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Developing a chaotic pattern of dynamic Hazmat routing problem☆

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ABSTRACT

The present paper proposes an iterative procedure based on chaos theory on dynamic risk definition to determine the best route for transporting hazardous materials (Hazmat). In the case of possible natural disasters, the safety of roads may be seriously affected. So the main objective of this paper is to simultaneously improve the travel time and risk to satisfy the local and national authorities in the transportation network. Based on the proposed procedure, four important risk components including accident information, population, environment, and infrastructure aspects have been presented under linguistic variables. Furthermore, the extent analysis method was utilized to convert them to crisp values. To apply the proposed procedure, a road network that consists of fifty nine nodes and eighty two-way edges with a pre-specified affected area has been considered. The results indicate that applying the dynamic risk is more appropriate than having a constant risk. The application of the proposed model indicates that, while chaotic variables depend on the initial conditions, the most frequent path will remain independent. The points that would help authorities to come to the better decision when they are dealing with Hazmat transportation route selection.

1. Introduction

As hazardous material (Hazmat for short) transportation covers a significant part of economic activities in most industrialized countries, management of Hazmat is a multimodal issue involving environmental, engineering, economical, societal as well as political concerns [1,2]. Determining the route for Hazmat, known as the Hazmat routing problem, is usually a double-sided problem, where local authorities move towards minimizing public risk, while the carriers are concerned about minimizing transport costs [3].

Many different attributes as well as operation research methods are currently utilized to solve the Hazmat routing problem. In terms of attributes, social problems alongside out-of-pocket costs are the main considerations in developing mathematical models for solving risk-oriented optimization problems [4]. While the risk of spreading Hazmat in accidents and transport cost are considered as the main attributes for developing a mathematical model to determine the optimal assignment of truck flow within transportation, minimizing the weighted combination of objectives is also observed in the literature [5]. Dependency of travel time and risk associated to road network upon road traffic measures [6], road geometric designs [7,4], weather condition [8] and safety regulations which are imposed on drivers and transport companies [9] are also known as attributes in Hazmat routing problems. Population [5,10,11], environment [8,11–13] and accident [1,5,7,9,14] are the main apprehensions to define risk associated with Hazmat incident impacts. Specified regional networks which have been assigned by local authorities for Hazmat transportation, particularly for better enforcement and improving road safety, lead researchers to solve Hazmat routing problem [15–18] under network constraints. In addition, the nature of hazardous materials [4,19] together with public security [17] have also been studied as important attributes in this area. Management issues such as road network and vehicle capacity planning [10,20], Hazmat delivery time [1,7], and eventually emergency response have been considered in the above problem [5,12,14,19].

In terms of operation research methods, a two-stage approach was utilized to solve the problem of daily routes as well as departure intervals for Hazmat transportation trucks [7]. Within the first stage, a set of minimum and equitable risk routes are defined for each Hazmat transportation in order to spread the risk equitably over the population, while in the second stage, one among the aforementioned routes and a departure time not less than the preferred one, are assigned to each Hazmat transportation route in order to assure that at any given time, every two vehicles must be sufficiently far apart to minimize the sum of the Hazmat shipment delays [7]. K shortest path algorithm was developed for stochastic and dynamic networks and focused on identifying the exact solution to lead to computational inexplicit in large networks [15]. Defining bi-level objective function is another technique to solve the Hazmat routing problem [9,11,16]. To give an estimate, Serafini [11] developed a bi-level objective function model for the specific problem and extended a dynamic programming

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model to use for larger classes of network. As a more common technique, developing a mathematical model based on different priorities of attributes is also utilized to define objective function [1,5,21], in which each attribute is weighted by own weight of importance for modeling.

Some researchers have made research works in to clear routing as well as scheduling problem of Hazmat transportation in this area. Travel time and consequence measures are considered inherently unreliable and stochastic because of their dependencies upon characteristics like visibility, traffic volumes and activity patterns [6], while in the terms of uncertainty, weather condition was a main parameter in Hazmat routing problem [8]. Population risk and travel time are defined using probability distribution function to develop a multi-objective function for routing Hazmat transportation [5] due to the variation of the traffic condition over the network. Solving routing problem together with schematic points, made by Androuspolous et al. [9] to develop a mathematical model, considers Hazmat transport problem regarding dynamic risk to daily time while safety regulation made drivers to stop and rest in pre-specified locations.

Although transport risk of hazardous material is usually quantified with a path evaluation function [22], uncertainty is still considered in Hazmat routing problem. For more detail, in a typical research work presented by Dadkar et al. [23], the probability of assassination attacks has been verified in due model using stochastic variables. In another research work, done by Reilly et al. [18], while having the case studied through GAME theory, improvement was achieved by defining the probability of assassination attack. They defined transport companies, terrorist bodies and local authorities as game players to develop their model. The other types of defining risk including stochastic, uncertainty and linguistic are also observed in the area of Hazmat routing problem in academic literature. Qiao et al. developed a fuzzy model to estimate frequency of Hazmat transport accidents using a qualitative approach where driver, road construction and truck characteristics are presented as membership function based on experts' experiences [24].

According to the above, the use of probability function would seem as an appropriate approach to define risk. However, Hazmat road accidents incorporating their impacts might be more complicated compared to chaotic patterns which seem to be in regard of defining their characteristics for Hazmat routing problem.

Despite hazardous material transport risks being usually quantified by distribution functions [16], lack of reliable data, particularly in developing countries, will hamper to achieve the best routes in Hazmat transport planning. In the present case, an appropriate way of data collection is suggested for gathering experts' comments on using linguistic variables and converting them to the crisp values to be used in the process of developing and running mathematical models.

Although road safety in hazardous material transportation is definitely found to be much more important compared to other aspects, population must be protected from natural disasters as soon as possible [25], so trip time cannot be underestimated as a main criterion for vital substances transportation. Because flammable liquids are known as hazardous materials according to the hazardous material classification [26], fuel could be named as a main substance to be viewed for emergency situations in which time is a major concern for local authorities.

The main concept behind the present paper is to propose a methodology to help decision makers to achieve the best routes for Hazmat transportation under the emergency situations as soon as possible, while the other aspects of critical management such as people evacuation or first aid services have not been considered. In other words, the main concept behind the proposed methodology is to find the route for Hazmat transportation in which national and/or local authorities would make correct decisions in emergency situations while people who are settled in the affected area reach the requirements as soon as possible but other residents are not involved by Hazmat transport impacts. An other problem is that there is no exact data on road accidents; travel time might be a more important concern for carrying vital substances of Hazmat in emergency situations and behavior in road accidents seems to follow dynamic patterns due to the complexity of road accidents. The present paper is articulated in three main parts. The first part attempts to check the dynamic behavior of road accidents based on chaos theory. In order to cover a wide-range of variation, risks are defined by the chaotic pattern of one-dimensional logistic map equation as well as having analyzed during an annual period. The novelty in this paper is focused on using the chaotic property of traffic incidents such as traffic flow theory [27] in the process of solving Hazmat routing and scheduling problem, while risk in Hazmat transportation corresponds to four components including accident, population, environment and infrastructure due to accident impacts of Hazmat. The second part consists of a main subject to gather experts' considerations of risk according to the above components of risk and converts them to crisp values. The last one is to propose an iterative methodology to generate dynamic risk of the above risk components followed by a mathematical model, validation process and sensitivity analysis.

Consequently, the article is arranged in seven sections including introduction, brief description of chaos theory and presence of chaos as well as extent analysis method. They are followed by defining procedure and developing mathematical model, case study and experimental data embracing linguistic variables and converting them to crisp values, running proposed procedure incorporating problem definition, comparing results and sensitively analysis and summary, conclusions and recommendations to future studies, respectively.

2. Chaos theory and presence of chaos

Edvard Lorenz introduced the concept of chaos theory in 1963. He found the chaotic attractions in complex systems of weather forecasting when he entered different values as starting points in a computer program [28]. Chaos has been studied within the engineering scientific and mathematical communities and found to be useful in many disciplines such as high-performance circuits and devices, collapse prevention of power systems and also information processing [29]. Some sudden and dramatic changes in nonlinear systems may give rise to the complex behavior called chaos [27]. A nonlinear system is said to be chaotic if it exhibits sensitive dependence on initial conditions. It may happen that small differences in the initial conditions produce very great ones in the final outputs [30]. Chaos theory is commonly applied for short-term prediction because of the existing property of "sensitive dependence upon initial condition", which would hamper the success of long-term prediction, in respect. It has been widely applied on various fields of science particularly in the area of traffic flow theory [27].

Since chaos theory is used to analyze complex systems and transportation systems are complex entities, it may be found useful for transportation applications. In transportation systems, legal and social constraints may bind behavior, allowing a researcher to more accurately predict human actions and system evolution [31,32].

Determining the presence of chaotic behavior is a very important step. The Lyapunov characteristic exponent, λ, is the clearest measure to prove the existence and to quantify chaos in a dynamical system or time series [33]. Calculating the largest Lyapunov exponent is a more common technique to determine the presence of chaos, which measures the divergence of nearby trajectories [31]. As the system evolves, the sum of a series of convergence in each dimension will converge or diverge. Lyapunov exponents measure the rate of convergences and divergences in each dimension. If the largest Lyapunov exponent is positive, it indicates that the system under investigation is sensitive to initial condition and is chaotic. Eq. (1) is used to determine the largest Lyapunov exponent, $\lambda_{\max}$, where $S(t)$ is the system situation in period (t) and $S'(t)$ is its nearest neighbor. In this research work, $S'(t)$ is $S(t-\Delta t)$ where Δt is time step and N is the number of time steps [31].

$$\lambda_{\max} = \frac{1}{N\Delta t}\sum_{t=0}^{N-1} \mathrm{Ln}\left(\frac{\left|S(t+\Delta t) - S'(t+\Delta t)\right|}{\left|S(t) - S'(t)\right|}\right) \tag{1}$$

In theory, as Δt increases, the value of the exponent will converge to its true value [31]. However, in practice, concerning finite data sets and noise, the exponent can be determined within a range of primed values. The ratio of daily accidents in Fars province (to be referred to as the case study) to the whole accidents all over the country is considered as a chaotic risk factor. The above chaotic factor is known as the situation in the proposed methodology. Therefore, $S(t)$ is the actual ratio of daily fatal accidents in the area of case study to the whole accident all over the country in period (t), and $S'(t)$ is the expected ratio for $S(t)$ which is considered as the system situation in the previous day. Fig. 1 shows the ratio of daily accidents to the whole accident over 365 days. As it is observed in Fig. 1, primed and unprimed data points are distinct but close, so Eq. (1) is appropriate to be used for determining the presence of chaos [31]. Time is set to 24 h for experimental data (Δt) and risk factor is restricted into interval (0, 1). Accident risk for each edge is calculated by multiplying the ratio of daily accidents in the case study by ranking factors obtained by calculating crisp values, discussed more in the fifth section. The well-known equation of logistic map [32,34] is applied to generate risk factors because of good adeptness to traffic behaviors. Normal form of logistic map equation is defined as Eq. (2) [27]:

$$P(t+1) = K \times P(t) \times (1 - P(t)) \tag{2}$$

where $P(t)$ is the ratio of accident happening in the case study to the whole accidents all over the country. Using experimental data over 365 days, K, identified as equation parameter and will be used as K_1 in Eq. (10), has been estimated to be 3.6219; minimizing mean square errors is considered as a criterion, in which experimental data and estimated ones have been compared using one-dimensional logistic map equation to estimate the ratio of accidents. $\lambda_{\max}$ in Eq. (1) is equal to 0.249 if $\Delta t = 1$ and 0.154 if $\Delta t = 7$ which means that the ratio of daily accidents has a chaotic behavior in the case study.

3. Defining procedure

Because of existing chaotic behavior in road accidents, daily risk factors, obtained by multiplying the ratio of accident by associated risk of converting linguistic variables to crisp values and assigned to the edges of network, will be different over the year. In order to cover a wide-range of variations, an iterative procedure is designed as the solving method. Total combination of priorities for both risk and travel time is defined as a criterion in objective function. Risk factors are updated over the specific number of iterations, while different priorities for both risk and travel time are also considered in the proposed procedure. Mathematical model runs until stopping criteria is met. Stopping criteria is met when the number of iteration is met and all different priorities for risk and travel time have been checked. The number of iterations depends on the time period considered for filling out the questionnaires to present local experts' point of view. If linguistic variables (will be discussed more in Section 5) are presented based on a year, the number of iterations is set to 365. If they are presented based on a season, the number of iteration is set to 90. Keeping all paths and comparing them will outline the most frequent path, which is considered as the best path for Hazmat transportation under emergency situation in which national or/and local authorities are worried about risk and travel time. The proposed procedure including the nine steps given below is shown in Fig. 2.

1– Setting network parameters including nodes, edges, uniformed travel time and risk factors.
2– Initialize number of iterations, starting point and O–D pairs of supply and demands. (Number of iterations corresponds to duration such as 365 days over a year and starting point is considered to check sensitivity analysis. Supply nodes are located in the certain supporting area while demand nodes are located in the affected area.)
3– Update risk factors for all edges in the next iteration (day) using the logistic map equation.
4– Running mathematical model to find path while different priorities of travel time and risk are considered as measures and keep them in each iteration.
5– If stopping criteria (the number of iterations) is met, stop and go to step 6, otherwise go back to step 3.
6– Choose the most frequent path for each O–D pairs.

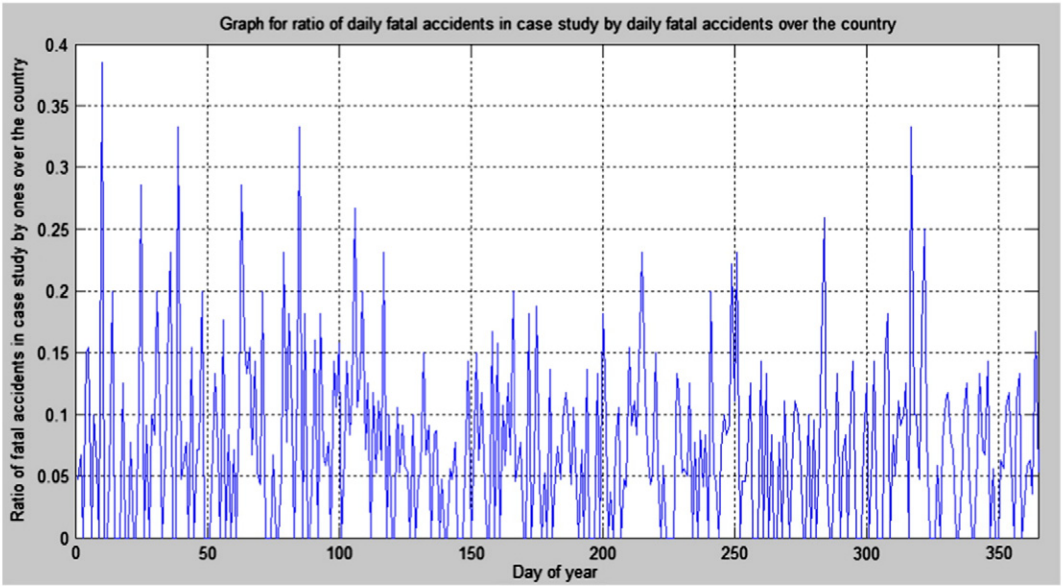

Fig. 1. Ratio of fatal daily accidents in Fars province to fatal daily accidents all over the country.

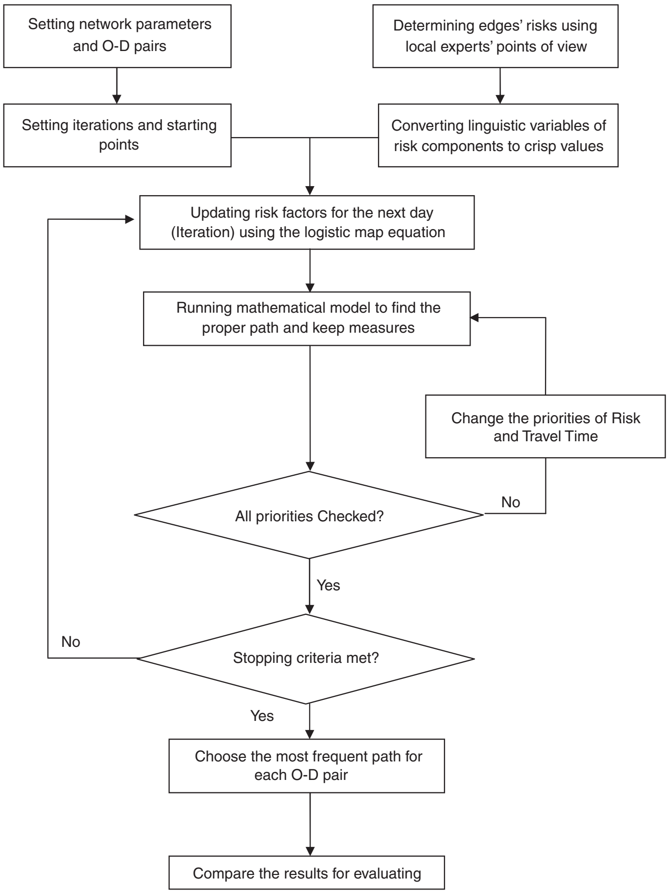

Fig. 2. Overall view of the proposed procedure.

7– Run steps 2–6 for all O–D pairs.
8– Solve the common transport problem to satisfy supplies and demands.
9– Compare and discuss the obtained results.

4. Developing mathematical model

Considering the concept of the proposed procedure, there are two levels of objective functions. Common transport problem and Hazmat routing problem are the main problems in the first and second levels, respectively. There are two defined risks regarding the proposed procedure. One of them is the ratio of daily accidents in Fars province to the whole daily accidents all over the country, and the other relates to the risk assigned to each edge. The number of daily accidents is estimated using crash prediction techniques [35,36] and risk is calculated by converting linguistic variables of four components of risk to crisp values which will be discussed in the next section.

Assume that graph G is the road network consisting a number of edges shown by notation (i, j). It indicates that an edge starts from node "i" and ends to node "j". N_s is a set of nodes defined as supplies and N_d is a set of nodes defined as demands. N_s is located in the supporting area while N_d is located in the affected area. (N_s, N_d) is defined as a set of O–D pairs in common transport problem. Therefore, the first level of objective function is defined by Eq. (3):

$$\mathrm{Min}Z = \sum_{(s,d)\in(N_s,N_d)} C_{sd} \times Y_{sd} \tag{3}$$

where "Y_{sd}" is the amount of Hazmat planned to transport from origin "s" to destination "d" that belong to "N_s" and "N_d" sets, respectively. Q_s is the maximum amount of Hazmat that should be transported from origin node "s", and Q_d is the minimum demand of required Hazmat transport to destination node "d". Because the overall model is a transport problem, the main constraints of meeting supply and demand are consequently defined by Eqs. (4) and (5):

$$\sum_{N_s} Y_{sd} \geq Q_d \quad \forall d \in N_d \tag{4}$$

$$\sum_{N_d} Y_{sd} \leq Q_s \quad \forall s \in N_s. \tag{5}$$

"C_{sd}" is the sum of uniformed risk and travel time priorities for all edges which are located in the most frequent path selected by the proposed procedure. It is obtained by selecting the most frequent path over a year (365 days) considering risk factors are updated by one-dimensional logistic map equation based on the concept of chaos theory [29]. In other words, C_{sd} is the value of objective function which belongs to the most frequent path, while $K_{sd}(t)$ is the obtained value of objective function in iteration (t).

The second level of objective function is defined by Eq. (6).

$$\mathrm{Min}K_{sd}(t) = \sum_{(i,j)\in G} \left[P_{ij}(t)\cdot P_r + TT_{ij}\cdot P_l\right]\cdot X_{ij}(t) \tag{6}$$
$$\forall t = 1,2,3,\ldots,365, (s,d)\in(N_s,N_d)$$

where $K_{sd}(t)$ is the sum of uniformed risk and travel time priorities for all edges located in the obtained path after running the developed mathematical model in iteration (t). $P_{ij}(t)$ is the uniformed risk in edge (i, j) and P_r is the priority of risk, TT_{ij} is uniformed travel time in edge (i, j), mainly represents cost, and P_l is the priority of travel time. In this situation, $P_r + P_l = 1$ means that the priorities for risk and travel time have contrary values. $X_{ij}(t) = 1$ if edge (i, j) is located in the selected path obtained by objective function determined by Eq. (6) in iteration (t) and 0 otherwise. In common Hazmat routing problem, nodes, which are located in the selected path, are defined by Eq. (7) [37]. Eq. (7) guarantees to keep the connection of nodes which are located in the selected path. Disconnection is acceptable if nodes had been set as origin or destination [37]. Objective function which is modeled as minimum equation guarantees that there is just one selected path in results in which origin and destination nodes are outlined.

$$\sum_{(i,j)\in G} X_{ij}(t) - \sum_{(i,j)\in G} X_{ji}(t) = \left\{\begin{array}{c} -1 \text{ if } j = \text{Origin} \\ 1 \text{ if } j = \text{Destination} \\ 0 \quad \text{otherwise} \end{array}\right\} \forall t = 1,2,3,\ldots,365 \tag{7}$$

$$(i,j),(j,i)\in G. \tag{8}$$

Constraint (8) guarantees the two-way road network, i.e. two opposite directions for each edge are available. Solving the mathematical model, some of the variables $X_{ij}(t)$ are assigned as 1 and the others as 0. Edge (i, j) is located in the selected path, if its corresponding variable is 1, and vice versa. Eq. (7) guarantees that there is no disconnection in the sequencing of links which are located in the path. Extracting the variables $X_{ij}(t)$, which are assigned by 1, obtains the selected path. To give an estimate, if links (1, 3), (3, 5), (5, 6), (6, 9), and (9, 10) in a network that consists of 10 nodes are equal to 1, the selected path is extracted as 1-3-5-6-9-10. Total risk for each edge (i, j) in each iteration, represented by $P_{ij}(t)$, is updated by Eq. (9) where $RF(t)$ is an identified risk by the ratio of accident in road network G by all accidents over the country, and $EO_{ij}(t)$ is a crisp value of risk obtained the using extent analysis method to convert linguistic variables of four components of defined risk for edge (i, j).

$$P_{ij}(t) = RF(t)\cdot EO_{ij}(t) \forall t = 1,2,3,\ldots,365. \tag{9}$$

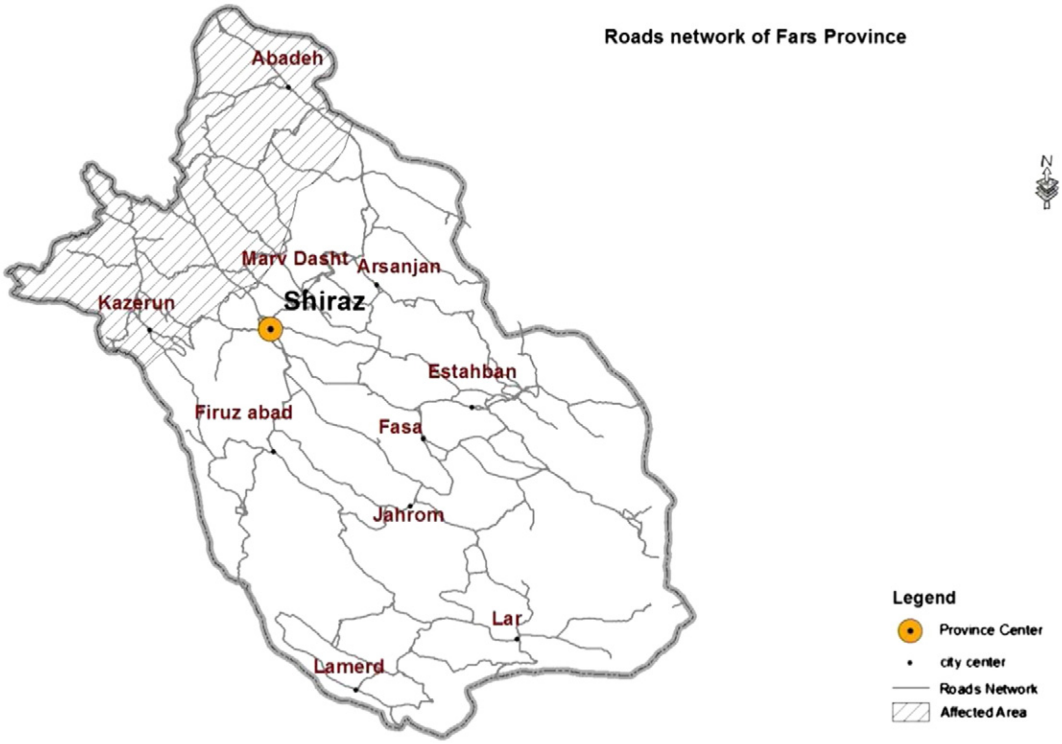

Fig. 3. The roads network of Fars province (case study).

One-dimensional logistic map equation [29], which is mainly used to generate chaotic patterns of variables shown in Eq. (2), is utilized for updating both risk factors in analytical process of the proposed mathematical model, where $RF_{ij}(t)$ and $EO_{ij}(t)$ are being updated in each iteration using Eqs. (10) and (11) in which K_1 and K_2 are logistic map equation parameters. Parameter K_1 is estimated by using well-known technique of minimizing mean square errors between experimental data and predicted data using logistic map equation as well as 4 is assigned to parameter K_2 [29].

$$RF(t) = K_1 * RF(t-1)\cdot(1-RF(t-1)) \;\; \forall t = 1, 2, 3, \ldots, 365 \tag{10}$$

$$EO_{ij}(t) = K_2 * EO_{ij}(t-1)\cdot\left(1-EO_{ij}(t-1)\right) \;\; \forall t = 1, 2, 3, \ldots, 365. \tag{11}$$

5. Case study and experimental data

Fars, the second largest province in Iran, is selected as the case study. Fig. 3 shows an overall view of the road network in Fars province. It is assumed that the north part of province is affected by natural disasters, shown by the shaded area in Fig. 3. It consists of fifty nine nodes and eighty two-way edges used without considering the affected area in another research work [38]. Some of the nodes are borders, which connect the research area to the neighboring provinces. Vehicles entering the affected area are supplies and those departing the other nodes are demands, respectively. Internal supplies and demands correspond to domestic nodes. The list of nodes are available in Appendix A.

The four components accident, population, environment, and infrastructure are defined as risk for each edge. Accident risk corresponds to the rate of crashes based on local experts' points of view. Population risk that corresponds to density of population might be affected if a Hazmat incident happened. Environmental risk is related to risk of impacts to environment, such as leaking Hazmat to the rivers. Finally, infrastructure risk belongs to the concerns of vital infrastructures, such as long bridges or tunnels which may be affected after Hazmat incidents. Due to defining linguistic variables for the above components which are presented by local experts, five states safe, medium safe, fair, medium risk and high risk are shown in Table 1, and fuzzy analytical hierarchy process (FAHP) [39] has been utilized to obtain risk.

For each edge, four criteria including accident, population, environment, and infrastructure risks have been identified by five decision makers using linguistic variables. Triangular fuzzy numbers (TFN) [40,41] assigned to each variable are calculated and shown in Table 1.

Based on the concept of FAHP, edges are considered as alternatives (A_1, A_2, … , A_{80}), risk components as criteria (accident (C_1), population (C_2), environment (C_3), and structure (C_4)), and five experts, who filled out questionnaires, as decision makers (P_1, P_2, … , P_5).

Linguistic variables for the importance weight of each criterion and the importance of criterion dependent on the experts' view points are identified in Tables 2 and 3, respectively. W in Table 3 is a vector corresponding to the weight vector of risk factors obtained by FAHP, observed in the literature [42].

Decision makers presented four criteria of risks by linguistic variables defined in column 2 of Table 1 and the four below steps of Chang's extent analysis [40,43] including defining value of fuzzy synthetic extent, calculating degree of possibility, calculating weighing factors, and obtaining uniformed weights have been utilized to convert linguistic variables to crisp values.

Table 1
Linguistic variable for edge risk component rating and TFN.

Safe	S	(0, 1, 3)
Medium safe	MS	(1, 3, 5)
Fair	F	(3, 5, 7)
Medium risk	MR	(5, 7, 9)
High risk	HR	(7, 9, 10)

Table 2
Linguistic variables for the importance weight of each criterion.

Very low	VL	(0, 0.1, 0.3)
Low	L	(0.1, 0.3, 0.5)
Medium	M	(0.3, 0.5, 0.7)
High	H	(0.5, 0.7, 0.9)
Very high	VH	(0.7, 0.9, 1)

Uniformed weighing numbers, non-fuzzy numbers, were considered as measures of total risk for edges and were used in the analytical process of finding the safest path in Hazmat routing problem. The weights obtained by the extent analysis method as well as the other variables such as travel time should be uniformed because of existing different scales of variables in the proposed mathematical model, therefore, Eq. (12) is used to uniform data into closed interval [0.05, 0.95]:

$$X_{new} = \frac{(X_{old} - X_{min})}{(X_{max} - X_{min})} * 0.9 + 0.05 \tag{12}$$

where X_{new} is the uniformed amount of variable X_{old}, while X_{max} and X_{min} are the maximum and minimum of variable X, respectively.

As an illustrative example, five experts presented their points of view for four components of risk by linguistic variables for edge (14, 15) which indicates the link (Polefasa, Sarvestan) in the network shown in Table 4. Node codes and their names are available in Appendix A. Triangular fuzzy numbers (TFN) of variables are also presented in the next rows of table according to Table 2. Using the extent analysis method [40,43], the total sum of TFN has been calculated for each risk component, while the portion of them regarding the whole amounts of network edges are shown in the next row in Table 4. Degree of possibilities has been calculated according to the minimum values of pair-wise comparison [42,43], then risk has been calculated using the weighing measures of importance criteria. While the minimum and maximum amounts of risk have been calculated as 0.2985 and 0.7278, respectively, using Eq. (12) uniformed value of risk for edge (14, 15) has been finalized to 0.6880.

6. Running model and discussion

6.1. Problem description

Six origin and two destination nodes have been selected for solving the problem, so twelve O–D pairs will be selected for running the proposed model. Origin nodes are located in the case study area, in which some of them known as boarders while destination ones are located in the pre-specified affected area, where they may be affected by heavy rain, snow or earthquake. Illustrative map is shown in Fig. 3 in which the affected area is shown by shaded area. The amounts of Hazmat supplies and demands required to transport from origins to destinations are briefly presented in Table 5. It is expected that the proposed mathematical model should be able to determine the amount of Hazmat, required to be carried from each origin nodes to the corresponding destination in the affected area.

Table 3
The importance of criterion.

	P_1	P_2	P_3	P_4	P_5	W
C_1	VH	VH	H	VH	H	0.4230
C_2	H	M	L	M	VL	0.2093
C_3	M	H	M	VL	M	0.2702
C_4	L	VL	VL	VL	L	0.0975

Table 4
Linguistic variables and TFN for edge (Polefasa, Sarvestan) (14, 15) as an illustrative example.

Measure for edge (14, 15)	Expert no.	Risk component			
		Accident	Population	Environment	Infrastructure
Experts' points of view by linguistic variables	P1	MR	MR	F	MS
	P2	MR	F	HR	MS
	P3	F	MR	F	F
	P4	MR	MR	MR	MR
	P5	F	F	MR	F
Triangular fuzzy numbers (TFN)	P1	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)	(1, 3, 5)
	P2	(5, 7, 9)	(3, 5, 7)	(7, 9, 10)	(1, 3, 5)
	P3	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	P4	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)
	P5	(3, 5, 7)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)
$\sum_P \text{TFN}$		(21, 31, 41)	(21, 31, 41)	(23, 33, 42)	(13, 23, 33)
$\sum_P \text{TFN} \Big/ \sum_{P,A} \text{TFN} \times 10^2$		(26, 29, 16)	(18, 12, 8)	(24, 25, 14)	(9, 11, 10)
Degree of possibility		0.725	0.458	0.612	0.356
Importance of criterion (W in Table 3)		0.4230	0.2093	0.2702	0.0975
Calculated risk		0.6026			
Uniformed risk		0.6880			

6.2. Dynamic versus constant risk in finding paths

The first stage is finding the most frequent path using the proposed procedure of dynamic risk and comparing results with the way of considering constant risks. Different priorities of time and risk are used in the analytical process, while the lengths of determined paths are the best criteria to compare results. Their priorities change when different values of P_r and P_l are assigned in Eq. (6). The order of priorities is shown in parenthesis in Tables 6 and 7. Time priority of "0" means there is no interest on travel time and risk priority of "0" means there is no interest on risk. Running the second level of mathematical model has been done for each origin–destination pair using different priorities of travel time and risk and results are presented in Tables 6 and 7. Five times of running the proposed model and constant risk factors using different amounts of both

Table 5
The amount of Hazmat required to be carried form origin to destination nodes (unit: truck/day).

	Origin (supply)						Destination (demand)	
Node	6	11	24	33	40	48	1	57
Amount	100	300	250	450	150	250	800	700

Table 6
Path lengths determined by constant risks (unit: km).

Origin	Destination	(Time, risk) priority, $(P_r + P_l = 1)$					Average
		(0, 1)	(0.25, 0.75)	(0.5, 0.5)	(0.75, 0.25)	(1, 0)	
6	1	312	312	252	252	252	276
11	1	288	288	288	288	288	288
24	1	429	429	369	369	357	390.6
33	1	987	773	659	629	607	731
40	1	634	634	482	482	482	542.8
48	1	451	451	423	423	423	434.2
6	57	344	344	344	344	344	344
11	57	254	254	132	132	132	180.8
24	57	461	461	311	311	272	363.2
33	57	761	463	409	409	409	490.2
40	57	284	284	284	284	284	284
48	57	225	225	225	225	225	225

Table 7
Most frequent path lengths determined by dynamic risks based on chaos theory[a] (unit: km).

Origin	Destination	(Time, risk) priority, ($P_r + P_l = 1$)					Average
		(0, 1)	(0.25, 0.75)	(0.5, 0.5)	(0.75, 0.25)	(1, 0)	
6	1	252	252	252	252	252	252
11	1	405	288	288	288	288	311.4
24	1	369	369	369	369	357	366.6
33	1	713	661	661	607	607	649.8
40	1	482	482	482	482	482	482
48	1	423	423	423	423	423	423
6	57	344	344	344	344	344	344
11	57	132	132	132	132	132	132
24	57	461	272	272	272	272	309.8
33	57	463	463	463	409	409	441.4
40	57	284	284	284	284	284	284
48	57	225	225	225	225	225	225

[a] One-dimensional logistic map equation is applied by $P(t) = K * P(t-1)(1 - P(t-1))$.

time and risk priorities from "0" to "1" step 0.25 have been done and the most frequent results are outlined for all twelve predefined O–D pairs. The main difference between results in Tables 6 and 7 is using constant risk for each edge in Table 5 and using dynamic risks obtained by defining chaotic pattern presented in Table 6.

As shown in Table 7, defining risks by chaotic pattern helps decision makers to achieve shorter paths for Hazmat transportation rather than using constant risks. Because of more relevance on cost and travel distance on transportation, it might be a good proposition for transport companies for Hazmat transport with lower cost. T-test is one of the common techniques to compare the means of two sample data. T-test for two average columns of Tables 6 and 7 demonstrates that $\mu_{constant} - \mu_{dynamic}$ # 0 with p-value = 0.0112 at a significant level of $\alpha = 0.05$ [44], so there is a significant difference between average distance for selected paths using dynamic rather than constant risks.

Results also show that there is a difference between the selected paths in short and long origin–destination paths in the case of using chaotic patterns of risks. Results in Table 7 revealed that the proposed methodology in the long-paths is more capable rather than the short-paths. The main reason for the difference between short and long paths seems to be the limitation of substituting routes, because the proposed model has a constraint to find more paths in short origin–destination pairs.

6.3. Hazmat transport planning

The last step of the proposed procedure is solving the common transport problem using defined amount of supplies and demands while total combination of travel time and risk priorities is considered as criterion in solving transportation problem. The amounts of Hazmat required to be carried from each origin to its corresponding destination are presented in Table 8, total length being used as the comparing measure. Table 8 also shows that, as time goes more

Table 8
Results on the final solution for different travel time and risk priorities.

Origin	Destination	(Time, risk) priority, ($P_r + P_l = 1$)									
		(0, 1)		(0.25, 0.75)		(0.5, 0.5)		(0.75, 0.25)		(1, 0)	
		Length	Quan.	Length	Quan.	Length	Quan.	Length	Quan.	Length	Quan.
6	1	252	100	252	100	252	100	252	100	252	100
11	1	405	50	288	200	288	200	288	200	288	200
24	1	369	250	369	250	369	250	369	250	357	250
33	1	713	0	661	0	661	0	607	0	607	0
40	1	482	150	482	0	482	0	482	0	482	0
48	1	423	250	423	250	423	250	423	250	423	250
6	57	344	0	344	0	344	0	344	0	344	0
11	57	132	250	132	100	132	100	132	100	132	100
24	57	461	0	272	0	272	0	272	0	272	0
33	57	463	450	463	450	463	450	409	450	409	450
40	57	284	0	284	150	284	150	284	150	284	150
48	57	225	0	225	0	225	0	225	0	225	0
Total length		551,250		544,950		544,950		520,650		517,650	

Quan. = Quantity.

Table 9
Results for most frequent paths using different initial seeds for travel time and risk priorities (0.75, 0.25).

Origin–destination		Seed = 0.05		Seed = 0.10		Seed = 0.15		Seed = 0.20		Seed = 0.25	
		Length	F.	Length	F.	Length	F.	Length	F.	Length	F.
6	1	252	365	252	365	252	365	252	365	252	365
11	1	288	326	288	327	288	325	288	323	288	325
24	1	369	270	369	269	369	265	369	267	369	269
33	1	607	158	607	176	607	179	607	178	607	178
40	1	482	280	482	280	482	282	482	281	482	281
48	1	423	321	423	327	423	326	423	328	423	329
6	57	344	341	344	348	344	346	344	344	344	346
11	57	132	221	132	223	132	225	132	225	132	223
24	57	272	275	272	252	272	257	272	253	272	263
33	57	409	280	409	279	409	276	409	284	409	280
40	57	284	365	284	365	284	365	284	365	284	365
48	57	225	365	225	365	225	365	225	365	225	365

F. = frequency of repeated path.

vital, the total length is decreased as well. In others words, when time priority leads to 100%, shorter paths are selected and vice versa.

6.4. Sensitivity analysis

Chaos theory is commonly applied for short-term prediction because of the existence of the property of 'sensitive dependence upon initial condition'. Initial condition may change the results of finding the safest path after some iteration, so it will hamper the success of long-term prediction [27]. Based on this concept, it may be questioned whether the proposed procedure is suitable for long term decision making or not. Actually, short and long term results depend on the initial seeds. In order to check the reliability of the proposed method, five initial seeds on ratio of daily accident to the whole accidents all over the country were used for sensitivity analysis. For each different seed the length of the most frequent path together with frequency are presented in Table 9 using time and risk priorities of 75% and 25%, respectively. In this table, symbol "F." indicates the frequency of path with specified length. Results reveal that although there are some differences between the numbers of frequencies for O–D pairs, paths are outlined without any changes in the cases of using different seeds.

Based on the abovementioned data, it is fair to say that, the proposed method is capable of being used for Hazmat routing problem in emergency situations while data in a longer period of a year are being utilized for parameters' calibration.

7. Summary and conclusion

In this research work, an iterative procedure, structured by basic principle of Hazmat routing problem and the concept of chaos theory in dynamic risk definition, has been applied in vital Hazmat transportation of fuel in emergency situations. Travel time is a main concern in decision making in emergency situations, so the model has been developed to be capable for considering both risk and time simultaneously. Risk is defined by four components including accident, population, environment and infrastructure concerns. Due to lack of data on road accidents, risks have been identified by linguistic variables and the extent analysis method was utilized to convert linguistic variables to crisp values corresponding to the risks of network edges. A provincial road network, including fifty-seven nodes and eighty edges in existing experimental data, has been used as a case study for analytical and evaluation process. To be more confident on using chaos theory, the presence of chaos in the experimental data has been checked and concluded that the ratio of daily accidents has chaotic behavior. Then, the principle of Hazmat routing problem and fundamental concerns of chaos theory for defining dynamic variables are combined to develop a mathematical model while risk and time are prioritized by corresponding coefficients. Risks are dynamically updated according to the one-dimensional logistic map equation, which is known to be a well-known equation in this area.

The proposed procedure and mathematical model have been run in pre-specified iterations and results revealed that the concept of chaos theory is capable of finding better routes compared to constant risks. They also show that considering high-priority risks leads decision makers to achieve longer paths, although total length is being decreased when travel time is taken as being more important. Sensitivity analysis also revealed that, although there are some differences between outlined paths by using various seeds, the most frequent paths remain independent from the initial conditions of risks.

It is highly recommended that further researches might be focused on the process of estimating risks while a combination of risk measures, such as weather condition and traffic, is needed to be considered in Hazmat routing problem.

Appendix A

List of nodes and their codes located in the Fars road network (case study).

Code	Name
1	Izadkhast
2	Abadeh
3	Sormagh
4	Safashahr
5	Bovanat
6	Sarvestan
7	Saadatshahr
8	Arsanjan
9	Naghsherostam
10	Jamalabad
11	Marvdasht
12	Soltanabad
13	Shiraz
14	Polefasa
15	Sarvestan
16	Dorahi-Estahban-Fasa
17	Fasa
18	Sahrarood
19	Fadshokoieh
20	Chaliyan
21	Dindarlo
22	Ghaleh ab barik
23	Neyriz
24	To Sirjan
25	Ich
26	Darab
27	Dolatabad
28	Khosvaieh
29	Hajiabad
30	Dorahi Lar Jahrom
31	Mansourabad
32	Lar
33	To Bandar-Abbas
34	Evaz
35	Lamerd
36	Chah eyni
37	Khonj
38	Qirokarzin
39	Simakan
40	Jahrom
41	Qotbabad
42	Firoozabad
43	Esmaeilabad
44	Fathabad
45	Kazeroon
46	Dashte-Arjan
47	Bazernegan
48	Konartakhteh
49	Qaemiyeh
50	Norabad
51	Serahi-Nourabad-Sepidan-Shiraz
52	Mosiri
53	Koshk
54	Asias
55	Eqlid
56	Sepidan
57	To Yasooj
58	Forg
59	Dehno

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