM) as

$$Y_B = -1/3 \frac{\epsilon_L}{g^*} d_{B-L},$$

³ Note, that $M^\dagger M$ is diagonalized using the RH mixing matrix.

⁴ $\tan \beta = 10$ is used in the model [15].

Table 3
The CP asymmetry ϵ_L , the dilution factor d_{B-L} and the baryon asymmetry Y_B for the five solutions

Solution	ϵ_L	d_{B-L}	Y_B
1	-6.5×10^{-7}	0.0064	6.1×10^{-12}
2	-6.6×10^{-5}	0.0074	7.1×10^{-10}
3	-7.4×10^{-5}	0.0088	9.5×10^{-10}
4	-1.3×10^{-6}	0.009	1.7×10^{-11}
5	-5.6×10^{-5}	0.06	4.9×10^{-10}

where $g^* = 228.75$ and d_{B-L} is the dilution factor due to inverse decay washout effects and lepton number violating scattering. It must be obtained by solving the corresponding Boltzmann equation. There are different approximate solutions in the literature. The frequently used approximate solution [17] is good only for

$$K = \frac{\tilde{m}_1 M_P}{1.7 \times 8\pi v^2 \sqrt{g^*}} = \frac{\tilde{m}_1 \text{ (eV)}}{1.08 \times 10^{-3} \text{ (eV)}} > 1,$$

where $\tilde{m}_1 = \frac{(M_D^\dagger M_D)_{11}}{M_1}$. In our model however, $K \approx 10^{-2}$.

Buchmüller et al. [6] studied recently in detail both cases $K > 1$ and $K < 1$. They found that for $K < 1$ one must take into account thermal correction due to the gauge bosons and the top quark. Hence, d_{B-L} depends on “initial conditions” and they found⁵ that for $K \approx 10^{-2}$.

$$10^{-4} \geq d_{B-L} \leq 10^{-2}.$$

Hirsch and King [18] give empirical approximate solutions for the case $K \ll 1$. The solution corresponding to our model is

$$\text{Log}_{10}(d_{B-L}) = 0.8 \times \text{Log}_{10}(\tilde{m}_1 \text{ eV}) + 1.7 + 0.05 \times \text{Log}_{10}(M_1/10^{10} \text{ GeV}).$$

I will use this expression to have a definite prediction. The results for the five solutions are given in Table 3.

This must be compared with the experimental results:

BOOMerANG and DASI [19]

$$0.4 \times 10^{-10} \leq Y_B \leq 1.0 \times 10^{-10}.$$

WMAP and Sloan Digital Sky Survey [20]

$$Y_B = (6.3 \pm 0.3) \times 10^{-10}.$$

Hence,

- Solutions 1 and 3 are probably excluded. The other solutions are consistent with the experimental observation, especially if the uncertainty in d_{B-L} is taken into account.
- All solutions have the right sign. This is the main prediction of the model in view of the uncertainty in d_{B-L} . I must emphasize that there is no ambiguity in the prediction of the sign because of the following reasons:
 - (a) The sign of M_1 must be positive because ϵ_i is calculated in terms of M_D which is the neutrino Dirac mass matrix in the basis where the RH neutrino mass matrix (7) is diagonal, real and positive;
 - (b) The parameters and especially the phases of M_ν^{Dirac} (6) are fixed without ambiguity for each one of the above solutions, although one cannot write explicitly their dependence on α . As was mentioned before, the entries to the mass matrices are solutions of non-linear equations in which the induced components of Φ_{126}

⁵ See Fig. 9 in their paper where d_{B-L} is called κ_f .

Table 4
The leptonic mixing matrix for the different solutions

Solution	1	2	3	4	5
Re(U_{PMNS}) ₁₁	-0.8583	0.8136	0.7465	0.8579	0.8740
Im(U_{PMNS}) ₁₁	0.000004	0.00034	-0.000001	-0.000001	0.000001
Re(U_{PMNS}) ₁₂	-0.5104	-0.5778	-0.6589	-0.5059	-0.4806
Im(U_{PMNS}) ₁₂	-0.000007	0.000007	-0.00027	-0.00021	-0.0002
Re(U_{PMNS}) ₁₃	-0.0526	-0.0644	0.0927	0.0897	0.0717
Im(U_{PMNS}) ₁₃	0.000002	0.00026	0.00042	0.00004	0.00003
Re(U_{PMNS}) ₂₁	-0.3496	-0.4869	-0.4653	-0.3754	-0.2492
Im(U_{PMNS}) ₂₁	0.00191	0.00190	0.00212	0.0017	0.00088
Re(U_{PMNS}) ₂₂	0.6567	-0.6168	-0.6167	-0.7364	-0.5670
Im(U_{PMNS}) ₂₂	-0.0030	0.0029	0.00260	0.0031	0.00018
Re(U_{PMNS}) ₂₃	-0.6682	-0.6185	-0.6350	-0.5628	-0.7829
Im(U_{PMNS}) ₂₃	0.0031	0.00285	0.0029	0.0026	0.0028
Re(U_{PMNS}) ₃₁	-0.3756	-0.3176	-0.4756	-0.3508	-0.4172
Im(U_{PMNS}) ₃₁	0.00082	0.00085	0.00216	0.0009	0.0011
Re(U_{PMNS}) ₃₂	0.5552	-0.6168	-0.4309	-0.4492	-0.6664
Im(U_{PMNS}) ₃₂	-0.00121	0.00127	0.0009	0.00097	0.0014
Re(U_{PMNS}) ₃₃	0.7421	0.7832	0.7669	0.82168	0.6179
Im(U_{PMNS}) ₃₃	-0.00163	-0.00204	-0.0020	-0.00213	-0.0016

Table 5
The CP violation invariant for the leptonic sector J_{leptons} and the effective neutrino mass for the neutrino-less double-beta decay for the five solutions

Solution	1	2	3	4	5
J_{leptons}	0.0092	0.000059	9.8×10^{-6}	7.8×10^{-6}	6.6×10^{-6}
$\langle m_{ee} \rangle$	0.0031	0.005	0.0068	0.0056	0.0029

(with the phase α) are involved. The physical value of α is then fixed by requiring that $J_{\text{Jarlskog}} \sim 10^{-5}$ to be $\alpha \sim 0.003$.⁶

To complete the predictions of the model let me use the complex lepton mixing matrix U_{PMNS} of Ref. [15] (see Table 4) to give the amount of CP violation in the neutrino oscillation

$$J_{\text{leptons}} = \text{Im}(U_{11}U_{22}U_{12}^*U_{21}^*)$$

and the value of $\langle m_{ee} \rangle$

$$\langle m_{ee} \rangle = \sum_{i=1}^3 (U_{e1})^2 m_i$$

relevant for the neutrino-less double-beta decay $\beta\beta_{0\nu}$.⁷ See Table 5.

⁶ In a recent paper Frampton, Glashow and Yanagida in Ref. [12] presented a model where the sign of the BAU can be related to the CP violation in neutrino oscillation experiments. In our model both CP violation in the neutrino oscillation as well as the sign of the BAU are predicted in terms of CP violation in the quark sector.

⁷ m_1 in our solutions is of $O(10^{-3} \text{ eV})$.

Conclusions

I presented in the Letter the following observations:

CP is naturally broken spontaneously at high energies in $SO(10)$ GUTs.

A phase is generated in a VEV and not in the Yukawa couplings, as it is usually done. This can be used as the only origin CP violation.

In the framework of a SUSY $SO(10)$ model that uses this idea, fitting to the observed CP violation, as it is reflected in the CKM matrix, fixes uniquely the CP breaking in the leptonic sector without free parameters. An explicit calculation of leptogenesis in this model gives solutions consistent with the range and sign of the observed BAU.⁸

Our model applies the conventional see-saw mechanism [22], it is possible however, to use a similar program for the type II see-saw [23] as well [24].

The large value of the RH neutrino mass can be incompatible with the gravitino problem if SUSY is broken in the framework of mSUGRA. Possible solutions are discussed in the literature. E.g., Ibe, Kitano, Murayama and Yanagida [25] presented very recently a nice solution based on anomaly mediated SUSY breaking.

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⁸ “A common origin for all CP violations” was suggested recently also by Branco et al. [21]. They use a non-SUSY SM extended by adding scalar Higgs, leptons and exotic vector-like quarks. The complex phase is generated spontaneously in the VEV of the heavy singlet scalar meson. The connection with the low energy CP violation in the hadronic sector is obtained only via mixing with the exotic quarks. They give also no explicit value for the leptogenesis.

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