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Inflation in entropic cosmology: Primordial perturbations and non-Gaussianities

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ABSTRACT

We investigate thermal inflation in double-screen entropic cosmology. We find that its realization is general, resulting from the system evolution from non-equilibrium to equilibrium. Furthermore, going beyond the background evolution, we study the primordial curvature perturbations arising from the universe interior, as well as from the thermal fluctuations generated on the holographic screens. We show that the power spectrum is nearly scale-invariant with a red tilt, while the tensor-to-scalar ratio is in agreement with observations. Finally, we examine the non-Gaussianities of primordial curvature perturbations, and we find that a sizable value of the non-linearity parameter is possible due to holographic statistics on the outer screen.

1. Introduction

As early as the study of black hole physics [1,2], the holographic thermodynamics was discovered to be related to the quantization of Einstein gravity as a nonperturbative quantum feature. In particular, the holographic principle was conjectured as a significant property of quantum gravity, stating that physics of a volume of space is encoded on its boundary, such as a gravitational horizon [3]. This principle was also applied in cosmology [4,5] and it was studied in detail in string theoretical background [6].

Based on these, an extended holographic picture was suggested by Verlinde [7] in which Einstein gravity is no longer a fundamental theory, but it emerges from a statistic effect of a holographic screen, while a similar scenario was discussed by Padmanabhan [8]. The cosmological application was extensively studied in the literature, for example see Refs. [9–11] and references thereafter. However, this theory involves the controversial issue of whether the uniqueness of gravity is preserved in such an emergent scenario. Therefore, a more explicit formulation of entropic gravity theory was suggested [12,13], in which Einstein gravity is still a fundamental theory but with a boundary term being introduced. Such a boundary term provides a holographic statistics and thus it leads to an entropic force in bulk physics. This model was soon applied to realize the current acceleration [12] and inflation [13] at early times, but it has also led to some criticism from the point of view of observations [14].

On the other hand, inflation has been widely considered as a remarkably successful theory in describing the very early universe [15]. In this paradigm, the primordial curvature perturbation caused by the quantum fluctuations of the inflaton field was found to be nearly scale-invariant and thus it is able to form the Large Scale Structure (LSS) of our universe [16]. Currently many observations, particularly the angular spectrum of the Cosmic Microwave Background (CMB) anisotropies [17] and the power spectrum of density fluctuations observed for the LSS [18], strongly support the compatibility of inflationary cosmology for describing the early universe.

Recently, an explicit scenario of realizing the inflationary period in entropic cosmology was proposed in [19], composed by two holographic screens. In particular, it was found that inflationary solutions can be achieved even in a radiation dominated universe, provided the two screens are not at thermal equilibrium. Such realizations of "thermal inflation" have become an interesting issue in recent studies of entropic inflationary cosmology.[1]

In the present work we are interested in investigating thermal inflation in double-screen entropic cosmology, both at the background as well as at the perturbations level. In particular, after showing the generality of inflationary solutions at high energy scales, we study the primordial curvature perturbations. As we will see, the main contribution arises from the holographic fluctuations generated on the outer screen, while the usual thermal fluctuations of the universe content is subdominant, and the resulting a power spectrum is nearly scale-invariant with a red tilt. Additionally, by examining the non-Gaussianities for holographic initial conditions, we find that a sizable non-linearity parameter could be obtained.

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[1] See also [20,21] for relevant discussion in Verlinde's framework.

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The outline of this Letter is as follows. In Section 2 we briefly review the scenario of entropic cosmology with two holographic screens, focusing on the background evolution, and in Section 3 we examine the realization of thermal inflation in this model. In Section 4 we perform an analysis of the cosmological perturbations generated during thermal inflation, which are mainly of holographic origin. Then, in Section 5 we estimate the non-Gaussianities that arise in the examined scenario. Finally, Section 6 is devoted to the summary of the obtained results.

2. A double-screen model of entropic cosmology

In this work we are interested in investigating thermal inflation in a scenario of entropic cosmology involving two holographic screens[19]. However, let us first remind the basic features of standard, one-screen, entropic cosmology, which is also called *EFS* scenario [12,13].

2.1. One-screen entropic cosmology

In usual entropic cosmology one incorporates a gravitational system including matter fields and surface terms of the form of

$$\mathcal{I} = \int_{M_b} \left(\frac{R}{16\pi G} + \mathcal{L}_m \right) + \oint_{\partial M_b} \mathcal{L}_b, \tag{1}$$

where R is the Ricci scalar of the whole spacetime, $\mathcal{L}_m$ is the Lagrangian of matter fields living in the bulk, and $\mathcal{L}_b$ is the corresponding Lagrangian describing the physics of the boundary. Clues from string theory and AdS/CFT indicate that the boundary terms should include the extrinsic curvature of the boundary and holographic dual gauge theories. Finally, throughout the Letter we use the convention $c = k_B = \hbar = 1$ and $M_p = 1/\sqrt{G}$.

By varying the action with respect to the metric, one obtains the Einstein field equation as follows,

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T_m^{\mu\nu} + J_b^{\mu\nu}, \tag{2}$$

in which the last term J_b is a current describing the exchange of energy and momentum between the bulk and the boundary. This term is determined by the holographic description of boundary physics, and so is a nonlocal effect which corresponds to an entropic force in the universe.

Assuming that the boundary physics can be described by thermodynamics satisfying a holographic distribution, the number of degrees of freedom on this holographic screen is proportional to its area, that is $N \propto A$. Thus, the classical holographic entropy on this screen is given by

$$S_b = \frac{A}{4G} = \frac{\pi}{G} r_b^2, \tag{3}$$

where r_b is the radius location of the boundary surface. Therefore, variation of energy with respect to the radius will provide us the entropic force [7]:

$$F_e = -\left(\frac{dE}{dr}\right)_b = -\left(T \frac{dS}{dr}\right)_b = -\frac{2\pi}{G} T_b r_b, \tag{4}$$

in which T_b is the temperature of the boundary of the system. Finally, due to the Unruh effect (when a test particle with mass m is located nearby the holographic screen the variation of the entropy on this screen with respect to the radius takes the form of $\frac{dS}{dr} = -2\pi m$) the above force yields an entropic acceleration a_e of the form [22]

$$a_e \equiv \frac{F_e}{m} = 2\pi T_b. \tag{5}$$

Note that the corresponding entropic pressure is negative $P_e = F_e/A_b = -T_b/2Gr_b$, and so it is expected to realize an accelerating process of the universe.

Let us apply the above results into a homogeneous and isotropic flat Friedmann–Robertson–Walker (FRW) universe described by the metric

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i. \tag{6}$$

In usual, one-screen entropic cosmology, the boundary r_b, that is the location of the holographic screen, is assumed to be near the Hubble horizon $r_H = H^{-1}$, where $H \equiv \dot{a}/a$ is the Hubble parameter of the universe. This non-complete coincidence is quantified by the parameter β [19], that is we write

$$r_b = (\beta H)^{-1}, \tag{7}$$

while the boundary temperature is

$$T_b = \frac{\beta H}{2\pi}. \tag{8}$$

Thus, substituting everything in the field equations, we obtain the modified Friedmann acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \beta^2 H^2, \tag{9}$$

where ρ and p are respectively the total energy density and pressure of the content of the universe. In this expression, the last term accounts for the cosmological acceleration due to the entropic force.

A final addition must be made, concerning the precise form of the horizon entropy. In particular, quantum gravitational and string theoretical considerations, taking into account higher order quantum corrections [23] and the holographic renormalization group flow [24], yield an improved relation for the entropy with leading order correction as:

$$S = \frac{1}{4G}\left(A + gG \ln \frac{A}{G} + \cdots\right), \tag{10}$$

where the coefficient g is determined by the specific environment and it is left as a free parameter. Thus, the Friedmann acceleration equation arising from this improved entropic relation reads [13]

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \beta^2 H^2 + \frac{gG\beta^4 H^4}{4\pi} + \cdots. \tag{11}$$

Although the aforementioned scenario is qualitatively very interesting, the above modified Friedmann equation (with β^2 of the order of $\mathcal{O}(1)$) cannot quantitatively describe the radiation and matter epochs. One interesting way out is the additional consideration of a second holographic screen.

2.2. Double-screen entropic cosmology

Since one-screen considerations exhibit difficulties in quantitatively describing the thermal history of the universe, a double-screen extension was introduced in [19]. Since the Hubble horizon (or a surface near it) is the natural choice for the outer boundary of the universe, one introduces an additional “inner” boundary, which is just the Schwarzschild horizon of the whole universe. The corresponding Schwarzschild radius r_S is given by

$$r_S = 2GM_{tot} = 2G \int_{M_b} \rho\, dV = \frac{8\pi G\rho}{3\beta^3 H^3}, \tag{12}$$

where we have used that the volume of the universe is $V = 4\pi r_b^3/3$. Its corresponding temperature is given by

$$T_S = \frac{1}{8\pi G M_{tot}} = \frac{3\beta^3 H^3}{32\pi^2 G\rho}, \tag{13}$$

and therefore its induced acceleration (with the simple entropy form) will be

$$a_e = 2\pi T_S, \tag{14}$$

but with direction towards the inner screen, that is opposite to the outer one.

In summary, in double-screen entropic cosmology, the induced acceleration is

$$a_e = 2\pi(T_b - T_S) = \beta H\left(1 - \frac{3\beta^2 H^2}{16\pi G\rho}\right), \tag{15}$$

that is it incorporates a competition of entropic effects from the outer and the inner screens. Consequently, the modified Friedmann acceleration equation in this scenario writes as [19]

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \beta^2 H^2\left(1 - \frac{3\beta^2 H^2}{16\pi G\rho}\right). \tag{16}$$

Finally, if instead of the simple entropy form we use the quantum corrected one (10), the modified Friedmann equation in double-screen cosmology becomes

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + f(\rho, H), \tag{17}$$

with the form of surface function being

$$f(\rho, H) = \beta^2 H^2\left(1 - \frac{3\beta^2 H^2}{16\pi G\rho}\right) + \frac{g_H G\beta^4 H^4}{4\pi}\left(1 - \frac{27 g_S \beta^6 H^6}{1024 g_H \pi^3 G^3 \rho^3}\right) + \cdots, \tag{18}$$

where g_H and g_S are the corresponding correction coefficient for each boundary.

Eq. (17) determines the cosmological evolution in double-screen cosmology. If the two holographic screens are in thermal equilibrium with $T_b = T_S$ and choosing the coefficient $\beta = \sqrt{2}$, one can recover the exact form of the traditional Friedmann equation. However, in general, Eq. (17) describes the evolution of the universe towards such an equilibrium. The cosmological system will close, as usual, by the consideration of the evolution equation of the total energy density ρ. In the case at hand, in which one may have flow through the boundaries, the corresponding equation is modified as [19]

$$\dot{\rho} + 3H(\rho + p) = \Gamma, \tag{19}$$

with the effective coupling term Γ being

$$\Gamma = \frac{27\beta^6 H^6}{1024\pi^3 G^3 \rho^3}\dot{\rho} + \frac{3\beta^2 H\dot{H}}{4\pi G}\left(1 - \frac{27\beta^4 H^4}{256\pi^2 G^2 \rho^2}\right), \tag{20}$$

at classical level. Again, when $T_b = T_S$ and $\beta = \sqrt{2}$, the coupling Γ vanishes and (19) takes its standard form.

We close this subsection by mentioning the following. At early cosmological times the aforementioned scenario holds as it is. However, for completeness we mention that at late times, in order to describe the dark-energy epoch and universe acceleration, one has to take into account the evaporation of the inner, Schwarzschild screen [19]. Since in the present work we are interested in very early times, that is in inflationary epoch, we will not make such a consideration in the following.

3. Thermal inflation at early universe

In the previous section we analyzed the basic features of double-screen entropic cosmology. Here we focus on the early-time universe evolution, and in particular we examine the inflation realization. Let us first show why such a realization is easily obtained in the model at hand.

In such early-time epochs, the universe is radiation dominated, and thus in the following we assume that the equation of state of the total universe content is $p = \rho/3$. Solving the equations of motion (17) and (19) up to leading order, considering the first order quantum correction to the entropy, one can obtain the following approximate solution for the Hubble parameter at early times [19]

$$H^2 = \frac{8\pi G}{3}\left[\rho + \frac{8g}{69}G^2\rho^2 + \cdots\right], \tag{21}$$

where we have introduced the coefficient $g = g_H - 4g_S$. Therefore, the standard Friedmann equation can be achieved when $g = 0$ at early times. An interesting property of this scenario is that when $g > 0$, the Hubble parameter is proportional to the energy density at high energy scales. In this case the ρ^2 term could make the early time inflation much easier to be realized, providing an implement of holographic inflation.

Let us now investigate the inflation realization in more detail. In the case of $g > 0$, at sufficiently early times the ρ^2-term in (21) will always dominate, and thus the universe will exhibit the inflationary epoch, which, since it is radiation dominated, we call thermal inflation. This is a difference from other examinations of inflation in one-screen entropic cosmology, in which the neglecting of thermal effects makes inflation difficult [13] or impossible [21]. As time passes and the universe grows, ρ will be decreasing, and when it reaches the critical value of $\rho_C \simeq 69/(8gG^2)$ the ρ-term will dominate, triggering the end of inflation. Solving the equations of motion one finds that the energy density of the universe evolves as [19]

$$\rho \simeq \sqrt{\rho_C^2 - \frac{512\pi^3 t}{27 g^{5/2} G^{9/2}}}. \tag{22}$$

In this relation the initial Big Bang time is set to be negative infinity, the observable entropic thermal inflation starts at a time $-\infty < t_i \ll 0$ (surely $-g^{3/2}G^{1/2} \lesssim t_i$ since only after that time the Hubble parameter becomes smaller than the Planck scale), while it ends at $t_C = 0$, after which ρ becomes smaller than ρ_C and standard post-inflationary cosmology begins.

Proceeding forward one finds that at early times ($t \ll 0$) the Hubble parameter behaves like

$$H(t) \simeq 24.25 \times \frac{(-t)^{1/2}}{(gG)^{3/4}}, \tag{23}$$

and thus the slow-roll parameter ϵ reads:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq 2.06 \times 10^{-2}\frac{(gG)^{3/4}}{(-t)^{3/2}}, \tag{24}$$

which is indeed much less than unity when $t \ll -\sqrt{gG}$. Thus, one can make an estimation for the efolding number $\mathcal{N}$, for the observable inflationary stage starting at t_i and ending at $t_C = 0$, as

$$\mathcal{N} \equiv \int_{t_i}^{t_C} H(t)\,dt \simeq 16.17 \times \frac{(-t_i)^{3/2}}{(gG)^{3/4}}. \tag{25}$$

We mention that this is an approximate result, since the relation (23) does not hold up to $t = t_C = 0$. Finally, note that in the above case $g \sim \mathcal{O}(10^{16})$ [19], as it is implied by the requirement that the inner holographic screen evaporates within the age of our universe so that one can obtain the late-time acceleration. The relevant observational constraints on this parameter will be studied in detail in near future.

4. Primordial perturbations in entropic cosmology

In the previous section we investigated the realization of thermal inflation in a double-screen entropic cosmology. The whole analysis remained at the background level, since it is the one that determines the basic features of the cosmological evolution. In the present section we extend our analysis at the perturbation level, since such an examination reveals important details of a cosmological scenario. More importantly, especially for the case of inflation, the perturbation analysis can be straightforwardly confronted by observations, leading to strong constraints or ruling out a specific inflationary model.

The standard mechanism of generating primordial perturbations is to require that the initial cosmological fluctuations emerge inside the Hubble radius, and subsequently they are transformed into classical perturbations, through decoherence, after exiting the Hubble radius. It is usually suggested that these initial fluctuations are generated as quantum vacuum perturbations. However, in the scenario of the present work, the universe is always filled with radiation, even at very early times. As a consequence, and as predicted by thermal field theory [25], the thermal fluctuations dominate the quantum ones, and thus their investigation is sufficient. Now, in our case the thermal fluctuations have two origins, one is the thermal particle fluctuation inside the bulk-universe, and the other is the holographic fluctuation on the two boundary screens. In the following we assume that the correlation between thermal particle fluctuation and holographic perturbation is negligible, and thus we calculate the contribution of each component independently.

Thermal fluctuations as the origin of the structure in the universe were considered in the context of an expanding universe, but it was concluded that a scale-invariant spectrum of cosmological perturbations could not be created from a usual thermal origin [26]. However, motivated by string gas cosmology [27], people have noticed that thermal fluctuations satisfying a specific holographic statistical distribution are able to provide a scale-invariant spectrum in certain backgrounds, namely in a Hagedorn phase [28], in an eternally expanding universe [29] and in bouncing cosmology [30]. As we will show, this is not the case in the scenario at hand, that is we can obtain a scale-invariant spectrum without the need of specific considerations.

4.1. The formalism

We are interested in studying primordial curvature perturbations originating both from the fluctuations of normal radiation and of boundary matter on the two holographic screens. We start by considering the perturbed flat FRW metric in longitudinal gauge, which takes the usual form

$$ds^2 = a(\tau)^2\big[(1+2\Phi)\,d\tau^2 - (1-2\Phi)\,dx^i\,dx^i\big], \tag{26}$$

where τ is the conformal time, and $\Phi(\tau, x^i)$ represents the metric fluctuation. Following the formula developed in [30], the key constraint equation relating matter and metric fluctuations is given by the time component of the perturbed Einstein equations, namely from

$$-3\mathcal{H}\big(\mathcal{H}\Phi + \Phi'\big) + \nabla^2\Phi = 4\pi G a^2 \delta\rho, \tag{27}$$

where $\mathcal{H} = aH$ is the conformal Hubble parameter, the prime denotes the derivative with respect to conformal time, and $\delta\rho$ is the fluctuation of energy density which contains thermal particle modes and holographic boundary ones. Finally, as usual, one transforms into Fourier space, and uses the corresponding modes as the relevant variables.

In summary, for a cosmological system filled with general thermal matter, the thermally originated power spectrum Φ_k can be expressed as [30]

$$P_\Phi(k) \equiv \frac{k^3}{2\pi^2}\langle\Phi_k^2\rangle = \frac{8G^2\langle\delta\rho^2\rangle}{H^4}\bigg|_{t_*(k)}, \tag{28}$$

up to a constant of order $\mathcal{O}(1)$, where $t_*(k)$ denotes the moment of Hubble crossing for the specific mode. In this expression, $\langle\delta\rho^2\rangle$ is the correlation function of density fluctuations in position space, within a sphere of radius $R(k)$, where $R(k)$ is the physical correlation length corresponding to the co-moving momentum scale k. Moreover, in a thermal system $\langle\delta\rho^2\rangle$ is given by

$$\langle\delta\rho^2\rangle = C_V\frac{T^2}{R^6}, \tag{29}$$

where $C_V \equiv \partial\langle E\rangle/\partial T$ is the heat capacity of radiation matter. We mention that in our scenario there exist two kinds of thermal matter, one being the normal radiation constituted by a gas of relativistic point particles, and the other being the boundary matter on the two holographic screens. In the following two subsections we study the curvature perturbations arisen from these two sources separately.

4.2. Fluctuations from normal radiation

In this subsection we consider primordial curvature perturbations induced by the radiation sector that fills the universe during thermal inflation. As it is known from thermodynamics, the radiation energy density as a function of the temperature is given by

$$\rho_r \sim T_r^4, \tag{30}$$

while the heat capacity of normal radiation reads [30]

$$C_V^r = g_r R_r^3 T_r^3, \tag{31}$$

where the subscript r stands for "radiation" and R_r is the radiation correlation length, given as usual from $R_r = c_s/H$, with c_s the sound speed. Additionally, the coefficient g_r characterizes the species of the relativistic point particles of the radiation sector, and it usually takes a value of the order $\mathcal{O}(1)$.

Inserting (31) in (29) and then in (28), with all quantities considered with a subscript r, one obtains the expression of the power spectrum for metric perturbations seeded by normal radiation, namely

$$P_\Phi^r = \frac{g_r\beta^5}{4\pi^5 c_s^3}G^2H^4. \tag{32}$$

Note that in the extraction of this relation we have assumed that the background temperature of the universe is $T_r = \beta H/2\pi$ near thermal equilibrium, that is it coincides with the temperature of the outer holographic screen given by (8). Specifically, in a realistic model with $c_s = 1/\sqrt{3}$ and $\beta = \sqrt{2}$, (32) yields,

$$P_\Phi^r = \frac{3\sqrt{6}g_r}{\pi^5}G^2H^4. \tag{33}$$

Therefore, as can be clearly seen from (32) or (33), the spectrum of curvature perturbation from radiation fluctuations is scale invariant during thermal inflation.

4.3. Fluctuations from outer holographic screen

In this subsection we consider primordial curvature perturbations induced by holographic fluctuations on the outer screen. In entropic cosmology the boundary terms satisfy a holographic statistics, which states that the fundamental degrees of freedom are bounded by their surface areas. According to the equipartition principle, one can acquire the total energy of the outer screen as $\langle E\rangle \sim r_b T_b/G$, with the boundary location r_b and temperature T_b given by (7) and (8) respectively. Correspondingly, the heat capacity of the holographic statistics on outer screen can be written as

$$C_V^b = c_v \frac{r_b^2}{G}, \tag{34}$$

where c_v is a constant of the order of $\mathcal{O}(1)$ determined by the detailed microscopic quantities of quantum gravity.

Having expressed the heat capacity, we repeat the steps of the previous subsection, that is we insert (34) in (29) and then in (28), with all quantities considered with a subscript b. We mention here that in this case the correlation length coincides with the holographic screen's radius r_b, as it has been shown from black-hole physics and its application to cosmology [28–30]. Assembling everything we extract the expression of the power spectrum for perturbations caused by holographic fluctuations on the outer screen, namely

$$P_\Phi^b = \frac{2c_v\beta^6}{\pi^2} G H^2. \tag{35}$$

Furthermore, in the specific model with $\beta = \sqrt{2}$, the power spectrum writes as

$$P_\Phi^b = \frac{16 c_v}{\pi^2} G H^2. \tag{36}$$

As we observe, the primordial curvature perturbations are also scale-invariant.

Finally, we mention that one should repeat these calculations for the inner screen, too. From (35) it is implied that the power spectrum of holographic fluctuations is almost proportional to one over the area of the screen, and thus the contribution of the inner screen might be significant. However, since the size of the inner screen is much smaller than the cosmological scale during inflation, its corresponding fluctuations only contribute to the sub-Hubble regime. Therefore, we can neglect the inner screen contribution to CMB observations.

4.4. Primordial perturbation spectrum

In the previous two subsections we extracted the expressions for the power spectrum for primordial curvature perturbations, generated by the radiation sector, as well as by the outer holographic screen, namely relations (32) and (35) respectively, neglecting possible interaction terms between the two perturbation sources and the inner screen contribution. Comparing the two results we can immediately find that the perturbation amplitude from radiation behaves like the square of the one from the outer holographic screen. Fitting with current CMB observations, one concludes that P_Φ^b is of the order of $\mathcal{O}(10^{-10})$, and therefore P_Φ^r is completely negligible. Thus, this is a significant difference of the model at hand from conventional cosmology, that is the main source of perturbation comes from the outer holographic screen and not from the radiation sector of the universe interior. This feature has some interesting physical implications.

Let us specify the discussion, considering a variable that is widely used, namely the curvature perturbation in comoving coordinates [16]:

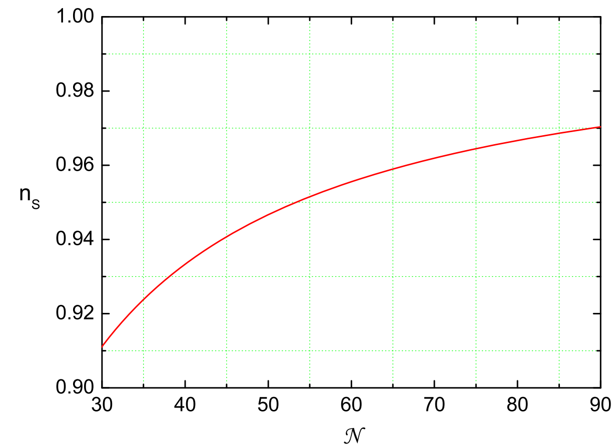

Fig. 1. The spectral index n_S of primordial curvature perturbation in the entropic scenario of thermal inflation as a function of the efolding number $\mathcal{N}$.

$$\zeta \equiv \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}(\Phi' + \mathcal{H}\Phi). \tag{37}$$

This variable can be computed from the gravitational potential Φ and background parameters. Since in inflation the metric perturbation is frozen at super-Hubble scales, we acquire the simple relation $\zeta \simeq \Phi/\epsilon$. As a consequence, and using (35), we obtain the primordial power spectrum of curvature perturbation during the thermally induced inflationary period in entropic cosmology:

$$P_\zeta \simeq \frac{16 c_v}{\epsilon^2\pi^2} G H^2. \tag{38}$$

Therefore, we can easily calculate the spectral index, which is the basic quantity in any relevant discussion [31], namely

$$n_S - 1 \equiv \frac{d\ln P_\zeta}{d\ln k} = -2\epsilon - 2\eta, \tag{39}$$

where $\eta \equiv \dot{\epsilon}/H\epsilon$. In the deviation of this relation we have used the usual relation $d\ln k \approx d\ln(aH) \approx d\ln(a)$ [16], which results from the fact that during inflation the variation of the scale factor is much larger than that of the Hubble parameter.

Interestingly, in the scenario at hand one can approximately extract a very simple relation connecting the spectral index of thermal inflation n_S with the efolding number $\mathcal{N}$. In particular, combining (24), (25) and (39), using t_i as a free variable and using the background parameter value $\beta = \sqrt{2}$, we result to

$$n_S \simeq 1 - \frac{8}{3\mathcal{N}}. \tag{40}$$

Thus, it is obvious that the longer time inflation lasts, the closer to scale-invariance will be the spectral index. Therefore, we can immediately construct the n_S–$\mathcal{N}$ graph, which is presented in Fig. 1. Indeed, at large $\mathcal{N}$ the resulting spectrum is very close to scale-invariance, with a red tilt, and the deviation from 1 is quantitatively in agreement with observations, which require that $n_S = 0.96 \pm 0.012$ at 2σ level [17]. This is a basic result of the present work, revealing that the dominance of holographic fluctuations not only does not affect the scale-invariant, conventional thermal ones, but it also improves the picture of the produced spectrum.

Finally, let us examine the tensor perturbations and their relation to the scalar ones examined above. Such a quantity, that is the tensor-to-scalar ratio, is the second measure, along with the spectral index, that characterizes the primordial fluctuations. In the scenario of thermal inflation in entropic cosmology, the primordial power spectrum for tensor perturbation coincides with that of the

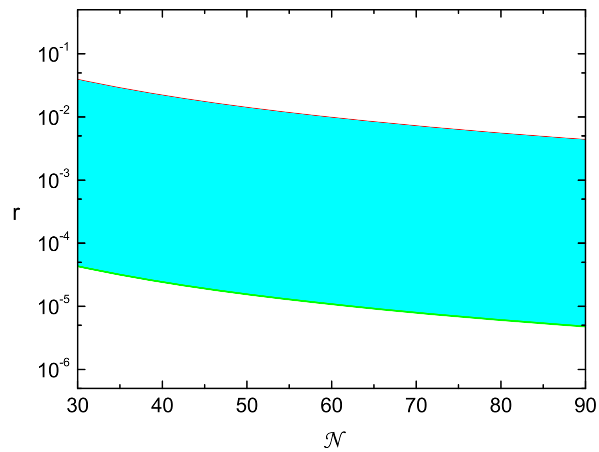

Fig. 2. The contour of the tensor-to-scalar ratio r in the entropic scenario of thermal inflation, as a function of the efolding number $\mathcal{N}$, in the value regime of $0.01 \leqslant c_v \leqslant 9$.

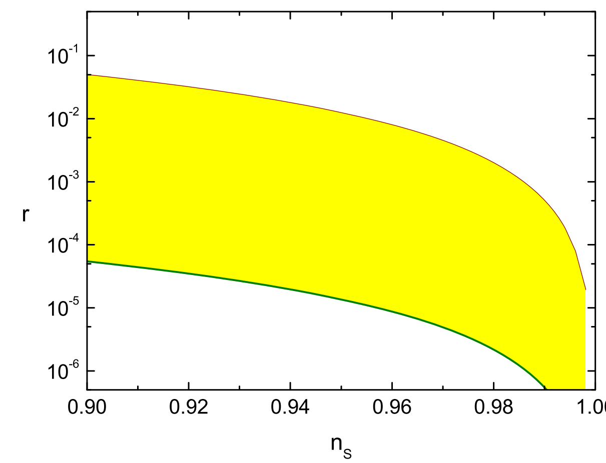

Fig. 3. The contour of the tensor-to-scalar ratio r in the entropic scenario of thermal inflation, as a function of the spectral index n_S, in the value regime of $0.01 < c_v < 9$.

usual slow-roll inflation, which reads $P_T = 16GH^2/\pi$ [31], since the holographic screens do not affect the tensor part of perturbation equations. Therefore, defining as usual the ratio r of tensor-to-scalar perturbation, we acquire

$$r \equiv \frac{P_T}{P_\zeta} = \frac{\epsilon^2\pi}{c_v}, \tag{41}$$

which is doubly suppressed by the slow roll parameter ϵ but may be enhanced by a small value of the holographic parameter c_v. This behavior is different from the usual inflationary scenario. In particular, we conclude that in the general case with $c_v \sim O(1)$ the primordial tensor perturbation is insensitive to current cosmological observations, but it is still possible to obtain sizable tensor modes if we fine-tune the value of c_v to be small enough.

To investigate these features in more detail we again combine (24) and (25) using t_i as a free variable and using the parameter value $\beta = \sqrt{2}$, resulting to the helpful relation $\epsilon \approx \frac{1}{3\mathcal{N}}$. Thus, insertion into (41) leads to the simple relation

$$r \approx \frac{\pi}{9c_v\mathcal{N}^2}. \tag{42}$$

In Fig. 2 we present the r–$\mathcal{N}$ graph taking the value regime of c_v between 0.01 and 9. As we observe, at large efolding $\mathcal{N}$ the resulting tensor-to-scalar ratio r acquires very small values. Furthermore, combining (40) with (42), we find

$$r \approx \frac{\pi(1-n_S)^2}{64c_v}. \tag{43}$$

The corresponding r–n_S graph is presented in Fig. 3, taking c_v in the interval from 0.01 to 9. Comparing this figure with latest cosmological data [17,32], it is clear that our results are compatible with current observations. Moreover, the smallness of r seems to be closer to observations comparing to the usual paradigm of chaotic inflationary models [33].

5. Non-Gaussianities

Recently, a lot of interest has been paid on the analysis of non-linear perturbations at early universe, under the scenarios of single field slow-roll inflation [34], brane inflation models [35–37], inflation models with general non-canonical form [38,39], curvaton configurations [40–42], ekpyrotic scenarios [43], phantom inflation [44], matter bounce cosmology [45], etc. (see [46,47] for recent reviews). Such a non-linear analysis is necessary in order to reveal the possible non-Gaussianity of the primordial fluctuations, which can be measured by cosmological observations [48]. Thus, along with the examination of the spectral index and the tensor-to-scalar ratio, the estimation of the non-Gaussianities that are produced by an inflationary scenario is a crucial step, since they can constrain or rule out the examined scenario.

In this section we investigate the non-linear perturbation of thermal inflation in double-screen entropic cosmology, by computing its non-Gaussianity estimator. This technique of incorporating non-Gaussianities in a thermal system has been applied in an inflationary model coupled to normal radiation [49], in the context of bounce cosmology [30], in a string gas scenario [50], and in a holographic universe [51].

For a perturbation mode with fixed k its non-Gaussianity estimator is given by the amplitude of the three-point correlation function over the square of the two-point one, and can be written as

$$f_{NL} = \frac{5\langle\zeta_k^3\rangle}{18k^{3/2}\langle\zeta_k^2\rangle^2}. \tag{44}$$

In the previous section we calculated the result of the two-point function, namely relation (29). Therefore, we need to calculate also the three-point correlator.

As we showed above, the dominant contribution of primordial curvature perturbation comes from the holographic fluctuations on the outer boundary surface. Thus, the key point is to calculate the three-point function of these holographic fluctuations. In an equilibrium ensemble, one obtains [30]

$$\langle\delta\rho_b^3\rangle\big|_{t_*} = \frac{T_b^2}{R_b^9}\frac{\partial}{\partial T_b}\left(C_V^b T_b^2\right) = \frac{2c_v T_b^3}{GR_b^7}, \tag{45}$$

which is calculated at the moment t_* of Hubble crossing in coordinate space. In the above relation we have used (34), and the fact that the correlation length R_b coincides with the holographic screen's radius r_b.

In summary, we can now insert (45) and (29) (applied with indices "b") into the expression of non-Gaussianity estimator (44). Using also the approximation $\zeta \simeq \Phi/\epsilon$, we finally obtain

$$f_{NL} \simeq \frac{5}{36\sqrt{2}\pi^2}\frac{\epsilon R_b H^2}{c_v T_b}. \tag{46}$$

We mention here that this result is similar to the one obtained in a thermal bouncing universe filled with Gibbons–Hawking radiation [30]. However, there exists a manifest difference between the two results, that is the non-Gaussianities may be suppressed by the slow-roll parameter ϵ in the present scenario of entropic thermal inflation, but not in the thermal bouncing cosmology. Thus, we deduce that this suppression behavior is a consequence and a physical reflection of inflation.

Proceeding forward, we insert in (46) the expression for R_b, that is for r_b, which is given by (7), and for T_b, which is given by (8), resulting to

$$f_{NL} \simeq \frac{5\epsilon}{36\sqrt{2}\pi c_v}. \qquad (47)$$

This relation provides the non-Gaussianity of thermal inflation in double-screen entropic cosmology, and as we observe it is scale-invariant.

We close this section by mentioning that although the non-Gaussianities of primordial curvature perturbation in the scenario at hand are suppressed by the slow-roll parameter ϵ, it is still possible to produce a sizable value of f_{NL} if $c_v \sim \epsilon$. Therefore, in order to complete a quantitative investigation, one needs to perform a detailed analysis of the microscopic properties of a holographic screen in entropic cosmology, which will lead to an estimation of c_v. However, such an analysis is a hard task under the present knowledge, and it lies outside the scope of the present work.

6. Conclusion

In this work we investigated a scenario of thermal inflation realized by two holographic screens in the context of entropic cosmology. We found that the realization of inflation is general, resulting from the system evolution from non-equilibrium to equilibrium. Going beyond the background evolution, we analyzed the primordial curvature perturbations arising from the universe interior, as well as the thermal fluctuations generated on the outer holographic screen. For both these contributions the power spectra are almost scale-invariant, however the amplitude of curvature perturbation arisen from holographic fluctuations on the outer screen is much larger than that of the universe interior. Furthermore, due to the thermal initial conditions for scalar-type metric perturbations, the consistency relation widely held in usual inflation models was found to be modified in the present scenario. In summary, the produced power spectrum is nearly scale-invariant with a red tilt.

Proceeding forward, we provided approximate analytic expressions for the tensor-to-scalar ratio as a function of the spectral index, with the one free parameter c_v determined by the detailed microscopic quantities of quantum gravity. As we saw, the corresponding contour plot is in agreement with observations, with even better quantitative features comparing to the usual paradigm of chaotic inflationary models.

Finally, we examined the non-Gaussian distribution of the inhomogeneities of primordial curvature perturbations, generated from the outer screen. Since these fluctuations satisfy the holographic statistics, the resulting non-linearity parameter is inversely proportional to c_v, and it is suppressed by the slow-roll parameter, while it is nearly scale-invariant. Therefore, a sizable value of the non-linearity parameter is possible due to holographic statistics on the outer screen, provided c_v is of the same order with the slow-roll parameter.

It is important to mention that our analysis involves a few uncertainties on the coefficients, since the detailed thermodynamics of holographic statistics on the boundary screens is still not well understood in current knowledge. This provides a wide parameter space to fit to current cosmological observations. Therefore, it may be far from conclusive to give strong constraints on the scenario of double-screen entropic thermal inflation. However, we still might be able to distinguish such a scenario from a normal model of slow-roll inflation by measuring the spectral indexes of primordial power spectra and examining their consistency relation in the coming experiments. Moreover, we expect that the scenario considered in this work can be theoretically developed along with the accumulating studies on holographic properties of entropic cosmology, so that it may be verified or ruled out by future cosmological data.

As an end, we would like to point out that a distinguishable feature of entropic cosmology with double holographic screens is the explanation of inflation and late time acceleration in a unified frame. In this work we focused on the predictions of inflation realized by holographic screens out of thermal equilibrium at early universe. However, at significantly late times the inner screen would evaporate and thus yield another acceleration epoch, which could explain the current dark-energy period. Therefore, we expect that the scenario at hand might be related to the holographic dark energy scenario, which incorporates the universe acceleration in consistency with the basic quantum gravitational requirements embedded in the holographic principle [52].

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References

[1] J.D. Bekenstein, Phys. Rev. D 7 (1973) 2333.
[2] S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[3] G. 't Hooft, arXiv:gr-qc/9310026.
[4] G.W. Gibbons, S.W. Hawking, Phys. Rev. D 15 (1977) 2738.
[5] R. Bousso, Rev. Mod. Phys. 74 (2002) 825.
[6] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.
[7] E.P. Verlinde, arXiv:1001.0785 [hep-th].
[8] T. Padmanabhan, Rep. Prog. Phys. 73 (2010) 046901;
T. Padmanabhan, Mod. Phys. Lett. A 25 (2010) 1129.
[9] R.G. Cai, L.M. Cao, N. Ohta, Phys. Rev. D 81 (2010) 061501.
[10] M. Li, Y. Wang, Phys. Lett. B 687 (2010) 243.
[11] Y. Zhang, Y.g. Gong, Z.H. Zhu, arXiv:1001.4677 [hep-th];
R. Banerjee, B.R. Majhi, Phys. Rev. D 81 (2010) 124006.
[12] D.A. Easson, P.H. Frampton, G.F. Smoot, arXiv:1002.4278 [hep-th].
[13] D.A. Easson, P.H. Frampton, G.F. Smoot, arXiv:1003.1528 [hep-th].
[14] U.H. Danielsson, arXiv:1003.0668 [hep-th];
R. Casadio, A. Gruppuso, arXiv:1005.0790 [gr-qc].
[15] A.H. Guth, Phys. Rev. D 23 (1981) 347;
A.D. Linde, Phys. Lett. B 108 (1982) 389;
A.J. Albrecht, P.J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220;
For earlier attempts see A.A. Starobinsky, Phys. Lett. B 91 (1980) 99.
[16] V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger, Phys. Rep. 215 (1992) 203.
[17] E. Komatsu, et al., arXiv:1001.4538 [astro-ph.CO].
[18] M. Tegmark, et al., Astrophys. J. 606 (2004) 702;
M. Tegmark, et al., Phys. Rev. D 74 (2006) 123507.
[19] Y.F. Cai, J. Liu, H. Li, Phys. Lett. B 690 (2010) 213.
[20] Y. Wang, arXiv:1001.4786 [hep-th].
[21] M. Li, Y. Pang, Phys. Rev. D 82 (2010) 027501.
[22] W.G. Unruh, Phys. Rev. D 14 (1976) 870.
[23] A. Strominger, C. Vafa, Phys. Lett. B 379 (1996) 99;
E. Halyo, B. Kol, A. Rajaraman, L. Susskind, Phys. Lett. B 401 (1997) 15;
S.N. Solodukhin, Phys. Rev. D 57 (1998) 2410.
[24] E. Alvarez, C. Gomez, Nucl. Phys. B 541 (1999) 441.
[25] U. Kraemmer, A. Rebhan, Rep. Prog. Phys. 67 (2004) 351.
[26] J. Magueijo, L. Pogosian, Phys. Rev. D 67 (2003) 043518.

[27] R.H. Brandenberger, C. Vafa, Nucl. Phys. B 316 (1989) 391;
R.H. Brandenberger, arXiv:0808.0746 [hep-th].
[28] A. Nayeri, R.H. Brandenberger, C. Vafa, Phys. Rev. Lett. 97 (2006) 021302.
[29] J. Magueijo, L. Smolin, C.R. Contaldi, Class. Quant. Grav. 24 (2007) 3691;
Y.S. Piao, Phys. Rev. D 76 (2007) 043509.
[30] Y.F. Cai, W. Xue, R. Brandenberger, X.m. Zhang, JCAP 0906 (2009) 037.
[31] M. Sasaki, E.D. Stewart, Prog. Theor. Phys. 95 (1996) 71.
[32] J.Q.Y. Xia, H. Li, G.B. Zhao, X. Zhang, Phys. Rev. D 78 (2008) 083524.
[33] A.D. Linde, Phys. Lett. B 129 (1983) 177.
[34] J.M. Maldacena, JHEP 0305 (2003) 013.
[35] M. Alishahiha, E. Silverstein, D. Tong, Phys. Rev. D 70 (2004) 123505.
[36] J.E. Lidsey, I. Huston, JCAP 0707 (2007) 002;
T. Kobayashi, S. Mukohyama, S. Kinoshita, JCAP 0801 (2008) 028.
[37] Y.F. Cai, H.Y. Xia, Phys. Lett. B 677 (2009) 226;
Y.F. Cai, W. Xue, Phys. Lett. B 680 (2009) 395.
[38] D. Seery, J.E. Lidsey, JCAP 0506 (2005) 003;
X. Chen, M.x. Huang, S. Kachru, G. Shiu, JCAP 0701 (2007) 002.
[39] F. Arroja, K. Koyama, Phys. Rev. D 77 (2008) 083517;
X. Gao, JCAP 0806 (2008) 029;
F. Arroja, S. Mizuno, K. Koyama, JCAP 0808 (2008) 015;
C.T. Byrnes, G. Tasinato, JCAP 0908 (2009) 016;
X. Gao, arXiv:1008.2123 [gr-qc];
G.L. Alberghi, R. Casadio, arXiv:1010.4395 [astro-ph.CO].
[40] D.H. Lyth, D. Wands, Phys. Lett. B 524 (2002) 5.
[41] S. Li, Y.F. Cai, Y.S. Piao, Phys. Lett. B 671 (2009) 423;
J. Zhang, Y.F. Cai, Y.S. Piao, JCAP 1005 (2010) 001;
Y.F. Cai, Y. Wang, arXiv:1005.0127 [hep-th].
[42] Q.G. Huang, Phys. Lett. B 669 (2008) 260;
A. Mazumdar, J. Rocher, arXiv:1001.0993 [hep-ph];
C.T. Byrnes, K. Enqvist, T. Takahashi, JCAP 1009 (2010) 026;
C.J. Feng, X.Z. Li, JCAP 1010 (2010) 027.
[43] K. Koyama, S. Mizuno, F. Vernizzi, D. Wands, JCAP 0711 (2007) 024;
J.L. Lehners, Adv. Astron. 2010 (2010) 903907.
[44] Y.S. Piao, Y.Z. Zhang, Phys. Rev. D 70 (2004) 063513;
Y.S. Piao, Phys. Rev. D 79 (2009) 083512;
C.J. Feng, X.Z. Li, E.N. Saridakis, Phys. Rev. D 82 (2010) 023526.
[45] Y.F. Cai, W. Xue, R. Brandenberger, X. Zhang, JCAP 0905 (2009) 011;
Y.F. Cai, T.t. Qiu, R. Brandenberger, X.m. Zhang, Phys. Rev. D 80 (2009) 023511.
[46] N. Bartolo, E. Komatsu, S. Matarrese, A. Riotto, Phys. Rep. 402 (2004) 103.
[47] X. Chen, Adv. Astron. 2010 (2010) 638979.
[48] A.P.S. Yadav, B.D. Wandelt, Phys. Rev. Lett. 100 (2008) 181301.
[49] B. Chen, Y. Wang, W. Xue, JCAP 0805 (2008) 014.
[50] B. Chen, Y. Wang, W. Xue, R. Brandenberger, arXiv:0712.2477 [hep-th].
[51] Y. Ling, J.P. Wu, Phys. Lett. B 675 (2009) 151.
[52] A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Rev. Lett. 82 (1999) 4971;
S.D.H. Hsu, Phys. Lett. B 594 (2004) 13;
M. Li, Phys. Lett. B 603 (2004) 1;
K. Enqvist, M.S. Sloth, Phys. Rev. Lett. 93 (2004) 221302;
M. Ito, Europhys. Lett. 71 (2005) 712;
B. Wang, Y. Gong, E. Abdalla, Phys. Lett. B 624 (2005) 141;
E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D 71 (2005) 103504;
E.N. Saridakis, Phys. Lett. B 660 (2008) 138;
E.N. Saridakis, JCAP 0804 (2008) 020;
M.R. Setare, E.N. Saridakis, Phys. Lett. B 671 (2009) 331.