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A Parking-State-Based Transition Matrix of Traffic on Urban Networks

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Abstract

The urban parking and the urban traffic systems are essential components of the overall urban transportation structure. The short-term interactions between these two systems can be highly significant and influential to their individual performance. The urban parking system, for example, can affect the searching-for-parking traffic, influencing not only overall travel speeds in the network (traffic performance), but also total driven distance (environmental conditions). In turn, the traffic performance can also affect the time drivers spend searching for parking, and ultimately, parking usage. In this study, we propose a methodology to model macroscopically such interactions and evaluate their effects on urban congestion.

The model is built on a transition matrix describing how, over time, vehicles in an urban area transition from one parking-related state to another. With this model it is possible to estimate, based on the traffic and parking demand as well as the parking supply, the amount of vehicles searching for parking, the amount of vehicles driving on the network but not searching for parking, and the amount of vehicles parked at any given time. More importantly, it is also possible to estimate the total (or average) time spent and distance driven within each of these states. Based on that, the model can be used to design and evaluate different parking policies, to improve (or optimize) the performance of both systems.

A simple numerical example is provided to show possible applications of this type. Parking policies such as increasing parking supply or shortening the maximum parking duration allowed (i.e., time controls) are tested, and their effects on traffic are estimated. The preliminary results show that time control policies can alleviate the parking-caused traffic issues without the need for providing additional parking facilities. Results also show that parking policies that intend to reduce traffic delay may, at the same time, increase the driven distance and cause negative externalities. Hence, caution must be exercised and multiple traffic metrics should be evaluated before selecting these policies.

Overall, this paper shows how a parking-state-based transition matrix, despite its simplicity, can be used to efficiently evaluate the urban traffic and parking systems macroscopically. The proposed model can be used to estimate both, how parking availability can affect traffic performance (e.g., average time searching for parking, number of cars searching for parking); and how different traffic conditions (e.g., travel speed, density in the system) can affect drivers ability to find parking. Moreover, the proposed model can be used to study multiple strategies or scenarios for traffic operations and control, transportation planning, land use planning, or parking management and operations.

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Keywords: macroscopic parking model; traffic congestion; parking-related traffic state; cruising-for-parking; transition matrix of traffic

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1. Introduction

The urban parking and the urban traffic systems are essential components of the overall urban transportation structure. The interactions between these two systems, can have both, long-term effects (i.e., parking policies can affect travel demand, and vice versa), and short-term effects (i.e., parking policies can affect traffic operations, and vice versa). While the long-term effects have attracted lots of research attention (e.g. Feeney, 1989; Young et al., 1991), the short-term effects have not been well researched yet. This is unfortunate, as the short-term interactions between parking and traffic can be highly significant and influential to the performance of both systems. For example, parking availability can affect the traffic composition on the network. Shoup (2005), based on the review of 16 studies of mostly American and European cities, concluded that cars searching for free parking spaces contribute to over 8% of the total traffic in a city, reaching 30% in business areas during rush hour. Although this part of traffic is caused by inefficient parking provision, its corresponding externalities are endured by the traffic system as a whole. Such externalities have been studied from an economic point of view (e.g. Arnott and Inci (2006)) and could have a significant influence on traffic performance, causing congested or hyper-congested traffic conditions (Geroliminis (2009)).

Studies like these, all provide some insights on how the urban parking system (both supply and policies) can influence traffic performance. Nevertheless, although different parking policies including pricing schemes have been analyzed, proposed and implemented; to the authors' knowledge, no study has provided yet a generalized methodology to macroscopically model the relation between parking demand, parking availability, and traffic conditions.

In this paper, we develop a parking-state-based transition matrix that aims to model macroscopically a dynamic urban parking system. Basic assumptions for the matrix include a traffic demand over a period of time (e.g., a day), the distribution of parking durations, the length and the traffic properties of the network. Within the matrix, the likelihood of a parking searcher to access/find parking spots in an urban network is estimated, as well as other transition events such as starting to search for parking and departing from it. The model then provides an approximation of the proportion of cars searching for parking, as well as the time cars spent searching for parking, or traveling through the system. Moreover, traffic density and travel speed are also estimated over time based on different background conditions. These results are useful to evaluate both, how traffic performance (e.g., speed, density, flow) affects drivers' ability to find parking; and how parking availability affects traffic performance.

The main contributions of this paper are twofold.

1. This study looks at the relation between parking and traffic performance macroscopically. Most of the existing research looks at the problem microscopically, modeling the parking behavior of individual agents. The agent-based studies can require huge amounts of data, and high levels of detail both on the demand and the supply side. In this paper we look at the problem macroscopically, and focus only on average values and probability distributions across the whole population. This is valuable, as all data requirements correspond to aggregated values at the network level and there is no data requirement for individual drivers or parking spots. This macroscopic approach, compared to microscopic methods, saves not only on data collection efforts (e.g., drivers preferences, individual driving routes, individual parking spots turnovers) but also reduces the computation costs significantly. Such efficiencies are especially useful when the network of interest is large and/or data is scarce.

2. This study allows us to model two dynamic systems interacting with each other. For the traffic system, the model is able to analyze overcrowded situations, where time-varying traffic conditions are provided as traffic performance indicators; they are also taken into consideration for the evolution of the matrix. For the parking system, the usage and the arrival/departure rates are all dynamically updated over time. Notice that in the existing literature, with the exception of a few studies, these elements are mostly assumed without regard for past conditions. Here, however, these variables are dynamically estimated based on changing conditions to better replicate reality.

The paper is organized as follows. Section 2 reviews the existing work on the interactions between parking system and traffic performance, highlighting the differences between the work presented in this paper and the existing studies. Section 3 introduces the concept/framework of the parking-state-based transition matrix. Section 4 contains the methodology to build up the transition matrix. Section 5 shows a numerical example to explore the use of the concept and methodology proposed. Section 6 summarizes the findings of this paper.

2. Literature Review

Currently, there are three general approaches that are used for understanding and estimating traffic-parking interactions: empirical, analytical, and multi-agent (MA) simulations tools.

Empirical studies rely on driver surveys (e.g. Axhausen et al., 1994; IBMs global parking survey, 2011), videotaping (e.g. King (2010)), driving test cars and searching for parking (e.g. Shoup (2006)), GPS data (e.g. van der Waerden et al., 2014; Montini et al., 2012), and parking occupancy data (e.g. Millard-Ball et al. (2014)).

Empirical data is often collected for local projects as it is mostly specific to an area or a city. Thus, since the data observed is based on localized conditions, it is difficult to draw generalized conclusions from it. For example, driver surveys generally stop people at intersections to ask if they are seeking parking, or ask people emerging from their cars about their experience finding a parking place. As one would expect, the results are then very much based on local drivers' preference for parking, and their value-of-time, as well as the time of day. The same is true for studies that rely on video or other visual techniques. Methods using GPS and parking occupancy data can be used for a wider range of cities as they can provide more generalized conclusions. However, GPS data extraction tools are still under development, so the precision and generality of conclusions drawn with them are not yet known. As for the use of parking occupancy data, this one typically does not include any traffic information and thus, the parking-caused traffic still needs to be derived through other methods. Therefore, a macroscopic model that does not require any physical devices and yet can provide both more generalized conditions and results is desirable.

Notable theoretical contributions on the interaction between parking and traffic include literature on economics and traffic assignment. The literature on economics includes Arnott and Rowse (1999), Arnott and Inci (2006; 2010). Based on two traffic assignment methods, user equilibrium and social optimal, the externalities of parking system on traffic congestion are presented (Arnott (1999)). However, the model does not represent traffic performance, e.g., a fixed value travel speed is assumed for all conditions. More connected to our study, Arnott and Inci (2006) defined different types of vehicles (moving, cruising, and parked), then provided very useful relations between these types. However, the model is based on stationary-state conditions, and cannot describe the dynamics of the system (i.e., time-varying conditions).

Gallo and D'Acierno (2011) proposed an assignment model on urban networks to simulate parking choice, and the impact of parking search on traffic congestion. The cost function of users included driving, parking, and walking. Then the traffic conditions induced by the parking search process could be found. However, within each interval of the traffic assignment (1 hour), the traffic and parking conditions remained steady, limiting the application of the model, as in reality both traffic and parking conditions can change rapidly. Bodenbender (2013) developed a model which considered the probability of not finding a parking. The model was used to test different parking policies based on the urban network of Zurich, Switzerland. Same as the previous study, the static traffic assignment neglected all the time-varying conditions such as the parking usage and traffic performance. In addition, the study assumed that travelers were fully informed about parking including the probability to find one in each link.

MA simulation tools are widely used to simulate travel behavior for a large number of users. These tools allow inputs such as a non-homogeneous network and personal preferences. The output of the simulations can contain very detailed results, e.g., parking search traffic and impact on traffic performance (Benenson et al., 2008; Waraich, 2012; Horni and Montini, 2012; Geroliminis, 2009). This method, though comprehensive and powerful, relies on many preliminary models such as car following models, route and parking choice models, etc. Therefore, the accuracy of the final output can be affected through many aspects. In addition, agent-based simulations require a large amount of very detailed data for the specified conditions, including the detailed network of the city and its parking system, as well as the travel behavior and searching habits of its citizens. Hence, the transferability of the approach (and results) across cities and/or population types is rather low.

3. Overall Methodology and Matrix Framework

3.1. Parking-state-based transition matrix

Consider a round-trip going into an urban area as a tour instead of 2 single trips. The vehicle may experience three parking-related states separated by five parking-related transition events. The three parking-related states are:

- Non-searching state: Vehicles in this state are not searching for parking, they either just entered the area (including through traffic) or have departed the parking facilities (not yet left the area).
- Searching state: Vehicles in this state are searching for parking.
- Parking state: Vehicles in this state are parked (i.e., staying in parking spots).

The states are shown in Figure 1, linked by the transition events. Figure 1(a) describes the transition events based on one single vehicle trip. Figure 1(b) describes the transition events for all vehicles (the overall traffic movements) in an urban area; the shaded part represents the parking system in this area, and the rest represents the traffic system. Notice that we assume the parking maneuvers (access/depart) are instantaneous although in reality they are not. More details on the specific effects of these maneuvers can be found in Cao and Menendez (2014; 2015).

The five parking-related transition events are also shown in Figure 1. They are:

- Enter the area: These vehicles enter the "non-searching" state.
- Start to search: These vehicles transition from the "non-searching" to the "searching" state.
- Access parking: These vehicles transition from the "searching" to the "parking" state.
- Depart parking: These vehicles transition from the "parking" to the "non-searching" state.
- Leave the area: These vehicles finish the "non-searching" state as they exit the area of interest.

Consider a very small time slice, *i*, (e.g., 1 minute), Table 1 shows the notation for the number of vehicles in each state at the beginning of the time slice and the number of vehicles experiencing each transition event during the time slice.

Table 1. Key variables in a time slice.

Notation	Definition
N^i_{ns}	Number of vehicles in the state "non-searching" at the beginning of time slice *i*.
N^i_s	Number of vehicles in the state "searching" at the beginning of time slice *i*.
N^i_p	Number of vehicles in the state "parking" at the beginning of time slice *i*.
$n^i_{/ns}$	Number of vehicles that enter the area and transition to "non-searching" during time slice *i* (enter the area).
$n^i_{ns/s}$	Number of vehicles that transition from "non-searching" to "searching" during time slice *i* (start to search).
$n^i_{s/p}$	Number of vehicles that transition from "searching" to "parking" during time slice *i* (access parking).
$n^i_{p/ns}$	Number of vehicles that transition from "parking" to "non-searching" during time slice *i* (depart parking).
$n^i_{ns/}$	Number of vehicles that leave the area and transition from "non-searching" during time slice *i* (leave the area).

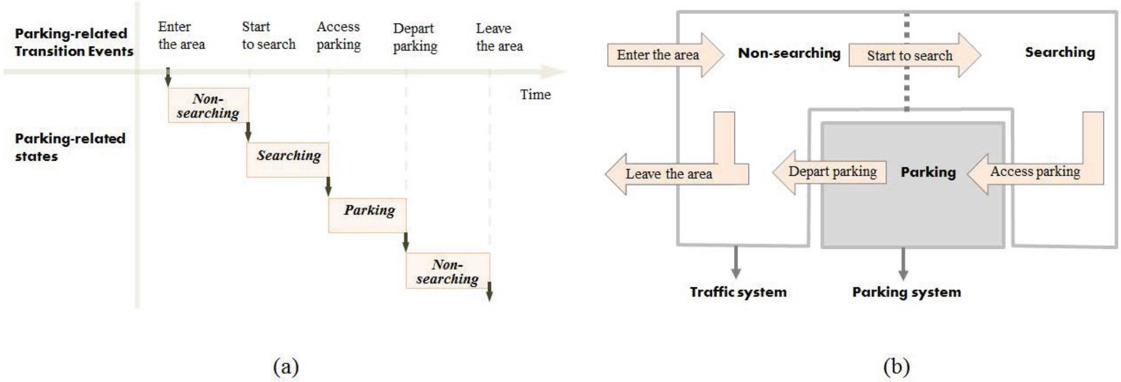

Fig. 1. Parking-related states and the parking-related transition events of vehicles in an area (a) for a single traveler (b) for all vehicles in the area.

3.2. Changes to the number of vehicles in each state

The transition of the whole system between consecutive time slices is shown in Figure 2 and formulated as Eq. 1, 2 and 3.

In Figure 2(a), the traffic composition (i.e., the number of vehicles in each parking-related state) at the beginning of time slice i is shown, as well as the transition events during time slice i. Based on them, the traffic composition at the beginning of the next time slice can then be obtained. When repeating the process described above, a queuing diagram for a longer period that consists of many time slices can be found (Figure 2(b)). In Figure 2(b), at a given time, the vertical distance between two neighboring curves indicates the number of vehicles in a state, e.g., at the beginning of time slice i, the vertical distance between the curves "start to search" and "access parking" is the number of vehicles in the "searching" state, N_s^i. Notice that the number of vehicles in the "non-searching" state, N_{ns}^i, includes two families of vehicles: new vehicles that just entered the area (on the top of the figure) and vehicles that are about to leave the area (on the bottom of the figure). For a given period of time, the average horizontal distance between two neighboring curves is the average time that vehicles spend in that state.

Eq. 1 updates the number of "non-searching" vehicles. During time slice i, vehicles that enter the area (i.e., $n^i_{/ns}$) and vehicles that depart parking (i.e., $n^i_{p/ns}$) join this state; vehicles that start to search (i.e., $n^i_{ns/s}$) and vehicles that leave the area (i.e., $n^i_{ns/}$) quit this state.

$$N_{ns}^{i+1} = N_{ns}^i + n^i_{/ns} + n^i_{p/ns} - n^i_{ns/s} - n^i_{ns/} \tag{1}$$

Eq. 2 updates the number of "searching" vehicles. During time slice i, vehicles that start to search (i.e., $n^i_{ns/s}$) join this state; vehicles that access parking (i.e., $n^i_{s/p}$) quit this state.

$$N_s^{i+1} = N_s^i + n^i_{ns/s} - n^i_{s/p} \tag{2}$$

Eq. 3 updates the number of "parking" vehicles. During time slice i, vehicles that access parking (i.e., $n^i_{s/p}$) join this state; vehicles that depart parking (i.e., $n^i_{p/ns}$) quit this state.

$$N_p^{i+1} = N_p^i + n^i_{s/p} - n^i_{p/ns} \tag{3}$$

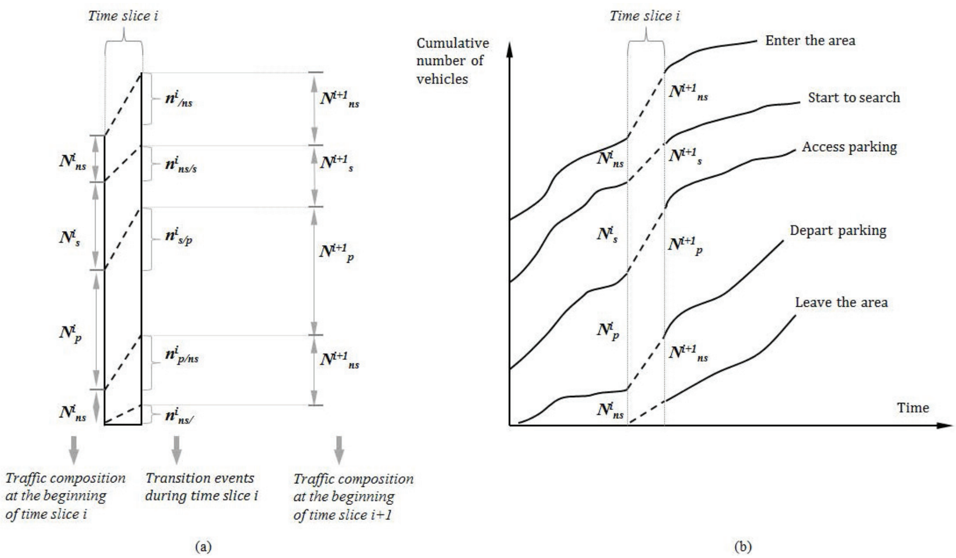

Fig. 2. (a) Future traffic composition based on current traffic composition and the quantification of the transition events (during one single time slice); (b) Construction of the queuing diagram of vehicles that have experienced each parking-related transition event based on a single time slice.

3.3. Changes to the number of vehicles going through each transition event

Time slices are sufficiently small. Hence, the trips can be generated simultaneously in each time slice. Vehicles can have only one parking-related state during one time slice. They may or may not transition to another state at the end of a time slice, such decision is endogenously generated within the model.

A framework showing all the interactions within urban traffic/parking systems is given in Figure 3. More important, it also indicates, conceptually, how the number of vehicles in each parking-related state affects the transition events.

As shown in Figure 3, in general, there are three ways that vehicles within each parking-related state may affect the transition events:

- The number of parked vehicles (i.e., within "parking" state) and the time they accessed parking, affects the number of vehicles "accessing parking" and "departing parking".

- The number of parking searchers (i.e., within "searching" state) and the time they started to search affects the number of vehicles "accessing parking".

- The number of vehicles driving (i.e., within the "non-searching" and "searching" states) and the time these vehicles joined the state affects the travel speed and further influences all the transition events except "enter the area".

In other words, based on the number of vehicles in each parking-related state and the time they joined that state, each transition event is modeled.

- For the "enter the area" transition, it is assumed to be the same as the travel demand (known or assumed). Details are explained in section 4.1.
- For the "start to search" transition, it is assumed to happen after a vehicle drives a given distance. The time needed for this transition depends on the corresponding traffic conditions (travel speeds) during that period. Details are explained in section 4.2.
- For the "access parking" transition, it is modeled based on the likelihood of finding an available parking spot based on the conditions during that time slice. Details are explained in section 4.3.
- For the "depart parking" transition, it is obtained based on the arrival time of vehicles to the parking facilities and the distribution of parking durations. Details are explained in section 4.4.
- For the "leave the area" transition, it is obtained also after a vehicle drives a given distance (two distances are assumed respectively for through traffic to leave the area and for parked cars to leave the area). Details are explained in section 4.5.

It should be noted that the transition matrix proposed in this paper is not a Markov Chain. In our model, the condition in the next stage does not only rely on the current stage, but also on earlier ones. For example, the number of vehicles that depart parking in the current time slice is not only related to the amount of currently parked vehicles, but also their accessing time to parking. That requires knowledge of the number of vehicles access parking in each and all previous time slices. Similarly, the modeling of start to search" and "leave the area" also need the number

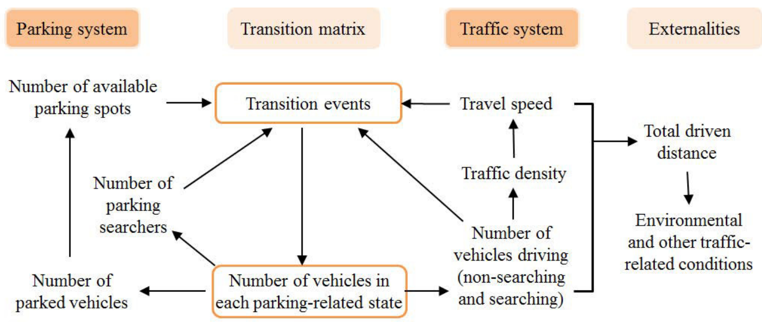

Fig. 3. Interactions between the urban parking and traffic systems.

of vehicles that enter the area and depart parking in each and all previous time slices. More details on modeling are explained in section 4.

The transition matrix can be programmed based on the definitions/equations/models provided here. Therefore, the system can either end when all traffic have left the area (this is the case for our numerical example), or any other time as needed.

3.4. Basic information for analytical model

Basic model assumptions, inputs, and expected outputs are briefly described below.

Assumptions

The network is relatively small, compact, and homogenous. On average, all existing parking spaces (not only the available ones) are uniformly distributed on the network. Moreover, they are all identical. We thus do not address the role of parking fees or walking distance when allocating more or less desirable parking spots to users. Having no consideration of fee, walking distance, allows us to avoid drivers preference on parking location and price, etc. This makes the model simple enough that one can focus on the searching process of travellers in the network. The basis for this assumption is that the network should not be too large so that the drivers can be more or less indifferent to parking spaces at different locations and take the first one they find.

The arrival rate of traffic into the area, the size of the network, and the distribution of parking durations and the traffic properties of the network are known. Trips are uniformly distributed along the network and the parking demand (vehicles that are searching for parking) is homogenously distributed within the overall driving traffic.

As vehicles that use parking garages do not typically search for parking (they treat the parking garage as the target destination), it is not realistic to model them as searching traffic. Considering that, we define a portion of travel demand as through traffic which represents trips that do not search for parking. They include users of off-street, dedicated/private parking facilities or vehicles that are simply driving through the network. In this way, the parking garages do not need to be modeled explicitly, but the vehicles using them are still taken into account.

Inputs

Corresponding to the assumptions described above. Table 2 shows all the model's independent variables.

Table 2. Independent variables (inputs).

Notation	Definition
$n^i_{/ns}$	New arrivals to the network during time slice i (i.e., travel demand).
β^i	Proportion of new arrivals during time slice i that corresponds to through traffic.
L	Size (length) of the network.
A	Total number of existing parking spots (for public use) in the area.
t	Length of a time slice.
t_d	Parking duration.
$f(t_d)$	The probability density function of the parking duration.
v	Free flow speed, i.e., maximum speed on the network.
Q_{max}	Maximum traffic flow rate that can be adopted on the network.
k_c	Optimal/critical traffic density on the network. If the traffic density is higher than this value, then congestion occurs.
k_j	Jam density.
$l_{ns/s}$	Distance that must be driven by a vehicle before it starts to search for parking.
$l_{/}$	Distance that must be driven by a vehicle before it leaves the area without parking.
$l_{p/}$	Distance that must be driven by a vehicle before it leaves the area after it has parked.
N^0_{ns}	The initial condition of non-searching state.
N^0_s	The initial condition of searching state.
N^0_p	The initial condition of parking state.

One can see four distinct sets of input variables.

The first set corresponds to the travel demand and supply, including the traffic demand, the proportion of through traffic, the distribution of the parking durations, and the parking supply in the area. These data can be assumed based on some historical data, e.g., traffic data on main roads to enter the network; parking data from one days data collection, etc.

The second set corresponds to the traffic network, including the total length, the traffic flow properties such as the saturation flow, free flow speed and jam density. These data can be estimated based on real measurements, the kinematic wave theory of traffic flow, the macroscopic fundamental diagram, and/or simulation results.

The third set corresponds to the distances one needs to drive before transitioning into the next state. These values can be assumed based on the length of the network, and other data collected from travellers.

The four set corresponds to the initial conditions of the parking-related states. These values can be measured, assumed or simulated.

Outputs

The model is able to provide, among others, the following outputs:

- Indicators for traffic conditions: vehicle accumulation on the network (traffic density), it includes vehicles in both non-searching and searching states; average travel speed, obtained based on the traffic density; total and average distance driven.

- Indicators for parking conditions: arrival to parking facilities (transition event “access parking”); departure of parking facilities; parking occupancy; available parking supply; parking demand (i.e., parking searchers).

Besides these, indicators specific to parking searchers can also be obtained, such as the average search time and distance; share of searching traffic and non-searching traffic, etc.

4. Analytical Formulations for Transition Events

For the modeling of the number of vehicles that go through a transition event in a time slice, we establish some more detailed assumptions. The urban network is abstracted as one ring road with cars driving in a single direction. The ring network represents networks which are homogenous. The assumption of a single travel direction simplifies the model without affecting the model results: the traffic demand can be seen as homogenously distributed on the network, whether vehicles travel in a single direction or two. We assume no overtaking takes place. This, although seems unrealistic, does not affect the model results: for any given number of available parking spaces and searchers, the average number of vehicles finding parking spaces in a time slice should not change even if cars can overtake each other.

During a given time slice, vehicles drive at the same speed. The available parking spots might be visited by several vehicles, but they only accommodate the first one that passes by. The vehicles that pass afterwards, see it full and continue searching for the next available parking spot.

Additionally, we define some new variables, they are listed in Table 3. They are used as the foundation to quantify the number of vehicles that experience each transition event in a time slice. Their functions are written as Eq. 4 - 10.

Table 3. Intermediate variables.

Notation	Definition
A^i	Number of available parking spots at the beginning of time slice i.
k^i	Average traffic density in time slice i.
v^i	Average travel speed in time slice i.
d^i	Maximum driven distance of a vehicle in time slice i.
s^i	Spacing between vehicles that are searching for parking at the beginning of time slice i.
m^i	Maximum number of vehicles that can pass by the same place on the network during time slice i.
d_r^i	Remainder of the division $\frac{d^i}{s^i}$ when $d^i > s^i$.

A^i is the parking capacity minus the number of parking spaces that are occupied at the beginning of a given time slice, $A^i \leq A$.

$$A^i = A - N_p^i \tag{4}$$

k^i is the total number of vehicles on the road network at the beginning of a given time slice divided by the length of the network.

$$k^i = \frac{N_s^i + N_{ns}^i}{L} \tag{5}$$

v^i is formulated based on a triangular Fundamental Diagram. Hence, it assumes that vehicles travel at speed v when traffic is not congested, and at a lower speed once the traffic density exceeds k_c (i.e., traffic starts to be congested). The form of v^i can be assumed differently in other cases if necessary. For a larger network, a macroscopic fundamental diagram (MFD) theory which takes into account aggregate conditions across the network might be more suitable. In case another network is used instead of a simple ring road, exchanging the triangular FD with an MFD, should not affect, however, any of the presented methodology.

$$v^i = \begin{cases} v & \text{, if } 0 \leq k^i \leq k_c \\ \dfrac{Q_{max}}{k_c - k_j} \cdot \left(1 - \dfrac{k_j}{k^i}\right) & \text{, if } k_c < k^i \leq k_j \end{cases} \tag{6}$$

d^i and s^i are formulated as explained by their definitions.

$$d^i = v^i \cdot t \tag{7}$$

$$s^i = \frac{L}{N_s^i} \tag{8}$$

Notice that, if k^i exceeds or equals k_j at a given time, a gridlock situation will be immediately generated. Since this moment, $v^i = 0$ and $d^i = 0$, i.e., no vehicle on the network is able to travel any further and no transition events can be reached (except for ”depart parking”). Such a situation may occur when a set of unfavorable conditions are met. For example, a combination of small parking supply, long parking durations and large parking demand, etc.

As the maximum number of vehicles that can pass by the same place on the network, m^i is formulated based on the maximum distance a vehicle can drive and the spacing between two consecutive vehicles. Note that, all locations on the network could be potentially visited by $m^i - 1$ cars.

$$m^i = \left\lceil \frac{d^i}{s^i} \right\rceil \tag{9}$$

d_r^i is formulated based on its definition.

$$d_r^i = d^i - \left\lfloor \frac{d^i}{s^i} \right\rfloor \cdot s^i \text{ for } d^i > s^i \tag{10}$$

Assume that at the beginning of time slice i, the starting point of a car is location x_c. Then the starting points of the vehicles behind this original car are locations $x_c - s^i$, $x_c - 2 \cdot s^i$, $x_c - 3 \cdot s^i$, etc. At the end of time slice i, depending on the ratio between d^i and s^i (i.e., the amount of overlap between vehicles' trajectories), one of those vehicles behind the original car will not be able to drive beyond $x_c + d_r^i$ (the vehicle with starting point $x_c - \lfloor \frac{d^i}{s^i} \rfloor \cdot s^i$). All the other vehicles with starting points between $x_c - \lfloor \frac{d^i}{s^i} \rfloor \cdot s^i$ and x_c will be able to drive further. In other words, within the area of $[x_c, x_c + d_r^i]$, a maximum of m^i cars can pass by (i.e., this is the area of maximum overlap between vehicles' trajectories); and within the area of $[x_c + d_r^i, x_c + s^i]$, a maximum of $m^i - 1$ cars can pass by (i.e., this is the area of minimum overlap between vehicles' trajectories). This will later be used to obtain $n_{s/p}^i$.

Based on these values, we can now find the number of vehicles that go through each transition event during time slice i.

4.1. Enter the area, $n^i_{/ns}$

$n^i_{/ns}$ is an input to the model. Within $n^i_{/ns}$, a percentage β^i is through traffic that will directly leave the area after driving a distance $l_/$, the rest will go through all transition events.

4.2. Start to search, $n^i_{ns/s}$

Vehicles start to search after driving a distance of $l_{ns/s}$ (since they enter the area). $l_{ns/s}$ can be fixed or drawn out of a given probability distribution function. Here, for simplicity, we assume it is fixed. The expression then for the number of vehicles starting to search for parking during time slice i is written as Eq. 11 and explained below.

$$n^i_{ns/s} = \sum_{i'=1}^{i-1} \underbrace{(1-\beta^{i'}) \cdot n^{i'}_{/ns}}_{\text{term 1}} \cdot \underbrace{\gamma^{i'}_{ns/s}}_{\text{term 2}} \tag{11}$$

$$\gamma^{i'}_{ns/s} = \begin{cases} 1, & \text{if } l_{ns/s} \leq \sum_{j=i'}^{j=i-1} d^j \text{ and } \sum_{j=i'}^{j=i-1} d^j \leq l_{ns/s} + d^{i-1}. \\ 0, & \text{if otherwise.} \end{cases}$$

$n^i_{ns/s}$ may consist of vehicles that entered the area in any time slice between 1 and $i-1$. Use i' to denote such a time slice, $i' \in [1, i-1]$ (notice that vehicles that enter the area in time slice i are not included as they already experience one transition event during this time slice). In time slice i', $n^{i'}_{/ns}$ vehicles entered the area. Term 1 in Eq. 11 represents the portion of those which need to park (i.e., all vehicles except through traffic). Term 2 is a binary variable (0 or 1) indicating whether these vehicles start to search for parking in time slice i. For $\gamma^{i'}_{ns/s}$ to be equal to 1, two conditions must be satisfied: the vehicles have driven enough distance to start searching, and they have not started the search before.

4.3. Access parking, $n^i_{s/p}$

After drivers start searching, their driving time/distance is not identical anymore (during the searching state). It depends on the current conditions (i.e., the density of available parking spaces, the density of searchers and traffic performance) and their luck finding an available parking spot (their own location, that of the available parking spots and the competitors). To find each vehicles' driving time/distance, one needs to record the location of all the cars and parking spots throughout the different time slices, this requires lots of additional details/efforts. However, we do not address who takes which parking space, but only the average number of travellers that access parking. Therefore, these efforts are saved. In other words, the model does not provide information about which vehicle parked, or which parking space was taken, or how far each vehicle drove before finding parking. We do know, however, the average number of vehicles that found parking spaces during this time slice, and the total/average searching distance driven during this time slice.

At the beginning of each time slice, the number of available parking spots and the number of parking searchers are found based on the matrix. These numbers are recorded over time. However, their locations are not tracked. The following two assumptions are used in the model: First, at the beginning of each time slice, the locations of the available parking spots are random. Second, at the beginning of each time slice, the locations of parking searchers are uniformly distributed on the network. The first assumption represents the stochasticity of the parking availability. The second assumption guarantees that the demand is homogeneously generated.

The second assumption is necessary, as during each time slice, the model provides an average amount of parking spots being taken. This average value only stands for a condition where, more or less, all searchers are uniformly distributed on the network. This, evidently, limits the model. For example, if in reality all searchers focus in one street where parking spots are scarce while parking spots are available somewhere else, then the model would most likely overestimate the real value of the amount of parking spots being taken. However, it does provide us an idea how the traffic might behave under general conditions where the parking demand (vehicles that are searching for parking) is homogenously distributed within the overall driving traffic. We are interest on whether there is on average at least one car that is able to take an available parking spot. This, does not require us to know exactly the actual location of each car.

Notice that, at the beginning of each time slice, the locations of parking searchers and available parking spots are reset (independently from the previous time slice). This guarantees that we obtain the average number of vehicles that find parking, without being influenced by the randomness of vehicles' location at that specific time.

To find then the value of $n^i_{s/p}$, we define three different scenarios based on the relation between d^i, s^i and L.

Scenario 1: if $d^i \in [0, s^i]$.

Under this scenario, the maximum driven distance of a vehicle is shorter than the spacing between two consecutive vehicles. Therefore, no two vehicles' trajectories will ever overlap during a single time slice. As a result, a parking spot can be visited at most by one car (recall Eq. 9).

Assume a parking spot is located at x and the rest $A^i - 1$ parking spots are located at x_r, for $r \in \{1, 2, ..., A^i - 1\}$. The searching vehicles' initial positions are x_c, for $c \in \{1, 2, ..., N^i_s\}$. Then, there are two conditions to guarantee that this parking spot at location x becomes occupied during time slice i.

- First, the parking spot must be within the reach of a car, i.e., $x \in [x_c, x_c + d^i]$ for any $c \in \{1, N^i_s\}$. The probability of that is $\sum_{c=1}^{c=N^i_s} \int_{x_c}^{x_c+d^i} \frac{1}{L} dx$.
- Second, there is no other parking spots between the car at location x_c and this parking spot at location x, i.e., $x_r \notin [x_c, x]$ for $r \in \{1, A^i - 1\}$. The probability of that is $\prod_{x_r=1}^{A_i-1} \left(1 - \int_{x_c}^{x} \frac{1}{L} dx_r\right)$.

Therefore, the probability of a random parking spot been taken during time slice i is the product of these two probabilities. As this is the same for all parking spots, the average number of parking spots been taken during time slice i equals to A^i times the product of the two probabilities detailed above; it is written as Eq. 12. A simplified equation for this scenario is written in Eq. 19.

$$\text{if } d^i \in [0, s^i], \quad n^i_{s/p} = A^i \cdot \sum_{c=1}^{c=N^i_s} \int_{x_c}^{x_c+d^i} \frac{1}{L} dx \cdot \prod_{x_r=1}^{A_i-1} \left(1 - \int_{x_c}^{x} \frac{1}{L} dx_r\right) \tag{12}$$

Scenario 2: if $d^i \in (s^i, L)$.

Under this scenario, vehicles' trajectories can overlap and a parking spot can be visited by more than one car (although it only accommodates the first one).

Assume a parking spot is located at x and the rest $A^i - 1$ parking spots are located at x_r, for $r \in \{1, 2, ..., A^i - 1\}$. The searching vehicles' initial positions are x_c, for $c \in \{1, 2, ..., N^i_s\}$.

To formulate the probability of this parking spot at location x been taken during time slice i, we define three sub-scenarios. They are based on the relation between A^i (i.e., the number of available parking spots) and m^i (i.e., the maximum number of searching vehicles that can pass by a spot).

- Sub-scenario 2.1: if $m^i > A^i$

Since in this scenario $d^i < L$, then according to Eq. 8 and 9, $N^i_s \geq m^i$. Therefore, there is more parking demand than supply ($N^i_s > A^i$). Recall also that any parking spot on the network could be potentially visited by $m^i - 1$ cars ($m^i - 1 \geq A^i$). Therefore, any available parking spot will be taken by one of these cars, as there are simply too many cars searching and they drive a distance that is long enough to reach all available parking spots. Hence, all the parking spots will be taken, and still some vehicles will remain searching at the end of the time slice. $n^i_{s/p}$ is written as Eq. 13.

$$\text{if } d^i \in (s^i, L) \text{ and } m^i > A^i, \quad n^i_{s/p} = A^i \tag{13}$$

- Sub-scenario 2.2: if $m^i = A^i$

Still, we use x as the location of the considered parking spot. As defined before,

- If $x \in [x_c, x_c + d^i_r]$, a number of m^i cars (A^i in this sub-scenario) could drive by that parking spot at x. If a parking spot is located within this area, it will be taken (see theory described in scenario 1). Thus, the probability of a parking spot located within this range and been taken is $\sum_{c=1}^{c=N^i_s} \int_{x_c}^{x_c+d^i_r} \frac{1}{L} dx$.

- If $x \in [x_c + d_r^i, x_c + s^i]$, a number of $m^i - 1$ cars ($A^i - 1$ in this sub-scenario) could drive by that parking spot at x. Denote $p_{f(n=m^i-1)}$ as the probability of this parking spot not being taken, i.e., the probability that all the cars that could reach location x park before arriving at x. Thus, the probability of a parking located within this range and been taken is $\sum_{c=1}^{c=N_s^i} \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i-1)}\right) dx$.

Combining these two ranges of x, for sub-scenario 2.2, the probability of a parking spot been taken can be written as $\sum_{c=1}^{c=N_s^i} \left\{ \int_{x_c}^{x_c+d_r^i} \frac{1}{L} dx + \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i-1)}\right) dx \right\}$. Since there are a number of A^i parking spots, $n_{s/p}^i$ can be written as Eq. 14.

$$\text{if } d^i \in (s^i, L) \text{ and } m^i = A^i, \quad n_{s/p}^i = A^i \cdot \sum_{c=1}^{c=N_s^i} \left\{ \int_{x_c}^{x_c+d_r^i} \frac{1}{L} dx + \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i-1)}\right) dx \right\} \tag{14}$$

where

$$p_{f(n)} = \underbrace{\sum_{z_n=n}^{A^i-1} C_{A^i-1}^{z_n} \cdot \left(\int_{-(n-1)s^i}^{x} \frac{1}{L} dx \right)^{z_n} \cdot \left(1 - \int_{-(n-1)s^i}^{x} \frac{1}{L} dx \right)^{A^i-1-z_n}}_{\text{term 1}} \cdot \underbrace{\prod_{j=n-1}^{1} p_{f_j}}_{\text{term 2}} \tag{15}$$

$$p_{f_j} = \sum_{z_j=j}^{z_{j+1}} C_{z_{j+1}}^{z_j} \cdot \left(\frac{\int_{-(j-1)s^i}^{x} \frac{1}{L} dx}{\int_{-j \cdot s^i}^{x} \frac{1}{L} dx} \right)^{z_j} \cdot \left(1 - \frac{\int_{-(j-1)s^i}^{x} \frac{1}{L} dx}{\int_{-j \cdot s^i}^{x} \frac{1}{L} dx} \right)^{z_{j+1}-z_j} \tag{16}$$

In Eq. 15, n stands for the number of vehicles that can potentially reach x. Within these n cars, the probability that the furthest vehicle (to x) parks before it arrives at x is shown in term 1; the probability that the rest $n - 1$ vehicles all park before they arrive at x is shown in term 2. A simplified equation of $n_{s/p}^i$ for this sub-scenario is written in Eq. 19.

- Sub-scenario 2.3: if $m^i < A^i$
Similar to sub-scenario 2.2, we define two ranges of x .

- If $x \in [x_c, x_c + d_r^i]$, a number of m^i cars could drive by that parking spot at x. Thus, the probability of a parking located within this range and been taken is $\sum_{c=1}^{c=N_s^i} \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i)}\right) dx$.
- If $x \in [x_c + d_r^i, x_c + s^i]$, a number of $m^i - 1$ cars could drive by that parking spot at x. Thus, the probability of a parking spot located within this range and been taken is the same as that defined in sub-scenario 2.2, i.e., $\sum_{c=1}^{c=N_s^i} \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i-1)}\right) dx$.

Combining these two ranges of x, for sub-scenario 2.3, the probability of a parking spot been taken is written as $\sum_{c=1}^{c=N_s^i} \left\{ \int_{x_c}^{x_c+d_r^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i)}\right) dx + \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i)}\right) dx \right\}$. Since there are a number of A^i parking spots, $n_{s/p}^i$ can be written as Eq. 17. A simplified equation of $n_{s/p}^i$ for this sub-scenario is written in Eq. 19.

$$\text{if } d^i \in (s^i, L) \text{ and } m^i < A^i, \quad n_{s/p}^i = A^i \cdot \sum_{c=1}^{c=N_s^i} \left\{ \int_{x_c}^{x_c+d_r^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i)}\right) dx + \int_{x_c+d_r^i}^{x_c+s^i} \frac{1}{L} \cdot \left(1 - p_{f(n=m^i)}\right) dx \right\} \tag{17}$$

Scenario 3: if $d^i \in [L, \infty)$
Under this scenario, each car can drive around the whole network at least once, so all cars will park if there are enough parking spots. Otherwise, all spots will be taken. The result is written as Eq. 18.

$$\text{if } d^i \in [L, \infty), \quad n_{s/p}^i = min\{A^i, N_s^i\} \tag{18}$$

The expression of $n_{s/p}^i$ for all scenarios described above is written as Eq. 19. For the convenience of the reader, here some of the equations have been further simplified with respect to what has been shown before for the description of each scenario.

$$n^i_{s/p} = \begin{cases} N^i_s \cdot \left[1 - (1 - \frac{d^i}{L})^{A^i}\right] & \text{, if } d^i \in \left[0, s^i\right] \\ \begin{cases} A^i & \text{, } if\ m^i > A^i \\ A^i \cdot (1 - \frac{N^i_s}{L^A} \cdot p_2) & \text{, } if\ m^i = A^i \\ A^i \cdot (1 - \frac{N^i_s}{L^A} \cdot p_1 - \frac{N^i_s}{L^{A^i}} \cdot p_2) & \text{, } if\ m^i < A^i \end{cases} & \text{, if } d^i \in \left(s^i, L\right) \\ min\{A^i, N^i_s\} & \text{, if } d^i \in [L, \infty) \end{cases} \tag{19}$$

Where

$p_1 = \sum_{i_m=m}^{A^i-1} \cdot C^{i_m}_{A^i-1} \cdot \left[\prod_{j=m-1}^{1}\left(\sum_{i_j=j}^{i_{j+1}} \cdot C^{i_j}_{i_{j+1}}\right)\right] \cdot \frac{L}{N^i_s}^{(i_m-i_1)} \cdot \int_0^{v^i \cdot t-(m-1)\frac{L}{N^i_s}} [(N^i_s - m + 1)\frac{L}{N^i_s} - x]^{A^i-1-i_m} \cdot x^i_1 dx$

$p_2 = \sum_{i_{m-1}=m-1}^{A^i-1} \cdot C^{i_{m-1}}_{A^i-1} \cdot \left[\prod_{j=m-2}^{1}\left(\sum_{i_j=j}^{i_{j+1}} \cdot C^{i_j}_{i_{j+1}}\right)\right] \cdot \frac{L}{N^i_s}^{(i_{m-1}-i_1)} \cdot \int_{v^i \cdot t-(m-1)\frac{L}{N^i_s}}^{\frac{L}{N^i_s}} [(N^i_s - m + 2)\frac{L}{N^i_s} - x]^{A^i-1-i_{m-1}} \cdot x^i_1 dx$

4.4. Depart parking, $n^i_{p/ns}$

As we know the number of vehicles accessing parking in all former time slices, we can find $n^i_{p/ns}$ based on the distribution of parking durations (an input to the model). Eq. 20 shows the number of vehicles that depart parking in time slice *i*.

$$n^i_{p/ns} = \sum_{i'=1}^{i-1} n^{i'}_{s/p} \cdot \int_{(i-i')\cdot t}^{(i+1-i')\cdot t} f(t_d)\, dt_d \tag{20}$$

$n^i_{p/ns}$ may consist of vehicles that accessed parking in any time slice between 1 and $i-1$. Use i' to denote such time slice, $i' \in [1, i-1]$. Notice that the vehicles that access parking during time slice *i* are not included, as they already experience one transition event during this time slice. The number of vehicles that accessed parking in time slice i' is $n^{i'}_{s/p}$. The probability that these vehicles depart parking in time slice *i* equals to the probability of the parking duration being between $(i - i') \cdot t$ and $(i + 1 - i') \cdot t$, i.e., $\int_{(i-i')\cdot t}^{(i+1-i')\cdot t} f(t_d)\, dt_d$.

Some distributions are more suitable to describe parking duration than others, see Richardson (1974), Lautso (1981) and Cao et al. (2013); although theoretically, any distribution can be used, e.g., negative binomial, poisson. Eq. 20, therefore, remains general enough to fit any distribution for describing parking duration in this model.

4.5. Leave the area, $n^i_{ns/}$

Vehicles leave the area after they drive for a given distance. The starting point for counting that distance corresponds to the moment they enter the area (if they are through traffic and do not park in the area), or the moment they depart the parking facilities. For these two cases, the required distances are $l_/$ and $l_{p/}$, respectively. They can be fixed values, or values drawn out of any given probability distribution function. Here, for simplicity, we assume they are fixed. Eq. 21 shows the number of vehicles that leave the area during time slice *i*.

$$n^i_{ns/} = \sum_{i'=1}^{i-1} \left(\beta^{i'} \cdot n^{i'}_{/ns} \cdot \gamma^{i'}_/ + n^{i'}_{p/ns} \cdot \gamma^{i'}_{p/}\right) \tag{21}$$

where

$$\gamma^{i'}_/ = \begin{cases} 1, & \text{if } l_/ \leq \sum_{j=i'}^{j=i-1} d^j \ and\ \sum_{j=i'}^{j=i-1} d^j \leq l_/ + d^{i-1} \\ 0, & \text{if otherwise.} \end{cases}$$

$$\gamma^{i'}_{p/} = \begin{cases} 1, & \text{if } l_{p/} \leq \sum_{j=i'}^{j=i-1} d^j \ and\ \sum_{j=i'}^{j=i-1} d^j \leq l_{p/} + d^{i-1} \\ 0, & \text{if otherwise.} \end{cases}$$

As shown in Eq. 21, $n^i_{ns/}$ consists of two parts, the vehicles that leave the area without parking, i.e., $\beta^{i'} \cdot n^{i'}_{/ns}$ for all time slices $i' \in [1, i-1]$ (through traffic); and the vehicles that leave the area after they have parked, i.e., $n^{i'}_{p/ns}$ for all time slices $i' \in [1, i-1]$. $\gamma_l^{i'}$ and $\gamma_{pl}^{i'}$ are binary variables (0 or 1) indicating whether these two groups of vehicles leave the area in time slice i. $\gamma_l^{i'}$ and $\gamma_{pl}^{i'}$ are equal to 1 if the vehicles that transitioned into the "non-searching" state in time slice i' have reached (during time slice $i-1$) the given distance required to leave the area.

5. Applications

In this section, a numerical example is provided to illustrate how to build the transition matrix and exploit the useful information it provides. Additionally, we use the numerical example to test different parking policies and evaluate their influence on the traffic system.

5.1. Numerical example

The total travel demand contains 200 trips. Each time slice lasts for 1 minute, i.e., t=1 min.

The entry time of the vehicles to the area obeys a gamma distribution, where the average arrival time is 20 minutes after the observation period starts (more precisely, the shape parameter is 4 and the scale parameter is 5). The parking durations also obey a gamma distribution, where the average duration is 10 minutes (more precisely, the shape parameter is 2 and the scale parameter is 5). Notice that, the application of the analytical model is not limited to a specific distribution, other distributions besides gamma could also be assumed. Although the model is suitable for cases with through traffic, for simplicity and due to space constraints, we assume there is no through traffic (i.e., $\beta^i = 0, \forall i$). Other inputs include: $L = 1$ km; $A = 21$ spaces; $v = 30$ km/h; $k_c = 60$ veh/km/lane; $k_j = 150$ veh/km/lane; $Q_{max} = 1800$ veh/h/lane; $l_{ns/s} = 0.5$ km and $l_{pl} = 0.5$ km. Note that for this specific case, $A = 21$ is the smallest integer where gridlock conditions are avoided, larger values are tested in section 5.2.

Section 5.1.1 provides some important outputs of the model/matrix. Section 5.1.2 explains these outputs and how to use them to better understand and evaluate the interactions between the urban parking and traffic systems.

5.1.1. Transition matrix and resulting queuing diagram

A queuing diagram is shown in Figure 4 with five curves, indicating the cumulative number of vehicles going through each parking-related transition event.

As mentioned before, the vertical distance between each pair of consecutive curves is the number of vehicles in a state (notice that the non-searching state consists of two parts). Also, the area between two consecutive curves is the total time vehicles spend within that state. The area between the curves "enter the area" and "start to search" is the total time vehicles spend within the network before they start to search for parking; the area between the curves "depart parking" and "leave the area" is the total time vehicles spend within the network after they depart their parking spots. The sum of these two areas constitutes the total time vehicles spend in the "non-searching" state. If there is no congestion and the average travel speed remains v, this area would equal to 400 vehicle-minutes (as each vehicle

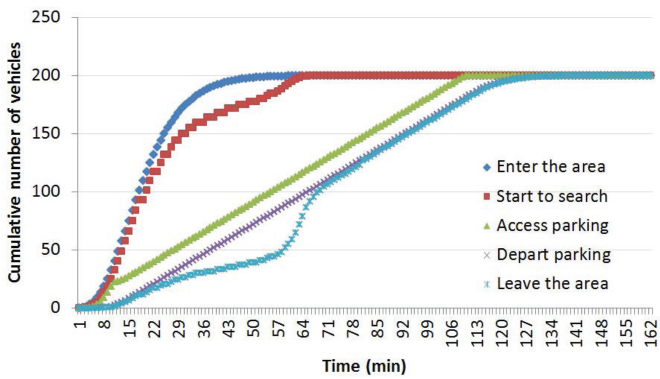

Fig. 4. Queuing diagram of vehicles in the area (numerical example).

would drive for a period of $\frac{l_{ns/s}+l_{p/}}{v}$ = 2 min). If there is congestion, then this area could be larger due to a lower travel speed. The obtained values are listed in Table 4.

Table 4. Average time vehicles spend in non-searching and searching states (numerical example).

State	Total time (vehicle-minutes)	Average time per vehicle (minutes)	Average delay per vehicle (minutes)
Non-searching state	2100	10.5	8.5
Searching state	6180	30.9	30.9
Total	8280	41.4	39.4

As shown in Table 4, the average time spent by a vehicle in the non-searching state is 10.5 minutes, i.e., a total time of 2100 vehicle-minutes. This contains an average delay (during non-searching state) of 8.5 minutes, i.e., a total delay of 1700 minutes (i.e., 28.3 hours).

Moreover, while searching, each vehicle spends on average 30.9 minutes. This is, evidently, an extreme case and not very realistic, as drivers spend three times longer searching for parking than actually parked. However, it illustrates the potential negative effects that a limited parking supply can have on the traffic system if the demand is not altered. In total, each vehicle is delayed for 39.4 minutes during driving (within the non-searching and the searching states).

Figure 5 shows both the proportion of traffic searching for parking, and the parking occupancy over time. It can be seen that the values of both of them are high for a large portion of the observation period. The peak of the parking occupancy starts earlier than that of the share of traffic searching for parking, indicating the causal relationship. Notice that once the parking occupancy reaches 100%, it stays there for most of the observation period, indicating that vacated parking spots get filled with new searchers right away within the same time slice. This is not surprising, as in this example the demand for parking is much larger than the supply.

5.1.2. Interactions between parking and traffic systems

In this section we illustrate these interactions with the results of the numerical example, e.g., the correlation between traffic speed, total driving traffic (non-searching and searching), and the total driven distance. These three values are shown in Figure 6.

(a)

(b)

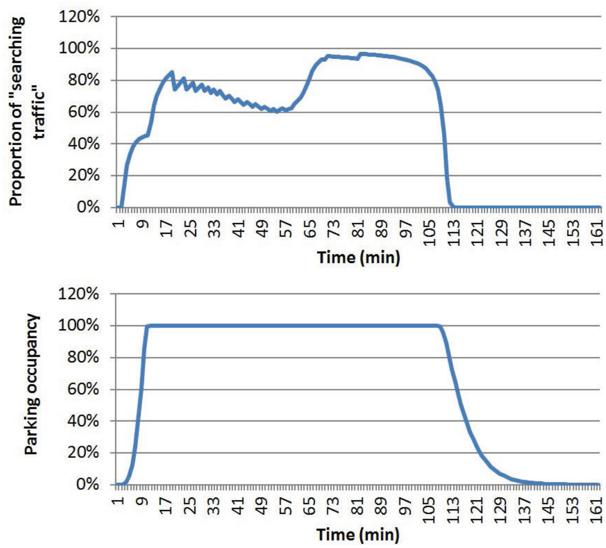

Fig. 5. (a) Proportion of traffic "searching for parking" over time. (b) Parking occupancy over time (numerical example).

(a) (b) (c)

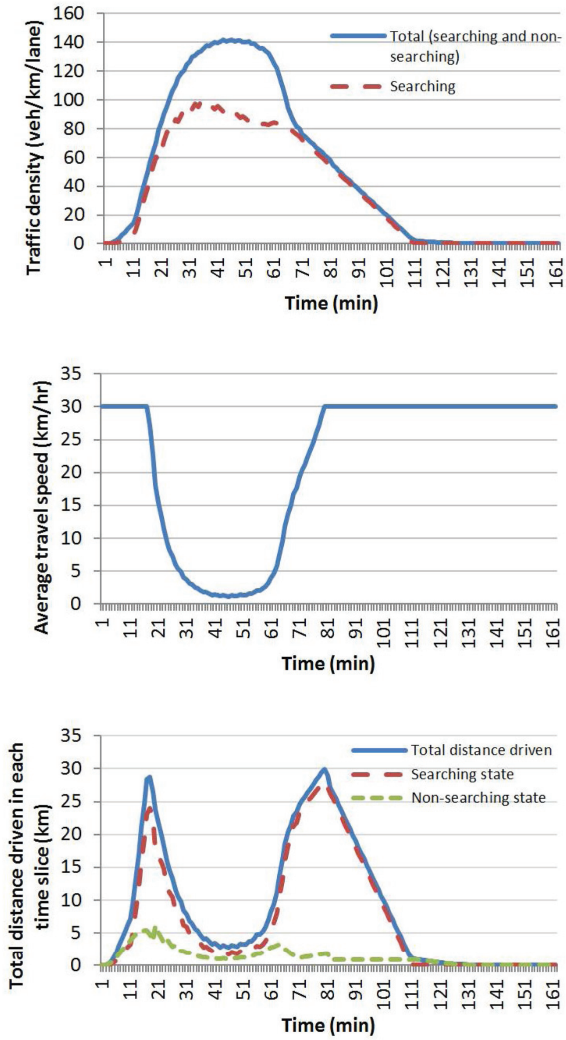

Fig. 6. Values of indicators for the traffic conditions. (a) Traffic density on the road network over time; (b) Average travel speed over time; (c) Total distance driven by vehicles over time (numerical example).

As seen in Figures 6 (a) and 6 (b), traffic congestion occurs between minutes 18 and 80 (i.e., the traffic density is higher than k_c and the average speed is lower than v). Additionally, near-gridlock conditions are reached between minutes 37 and 55 as the speed drops below 2 km/h.

Figure 6 (a) shows a continuous growth of traffic density before minute 40, there are three reasons contributing to this. First, the parking system starts to saturate, thus vehicles take longer to find a parking space (i.e., they spend more time in the “searching” state). Second, as the traffic becomes more congested, the vehicles can drive a smaller distance within a time slice, and this also influences their ability to find parking. Third, the distance vehicles can drive within a time slice becomes smaller, vehicles in the “non-searching” state (after parking) need a longer time to leave the area. Notice that the congestion also reduces the number of vehicles transitioning between the states of “non-searching” and “searching”; this influences the traffic composition, but it does not affect the overall traffic density.

Figure 6 (c) shows the total distance driven within each time slice. There are two peaks on this curve, the first one occurs approximately at minute 18 and the second one occurs approximately at minute 80. Notice that these two times correspond to the moments when the average travel speed starts to drop from 30 km/h, and when it reaches back 30 km/h, respectively. Before the first peak and after the second peak, the average travel speed remains at the maximum level, and the curve of the total driven distance follows the same pattern as that of the traffic density. Between the two peaks, the traffic density reaches the critical density, leading to a rather low speed across the system. Then the speed changes become more significant than the density changes, and the curve of the total driven distance follows the same pattern as that of the travel speed(Figure 6 (b)).

Table 5 shows the average and total driven distance within the non-searching and the searching states.

Table 5. Total driven distance and average driven distance per vehicle (numerical example).

State	Total driven distance	Average driven distance
Non-searching state	219 km	1.1 km/veh
Searching state	1175 km	5.9 km/veh
Total	1394 km	7.0 km/veh

Recall that vehicles can only transition into the next state at the end of each time slice (not immediately); therefore, the average driven distance for non-searching vehicles is 1.1 km/veh, slightly higher than that assumed (i.e., $l_{ns/s}$+$l_{p/}$=1 km). Nevertheless, the distance driven by non-searching vehicles constitutes a small portion of the total driven distance. The average driven distance of searching vehicles is 5.9 km/veh (over four times more than that of non-searching vehicles).

Overall, vehicles drive 1394 km (recall that the size of the network is 1km and there are only 200 trips). This distance can be used to measure energy consumption, air pollution, and other externalities caused by parking issues.

5.2. Assessment of alternative parking policies

In the numerical example, the traffic problems observed are highly related to the parking supply. Hence, in this section, we test two sets of parking policies and compare them by quantifying their effects on the total system delay and driven distance. The two sets of policies are (1) increasing the parking supply, and (2) limiting the maximum parking duration. They are independent of each other. Notice that the specifics of these policies as tested here are very simplistic (e.g., a 10 minute maximum parking duration is not realistic for most networks). However, they are only used to illustrate the effects that such policy types may have on traffic, and not to draw specific conclusions about their optimal values.

The results indicate that a proper time control scheme can highly improve the system without enlarging parking supply, which is typically harder to implement as it is a more expensive and controversial policy.

Increasing the parking supply

A1: provide 22 parking spaces instead of 21.

A2: provide 23 parking spaces instead of 21.

Limiting the maximum parking duration

B1: the longest parking duration is 20 minutes. Vehicles who wish to park shorter than 20 minutes are not affected. Vehicles who wish to park longer than 20 minutes have to leave at the end of the 20 minutes maximum parking duration.

B2: the longest parking duration is 10 minutes. Vehicles who wish to park shorter than 10 minutes are not affected. Vehicles who wish to park longer than 10 minutes have to leave at the end of the 10 minutes maximum parking duration.

Table 6 shows the comparison between the original conditions and policies A1, A2, B1 and B2. Values within parenthesis indicate the percentage change driven by the different policies with respect to the original conditions.

Not surprisingly, it can be seen that the non-searching time, searching time and delay per vehicle can be reduced both by increasing the parking supply and by limiting the maximum parking duration. Also, stronger policies are more effective at reducing delays, i.e., A2 reduces delay by an additional 7% compared to A1; B2 reduces delay by an additional 29% compared to B1.

Interestingly, not all the policies reduce the total driven distance. As a matter of fact, compared to the original conditions, policies A1 and A2 result in longer distances (3.9% and 3.8% longer respectively) despite reducing the

Table 6. Traffic effects of different parking policies (numerical example).

Policy	Non-searching time (min/veh)	Searching time (min/veh)	Delay (min/veh)	Total driven distance (km)
Original	10.5	30.9	39.4	1394
A1 (22 parking spaces)	7.9 (-24.8%)	29.5 (-4.5 %)	35.4 (-10.2 %)	1449 (+3.9 %)
A2 (23 parking spaces)	6.7 (-36.2%)	27.9 (-9.7 %)	32.6 (-17.3%)	1447 (+3.8 %)
B1 (20 minutes maximum)	9.7 (-7.6 %)	29.3 (-5.2 %)	37.0 (-6.1%)	1365 (-2.1 %)
B2 (10 minutes maximum)	5.4 (-48.6%)	22.2 (-28.2%)	25.6 (-35.0%)	1295 (-7.1 %)

delay. This is important to notice as it highlights the need for estimating/optimizing multiple traffic metrics (besides delay) when evaluating different parking policies from the traffic prospective.

To understand these seemingly controversial results, Figure 7 is provided to show the traffic density, the average travel speed and the total driven distance over time based on the tested policies. Figure 7 (a) compares policies A1 and A2 to the original conditions; Figure 7 (b) compares policies B1 and B2 to the original conditions.

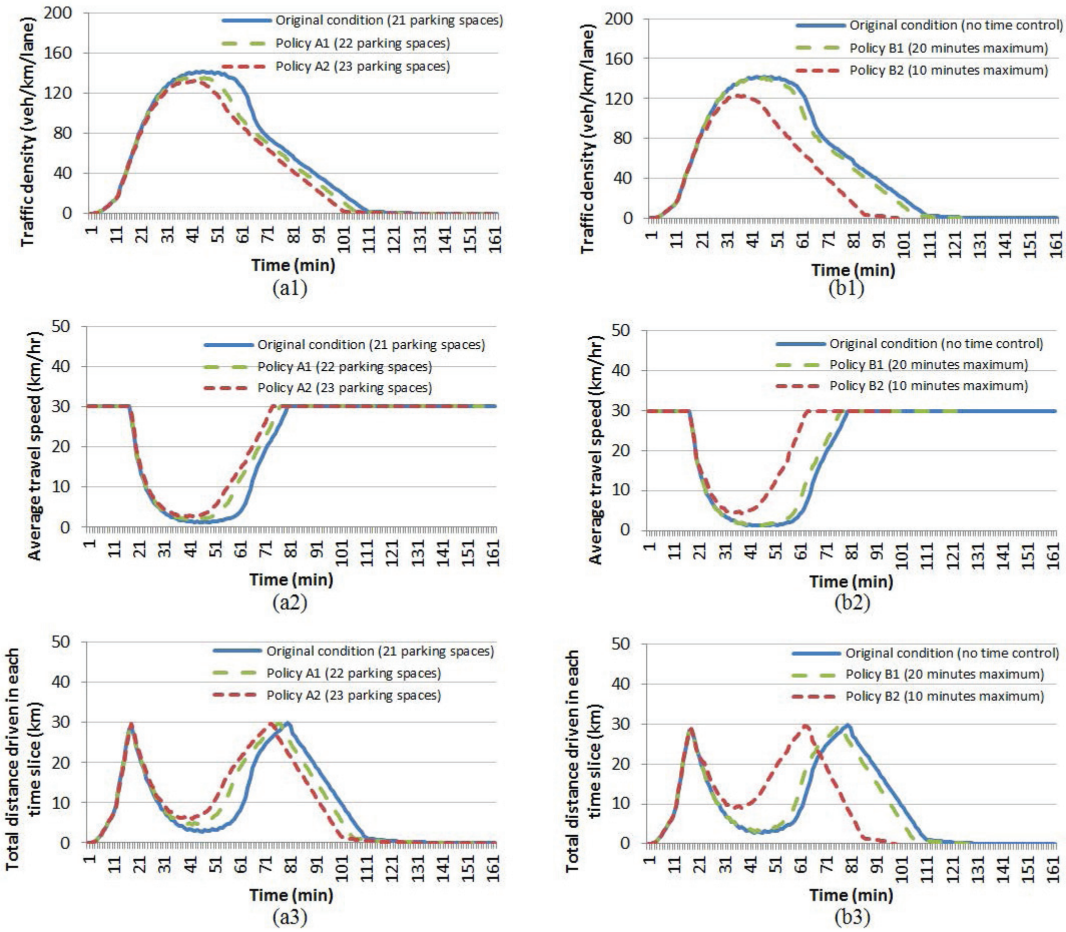

Fig. 7. Traffic conditions obtained from the transition matrix based on different parking policies, including the traffic density, the travel speed and the total driven distance in each time slice. (a) Policies A1, A2 and original conditions (b) Policies B1, B2 and original conditions (numerical example).

As shown in Figure 7 (a3) and (b3), the values of total driven distance in some time slices are larger than for the original conditions whereas for other time slices they are smaller. Increases in driven distance are caused by the higher average speeds on the network (Figure 7 (a2) and (b2)), and the still relatively high traffic density (Figure 7 (a1) and (b1)). Reductions in driven distance happen when congestion disappears earlier.

For policies A1 and A2, the increased driven distance during the congested period is larger than the saved driven distance after congestion (notice that the congested period finishes only a couple of minutes earlier compared to the original conditions). Even though the speed is higher than with the original conditions, the ability of vehicles to find parking spots (during congestion) is kept low as the parking occupancy is high. In other words, these drivers take the same time to find parking, but since the speed is faster than with the original conditions, they drive longer distances.

On the contrary, for policies B1 and B2, the increased driven distance during the congested period is smaller than the saved driven distance after congestion. Therefore, these two policies are more effective in reducing both the delay and the total driven distance in this case.

These findings are relevant, as they highlight the importance of estimating multiple metrics when designing or evaluating new parking policies. A good parking policy should be aiming to (at least) not only enhance the traffic performance but also the total driven distance, and this is not automatically achieved, as these two metrics might react to the policy in very opposite directions.

6. Conclusions

In this study, we develop a macroscopic model to analyze the interactions between the urban parking and traffic systems. Based on the transition matrix of vehicles between different parking-related states within the urban area, a queuing diagram can be provided. This can show the cumulative number of vehicles that go through each parking-related transition event as a function of time, as well as the number of vehicles within each state at any given time. The model can also furnish other traffic related metrics, such as total time vehicles spend in each state, total distance driven, and total delay.

The whole framework/model provides a new perspective for looking at parking systems and their interactions with traffic. Below we highlight some advantages of the model:

1. In comparison to microscopic models or MA simulation tools which are typically used when analyzing parking-caused traffic issues, the macroscopic model proposed here has several advantages.

   - The model has very little data requirements, while most of the tools used nowadays to analyze parking-related traffic require a lot of detailed data that is really hard to get. Our model, on the other hand, is macroscopically built and only needs some general inputs, distributions and probability theory.
   - This macroscopic model allows us to compute the results without the use of complex simulations, as it can be easily solved with a simple numerical solver such as excel or matlab. This is in part possible because we only have a few parameters, and all of them have a physical interpretation. Moreover, they can all be obtained from field data. In addition, there is no need to run the model many times in order to account for its stochasticity, as it is based on probability functions (i.e., the stochasticity is already implicit within the model formulations).
   - The simpler form of the macroscopic model might provide additional insights that cannot be delivered by microscopic models (e.g., insights into the mathematical relation between parking availability and traffic speeds).

2. The proposed model represents a dynamic system, where the time-varying conditions can be considered, and the time-based results or average values across time can be found. This gives it clear planning and operational applications (e.g., provide short term forecasting of traffic conditions based on the parking system, or parking usage based on traffic conditions; evaluate the total/average effect of different parking policies onto the traffic system over time and vice versa).

   - The model provides the proportion of vehicles looking for parking on an urban network. In reality, vehicles looking for parking are hard to distinguish from normal driving vehicles, hence significant investments must be made to collect empirical data through the use of GPS and other devices. This model, however, provides some analytical results, which could be very helpful for cities to estimate their parking search condition with very limited investment.

- The model provides a method to find the influence of parking searchers (or the parking system) on the non-searching vehicles (e.g., through traffic). This is interesting, as city governments and individual travelers often do not realize that parking can be a source of traffic jams (general congestion instead of a distinctive bottleneck). To this end, the model helps to detect the portion of traffic congestion which is caused by parking issues (i.e., detect parking-caused problems) as well as the magnitude of such negative effects.
- The model provides the total distance driven on the network, including the extra distance driven due to the search for parking. Even if there is no traffic jam, considering the same amount of trips, the longer the distance travelled, the worse it is for the environment (i.e., more air pollution). Based on this model, this part can be estimated as well, and further taken into account for policies such as pricing, etc.
- The model provides new insights and tools to evaluate the performance of parking systems over time (i.e., considering dynamic conditions). In other words, it also provides new aspects for parking systems to consider and new goals for them to reach when they are being planned and designed. Eventually, it can assess and assist parking provision such as dynamic pricing schemes and time control policies, to avoid the traffic deterioration caused by parking systems.

Overall, the usage/application of the proposed model is far beyond what we have illustrated in the numerical example. The model can provide the relation between the proportion of through traffic, the traffic conditions, and the likelihood of vehicles to access parking, for example. This is not included in the paper as the through traffic was assumed to be zero for simplification purposes. Also, the values of the driven distances needed for certain transition events (i.e., $l_n s/s$, $l_/$, and $l_p/$) can be generated by distributions to better duplicate reality, instead of using fixed values. In addition, the off-street parking facilities can be modeled more explicitly, such as multiple parking spots at the same location; so the network can be easily expanded to include different kinds of parking supplies. All of these extensions, although not directly presented here, can be achieved easily based on the current model. The future research work could incorporate, however, a non-homogeneous environment (e.g., where both, the parking demand and supply are inhomogeneously distributed temporally and spatially); and incorporate different adjacent networks, where parking decisions can be made based on the conditions of more than one network. Also, in future studies, the bottleneck and delays caused by on-street parking maneuvers to traffic flow will be taken into account as well based on some other studies by the authors (Cao and Menendez (2014; 2015)).

In summary, the proposed model, despite its simplicity, can be used to efficiently evaluate the urban traffic system macroscopically. The parking-state-based transition matrix for traffic can be used to estimate both, how parking availability can affect traffic performance (e.g., average time searching for parking, number of cars searching for parking); and how different traffic conditions (e.g., travel speed, density in the system) can affect drivers' ability to find parking. Moreover, the proposed model can be further exploited to study multiple strategies or scenarios for traffic operations and control, transportation planning, land use planning, or parking management and operations (e.g., evaluation of parking time/pricing controls, location and number of parking stalls). The numerical example in the paper, in spite of being rather simple, shows very optimistic results for the use of the proposed model. It provides an idea on how different parking policies can affect traffic in the short-term. It is also evident from the presented results that multiple traffic metrics should be considered when studying the potential impacts of parking policies, e.g., a given policy can reduce traffic delay (i.e., by increasing the travel speed), but simultaneously increase total distance travelled. This is a very interesting fact, and relevant as well, as it shows the importance of considering the total driven distance as an indicator of the policy/system, in addition to the direct traffic indicators such as speed. This could guide policy makers into a more sustainable direction, rather than short-sighted decisions which could be detrimental to the environment.

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