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On the dynamic balancing of a planetary moving rotor using a passive pendulum-type device

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Abstract

The possibility of using self-balancing devices to reduce the vibration amplitudes in planetary moving rotors is investigated analytically using the averaging method for a partially strongly damped system. Self-balancing was shown to be effective for a planetary moving rotor in the overcritical speed domain. However, the centrifugal forces caused small vibrations of the pendulum balancers, which increased with the increase of the velocity of the planetary motion. The analytic results match the numeric simulations very well as long as the velocity of the planetary motion is sufficiently small. To avoid high amplitudes while passing through resonance, a special design of the switchable pendulums that are controlled by centrifugal force is suggested. This design utilizes the only stable stationary orientation of the pendulums in the undercritical domain to position them in an appropriate way before coming into resonance. The results obtained provide new possibilities for the design of self-balancing devices for high precision applications.

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Keywords: Singular perturbation; averaging; non-linear resonance; Sommerfeld effect

1. Introduction

The effect of self-balancing devices providing the perfect balancing of rigid rotors in the overcritical speed range is well known and is investigated for rotors with fixed bearings [1,2]. There are two classical types of such systems. In the first type, a hollow disc is placed on the axis of the rotor. The disk has a circular channel where balls can move

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[3,4,5]. In the second type, two pendulums are suspended on the rotor [6]. The balls or the pendulums respond to the small oscillations of the rotor and adjust themselves in the overcritical domain of the rotation speeds so that the unbalance of the rotor is completely compensated. Such systems are primarily used in washing machines [7], electrical hand tools and CD and DVD drives [8]. All of these devices have stationary axes of rotation. However, in some technical devices, rotors perform complex motions. An example of such a system is a computed tomography scanner. One of the main components of a CT-Scanner is the X-ray tube. There is a cathode in the housing that emits electrons towards an anode. As soon as the electrons reach the anode, they decelerate and generate X-rays. The energy of the electrons is so high that they heat the target point of the anode to a temperature above its melting point. Therefore, the anode rotates very fast to achieve sufficient cooling. At the same time, the X-ray tube itself rotates rather slowly around the patient's body. It is very important for a CT scanner to maintain the minimal level of vibration that is possible to obtain a good image quality.

The objectives of this work are the following:

- to investigate how and to what extent self-balancing devices can be used for reducing vibrations in a planetary moving rotor in the overcritical speed range;
- to suggest a mechanical solution enabling the reduction of rotor vibrations while passing though resonance without decreasing the performance of the balancers in the overcritical domain.

The paper is structured as follows. The model of the self-balancing system for a planetary moving rotor is introduced in section 2. The stationary solutions of the system are first investigated analytically and are then compared to the numerical simulations in section 3. Then, passage through resonance is discussed and an improved design of the pendulum balancers is suggested in section 4. Finally, the main results are summarized in section 5.

2. The simplest model of a planetary moving rotor with a pendulum-type self-balancing device

Consider the system in Figure 1 that represents the simplest model of a planetary moving rotor mounted on a rigid carrier.

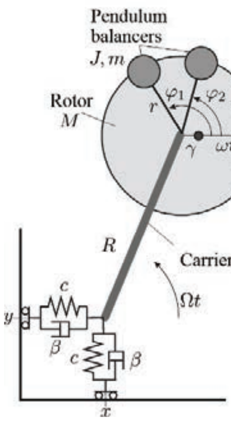

Fig. 1. Unbalanced rotor with two pendulums mounted on the rigid carrier.

A rotor of mass M is fixed to the end of the carrier, which has length R. The other end of the carrier is elastically suspended with two radial spring-dampers of a certain stiffness c and damping β. The carrier rotates around its point of suspension with a constant velocity Ω. At the same time, the rotor rotates around its symmetry axis with a given velocity ω. Its center of mass has an offset γ relative to the rotation axis. Two pendulum balancers of mass m, moment of inertia J and length r are placed on the rotation axis of the rotor.

2.1. Equations of motion and the non-dimensional parameters

The equations of motion of the whole system (1) can be split into two groups. The first two equations describe the radial vibrations of the carrier system, the last two equations describe the phases of the pendulums:

$$
\begin{aligned}
&(M+2m)\ddot{x}+\beta\dot{x}+cx= \\
&\qquad (M+2m)R\Omega^2\cos\Omega t+M\gamma\omega^2\cos\omega t+mr\left(\dot{\varphi}_1^{\,2}\cos\varphi_1+\ddot{\varphi}_1\sin\varphi_1+\dot{\varphi}_2^{\,2}\cos\varphi_2+\ddot{\varphi}_2\sin\varphi_2\right), \\
&(M+2m)\ddot{y}+\beta\dot{y}+cy= \\
&\qquad (M+2m)R\Omega^2\sin\Omega t+M\gamma\omega^2\sin\omega t+mr\left(\dot{\varphi}_1^{\,2}\sin\varphi_1-\ddot{\varphi}_1\cos\varphi_1+\dot{\varphi}_2^{\,2}\sin\varphi_2-\ddot{\varphi}_2\cos\varphi_2\right), \\
&\left(J+mr^2\right)\ddot{\varphi}_1+\beta_\varphi\left(\dot{\varphi}_1-\omega\right)=mrR\Omega^2\sin\left(\Omega t-\varphi_1\right)+mr\left(\ddot{x}\sin\varphi_1-\ddot{y}\cos\varphi_1\right), \\
&\left(J+mr^2\right)\ddot{\varphi}_2+\beta_\varphi\left(\dot{\varphi}_2-\omega\right)=mrR\Omega^2\sin\left(\Omega t-\varphi_2\right)+mr\left(\ddot{x}\sin\varphi_2-\ddot{y}\cos\varphi_2\right).
\end{aligned}
\tag{1}
$$

The first step is to convert the system into a non-dimensional form. The following non-dimensional parameters and variables can be introduced:

- $\tau=\omega t$ is the new fast time;
- $\xi=x/r$ and $\theta=y/r$ describe the radial vibrations of the carrier system;
- $\delta=\Omega/\omega$ is the relation between the rotational speed of the rotor itself and the speed of the planetary motion;
- $\varepsilon=\gamma/r$ describes the measure of eccentricity; and
- $\mu=m/(M+2m)$ characterizes the relation between the mass of the pendulum and a mass of the whole system.

μ, ε and δ^2 are considered to be small. The other non-dimensional parameters are $p=R/r$, $q=\left(J/mr^2+1\right)^{-1}$, $\eta^2=\dfrac{c}{(M+2m)\omega^2}$, $\sigma=\dfrac{\beta}{(M+2m)\omega}$ and $b=\dfrac{\beta_\varphi}{\left(J+mr^2\right)\omega}$. Form this point forward, it is assumed that the damping both σ in the oscillation subsystem and b in the rotational degrees of freedom for the pendulums is not small.

The non-dimensional equations of motion are:

$$
\begin{aligned}
&\xi''+\sigma\xi'+\eta^2\xi=p\delta^2\cos\delta\tau+(1-2\mu)\varepsilon\cos\tau+\mu\left(\varphi_1'^{\,2}\cos\varphi_1+\varphi_1''\sin\varphi_1+\varphi_2'^{\,2}\cos\varphi_2+\varphi_2''\sin\varphi_2\right), \\
&\theta''+\sigma\theta'+\eta^2\theta=p\delta^2\sin\delta\tau+(1-2\mu)\varepsilon\sin\tau+\mu\left(\varphi_1'^{\,2}\sin\varphi_1-\varphi_1''\cos\varphi_1+\varphi_2'^{\,2}\sin\varphi_2-\varphi_2''\cos\varphi_2\right), \\
&\varphi_1''+b\left(\varphi_1'-1\right)=q\left(p\delta^2\sin\left(\delta\tau-\varphi_1\right)+\xi''\sin\varphi_1-\theta''\cos\varphi_1\right), \\
&\varphi_2''+b\left(\varphi_2'-1\right)=q\left(p\delta^2\sin\left(\delta\tau-\varphi_2\right)+\xi''\sin\varphi_1-\theta''\cos\varphi_2\right).
\end{aligned}
\tag{2}
$$

These equations are now suitable for asymptotic analysis.

3. Steady state solutions of the planetary moving rotor with two pendulum balancers

3.1. Asymptotic analysis

The obtained equations of motion (2) can be analyzed using the averaging method for a partially strongly damped system [9]. This method says that if some strongly damped variables are weakly excited, these variables can be simply neglected, whereas first order averaging of the governing equations can be used for weakly damped variables. The solutions obtained by this operation are asymptotically close to the solutions of the original system after a short transient process at the beginning. However, to use this approach, equation (2) has to be converted to the standard form for a partially strongly damped system:

$$\begin{aligned} &x' = \varepsilon X\left(x, y, t\right), \\ &y' = -Ky + \varepsilon Y\left(x, y, t\right), K = diag\left\{k_i\right\}, \, k_i > 0, \, \varepsilon << 1. \end{aligned} \tag{3}$$

This conversion can be achieved in three steps. First, the equations for the carrier system can be transformed as follows. Assuming that the rotation of the pendulums is almost uniform, the equations for the carrier system can be significantly simplified to

$$\begin{aligned} &\varphi_i = \tau + \psi_i; \, \varphi_i' = 1 + O\left(\varepsilon\right), \; i = 1, 2, \\ &\xi'' + \sigma\xi' + \eta^2\xi = p\delta^2 \cos\delta\tau + \varepsilon\cos\tau + \mu\left(\cos\varphi_1 + \cos\varphi_2\right), \\ &\theta'' + \sigma\theta' + \eta^2\theta = p\delta^2 \sin\delta\tau + \varepsilon\sin\tau + \mu\left(\sin\varphi_1 + \sin\varphi_2\right). \end{aligned} \tag{4}$$

The corresponding pure forced solution is

$$\begin{aligned} &\xi_p = p\delta^2\left(A_\delta \sin\delta\tau + B_\delta \cos\delta\tau\right) + \varepsilon\left(A_0 \sin\tau + B_0 \cos\tau\right) + \mu\sum_{i=1}^{2}\left(A_i \sin\left(\tau + \psi_i\right) + B_i \cos\left(\tau + \psi_i\right)\right), \\ &\theta_p = p\delta^2\left(C_\delta \sin\delta\tau + D_\delta \cos\delta\tau\right) + \varepsilon\left(C_0 \sin\tau + D_0 \cos\tau\right) + \mu\sum_{i=1}^{2}\left(C_i \sin\left(\tau + \psi_i\right) + D_i \cos\left(\tau + \psi_i\right)\right). \end{aligned} \tag{5}$$

Here, the explicit expressions for the coefficients A, B, C, D can be easily obtained as the forced solutions of the corresponding 1 DOF oscillator. Note that all of these coefficients are of the order magnitude of the small parameter. Expressions (5) can be used to transform equations (4) into the desired form (3):

$$\begin{aligned} &\xi = \xi_p + \rho_\xi \sin\left(\eta_1\tau\right) + \nu_\xi \cos\left(\eta_1\tau\right) \\ &\theta = \theta_p + \rho_\theta \sin\left(\eta_1\tau\right) + \nu_\theta \cos\left(\eta_1\tau\right) \end{aligned}; \; \eta_1 = \sqrt{\eta^2 - \sigma^2/4} \tag{6}$$

The equations for the new variables $\rho_\xi, \nu_\xi, \rho_\theta, \nu_\theta$ have the following form (3):

$$\begin{aligned} &\rho_\xi' = -\frac{\sigma}{2}\rho_\xi - \frac{\varepsilon}{\eta_1}\Phi_\xi \sin\left(\eta_1\tau\right), \;\; \nu_\xi' = -\frac{\sigma}{2}\nu_\xi + \frac{\varepsilon}{\eta_1}\Phi_\xi \cos\left(\eta_1\tau\right), \\ &\rho_\theta' = -\frac{\sigma}{2}\rho_\theta - \frac{\varepsilon}{\eta_1}\Phi_\theta \sin\left(\eta_1\tau\right), \;\; \nu_\theta' = -\frac{\sigma}{2}\nu_\theta + \frac{\varepsilon}{\eta_1}\Phi_\theta \cos\left(\eta_1\tau\right). \end{aligned} \tag{7}$$

The explicit expressions for Φ_ξ and Φ_θ are cumbersome and are omitted here because the only fact that is important for further analysis is that they are limited. This means that, to a first order approximation, the variables (7) can be set to zero:

$$\rho_\xi \approx \nu_\xi \approx \rho_\theta \approx \nu_\theta \approx 0; \quad \xi \approx \xi_p; \theta \approx \theta_p. \tag{8}$$

Now, the equations for the pendulum balancers can be simplified and rewritten as

$$\psi_i'' + b\psi_i' = qp\delta^2 \sin\left(\left(\delta - 1\right)\tau - \psi_i\right) + f_i, \quad f_i\left(\psi_1, \psi_2, \tau\right) = q\left(\xi_p'' \sin\left(\tau + \psi_i\right) - \theta_p'' \cos\left(\tau + \psi_i\right)\right), \; i = 1,2 \tag{9}$$

The strongly and the weakly damped variables in (9) can be separated by the following change of variables:

$$\begin{aligned} &\psi_i' + b\psi_i = u_i; \; \psi_i = \alpha_i + \frac{u_i}{b}, \\ &\alpha_i' + b\alpha_i = -\frac{1}{b} f_i - \frac{1}{b} qp\delta^2 \sin\left(\left(\delta - 1\right)\tau - \alpha_i - \frac{u_i}{b}\right), \\ &u_i' = f_i + qp\delta^2 \sin\left(\left(\delta - 1\right)\tau - \alpha_i - \frac{u_i}{b}\right). \end{aligned} \tag{10}$$

The variables α_i are also strongly damped and thus can be neglected (to a first order approximation). Therefore, only the equations for u_i remain, which, after neglecting the higher order terms, are as follows:

$$\begin{aligned} &u_i' = -f_i - qp\delta^2 \sin\left(\tau_1 - \frac{u_i}{b}\right), \; i = 1,2, \\ &\tau_1 = \left(\delta - 1\right)\tau. \end{aligned} \tag{11}$$

These equations are in the standard form for averaging because $f_i = O\left(\varepsilon\right)$. Thus the first order approximation can be considered:

$$\begin{aligned} &\bar{u}_1' = P_1\left(\bar{u}_1, \bar{u}_2\right); \; \bar{u}_2' = P_2\left(\bar{u}_1, \bar{u}_2\right), \\ &P_i\left(\bar{u}_1, \bar{u}_2\right) = \frac{1}{2\pi} \int_0^{2\pi} \left(-f_i - qp\delta^2 \sin\left(\tau_1 - \frac{\bar{u}_i}{b}\right)\right) d\tau_1. \end{aligned} \tag{12}$$

Therefore, the dynamics of the system that has 4 DOF, and thus is described by the 8th order system, can be reduced to two first order differential equations.

3.2. Steady state configurations of the system and their stability

Based on the averaged equations (12), the stationary solutions corresponding to the steady state configurations of the whole system can be investigated. There are four groups of stationary solutions.

$$\bar{u}_1^{*(a)} = -b\arctan\left(\sqrt{4\mu^2/\varepsilon^2 - 1}\right),\ \bar{u}_2^{*(a)} = b\arctan\left(\sqrt{4\mu^2/\varepsilon^2 - 1}\right),$$
$$\bar{u}_1^{*(b)} = \bar{u}_2^{*(b)} = b\arctan\left(\frac{2\mu\sigma\left(1-\eta^2\right)+\sigma H}{-2\mu\sigma+\left(\eta^2-1\right)H}\right),\ H = \sqrt{\varepsilon^2\left(\left(1-\eta^2\right)^2+\sigma^2\right)-4\mu^2\sigma^2}, \tag{13}$$

$$\bar{u}_1^{*(c)} = b\arctan\left(\frac{\sigma}{1-\eta^2}\right),\ \bar{u}_2^{*(c)} = \pi + \bar{u}_1^{*(c)},\ \bar{u}_1^{*(d)} = \bar{u}_2^{*(d)} = b\arctan\left(\frac{2\mu\sigma\left(1-\eta^2\right)-\sigma H}{-2\mu\sigma-\left(\eta^2-1\right)H}\right). \tag{14}$$

An analysis of the stability of these solutions can be performed based on the averaged equations (12). A sufficient condition for asymptotic stability is for the real parts of the eigenvalues of the Jacobian matrix to be negative, i.e.,

$$\begin{vmatrix} \frac{\partial P_1}{\partial \bar{u}_1} - \lambda & \frac{\partial P_1}{\partial \bar{u}_2} \\ \frac{\partial P_2}{\partial \bar{u}_1} & \frac{\partial P_2}{\partial \bar{u}_2} - \lambda \end{vmatrix} = 0,\ \mathrm{Re}\{\lambda_i\} < 0. \tag{15}$$

Figure 2 shows the possible stationary geometrical configurations of the rotor with the self-balancing device. Only the first configuration (a), which is stable in the overcritical domain, provides a complete compensation for the unbalance. The second configuration (b) is stable in the undercritical domain. This configuration provides the maximal possible increase of the unbalance. The third and the fourth configurations (c) and (d) are always unstable.

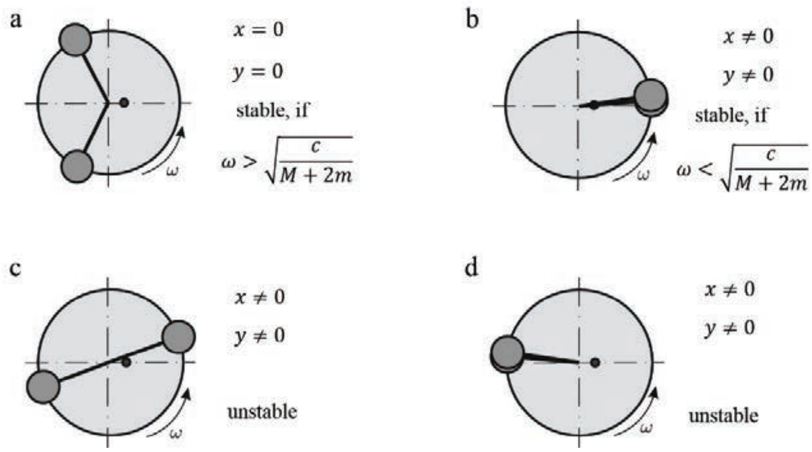

Fig. 2. The steady state configurations of the system: (a) the balanced configuration is stable in the overcritical speed range; (b) the exciting configuration is stable in the undercritical speed range; (c) in the unstable configuration, the pendulums compensate for each other; (d) the unstable configuration in which the pendulums overcompensate for the unbalance.

These configurations correspond to the solutions of the system with a stationary rotational axis[1]. However, these configurations do not demonstrate how the centrifugal forces caused by the planetary motion affect the behavior of the system, which is one of the principal questions of this investigation. Therefore, the small oscillations near the stationary solution have to be analyzed.

3.3. Improved first order approximation and the effect of the planetary motion

The improved first order approximation to (12) can be obtained directly as

$$\tilde{u}_i = \int\left(u_i' - \bar{u}_i'\right) d\tau_1 = \int\left(-f_i - qp\delta^2 \sin\left(\tau_1 - \frac{\bar{u}_i}{b}\right) - P_i\left(\bar{u}_1, \bar{u}_2\right)\right) d\tau_1. \tag{16}$$

However, this approximation does not describe the small oscillations of the pendulums phases completely. The residual oscillations of the strongly damped variables α_i (10) have to be taken into account as well:

$$\tilde{\alpha}_i = -\frac{1}{b^2} f_i\left(\bar{u}_1, \bar{u}_2\right) - \frac{1}{b^2} qp\delta^2 \sin\left(\left(\delta - 1\right)\tau - \frac{\bar{u}_i}{b}\right). \tag{17}$$

By gathering all of the oscillating components, the final expressions for the phases of the pendulums can be obtained:

$$\varphi_i = \tau + \frac{\bar{u}_i^*}{b} + \frac{\tilde{u}_i}{b} + \tilde{\alpha}_i. \tag{18}$$

These results are compared with the results of the direct numerical simulations of the original system (2) in Figures 3 and 4.

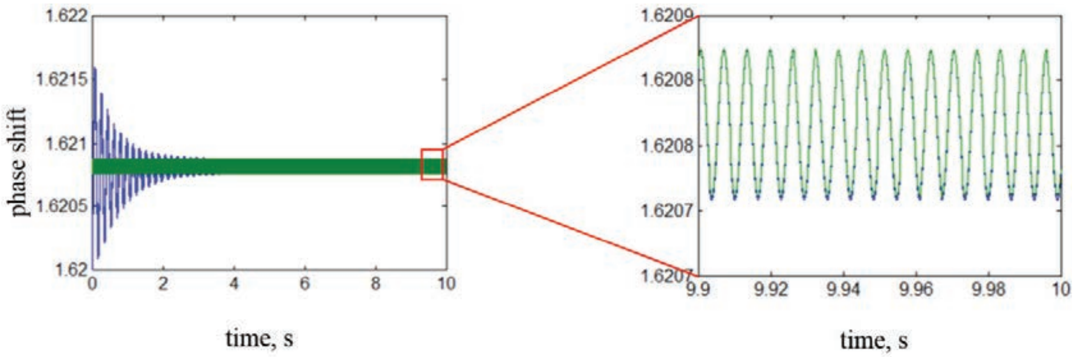

Fig. 3. Comparison between the asymptotic approximation (green line) and the results of the direct numerical simulations (blue line).

Notice that the initial discrepancy between the analytical and numerical solutions (Figure 3, left-hand side) disappears after a short transient process at the beginning. The steady state solutions are so close to each other that there is no visible difference between them (Figure 3, right-hand side).

The influence of the planetary motion parameters on the oscillation amplitude of the pendulums is shown in Figure 4. The amplitude increases linearly with the length of the carrier and quadratically with the velocity of the planetary motion. The analytic results match very well with the numeric simulations as long as the velocity of the planetary motion is sufficiently small.

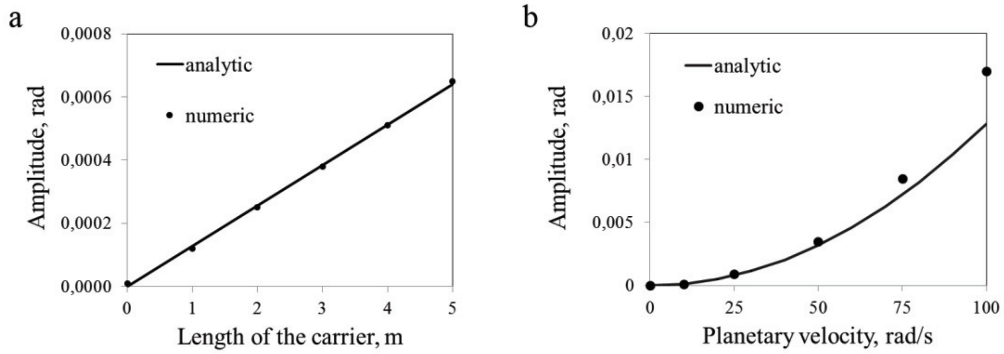

Fig. 4. Comparison between the asymptotic approximation (solid line) and the results of the direct numerical simulations (dots).

Note that the simulations have been performed for the angular velocity of the rotor $\omega = 1000$ rad/s and for the velocities of the planetary motion $\Omega = 0\ldots100$ rad/s.

4. Passage through the resonance of the system with the pendulum balancers

Due to the disadvantageous configuration of the system in the undercritical speed range (Figure 2 (b)), the self-balancing system can cause problems during passage through the resonance of the carrier. The system under consideration is still the same. However, instead of a constant rotational speed of the rotor, the engine torque is introduced. To describe the engine, a static motor characteristic is used [10], giving

$$\begin{aligned}
&(M+2m)\ddot{x}+\beta\dot{x}+cx=(M+2m)R\Omega^2\cos\Omega t+\\
&\qquad M\gamma\left(\dot{\phi}^2\cos\phi+\ddot{\phi}\sin\phi\right)+mr\left(\dot{\varphi}_1{}^2\cos\varphi_1+\ddot{\varphi}_1\sin\varphi_1+\dot{\varphi}_2{}^2\cos\varphi_2+\ddot{\varphi}_2\sin\varphi_2\right),\\
&(M+2m)\ddot{y}+\beta\dot{y}+cy=(M+2m)R\Omega^2\sin\Omega t+\\
&\qquad M\gamma\left(\dot{\phi}^2\sin\phi-\ddot{\phi}\cos\phi\right)+mr\left(\dot{\varphi}_1{}^2\sin\varphi_1-\ddot{\varphi}_1\cos\varphi_1+\dot{\varphi}_2{}^2\sin\varphi_2-\ddot{\varphi}_2\cos\varphi_2\right),\\
&\left(J+mr^2\right)\ddot{\varphi}_1+\beta_\varphi\left(\dot{\varphi}_1-\dot{\phi}\right)=mrR\Omega^2\sin\left(\Omega t-\varphi_1\right)+mr\left(\ddot{x}\sin\varphi_1-\ddot{y}\cos\varphi_1\right),\\
&\left(J+mr^2\right)\ddot{\varphi}_2+\beta_\varphi\left(\dot{\varphi}_2-\dot{\phi}\right)=mrR\Omega^2\sin\left(\Omega t-\varphi_2\right)+mr\left(\ddot{x}\sin\varphi_2-\ddot{y}\cos\varphi_2\right),\\
&\left(J_{rot}+M\gamma^2\right)\ddot{\phi}+\beta_\varphi\left(2\dot{\phi}-\dot{\varphi}_1-\dot{\varphi}_2\right)=T_m+MR\gamma\Omega^2\sin\left(\Omega t-\phi\right)+M\gamma\left(\ddot{x}\sin\phi-\ddot{y}\cos\phi\right),\\
&T_m=L\left(\dot{\phi}\right)-R\left(\dot{\phi}\right)=U-V\dot{\phi}.
\end{aligned} \tag{19}$$

Indeed, the speeding-up simulation shows that the resonance amplitude of the system with the self-balancing device is much greater than without it (cf. Figure 5 (a)). To avoid a high amplitude at resonance, switchable pendulums can be used. The switchable pendulum consists of a rigid frame and a mass within it. The mass is attached to the preloaded spring in such a way that the pendulum itself is perfectly balanced in the undercritical speed domain (Figure 5 (b)). The amplitude curve of the system with the pendulum balancers follows the curve of the system without the self-balancing device. In the overcritical speed range, the mass would be pushed outwards by the centrifugal force, which provides the unbalance (Figure 5 (c)). Now, the unbalance of the rotor can be compensated. However, a quick change of the system parameters causes a short transient process with a rather high vibration amplitude (Figure 6 (d)).

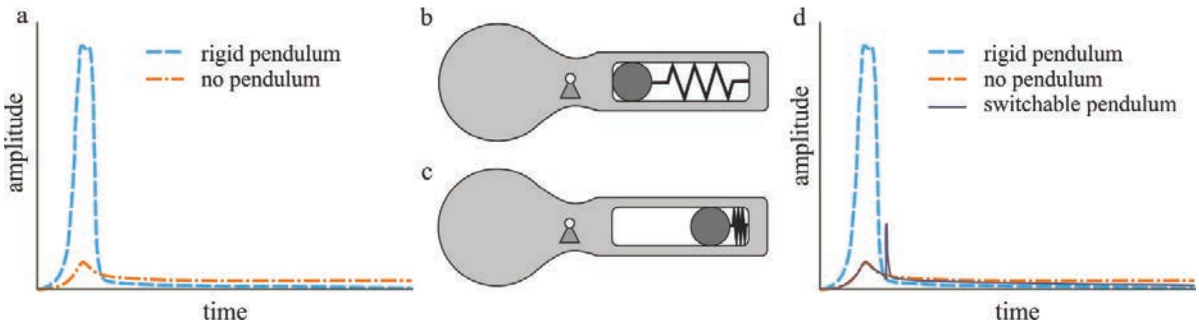

Fig. 5. (a) Passage through the resonance of the system with and without balancing pendulums; (b) the switchable pendulum in the undercritical speed domain; (c) the switchable pendulum in the overcritical speed domain; (d) passage through the resonance of the system with the switchable pendulums compared with the previous two curves.

However, the peak amplitude of this transient process strongly depends on the orientation of the balancing pendulums to each other just before coming into the resonance zone (see Figure 6). The orientation in the opposite directions (Figure 6 (b)) appears to be advantageous.

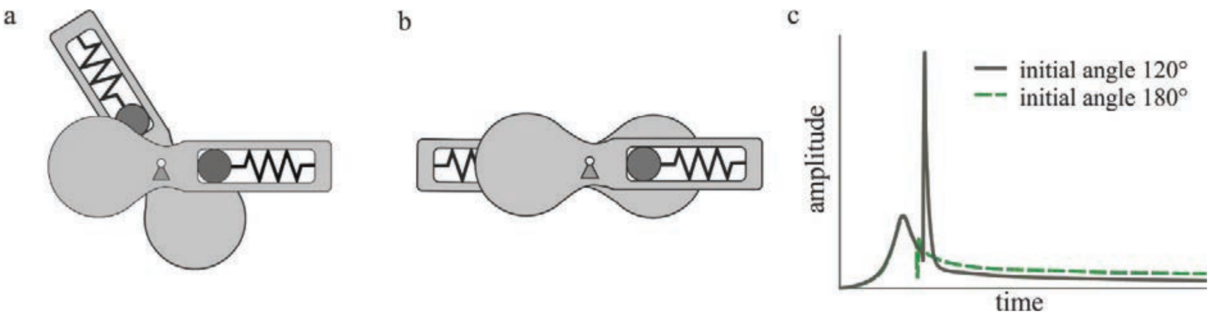

Fig. 6. (a) Balancing pendulums with a relative angle of 120°; (b) balancing pendulums with a relative angle of 180°; (c) passage through resonance of the system with the different orientations of the switchable pendulums.

In this configuration, the pendulums compensate each other, leading to a significant reduction of the peak's amplitude. The question is the following: how can one ensure that this configuration is achieved just before the pendulums are activated? The idea is to use the natural dynamics of the system in the undercritical domain. As previously shown (Figure 2 (b)), the pendulums intensify the unbalance of the rotor in the undercritical domain and reach the same phase position. Then, if one pendulum is designed with a small permanent unbalance in the direction of the main switchable unbalance and if the other pendulum is designed with the small permanent unbalance in the opposite direction, the primary pendulums will reach the paraphrase orientation while accelerating in the undercritical domain and compensate each other in the moment of activation. The small unbalance increases the amplitude at resonance slightly, but helps avoid a much greater amplitude due to the activation of the main pendulums (Figure 7).

The numerical simulations show that this behavior is robust with respect to the velocity of the planetary motion.

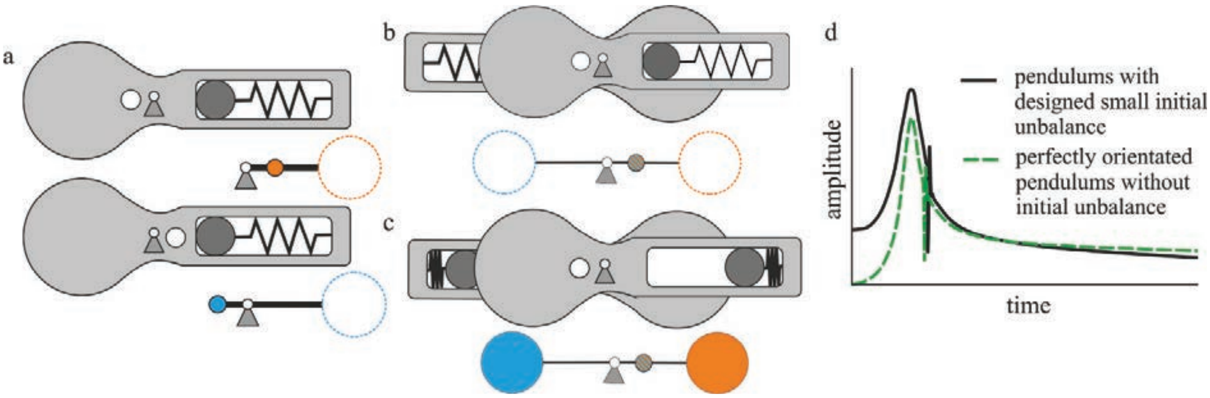

Fig. 7. (a) Balancing pendulums with the designed small permanent unbalance (the white circular holes in the picture); (b) balancing pendulums achieve a stable orientation in the undercritical domain (maximizing the unbalance of the rotor); (c) in the achieved orientation, the main unbalances switch on in the opposite directions; (d) passage through the resonance of the system with the switchable pendulums with and without the designed permanent unbalance.

5. Conclusions

The model of a self-balancing system for a planetary moving rotor has been investigated analytically using the averaging method for a partially strongly damped system. Self-balancing has been shown to be effective for a planetary moving rotor in the overcritical speed domain. However, the centrifugal forces cause small vibrations of the balancers that increase with an increase in the velocity of the planetary motion. The analytic results agree very well with the numeric simulations as long as the velocity of the planetary motion is sufficiently small. To avoid high amplitudes while passing through resonance, a special design of the pendulums has been suggested.

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