Topology and its Applications 13 (1982) 255–262
North-Holland Publishing Company

PMEA IMPLIES PROPOSITION P

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Received 17 June 1980
Revised 10 July 1981

The Product Measure Extension Axiom (PMEA) asserts that for every set A, Haar measure on 2^A can be extended to all subsets of 2^A. PMEA implies the normal Moore space conjecture. Proposition P is the statement that every point-finite analytic-additive family of subsets of a metrizable space is σ-discretely decomposible. Proposition P is useful in nonseparable Borel theory. We show in this paper that PMEA implies Proposition P.

AMS (MOS) Subj. Class. (1980): Primary 03E35; Secondary 03E55, 54H05

Product Measure Extension Axiom	Proposition P
σ-discretely decomposible	analytic additive

1.

In this section we sketch the proof of our main theorem. The main body of this paper, Section 2, contains the definitions and statements of the lemmas used. Lemma 1 is proved in [3]; Lemmas 2 and 3(a) are proved in [4]; Lemmas 3(b) and 4 are proved in Section 2. The consistency of the axioms used is discussed in Section 3. Section 4 contains some short remarks about [2].

Because we use some technical notions from several fields, there is a danger that the proof of the main theorem will be lost in the preliminaries. For expository purposes, we present now our theorems and their proofs, deferring definitions and proofs of lemmas to Section 2.

Theorem 1. *PMEA implies Proposition P.*

* Partially supported by NSF Grant MCS 79-01848.
** Work done while visiting University of Pittsburgh on a Mellon Postdoctoral Fellowship.

Proof. We are given a point-finite analytic-additive family, $\mathscr{L}$, of subsets of a metrizable space X; we must show that, assuming PMEA, $\mathscr{L}$ is σ-discretely decomposible. Since $\mathscr{L}$ is analytic-additive, there is, for each $\mathscr{M} \subset \mathscr{L}$, a family, $\mathscr{G}(\mathscr{M}) = \{G_s(\mathscr{M}): s \in S\}$, of open subsets of X such that $\mathscr{A}(\mathscr{G}(\mathscr{M})) = \bigcup \mathscr{M}$. By Lemma 1, we may assume that each $\mathscr{G}(\mathscr{M})$ is an orderly family. Being of weak character below $\mathfrak{c}$ is a much weaker notion than being metrizable, so we may apply Lemma 3(b) to X and $\mathscr{L}$ to conclude that $\mathscr{L}$ has a θ-companion. (It is to prove Lemma 3(b) (and Lemma 2 on which Lemma 3 depends) where we use PMEA.) Finally, because X is perfect, Lemma 4 yields that $\mathscr{L}$ is σ-discretely decomposible.

Theorem 2 is merely a statement about which consequences of the metrizability of X were used in the above proof. Theorem 3 is the statement of what the proof, not including the last step of Lemma 4, yields in the special case where $\mathscr{L}$ is the family of singleton subsets of X.

Theorem 2 (PMEA). *Perfect spaces of weak character below* $\mathfrak{c}$ *have Property* P.

Theorem 3 (PMEA). *If every subset of a space* X *of weak character below* $\mathfrak{c}$ *is analytic, then* X *is the union of countably many (relatively) discrete subspaces.*

2.

For two sets A and B, we denote by ${}^AB$ the set of all functions from A to B. An ordinal is the set of smaller ordinals; ω is the first infinite ordinal. We denote the cardinal of the continuum by $\mathfrak{c}$. We denote the cardinality of the set A by $|A|$. We denote by S the set $\bigcup_{n \in \omega} {}^n\omega$. The $\mathscr{A}$-operation assigns to a family $\mathscr{G} = \{G_s: s \in \omega\}$ of sets the set

$$\mathscr{A}(\mathscr{G}) = \bigcup_{f \in {}^\omega\omega} \bigcap_{n \in \omega} G_{f|n}.$$

By an *analytic* subset of a topological space, we mean a set which can be obtained from the family of open sets by the A-operation. Often, the closed sets are used instead of open sets to define the analytic sets; for *perfect* spaces (i.e., spaces in which every closed set is a G_δ-set), both ways of defining analytic sets yield the same class of sets.

Let $\mathscr{L}$ be a family of subsets of a space X. For $x \in X$, we define $(\mathscr{L})_x$ to be the set $\{L \in \mathscr{L}: x \in L\}$, and $F(\mathscr{L})$ to be the set $\{x \in X: (\mathscr{L})_x \text{ is finite}\}$. We say that $\mathscr{L}$ is *point-finite* if $F(\mathscr{L}) = X$. We say that $\mathscr{L}$ is *analytic-additive* if, for each $\mathscr{M} \subset \mathscr{L}$, $\bigcup \mathscr{M}$ is analytic. We say that $\mathscr{L}$ is σ-discretely decomposible if there exists a family $\{D_n(L): n \in \omega, L \in \mathscr{L}\}$ of subsets of X such that

$$\textit{for each } L \in \mathscr{L},\ L = \bigcup_{n \in \omega} D_n(L), \tag{1}$$

$$\text{for each } x \in X \text{ and } n \in \omega, \text{ there is a neighborhood of } x \text{ meeting at most one element of } \{D_n(L) : L \in \mathscr{L}\}. \tag{2}$$

We say that a space X has *property P* if every point-finite, analytic additive family of subsets of X is σ-discretely decomposible. *Proposition P* [2] is the statement that every metrizable space has property P.

The Product Measure Extension Axiom (PMEA) [8] is the statement that for any cardinal number κ, there exists a $\mathfrak{c}$-additive measure μ_κ, defined on *all* subsets of 2^κ, extending the usual product measure (i.e., Haar measure). PMEA is equivalent (in ZFC) with the assertion that for any set A, there exists a non-negative real-valued function μ defined for all families of subsets of A and satisfying the following two conditions

$$\text{If } \mathscr{J} \text{ is a collection of pairwise disjoint families of subsets of } A \text{ and } |\mathscr{J}| < \mathfrak{c}, \text{ then } \mu(\bigcup \mathscr{J}) = \sum \{\mu(\mathscr{B}) : \mathscr{B} \in \mathscr{J}\}. \tag{3}$$

$$\text{If } c \text{ and } d \text{ are disjoint finite subsets of } A \text{ and } n = |c \cup d|, \text{ then } \mu\{B \subset A : c \in B \text{ and } d \in A - B\} = 2^{-n}. \tag{4}$$

We will call μ a *product measure extension for A* if (3) and (4) hold.

We will think of the elements of $\mathscr{G}$ as approximation to $\mathscr{A}(\mathscr{G})$. We introduce here a way of getting better approximations.

First, note that ${}^0\omega = \{\emptyset\}$ and that for $s \in S$, the domain of s is $|s|$. For each $n \in \omega$, let $\leq^n$ be the product partial order on ${}^n\omega$. That is, for s, $t \in \omega^n$, $s \leq^n t$ iff for all $k < n$, $s(k) \leq t(k)$. Set $\leq$ equal to $\bigcup_{n \in \omega} \leq^n$. Thus, for s, $t \in S$, $s \leq t$ implies $|s| = |t|$.

We say that a family $\mathscr{G} = \{G_s : s \in S\}$ of a set X is an *orderly family* if $G_\emptyset = X$ and, for all s, $t \in S$

$$\text{if } s \subset t, \text{ then } \quad G_s \supset G_t \tag{5}$$

$$\text{if } s < t, \text{ then } \quad G_s \subset G_t. \tag{6}$$

Thus, to get a better larger approximation than G_s, take G_t, where $s < t$; to get a better smaller approximation than G_s, take G_t, where $s \subset t$. (The partial order on S generated by $\subset$ and $<$ is implicit in [3] but will not be used in this paper.)

Lemma 1 [3]. *Every analytic subset of a topological space can be obtained by the $\mathscr{A}$-operation from an orderly family of open subsets of the space.*

Although Proposition P mentions only metrizable spaces, our proof, assuming PMEA, will show that spaces in a much larger class have Property P.

We say that a space X has *weak character below* $\mathfrak{c}$ if there exists a collection $\{\mathscr{F}_x : x \in X\}$ of filterbases on X, each of cardinality less than $\mathfrak{c}$, such that, for every subset G of X, G is open in X iff for each $x \in G$ there is $F \in \mathscr{F}_x$ such that $x \in F \subset G$.

Let us recall how Nyikos used PMEA [8]. Given a collection $\mathscr{L}$, normalized by $\{U(\mathscr{M}) : \mathscr{M} \subset \mathscr{L}\}$, he obtained, for each $L \in \mathscr{L}$, an open set $V(L) \supset L$ by choosing, for each $x \in L \in \mathscr{L}$, a neighborhood $V(x)$ such that $V(x) \subset U(\mathcal{M})$ for "most" $\mathcal{M}$, "most" being given a precise meaning by a product measure extension for $\mathscr{L}$. Nyikos' results were extended in [5] from character to weak character by the following paradoxical observation. If for each $x \in L$ we can choose a neighborhood, then we don't have to choose a neighborhood because the set of all points which are in $U(\mathcal{M})$ for "most" $\mathcal{M}$ is automatically open.

We introduce machinery to make the ideas above precise.

Convention. Given a space X, a set A, a function G from the set of subsets of A to the set of open sets of X, and μ, a product measure extension for A, we set, for each $a \in A$ and $n \in \omega$.

$$\mathscr{B}_{a,x} = \{B \subset A: \text{if } a \in B, \text{ then } x \in G(B)\},$$

$$U_n(a) = \{x \in X: \mu(\mathscr{B}_{a,x}) > 1 - 2^{-n}\}.$$

When the set, A, is a collection, $\mathscr{L}$, and the function G has a subscript, t, we write $U_{n,t}(L)$ for $U_n(a)$.

Lemma 2 [4]. *Let X, A, G and μ be as in the convention. If X is of weak character below $\mathfrak{c}$, then for each $a \in A$ and $n \in \omega$, the set $U_n(a)$ is open in X.*

We want a precise notion of how a collection of subsets of a space X can be a good approximation to another collection of subsets of X. We say that $\mathscr{U} = \{U_j(L): j \in J, L \in \mathscr{L}\}$ is a *θ-companion* of $\mathscr{L}$ if

$$J \text{ is countable,} \tag{7}$$

$$\text{for each } j \in J \text{ and each } L \in \mathscr{L},\ U_j(L) \text{ is open in } X \tag{8}$$

$$\text{for each } x \in F(\mathscr{L}), \text{ there is } j \in J \text{ such that } (\mathscr{L})_x = \{L \in \mathscr{L}: x \in U_j(L)\}. \tag{9}$$

If, additionally,

$$\text{for each } L \in \mathscr{L} \text{ and each } j \in J,\ L \subset U_j(L), \tag{10}$$

we say that $\mathscr{U}$ is a *θ-expansion* of $\mathscr{L}$.

Lemma 3. *Let X be a space of weak character below $\mathfrak{c}$, and for each $k \in \omega$, (or $s \in S$) let X, $\mathscr{L}$, G_k and μ be as in the convention.*

(a) [4] *If for all $\mathcal{M} \subset \mathscr{L}$, $\{G_k(\mathcal{M}): k \in \omega\}$ is a descending sequence of open sets whose intersection is $\bigcup \mathcal{M}$, then $\{U_{n,k}(L): L \in \mathscr{L}\}$ is a θ-expansion of $\mathscr{L}$.*

(b) *If for all $\mathcal{M} \subset \mathscr{L}$, $\mathscr{G}(\mathcal{M}) = \{G_s(\mathcal{M}): s \in S\}$ is an orderly family with $\mathscr{A}(\mathscr{G}(\mathcal{M})) = \bigcup \mathcal{M}$, then $\{U_{n,s}(L): L \in \mathscr{L}, n \in \omega, s \in S\}$ is a θ-companion of $\mathscr{L}$.*

Proof of (b). Fix $x \in F(\mathscr{L})$; let $m = |(\mathscr{L})_x| + 2$. For each $n \in \omega$ and each $s \in {}^n\omega$, define $R_s = \{f \in {}^\omega\omega: f|n <^n s\}$. For each $f \in {}^\omega\omega$ and $\mathcal{M} \subset \mathscr{L}$, set

$$G_f(\mathcal{M}) \bigcap_{n \in \omega} G_{f|n}(\mathcal{M}).$$

Our first goal is to find $f \in {}^{\omega}\omega$ such that

$$\mu(\{\mathcal{M} \subset \mathcal{L}: \text{if } x \in \bigcup \mathcal{M}, \text{ then } x \in G_f(\mathcal{M})\}) > 1 - 2^{-m}.$$

For each $s \in S$, let

$$\mathscr{C}_s = \{\mathcal{M} \subset \mathcal{L}: \text{if } x \in \bigcup \mathcal{M}, \text{ then } (\exists f \in R_s)(x \in G_f(\mathcal{M}))\}.$$

Then for each $n \in \omega$ and each $s \in {}^{n}\omega$, $\mathscr{C}_s = \bigcup \{\mathscr{C}_t : t \in {}^{n+1}\omega \text{ and } s \subset t\}$. As the families $\mathscr{G}(\mathcal{M})$ are orderly, we have $\mathscr{C}_s \subset \mathscr{C}_t$ whenever $s < t$. It follows that we can inductively define $s(n) \in {}^{n}\omega$, such that, for each $n \in \omega$, $s(n) \subset s(n+1)$ and $\mu(\mathscr{C}_{s(n)} \sim \mathscr{C}_{s(n+1)}) < 2^{-m-n-1}$. Set

$$f = \bigcup_{n \in \omega} s(n); \qquad \mathscr{C}_f = \bigcap_{n \in \omega} \mathscr{C}_{s(n)}.$$

Noting that $s(0) = \emptyset$, and hence $\mu(\mathscr{C}_{s(0)}) = 1$, we have

$$\mu(\mathscr{C}_f) = \mu\Big(\bigcap_{n \in \omega} \mathscr{C}_{s(n)}\Big) = \mu(\mathscr{C}_{s(0)}) - \sum_{n \in \omega} \mu(\mathscr{C}_{s(n)} \sim \mathscr{C}_{s(n+1)})$$

$$> 1 - \sum_{n \in \omega} 2^{-m-n-1} = 1 - 2^{-m}.$$

To see that f has the required property, it suffices to observe that

$$\bigcap_{n \in \omega} \mathscr{C}_{s(n)} \subset \{\mathcal{M} \subset \mathcal{L}: \bigcup \mathcal{M} \subset \bigcap G_{f|n}(\mathcal{M})\}.$$

This observation can be easily made by noting that for all $n \in \omega$ and $\mathcal{M} \subset \mathcal{L}$, if $\mathcal{M} \in \mathscr{C}_{s(n)}$ and $x \in \bigcup \mathcal{M}$, then it follows from the orderliness of $\mathscr{G}(\mathcal{M})$ that $x \in G_{s(n)}(\mathcal{M})$.

For every $n \in \omega$, set

$$\mathscr{E}_n = \{\mathcal{M} \subset \mathcal{L}: (X - \bigcup \mathcal{M}) \subset (X - G_{s(n)}(\mathcal{M}))\}.$$

Note that $\bigcup_{n \in \omega} \mathscr{E}_n = \{\mathcal{M}: \mathcal{M} \subset \mathcal{L}\}$, and that for each $n \in \omega$, $\mathscr{E}_n \subset \mathscr{E}_{n+1}$. It follows that there is $k \in \omega$ such that $\mu(\mathscr{E}_k) > 1 - 2^{-m}$. Let $t = s(k)$.

We complete the proof of the lemma by showing that, for each $L \in \mathcal{L}$, $x \in L$ iff $x \in U_{m,t}(L)$.

Case 1. $x \in L$. It is easy to see that $\mathscr{C}_t \subset \mathscr{B}_{x,t}$; hence, $\mu(\mathscr{B}_{x,t}(L)) > 1 - 2^{-m}$; and so $x \in U_{m,t}(L)$.

Case 2. $x \notin L$. Set

$$\mathscr{Q} = \{\mathcal{M} \subset \mathcal{L}: L \in M \text{ and } \mathcal{M} \cap (\mathcal{L})_x = \emptyset\}.$$

Note that $\mu(\mathscr{Q}) = 2^{-m+1}$. We show that $(\mathscr{Q} \cap \mathscr{E}_k) \cap \mathscr{B}_{x,t}(L) = \emptyset$. Let $\mathcal{M} \in \mathscr{Q} \cap \mathscr{E}_k$. Since $\mathcal{M} \in \mathscr{Q}$, we have $L \in \mathcal{M}$ and $x \notin \bigcup \mathcal{M}$. Since $x \notin \bigcup \mathcal{M}$ and $\mathcal{M} \in \mathscr{E}_k$, we have $x \notin G_{s(k)}(\mathcal{M}) = G_t(\mathcal{M})$. Finally since $L \in \mathcal{M}$ and $x \in G_t(\mathcal{M})$, we have $\mathcal{M} \notin \mathscr{B}_{x,t}(L)$, establishing $(\mathscr{Q} \cap \mathscr{E}_k) \cap \mathscr{B}_{x,t} = \emptyset$.

It follows that $\mu(\mathcal{B}_{x,t}(L)) \leq 1-\mu(\mathcal{Q} \cap \mathcal{E}_k)$. We have $\mu(\mathcal{Q}) = 2^{-m+1}$ and $\mu(\mathcal{E}_k) > 1-2^{-m}$; hence $\mu(\mathcal{Q} \cap \mathcal{E}_k) > 2^{-m}$. Consequently, $\mu(\mathcal{B}_{x,t}(L)) \leq 1-2^{-m}$; in other words, $x \notin U_{m,t}$. □

Our final lemma relates θ-companions and σ-discrete decompositions.

Lemma 4. *Let $\mathcal{L}$ be a point-finite family of subsets of a perfect space X. If $\mathcal{L}$ has a θ-companion, $\{U_j(L): L \in \mathcal{L}, j \in J\}$, then $\mathcal{L}$ is σ-discretely decomposible.*

Proof. For each $n \in \omega$, let $[\mathcal{L}]^n$ be the family of n-element subsets of $\mathcal{L}$. For each $K \in [\mathcal{L}]^n$, let the elements of K be K_m, $m<n$. For each $j \in J$, $n \in \omega$, $m<n$, and $K \in [\mathcal{L}]^m$, set

$$Z(K,j) = \{x \in X: (\mathcal{L})_x = K = \{L: x \in U_j(L)\}\},$$

$$D_{j,m,n}(L) = \bigcup\{Z(K,j): L = K_m \in [L]^n\},$$

$$\mathcal{D}_{j,m,n} = \{D_{j,m,n}(L): L \in \mathcal{L}\}.$$

First, note that $\bigcup\{Z(K,j): j \in J, K \in [\mathcal{L}]^n, n \in \omega\} = X$. Second, note that for each $j \in J$, $n \in \omega$, $m<n$ and $K \in [\mathcal{L}]^n$

$$(\bigcup \mathcal{D}_{j,m,n}) \cap (\bigcap\{U_j(L): L \in K\}) = Z(K,j) \cap K_m,$$

so $\mathcal{D}_{j,m,n}$ is disjoint, and $H_{j,m,n}$, the set of points of X which do not have a neighborhood meeting at most one element of $\mathcal{D}_{j,m,n}$, is a closed set disjoint from $\bigcup \mathcal{D}_{j,m,n}$. Since X is perfect, there is a family $\{V_{j,m,n,i}: i \in \omega\}$ whose intersection is $H_{j,m,n}$.

Finally, we set $D_{j,m,n,i}(L) = D_{j,m,n}(L) - V_{j,m,n,i}$. The family $\{D_{j,m,n,i}(L): L \in \mathcal{L}\}$ shows that $\mathcal{L}$ is σ-discretely decomposible. □

The proof above can be used to show that weakly θ-refinable perfect spaces are subparacompact [1]. The ideas are from Theorem 4 of [10].

3.

Proposition P has been proved under two separate assumptions – PMEA and (SC ω_2 + $\forall S \diamond_S$). Let us discuss the similarities and differences.

One important similarity is that large cardinals are involved. To establish Con(ZFC + PMEA), Kunen assumed Con(ZFC + ∃ strong compact cardinal); to establish Con(ZFC + SC ω_2 + $\forall S \diamond_S$), Con(ZFC + ∃ supercompact cardinal) was assumed in [2]. It is known that each of Con(ZFC + PMEA) and Con(ZFC + SC ω_2) implies Con(ZFC + ∃ measurable cardinal).

There is an important difference between PMEA and SC ω_2 + $\forall S \diamond_S$. The latter implies CH while the former implies not CH is false in a strong way: PMEA implies that $\mathfrak{c}$ is weakly Mahlo. Another difference is that PMEA implies NMSC (the normal Moore space conjecture), while CH implies that NMSC is false.

Let us formulate some of the questions suggested by the above discussion.

(1) Is there an axiom, A, such that PMEA $\to A$, SC $\omega_2 + \forall S\, \diamondsuit_S \to A$ and $A \to$ Prop P? (Of course, we seek an axiom more natural than "either PMEA or SC $\omega_2 + \forall S\, \diamondsuit_S$".)

(2) Can Prop P be proved without large cardinal assumptions? In particular, does $V = L$ imply Prop P?

(3) Does NMSC imply Prop P?

4.

In this section we briefly comment on some aspects of [2].

R. Pol [9] proved (in ZFC) that a point-countable Borel-additive family of subsets of weight $\leqslant \omega_1$ of a complete metric space has a σ-discrete refinement. Assuming $\forall S\, \diamondsuit_S$, we may omit "complete" in the hypothesis; assuming further SC ω_2 we may omit "of weight $\leqslant \omega_1$". Pol's application of his result to generalize the Kuratowski–Ryll–Nardzewski selection theorem to separable-valued maps can be altered similarly.

Several simplifications of the work in [2] have been pointed out. First, in the consistency proof, the use of elementary embeddings can be avoided by using the "second-order Lowenheim–Skolem" characterization of supercompact cardinals [7]. Second, in extending the results from ultrametric spaces to general metric spaces, the following observation [5] can be used instead of perfect maps: If $(X, \mathcal{T})$ is a σ-space, then there is a finer ultrametric topology $\mathcal{T}'$ on X such that every open set $U \in \mathcal{T}'$ is an F_σ in the sense of $\mathcal{T}$. Third, the formulation of SC ω_2 is too complex; the following axiom TC ω_2 (proved consistent in the same way) is simpler and has more applications.

For $\mathcal{T} = (T, \leqslant_T)$, a tree, let $[T]\!\downarrow\!\omega$ be the set of countable initial segments of $\mathcal{T}$. We say that $(\mathcal{T}, g)$ is a *tree with local tasks* if

(1) $\mathcal{T}$ is a tree of height $\leqslant \omega_1$.

(2) g is a function with domain $\{(y, a)\colon y \in a \in [T]\!\downarrow\!\omega\}$.

(3) If $f \in g(y, a)$, then f is a function with domain $\{t \in T\colon t <_T y\}$.

(4) For each $y \in T$ and $a \in [T]\!\downarrow\!\omega$, $|g(y, a)| \leqslant \mathfrak{c}$.

We say that $(\mathcal{T}, g)$ is *satisfiable* if there is a function Γ with domain T such that for all $y \in T$,

$$\Gamma|\{t \in T\colon t <_T y\} \in \bigcup \text{range } g.$$

We say that $(\mathcal{T}, g)$ is ω_2-*satisfiable* if for all initial segments S of $\mathcal{T}$, if $|S| < \omega_2$, then $((S, <_T S^2), g|(S \times [S]\!\downarrow\!\omega))$ is satisfiable.

Axiom TC ω_2 is the assertion that if a tree with local tasks is ω_2-satisfiable, then it is satisfiable.

One way in which TC ω_2 differs from SC ω_2 is that satisfiable does not imply ω_2-satisfiable. (It is not required that $\Gamma|\{t \in T: t <_T y\} \in g(y, a)$). The following application illustrates this difference.

Lemma (TC ω_2). *Let A be a subspace of an ultrametrizable space X. If for every subspace Y of X, weight $Y < \omega_2$ implies that $A \cap Y$ has the Baire Property in Y, then A has the Baire Property in X.*

Notice that the fact that A has the Baire Property in a space X does not imply that, for $Y \subset X$, $A \cap Y$ has the Baire Property in Y. Thus, it seems that the lemma cannot follow directly from SC ω_2.

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