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The ionization of H, He or Ne atoms using neutrinos or antineutrinos at keV energies

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Abstract

We calculate the ionization cross sections for H, He or Ne atoms using ν_e and $\bar{\nu}_e$ scattering at keV energies. Such cross sections are useful for e.g., $\bar{\nu}_e$-oscillation experiments using a tritium source. Using realistic atomic wave functions, we find that for $E_\nu \lesssim 10$ keV the atomic ionization cross sections, normalized to one electron per unit volume, are smaller than the corresponding free electron ones, and that they approach it from below as energies of 20 keV are reached.

The scattering of electron-type neutrinos or antineutrinos from electrons gives a small cross section which has been studied in refined experiments at rather high energies. As the energy decreases though, these cross sections become smaller, making their measurement increasingly difficult. Nevertheless, low energy neutrino cross sections have been measured in reactor- and solar-neutrino experiments. Very low energy reactor experiments started with searches for neutral currents where a threshold of 1.0 to 2.0 MeV was set [1], and developed into numerous oscillation experiments [2]. The solar neutrino experiments use a calculated flux from the sun and look at reactions with a low energy threshold of about 0.2 MeV [3,4].

Here we wish to emphasise that ν_e or $\bar{\nu}_e$ with energies of keV, may allow to study in terrestrial experiments oscillations that up to now have only been observed in neutrinos coming from the Sun. In addition, keV-energy neutrinos may be useful for improving the present constraints on, e.g., the neutrino anomalous magnetic moment [5].

In a realistic experiment of this kind, we need to produce neutrinos (or antineutrinos) at a source, then let them travel a distance comparable to their oscillation length, so that a sufficient decrease of the original flux becomes observable. Starting from the oscillation length

$$\frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{L}{\text{km}} \frac{\Delta m^2}{\text{eV}^2} \frac{\text{GeV}}{E_\nu} = \frac{\pi}{2},$$

and assuming that the presently favoured LMA solar neutrino solution with $\Delta m^2 \simeq 4.5 \times 10^{-5}$ eV^2, is realized in nature [6], we are led to expect a ν_e oscillation length $L \simeq 27.5$ m and 275 m for neutrino energies of 1 and 10 keV, respectively.

The situation may become even more interesting if $\Delta m^2 \simeq 5 \times 10^{-4}$ eV^2, which is still consistent with present measurements [6]. In this case $L = 2.5$ m and $L = 25$ m for neutrino energies of 1 and

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10 keV, respectively. A requirement for observing such oscillations, is the study of the cross sections

$$\nu_e(\bar{\nu}_e)\, e^- \to \nu_e(\bar{\nu}_e)\, e^-, \tag{1}$$

at very low energies, where the binding of the electrons to atoms cannot be ignored.

As an example of such an experiment, one can consider the case where a source of tritium provides a beam of antineutrinos through the decay $^3\mathrm{H} \to {}^3\mathrm{He}\, e^- \bar{\nu}_e$. The surrounding or nearby volume is filled with a gas (like He or Ne) at atmospheric pressure. As the antineutrinos (whose energy spectrum is peaked at about 15 keV) travel through this medium, they will scatter on the atomic electrons, ionizing the atoms [7]. The produced electrons will then be detected by counters located on the walls of the surrounding volume.

Since the decay of tritium is known, the only other limitation is the ability of the experiment to measure the scattering cross section of $\bar{\nu}_e$'s on atomic electrons at an average energy of 15 keV. To lowest order in the Fermi coupling, the antineutrino or neutrino cross section for producing electrons [1] is given by the incoherent sum of the individual atomic electron cross sections

$$\begin{aligned} &\mathrm{d}\sigma\big(\nu_e[\bar{\nu}_e] + \mathrm{Atom} \to \nu_e[\bar{\nu}_e]\, e^- + \mathrm{Ion}\big) \\ &\quad = Z\, \mathrm{d}\sigma\big(\nu_e[\bar{\nu}_e] + e^- \to \nu_e[\bar{\nu}_e]\, e^-\big). \end{aligned} \tag{2}$$

The neutrino (antineutrino) ionization cross sections of hydrogen-like atoms have already been considered in [8], where it is stated that the ionization cross section per electron, exceeds the free electron cross section by a factor of 2 or 3 for neutrino energies of $E_\nu \sim Z\alpha mc^2$. Subsequent studies computed the electron spectra from inelastic scattering of neutrinos by atomic electrons (ionization) [9]. For $^{19}\mathrm{F}$ and $^{96}\mathrm{Mo}$ these studies found that the electron spectra differ significantly from the scattering on a free electron.

It is thus worthwhile to reconsider the neutrino scattering from atomic structures, in order to determine whether special effects exist that might justify such an enhancement. Below we present a detail derivation of the neutrino ionization cross section of H, He and Ne atoms, treating the atomic electrons non-relativistically. This is justified for light and medium-light atoms, where the average momenta of the bound electrons are small. For the neutrinos and the final electrons however, full relativistic kinematics are retained. Since for light and medium atoms, the average potential energy of the final electrons are much smaller then their kinetic energy, we ignore the Coulomb wave function correction for the final electrons. Finally, numerical applications are given and the results are discussed.

The range of the electron–neutrino interaction at very low energies is determined by the W or Z mass as

$$\lambda_W \sim \frac{1}{m_Z} \sim \frac{1}{m_W} \sim 1.5 \times 10^{-16}\ \mathrm{cm},$$

which is eight orders of magnitude smaller than the interelectron distances within an atom. Even if ν_e (or $\bar{\nu}_e$) is taken to be a plane wave which is spread over the whole target region, there can never be any interference or coherence phenomenon in an ionization process in which the state of the target is changed and the outgoing electron is looked at. Only in elastic processes, where the target remains intact, can interference phenomena appear, as, e.g., in the MSW effect [2] [10]. Thus, in an ionization process the incident neutrino interacts with only one electron at a time.

At very low energies, after integrating out the W and Z fields, the Standard Model dynamics described by the diagrams in Fig. 1 induce the local effective interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{e\nu_e} = -\frac{G_F}{\sqrt{2}}\Big[\bar{\nu}_e\gamma^\mu\frac{(1-\gamma_5)}{2}\nu_e\Big] \\ \times\big[v_e\bar{e}\gamma_\mu e - a_e\bar{e}\gamma_\mu\gamma_5 e\big], \end{aligned} \tag{3}$$

describing the ν_e and $\bar{\nu}_e$ interactions with electrons. Here, $v_e = 1 + 4s_W^2$, $a_e = 1$, and G_F is the usual Fermi coupling. This Lagrangian is used below to calculate the invariant amplitude squared $|F|^2$, summed over all initial and final electron spin-states for the process

$$\nu_e(p_1)\, e^-(p_2) \to \nu_e(p_3)\, e^-(p_4), \tag{4}$$

[1] In principle, electrons could also be produced from the scattering of keV ν_e off the nuclei of the atoms, provided that the isotopes considered are close to being unstable under beta decay. We are not interested in this case here, and we thus assume that the nuclear binding of all isotopes involved is sufficiently strong.

[2] We come back to this at the end of the Letter.

where the four-momenta are indicated in parentheses and the corresponding energies are denoted by E_j. The standard variables $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, $u=(p_1-p_4)^2$ will be used.

We first consider the case where the initial electron is free, so that $p_1^2=p_3^2=0$, $p_2^2=p_4^2=m^2$, with m being the electron mass. Summing over all initial and final electron spin states, we have

$$\begin{aligned}&\left|F\left(\nu_e e^- \to \nu_e e^-\right)\right|^2_{\text{free}}\\&\quad=2G_F^2\big\{(v_e+a_e)^2\big(s-m^2\big)^2\\&\qquad+(v_e-a_e)^2\big(u-m^2\big)^2+2m^2\big(v_e^2-a_e^2\big)t\big\}.\end{aligned}\tag{5}$$

In the lab system where the initial electron is at rest $(E_2=m)$, the differential cross section describing the energy distribution of the final electron is

$$\begin{aligned}&\left.\frac{\mathrm{d}\sigma\left(\nu_e e^- \to \nu_e e^-\right)}{\mathrm{d}E_4}\right|_{\text{free}}=\frac{mG_F^2}{8\pi E_1^2}\big\{(v_e+a_e)^2E_1^2\\&\quad+(v_e-a_e)^2(E_1+m-E_4)^2\\&\quad+m\big(v_e^2-a_e^2\big)(m-E_4)\big\}.\end{aligned}\tag{6}$$

Integrating (6) over the allowed range

$$m<E_4<m+\frac{2E_1^2}{m+2E_1},\tag{7}$$

we then obtain

$$\begin{aligned}&\sigma^{\nu}_{\text{free}}\equiv\sigma\left(\nu_e\, e^-\to\nu_e\, e^-\right)\big|_{\text{free}}\\&\quad=\frac{mG_F^2E_1}{8\pi}\left\{(v_e+a_e)^2\frac{2E_1}{m+2E_1}\right.\\&\qquad+\frac{1}{3}(v_e-a_e)^2\left[1-\frac{m^3}{(m+2E_1)^3}\right]\\&\qquad\left.-\big(v_e^2-a_e^2\big)\frac{2mE_1}{(m+2E_1)^2}\right\},\end{aligned}\tag{8}$$

which agrees with the result quoted in [8].

For antineutrino scattering, crossing symmetry implies that $|F(\bar\nu_e e^-\to\bar\nu_e e^-)|^2_{\text{free}}$ is obtained from (5) by interchanging $s\leftrightarrow u$. Because of the structure of (5), such an interchange is equivalent to the substitution $a_e\to -a_e$. Thus the differential and integrated cross sections for antineutrino scattering off free electrons may be obtained from (6) and (8), respectively, by substituting $a_e\to -a_e$.

We turn next to the discussion of the neutrino ionization cross section [11], where the basic process is again given by (4), but now the energy of the initial electron is fixed as

$$E_2=m+\epsilon<m,\tag{9}$$

where ϵ is its binding energy, while its "squared-momentum"

$$p_2^2\equiv\tilde m^2=E_2^2-\vec p_2^{\,2},\tag{10}$$

necessarily goes slightly[3] off-shell as $|\vec p_2|$ varies according to the distribution dictated by the atomic wave function. Using again $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, $u=(p_1-p_4)^2$, $p_1^2=p_3^2=0$, $p_4^2=m^2$, and summing over all initial and final electron spin states, we find for the case when the initial electron is bound to an atom that

$$\begin{aligned}&\left|F\left(\nu_e e^-\to\nu_e e^-\right)\right|^2\\&\quad=2G_F^2\big\{(v_e+a_e)^2\big(s-m^2\big)\big(s-\tilde m^2\big)\\&\qquad+(v_e-a_e)^2\big(u-m^2\big)\big(u-\tilde m^2\big)\\&\qquad+2m^2\big(v_e^2-a_e^2\big)t\big\}.\end{aligned}\tag{11}$$

When $\tilde m^2\to m^2$, this expression coincides with the free electron expression appearing in (5). As before, $|F(\bar\nu_e e^-\to\bar\nu_e e^-)|^2$ for bound initial electrons is obtained from (11), by interchanging $s\leftrightarrow u$, which is also equivalent to the simple substitution $a_e\to -a_e$ in (11). This later substitution may then be used for obtaining the antineutrino cross sections from the neutrino ones given below. We will, therefore, discuss from here on only the derivation of the neutrino cross section, and simply quote the results for antineutrinos.

To present the subsequent steps of the calculation of the neutrino ionization cross sections, it is convenient to concentrate first to the He-atom. In the laboratory frame, defined as the one where the atom is at rest, we assume that the two He electrons are in a singlet spin state described by the same momentum wave function $\Psi_{n00}(|\vec p_2|)$; where n is the usual principal quantum number, and the orbital angular momentum quantum numbers are zero. If we neglect the repulsion between the two electrons, each of the bound electrons has a fixed binding energy given by the usual Balmer

[3] Since the bound electron is non-relativistic to a very good approximation, it cannot go far off-shell.

formula

$$\epsilon = -\frac{m(Z\alpha)^2}{2n^2}, \tag{12}$$

in which, for a He-atom in the ground state, $n = 1$ should be used (see (9)).

Denoting by $E_1 = E_\nu$, the incoming neutrino energy in the laboratory frame, we write the ($\nu_e \, \mathrm{He}_{\mathrm{atom}} \to \nu_e \, e^- \, \mathrm{He}_{\mathrm{ion}}$) ionization cross section as

$$\sigma^{\nu e}_{\mathrm{He}} = \frac{1}{2}\sigma^{\nu}_{\mathrm{He}} = \frac{1}{8E_1E_2}\int \frac{d^3p_2}{(2\pi)^3}\left|\Psi_{n00}(|\vec{p}_2|)\right|^2 |F|^2 \times (2\pi)^4 \, \mathrm{d}\Phi_2(p_1, p_2; p_3, p_4), \tag{13}$$

where $\sigma^{\nu}_{\mathrm{He}}$ denotes the neutrino–He cross section normalized to one He-atom per unit volume, while $\sigma^{\nu e}_{\mathrm{He}}$ is the same cross section normalized to one electron per unit volume. Moreover, $|F|^2$ is given in (11), the momentum wave functions are normalized as

$$\int \frac{d^3p_2}{(2\pi)^3}\left|\Psi_{n00}(|\vec{p}_2|)\right|^2 = 1,$$

and $\mathrm{d}\Phi_2(p_1, p_2; p_3, p_4)$ is the usual 2-body phase space satisfying [12]

$$(2\pi)^4 \, \mathrm{d}\Phi_2(p_1, p_2; p_3, p_4) = \frac{1}{8\pi(s - \tilde{m}^2)}\,\mathrm{d}t. \tag{14}$$

Comparing (13) to the corresponding neutrino cross section from a *free* electron, one identifies three differences. These are first the "off-shell" effect in $|F|^2$ which has been already discussed (compare (5), (11)); while the other two are the appearance in (13) of the momentum wave function and the atom-related flux factor.

For a spherically symmetric wave function as in the case of $\Psi_{n00}(|\vec{p}_2|)$, the angular part of the bound electron integral in (13) can be done immediately. Denoting the magnitude of its space momentum as $k \equiv |\vec{p}_2|$ and describing the Euler angles of $\vec{p}_2$ as $(\theta_2, \ \phi_2)$, we obtain from (14), (13)

$$\sigma^{\nu e}_{\mathrm{He}} = \frac{1}{64\pi E_1E_2}\int \frac{2\pi \, \mathrm{d}\cos\theta_2 \, k^2 \, \mathrm{d}k}{(2\pi)^3(s - \tilde{m}^2)} \times \left|\Psi_{n00}(k)\right|^2 |F|^2 \, \mathrm{d}t, \tag{15}$$

where, using (10), (9), (12), we write

$$s = \tilde{m}^2 + 2E_1(E_2 - k\cos\theta_2), \tag{16}$$

$$\tilde{m}^2 = (m + \epsilon)^2 - k^2. \tag{17}$$

As seen from these equations, the centre of mass energy of the neutrino-atomic electron system varies with k.

According to (15), only the k-integration depends explicitly on the detail form of the electron wave function. The t and θ_2 integrations are not affected by it, and their ranges are given by [4]

$$t_{\min} \equiv -s + m^2 + \tilde{m}^2 - \frac{m^2\tilde{m}^2}{s} < t < 0,$$
$$-1 < \cos\theta_2 < 1. \tag{18}$$

Therefore, it is convenient to carry out these two integrations and define the quantities

$$\Sigma(Z, n) = \frac{1}{64\pi E_1E_2}\int_{-1}^{1}\frac{\mathrm{d}\cos\theta_2}{2(s - \tilde{m}^2)}\int_{t_{\min}}^{0}\mathrm{d}t|F|^2 = \frac{G_F^2}{32\pi E_1E_2}\left\{(v_e + a_e)^2\Sigma_1 + (v_e - a_e)^2\Sigma_2 + 2m^2\left(v_e^2 - a_e^2\right)\Sigma_3\right\}, \tag{19}$$

with

$$\Sigma_1 = 4E_1^2\left(E_2^2 + \frac{k^2}{3}\right) + 2E_1E_2\left(\tilde{m}^2 - 2m^2\right) + m^4 - \frac{m^4\tilde{m}^2}{4E_1k}\ln\left(\frac{\bar{s} + 2E_1k}{\bar{s} - 2E_1k}\right), \tag{20}$$

$$\Sigma_2 = \frac{4E_1^2}{9}\left(k^2 + 3E_2^2\right) - E_1E_2\left(m^2 - \tilde{m}^2\right) + \frac{m^4\tilde{m}^2}{6\left(\bar{s}^2 - 4E_1^2k^2\right)}\left(m^2 + 3\tilde{m}^2\right) - \frac{\bar{s}m^6\tilde{m}^4}{3(\bar{s}^2 - 4E_1^2k^2)^2} + \frac{m^4}{24E_1k}\left(m^2 - 3\tilde{m}^2\right)\ln\left(\frac{\bar{s} + 2E_1k}{\bar{s} - 2E_1k}\right), \tag{21}$$

[4] There is a caveat concerning the θ_2 integration, related to (16). In order to have $s > m^2$ for the whole range $-1 < \cos\theta_2 < 1$, we must ensure that k always remains sufficiently small, which is in fact guaranteed by the consistency of the non-relativistic treatment of the bound electron.

$$\Sigma_3 = -\frac{m^2(m^2+2\tilde{m}^2)}{8E_1 k}\ln\left(\frac{\bar{s}+2E_1 k}{\bar{s}-2E_1 k}\right) + \frac{m^4\tilde{m}^2}{2(\bar{s}^2-4E_1^2k^2)} - E_1E_2 + m^2, \quad (22)$$

where Eqs. (16), (17), (11), (9), (12) and the definition

$$\bar{s} = \tilde{m}^2 + 2E_1E_2 \quad (23)$$

are used. We also note here that $\Sigma(Z,n)$ in (19) has been so normalized that at $k=0$ and $\tilde{m}=E_2=m$ it becomes identical to the free electron cross section appearing in (8). This guarantees that the neutrino cross section from a bound electron will always coincide with the free electron one, as soon as E_1 becomes much larger than the average k-momenta of the atomic electrons. It may also be worth mentioning that the dependence of $\Sigma(Z,n)$ on Z and n is induced by its dependence on the binding energy ϵ entering the definitions of E_2 and $\tilde{m}$ (compare with Eqs. (12), (9), (10)).

Combining (15) and (19) we write the ionization cross section for a ground state He atom, normalized to one electron per unit volume, as

$$\sigma_{\mathrm{He}}^{\nu e} = \frac{1}{2}\sigma_{\mathrm{He}}^{\nu} = \int_0^{k_{\max}} \frac{4\pi k^2\,\mathrm{d}k}{(2\pi)^3}\left|\Psi_{100}(k)\right|^2\Sigma(2,1), \quad (24)$$

where [11]

$$\Psi_{100}(k) = \frac{8\sqrt{\pi}\,\beta^{5/2}}{(k^2+\beta^2)^2}, \quad (25)$$

and $\beta \equiv Zm\alpha$ determines the range of k-values.

For the helium wave function, we use the hydrogen-like wave function obtained from (25), for an effective atomic number $Z_{\mathrm{eff}} = 2$–$5/16$ derived from variational calculations.[5] Using (12) for Z_{eff}, we obtain the total He binding energy as $E_{\mathrm{He}} = -77.4$ eV, which is very close to the experimental total binding energy of $E_{\mathrm{He,exp}} = -78.975$ eV. The implied single electron binding energy is $\epsilon_{\mathrm{He}} \simeq -24.6$ eV, which is the value we have used in the actual calculations. The results are insensitive to the exact magnitude of this value.

The upper bound of the integration in (24) is determined by the requirement that $s > m^2$, which according to (16) leads to

$$k_{\max} = \sqrt{E_1(E_1+2m+2\epsilon)+\epsilon(\epsilon+2m)} - E_1. \quad (26)$$

The corresponding ionization cross section from an unpolarized hydrogen atom in its ground state is written in analogy to (24) as

$$\sigma_{\mathrm{H}}^{\nu e} = \sigma_{\mathrm{H}}^{\nu} = \int_0^{k_{\max}} \frac{4\pi k^2\,\mathrm{d}k}{(2\pi)^3}\left|\Psi_{100}(k)\right|^2\Sigma(1,1), \quad (27)$$

where $Z=1$ is used.

Finally, for the Ne ionization from its ground state, we have to remember that there are ten electrons in this case, in the configuration $1s^22s^22p^6$. For the wave functions we use the exponentials suggested in [13], which reproduce the observed total binding energy of the atom. For the binding energies of each electron in the various bound states we use the values $\epsilon_{1s} = -870$ eV, $\epsilon_{2s} = -48.5$ eV, $\epsilon_{2p} = -21.7$ eV. The Fourier transforms to the momentum space are straight-forward and will not be given here. The range of momenta implied by the wave functions in [13] is much smaller than $Zm\alpha$.

The ν_eNe ionization cross section, normalized to one electron per unit volume is given by

$$\sigma_{\mathrm{Ne}}^{\nu e} = \frac{1}{10}\sigma_{\mathrm{Ne}}^{\nu} = \frac{1}{10}\int_0^{k_{\max}} \frac{4\pi k^2\,\mathrm{d}k}{(2\pi)^3}\Big[2\left|\Psi_{100}(k)\right|^2\Sigma(10,1) + 2\left|\Psi_{200}(k)\right|^2\Sigma(10,2) + \frac{6}{4\pi}\left|R_{21}(k)\right|^2\Sigma(10,2)\Big]. \quad (28)$$

We note that in the last term in (28), only the radial part $R_{21}(k)$ of the Ψ_{21m}-wave function appears. This is because the angular dependence of the wave function disappears when the contributions from all six electrons in the $(n=2,\ l=1)$-shell are added.

We present in Fig. 2(a) the neutrino ionization cross sections of the H, He and Ne atoms, normalized to one electron per unit volume, as well as the $\nu_e e^- \to \nu_e e^-$ cross section for the free electron case; while in Fig. 2(b) the ratios of the same atomic cross sections to the free electron one are presented. In both cases the neutrino energies are at the keV-range.

[5] See, e.g., any of the books in [11].

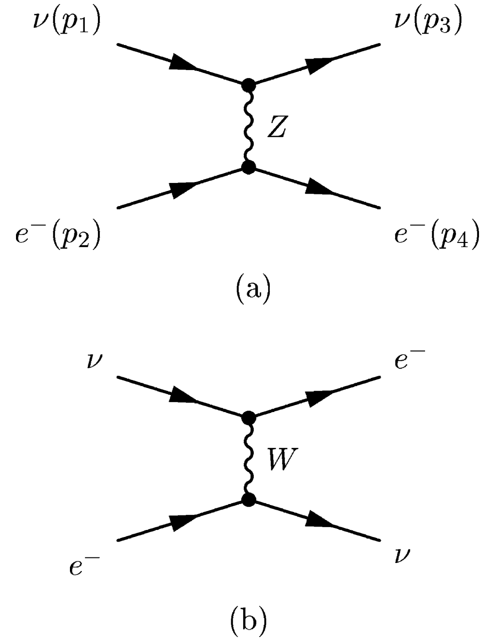

Fig. 1. Neutrino–electron Feynman diagrams

The corresponding results for the antineutrino case are presented in Fig. 3(a), (b).

As seen in Figs. 2 and 3, the cross sections for bound electrons are close to (but smaller than) the cross sections for free electrons. For energies larger than 10 keV their difference is less than 5%. These results are also rather insensitive to the exact magnitude of the values of the binding energies. The same pattern is repeated for antineutrinos, as seen in Fig. 3. As an example we note that at 15 keV the neutrino or antineutrino free electron cross sections, as well as the cross sections of the H or He atoms per electron, are all in the range of $\sim 6 \times 10^{-48}$ cm^2, while the Ne ones are slightly smaller. We have thus to conclude that we cannot reproduce the results of [8] for the H, He and Ne atoms.

The structure of the results in Figs. 2 and 3 can be understood intuitively. It just indicates that as Z increases, the binding of the atomic electrons is also increasing, obstructing the atom ionization through neutrinos of keV energies. This binding effect is rather small though, so that as the neutrino energy increases, the atomic ionization cross section rapidly approaches the free electron one.

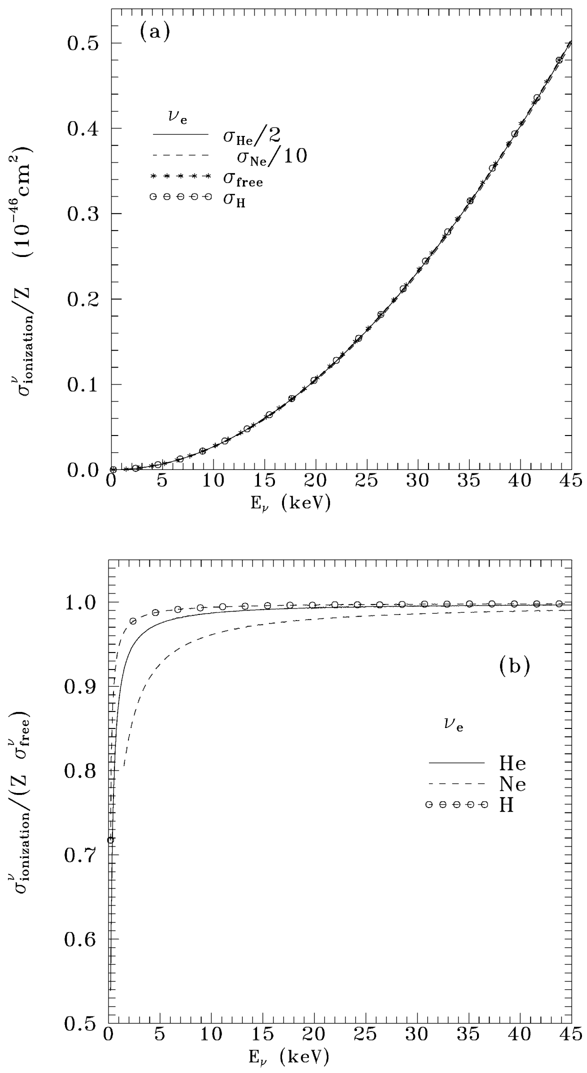

Fig. 2. The ν_e ionization cross sections for the H, He and Ne atoms divided by z, and the neutrino free electron cross section as functions of the neutrino energy E_ν (a); as well as the ratios of the atomic to free electron cross sections (b).

To summarize, we can claim that as soon as the neutrino energy passes the 20 keV region, the ionization cross sections for H, He and even the Ne atoms (normalized to one electron per unit volume) become virtually identical to the free electron cross section. In fact, on the basis of Figs. 2, 3 we could also claim that for a tritium experiment like the one suggested in [7],

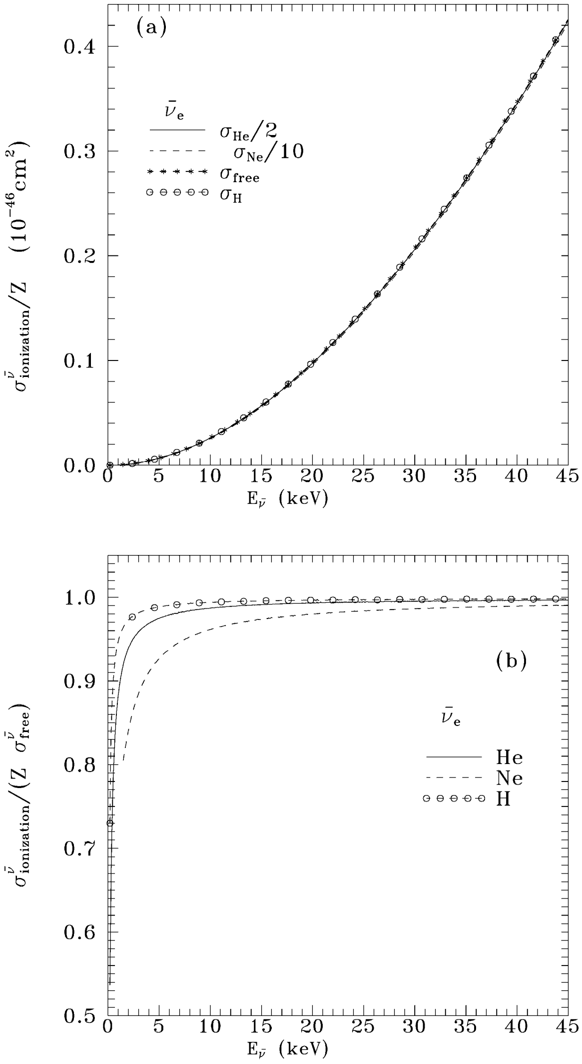

Fig. 3. The $\bar{\nu}_e$ ionization cross sections for the H, He and Ne atoms divided by Z, and the antineutrino free electron cross section as functions of the neutrino energy $E_{\bar{\nu}}$ (a), as well as the ratios of the atomic to free electron cross sections (b).

it would be probably impossible to discriminate these ionization cross section from the free electron one.

Finally, a comment should be added on the conditions under which coherent effects may appear in neutrino scattering. We have stressed above that there is no coherence phenomenon affecting the magnitude of the neutrino ionization cross section. It should be remembered however, that a coherent MSW [10] effect at keV energies, will always be induced by the forward elastic scattering of neutrinos (or antineutrinos) from the electrons bound in the atoms. Since the electron binding is not expected to play an important role in the forward elastic process, this MSW effect is essentially given by the forward free electron elastic amplitude convoluted with the square of the electron wave function. Thus, at keV neutrino energies the MSW effect may have some additional energy dependence compared to the standard one in [10]; but it should soon assimilate it as the energy approaches, e.g., the 20 keV range. The detail study of this phenomenon is beyond the scope of the present paper.

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References

[1] H.S. Gurr, F. Reines, H.W. Sobel, Phys. Rev. Lett. 28 (1972) 1406.
[2] M. Apollonio et al., CHOOZ Collaboration, Phys. Lett. B 446 (1999) 415.
[3] Y. Fukuda et al., Superkamiokande Collaboration, Phys. Rev. Lett. 82 (1999) 2644.
[4] A. Ianni, for the Borexino Collaboration, Nucl. Phys. A 663 (2000) 791.
[5] J.F. Beacom, P. Vogel, Phys. Rev. Lett. 83 (1999) 5222;
P. Vogel, J. Engel, Phys. Rev. D 39 (1989) 3378.
[6] G.L. Fogli, E. Lisi, D. Montanino, A. Palazzo, hep-ph/0106247;
J.N. Bahcall, M.C. Gonzalez-Garcia, C. Peña-Garay, hep-ph/0106258;
V. Barger, D. Marfatia, K. Whisnant, hep-ph/0106207.
[7] J. Bouchez, I. Giomataris, CEA/Saclay internal note, DAPNIA/01-07, June 2001;
Y. Giomataris, Ph. Rebourgeard, I.P. Robert, G. Charpak, Nucl. Instrum. Methods A 376 (1996) 29;
J.L. Collar, I. Giomataris, Low-background applications of Micromegas detector technology, presented at IMAGING

2000 Conference, Stockholm, 28 June 2001 – 1 July 2001, DAPNIA/00-06.

[8] Yu.V. Gaponov, Yu.L. Doprynin, V.I. Tikhonov, Sov. J. Nucl. Phys. 22 (1976) 170.

[9] S.A. Fayans, V.Yu. Dobretsov, A.B. Dobrotsvetov, Phys. Lett. B 291 (1992) 1;
V.Yu. Dobretsov, A.B. Dobrotsvetov, S.A. Fayans, Sov. J. Nucl. Phys. 55 (1992) 1180.

[10] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369;
L. Wolfenstein, Phys. Rev. D 20 (1979) 2634;
S.P. Mikheyev, A.Yu. Smirnov, Yad. Fiz. 42 (1985) 1441, Sov. J. Nucl. Phys. 42 (1985) 913;
For coherent scattering of a baryon-neutral current on nuclei see: J.J. Sakurai, L.F. Urritia, Phys. Rev. D 11 (1975) 159.

[11] The treatment parallels the ionization of atoms in the photoelectric effect; see, for instance:
H. Bethe, R.W. Jackiw, Intermediate Quantum Mechanics, Benjamin, Reading, MA, 1968;
K. Gottfried, Quantum Mechanics, Vol. 1, Benjamin, New York, 1966.

[12] J.D. Jackson, Particle Data Group, Eur. Phys. J. C 15 (2000) 211.

[13] A. Tubis, Phys. Rev 102 (1956) 1049, gives the parameters for the wave functions we use. A detailed discussion of their origin is given by J.C. Slater, in: Quantum Theory of Atomic Structure, Vol. I, McGraw–Hill, New York, 1960, pp. 348–355.