Optimization of Signal Timing at Critical Intersections for Evacuation

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Abstract

Aimed to make the evacuated vehicles leave the evacuation zone as quickly as possible, a bi-level programming model is proposed to determine the signal timing at critical intersections (CI) by means of dynamic traffic simulation. The upper-level designs the optimal signal timing plan with the objective that minimize delays of critical intersection approaches inside of the evacuation zone based on known traffic distribution, while the lower-level simulates the dynamic propagation process of evacuated vehicles in the evacuation zone based on CTM and the principle of User Equilibrium. To demonstrate the effectiveness of the proposed method, we apply the model and algorithm based on the survey of parking lot, the urban street network around Nanjing Olympic Sports Center, to obtain the traffic volume of each road section, arrival distributions of evacuated vehicles at CIs, and optimal signal timing plan of each CI. It can be seen that the proposed method can effectively solve the integrated problems of signal timing optimization and dynamic traffic simulation.

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Keywords: Urban traffic; signal timing; bi-level programming; critical intersections for evacuation; dynamic traffic simulation

1. Introduction

A favorable signal timing scheme can increase the traffic capacity of entering or leaving arterial roads from minor roads, and eliminate bottlenecks at connections [1]. To find an optimal cycle and appropriate duration for green time in each phase, researchers often aimed at minimizing the delay [2] or the queue length [3, 4]. Dion et al.
[5] compared the delays controlled in fixed-time and operated in a range of conditions extending from under-saturated to highly saturated; Liu et.al [6, 7] made a classification of signalize intersection approach according to its traffic condition, namely insaturation, critical saturation and oversaturation, and analyzed the delay at each class of approach; Ban et.al [8] estimated real time queue lengths at signalized intersections using travel times. In consideration of the problem of dynamic traffic simulation, Cell Transmission model (CTM) is frequently used in researches on signal timing optimization. Lo and Szeto [9] transformed CTM to a set of mixed-integer constraints and subsequently cast the dynamic signal-control problem to a mixed-integer linear program; Wang et.al [10] designed a bi-level programming method to optimize phase scheme and green time by integrated genetic algorithm and chaotic algorithm; Lian et.al [11, 12] presented a generalized bi-level programming model of combined dynamic traffic assignment and traffic signal control; Wang et.al [13] used randomly distributed saturation flow rates and arrival rates to estimate the average delay and search for an optimal traffic signal timing plan.

For evacuation, the traffic control of intersection and critical evacuation passageway can achieve pre-set goals such as minimizing conflicts and weaving points, maximum use of traffic corridor and so on. The major road signal coordination is an important measure for evacuation traffic control according to many references. At present, the effectiveness of a signal timing plan for evacuation is evaluated by simulation software in majority of researches. Chen [14] and Sisiopiku et.al [15] used CORSIN which was a micro simulation software to test different evacuation plannings and evaluate the influence on evacuation planning performance in line with signal timing optimizing. McHale and Collura [16] used another signal optimization simulation software named TRANSYT-7F to build optimal signal timing scheme. They also used CORSIM to evaluate emergency vehicles preemption. Overall, the existing researches about traffic organization and control measures for evacuation focus on the evacuations under the conditions of hurricane and flood, and organization of evacuation traffic plans depends on simulation technology. Actually, using CTM to describe the dynamic propagation characteristics of traffic flow in an evacuation situation were researched further. Dixit and Radwan [17] studied on the population evacuation before the landing of hurricane based on CTM. Chen and Li [18] designed a quasi-dynamic traffic assignment method for emergency, combing the static multi-routes traffic assignment and CTM. Zhao et.al [19] simulated the propagation process of traffic flow on network under the condition of evacuation by CTM as well. Therefore, this paper will continue using the CTM to simulate the dynamic evacuation process.

It is known that the legs are usually one-way [20] in evacuation zone. The terminal points of evacuation zone are usually intersections. As to this kind of intersections, which are critical points between evacuation zone and the safe zone, and consequently some legs of whose are in the evacuation zone, but others are outside, is defined as critical intersections. In the situation of the overlarge delay at critical intersections, the traffic of upstream road sections and intersections in the evacuation zone may be negatively affected, and the evacuated vehicles are not able to depart from evacuation zone rapidly. Therefore, optimizing the signal timing to minimize delays of critical intersection approaches inside of the evacuation zone is an important link to improve the evacuation efficiency.

In this study, a bi-level programming model is proposed to determine the signal timing at critical intersections by means of dynamic traffic simulation to make the evacuated vehicles leave the evacuation zone as quickly as possible. It is supposed that the evacuation zone which is influenced by the emergency is determinate; accordingly, the evacuation demand and loading time in the evacuation zone are known. The upper-level designs the optimal signal timing plan based on known traffic distribution when minimizing delays of critical intersection approaches inside of the evacuation zone, while the lower-level applies the dynamic evacuation simulation model based on CTM and User Equilibrium to simulate the dynamic propagation process of evacuated vehicles in the evacuation zone, thus the arrival distributions of evacuated vehicles at critical intersections are acquired.

2. Modeling

There is at least one leg of critical intersection (CI) inside of evacuation zone (EZ), and the legs are usually one-way [20]. Meanwhile, there is at least one leg outside of EZ. It can be found from Figure 1 that these two kinds of intersections both have crossing conflicts. Due to the high volume of traffic in evacuation situations, CI is usually signalized. The phase setting of signalized intersection is various, involving two phases, three phases, four phases and more.
2.1. Upper-level: Delay function at CI

Based on the route choice result of lower-level programming, the arrival distributions at CIs will change. Vehicles at signalized intersection approaches queue in line during red time and travel during green time. The intersection approaches can be classified according to the traffic conditions by the value of saturation [6, 7]. Hereinafter, \( s \) is saturation flow rate of an approach, veh/s; \( q(k) \) is vehicle arrival rate of the approach in the interval \( k \), veh/s; \( g(k) \) is effective green time in the interval \( k \), s; \( C(k) \) is duration of the interval \( k \), s; \( y_i(k) \) is traffic volume from approach \( i \) to destination \( S \), veh/s; \( t \) is time interval of CTM, s. Takes the three-phase signalized intersection as example to model delay.

- Delay of three-phase signalized intersection approach

Directing at the situations of different saturations, delay at the approach in the first phase of the interval \( D(k) \) and number of remaining vehicles of this approach at the end of the internal \( n(k) \), can be calculated as functions (1) and (2). Function (1) corresponds to the situation of \( s \cdot g(k) \geq q(k) \cdot g(k) + n(k-1) \), and function (2) to the situation of \( s \cdot g(k) < q(k) \cdot g(k) + n(k-1) \).

\[
\begin{align*}
D(k) &= \frac{n^2(k-1)}{2[s-q(k)]} + \frac{1}{2} q(k) \cdot r^2(k) \\
n(k) &= q(k) \cdot r(k)
\end{align*}
\]  
(1)

\[
\begin{align*}
D(k) &= n(k-1) \cdot C(k) + \frac{1}{2} [q(k) \cdot C^2(k) - s \cdot g^2(k)] - s \cdot g(k) \cdot r(k) \\
n(k) &= n(k-1) + q(k) \cdot C(k) - s \cdot g(k)
\end{align*}
\]  
(2)

Similarly, the delays at approaches in the third phase can be calculated by following functions (3)-(5) in the case of \( s \cdot g(k) > q(k) \cdot C(k) \) & \( n(k-1) = 0 \), \( s \cdot g(k) > n(k-1) + q(k) \cdot C(k) \) & \( n(k-1) \neq 0 \), \( s \cdot g(k) \leq q(k) \cdot C(k) \) & \( n(k-1) = 0 \) or \( s \cdot g(k) \leq n(k-1) + q(k) \cdot C(k) \) & \( n(k-1) \neq 0 \) respectively.
\[
D(k) = g(k) \cdot s \cdot r^2(k) / 2[s - q(k)]
\]
\[
n(k) = 0
\]

\[
D(k) = n(k-1) \cdot r(k) + \left[ q(k) \cdot r(k) + n(k-1) \right] \left[ s \cdot r(k) + n(k-1) \right] / 2[s - q(k)]
\]
\[
n(k) = 0
\]

\[
D(k) = n(k-1) \cdot C(k) + \frac{1}{2} q(k) \cdot C^2(k) - s \cdot g^2(k)
\]
\[
n(k) = n(k-1) + q(k) \cdot C(k) - s \cdot g(k)
\]

Aimed at different situations, Function (6) corresponds to the delay in the second phase under situation of
\[s \cdot g(k) > q(k) \cdot \left[ r_i(k) + g(k) \right] + n(k-1),\] and function (7) to the situation of
\[s \cdot g(k) \leq q(k) \cdot \left[ r_i(k) + g(k) \right] + n(k-1).
\]

\[
D(k) = r_i(k) \cdot n(k-1) + \left[ n(k-1) + r_i(k) \cdot q(k) \right] \left[ n(k-1) + r_i(k) \cdot s \right] / 2[s - q(k)] + \frac{1}{2} q(k) \cdot r_i^2(k)
\]
\[
n(k) = q(k) \cdot r_i(k)
\]

\[
D(k) = C(k) \cdot n(k-1) + \frac{q(k) \cdot C^2(k) - s \cdot g^2(k)}{2} - s \cdot g(k) \cdot r_i(k)
\]
\[
n(k) = n(k-1) + q(k) \cdot C(k) - s \cdot g(k)
\]

- **Cycle length**

  According to HCM 2010, cycle length \(C(k)\) can be defined:

  \[
  C(k) = \frac{LX_c}{X_c - \sum_{i=IN} q_i / s_i}
  \]

  where \(L\) is cycle lost time (s); \(X_c\) is critical intersection volume-to-capacity ratio; \(IN\) is all approaches inside the evacuation zone; \(q_i\) is flow rate for lane group \(i\) (veh/h); \(s_i\) is saturation flow rate for lane group \(i\) (veh/h).

- **Average delay per vehicle**

  The delay at intersection is the sum of delay values of the intersection approaches. However, to obtain the minimum delay at CI, estimate the delay value only considering the traffic flows of approaches which are inside of EZ.

  Based on the above models, the objective function of upper-level is
where, \( d \) is average delay per vehicle, s; \( K \) is cycle numbers of the intersection during evacuation; \( IN \) is the set of all approaches inside the evacuation zone; \( D_i(k) \) is the total delay of approach \( I \) in cycle \( k \), s; \( q_i(k) \) is flow rate for lane group \( i \) in cycle \( k \), veh/s.

### 2.2. Lower-level: Dynamic Evacuation Simulation Model

In this paper, we apply the dynamic evacuation simulation model based on CTM and the User Equilibrium traffic assignment method to simulate the dynamic propagation process of evacuated vehicles on the network in \( EZ \); the arrival distributions of evacuated vehicles at critical intersections are gained.

Ran & Boyce [21] and Chen [22] gave a consistent definition to Dynamic User Optimal. Namely, the lower-level is equivalent to finding vectors \( \tilde{f}_{rs}^{ts}(t) \) satisfying the following equation:

\[
\begin{align*}
\left\{ \begin{array}{l}
\tilde{f}_{rs}^{ts}(t) > 0, \quad \tilde{c}_{rs}^{ts}(t) = \tilde{c}_{\min}^{rs}(t) \\
\tilde{f}_{rs}^{ts}(t) = 0, \quad \tilde{c}_{rs}^{ts}(t) > \tilde{c}_{\min}^{rs}(t)
\end{array} \right.
\end{align*}
\]  

(10)

where \( f_{rs}^{ts}(t) \) is traffic volume of path \( K \) between \( OD \) pair \( rs \) loading at origin \( r \) at time interval \( t \); \( c_{rs}^{ts}(t) \) is the impedance in time units of route \( K \) between \( OD \) pair \( rs \) at time interval \( t \).

The propagation of traffic flow on the evacuation network is embodied in the road sections and intersections. The traffic propagation rule at intersections can be reflected through different constraints of the first and last cells of the link [19]: the first cell may be an ordinary cell or a merging cell [9], while the end cell may be an ordinary cell or a diverging cell with a fixed signal phase.

\[
f = \min \left\{ V_k, Q, W \left(k_{\text{jam}} - k\right) \right\}
\]  

(11)

where \( f \) is traffic volume, veh/s; \( k \) is traffic flow density, veh/m; \( k_{\text{jam}} \) is jam density of traffic flow, veh/m; \( Q \) is traffic capacity, veh/s; \( V \) is free-flow speed, m/s; \( W \) is backward speed of traffic flow, m/s.

### 3. Solution Method

#### 3.1. Upper-level

Aimed at the three-phases setting of CI with One-way for one direction, vehicles on approach inside of \( EZ \) are allowed to go through in the first phase and the green light duration

\[
g_1(k) = C(k) - r(k) - Y(k)
\]  

(12)

Where,

\[
r(k) = g_2(k) + g_3(k) + g_4(k) + g_5(k) \geq 2 \left[ \min g(k) + Y(k) \right]
\]  

(13)

that is to say,
\[ \text{max } g_1(k) = C(k) - 2 \cdot \min g(k) - 3 \cdot Y(k) \]  

(14)

Aimed at the three-phases setting of CI with One-way for two directions, vehicles on approaches inside of EZ are allowed to go through in the first phase and the third phase respectively, and the sum of the two green light durations

\[ g_1(k) + g_3(k) = C(k) - g_2(k) - 3 \cdot Y(k) \]  

(15)

And

\[ \text{max} \left[ g_1(k) + g_3(k) \right] = C(k) - \min g(k) - 3 \cdot Y(k) \]  

(16)

Use the branch and bound method to acquire values of \( g_1(k) \) and \( g_3(k) \) which satisfy the condition (16) and make the delays of two approaches inside of the evacuation zone minimal.

3.2. Lower-level: MSA

Step 0: Initialization. Set \( n=1 \). Record the initial traffic volume of route \( K \) between OD pair \( rs \) during time interval \([t, t+1]\),

\[ f^{(i)} = (f^{(i)}_K(t), K \in K_{rs}, t \in T) \]  

(17)

Step 1: The number of iterations is \( n \). Update the traffic volume of route \( K \) between OD pair \( rs \) loading at the source \( r \) at time interval \( t \),

\[ f^{(n)}_K(t) = f^{(n-1)}_K(t) + \left( \frac{1}{n-1} \right) (\gamma^{(n-1)}_K(t) - f^{(n-1)}_K(t)) \]  

(18)

Step 2: Update the impedance of route \( K \) at time interval \( t \), \( \gamma^{(n)}_K(t) \) [19].

Step 3: Calculate the auxiliary traffic volume of route \( K \) between OD pair \( rs \) loading at the source \( r \) at time interval \( t \), \( \gamma^{(n)}_K(t) \) based on User Equilibrium.

Step 4: If convergence is attained, stop, \( f^{(n)}_K(t) = f^{(n)}_K(t) \); If not, set \( n = n+1 \) and go to step 1.

Convergence criterion:

\[ \sqrt{\sum_{n=1}^{N} \sum_{rs} \sum_{K \in K_{rs}} \sum_{t=0}^{T} \left[ f^{(n)}_K(t) - \gamma^{(n)}_K(t) \right]^2} \leq \sum_{rs} \xi(K_{rs} - 1), \xi \leq 0.2 \]  

(19)

4. Case Study

Based on the survey of parking lot, the urban street network around Nanjing Olympic Sports Center, we apply the proposed method and above-mentioned algorithm to obtain the optimization plan of signal timing of CIs in an incident scenario to verify the validity of the proposed method.
4.1. Scenario Assumption

The incident was assumed to occur at intersection 1 during a large-scale event which was held in Nanjing Olympic Sports Center. According to the usage data supplied by Traffic Administration Bureau, the Olympic Center had an underground parking lot which can park 3000 vehicles. All of these parking vehicles had to evacuate away from the EZ. The traffic network and the specific parameters of road infrastructures around the Olympic Sports Center are shown in Figure 2 and Table 1. The circles represent intersections, and the gray circle represent starting point.

![Fig. 2. Street network.](image)

Table 1. Basic information of the street section.

<table>
<thead>
<tr>
<th>Street</th>
<th>Length (m)</th>
<th>Amount of Lanes</th>
<th>Street</th>
<th>Length (m)</th>
<th>Amount of Lanes</th>
<th>Street</th>
<th>Length (m)</th>
<th>Amount of Lanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>ba</td>
<td>300</td>
<td>10</td>
<td>da</td>
<td>600</td>
<td>10</td>
<td>ea</td>
<td>450</td>
<td>10</td>
</tr>
<tr>
<td>ga</td>
<td>1050</td>
<td>11</td>
<td>ia</td>
<td>1050</td>
<td>8</td>
<td>ka</td>
<td>1050</td>
<td>6</td>
</tr>
<tr>
<td>ma</td>
<td>300</td>
<td>8</td>
<td>pa</td>
<td>600</td>
<td>8</td>
<td>xa</td>
<td>900</td>
<td>11</td>
</tr>
<tr>
<td>ya</td>
<td>900</td>
<td>8</td>
<td>tc</td>
<td>1050</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Considering the driver behavior characteristics in an evacuation situation and based on the exiting studies, we set the values of all the above parameters as Free-flow speed is 54km/h (i.e., 15m/s); Road capacity is 2160veh/h/lane; Carrying capacity of a cell is 15veh/lane; Length of time interval is 5s; Jam density is 0.2veh/m; Backward propagation speed is 6m/s.

4.2. Calculations

- Traffic volume of each road section

After multiple iterations, the traffic volume of each road section with consideration of delays at CIs is acquired based on CTM and UE. The distribution outcomes are as shown in Table 2.

Table 2. Traffic volume of each road section.

<table>
<thead>
<tr>
<th>Street</th>
<th>Traffic volume (veh)</th>
<th>Street</th>
<th>Traffic volume (veh)</th>
<th>Street</th>
<th>Traffic volume (veh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ba</td>
<td>1619</td>
<td>da</td>
<td>927</td>
<td>ea</td>
<td>153</td>
</tr>
<tr>
<td>ga</td>
<td>1381</td>
<td>ia</td>
<td>692</td>
<td>ka</td>
<td>774</td>
</tr>
<tr>
<td>ma</td>
<td>517</td>
<td>pa</td>
<td>952</td>
<td>xa</td>
<td>864</td>
</tr>
<tr>
<td>ya</td>
<td>257</td>
<td>tc</td>
<td>153</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Arrival distributions of evacuated vehicles at CIs

As shown in Figure 2, the approaches of a CI inside of EZ could be more than one such as intersection 4; therefore, take one approach of the CI as an object to study the evacuated vehicles arrival. Based on the dynamic evacuation model, the distributions of arrival of evacuated vehicles at CI approaches can be shown as in Figure 3.

Fig. 3. The arrival of evacuated vehicles at CIs.

• Delays at CIs

Delay values of each approach of CIs inside of EZ are shown in Table 3.

Table 3. Delays at CIs.

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Approach</th>
<th>Delay at Approach (s)</th>
<th>Cycle length (s)</th>
<th>Green time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>xa</td>
<td>47.045</td>
<td>68</td>
<td>55</td>
</tr>
<tr>
<td>8</td>
<td>pa</td>
<td>42.448</td>
<td>97</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>ka</td>
<td>42.876</td>
<td>97</td>
<td>37</td>
</tr>
<tr>
<td>9</td>
<td>ya</td>
<td>38.377</td>
<td>71</td>
<td>42</td>
</tr>
<tr>
<td>11</td>
<td>tc</td>
<td>0</td>
<td>60</td>
<td>47</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

Aimed to make the evacuated vehicles leave the evacuation zone as quickly as possible, a bi-level programming model is proposed to determine the signal timing at critical intersections (CI) by means of dynamic simulation. The upper-level designs the optimal signal timing plan based on known traffic distribution, while the lower-level simulates the dynamic propagation process of evacuated vehicles in the evacuation zone.

For the bi-level model, it is supposed that the evacuation zone which is influenced by the emergency is determinate; accordingly, the evacuation demand and loading time in the evacuation zone are known parameters. In the upper-level, we establish delay functions at approaches with different saturations, and with the objective of minimizing delays of critical intersection approaches inside of the evacuation zone, thus we can acquire cycle length and green light durations at CI; The lower-level programming establishes the dynamic evacuation route choice model to simulate the evolution process of the traffic flow on the network and the route choice in an evacuation situation under determinate road network, signal design and OD demand. Based on the CTM model for evacuation and the principle of User Equilibrium, we can acquire the traffic distributions.

To demonstrate the effectiveness of the proposed method, we apply the proposed method and iterative algorithm based on the survey of parking lot, the urban street network around Nanjing Olympic Sports Center, to obtain the traffic volume of each road section, arrival distributions of evacuated vehicles at CIs, and optimal signal timing plan
of each CI. It can be seen that the proposed method can effectively solve the integrated problems of signal timing optimization and dynamic traffic simulation.

There are some directions for further research in this paper. On one hand, in this paper, the OD demand table is known a priori, then the estimation of OD demand should be constructed as part of the modeling effort; On the other hand, during evacuation, road users are not bound to aware of road and transportation conditions clearly or choosing their routes according to the User Equilibrium, therefore, taking stochastic character into route choice behavior simulation is considerable.

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References