On the Landau–Ginzburg theory of MAG projected $SU(2)$ lattice gauge theory

Kurt Langfeld, Hugo Reinhardt [1]

Institut für Theoretische Physik, Universität Tübingen, D-72076 Tübingen, Germany

Abstract

Maximal Abelian gauge fixing and subsequent Abelian projection of $SU(2)$ lattice gauge theory defines closed trajectories of magnetic monopoles. These trajectories can be interpreted in terms of an effective scalar field theory of the MAG monopoles using the world line representation of the functional determinants. Employing the monopole world lines detected in the numerical simulation, we show that a scalar bound state exists. The screening mass m of this state properly scales towards the continuum limit. We find $m \approx 1.4$ GeV when the string tension $\sqrt{\sigma} = 440$ MeV is used as reference scale.
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1. Introduction

Lattice calculations [1–7] performed in the so-called "Abelian gauges" [8,9] have provided evidence that a condensate of magnetic monopoles exist in the Yang–Mills vacuum. Consequently, the vacuum expels color-electric flux by virtues of the dual Meissner effect and produces confinement. In this sense, the vacuum represents a dual superconductor. On a phenomenological level a superconductor can be described by a Ginzburg–Landau theory. There have been attempts to construct the pertinent dual Ginzburg–Landau theory for the QCD vacuum or to extract it from lattice gauge simulations [10–13]. The difficulty seems to consist in the mapping of the monopole degrees of freedom to the one of the scalar Ginzburg–Landau field.

The Ginzburg–Landau theory describes a complex scalar field interacting with an electromagnetic gauge field. The (dual) Abelian electromagnetic field can be, in principle, be integrated out yielding an effective theory of a complex self-interacting scalar field. In this Letter, we will extract the effective scalar field theory describing the dual superconductor of the QCD vacuum with the help of lattice gauge simulations. To this end we firstly determine

the ensemble of magnetic monopole loops of the vacuum by performing a lattice calculation in the maximum Abelian gauge, performing the Abelian projection and identifying the magnetic monopole loops by the method of DeGrand and Toussaint [14]. The obtained ensemble of closed (magnetic monopole) trajectories is then described in terms of an effective scalar field theory by using the world line formalism [15,16]. Our lattice simulations will show that a scalar–antiscalar bound state survives in the continuum limit.

2. The Ginzburg–Landau theory of MAG monopole trajectories

The central idea of the present Letter is that the theory of closed monopole trajectories arising from MAG projected $SU(2)$ lattice gauge theory is equivalent to a theory of a charged scalar field. This scalar field theory designed to describe the monopole properties of the Abelian projected $SU(2)$ Yang–Mills theory necessarily inherits the scaling laws from the underlying $SU(2)$ Yang–Mills theory and, in particular, the property of asymptotic freedom by construction (provided that $SU(2)$ monopole theory properly scales towards the continuum limit). It was pointed out by Zakharov [18] that the scalar field theory which emerges from $SU(2)$ monopole loops is an interesting candidate for avoiding the so-called fine tuning problem, which is generic in (4-dimensional) scalar field theory equipped with the standard ϕ^4 potential. Since scalar field theories with local interactions of the scalar field possess an infra-red fixed point, the action term is presumably not a polynomial of finite order. One might argue that the increase of complexity due to the non-local interactions prohibit the access to such theories at a practical level (e.g., the numerical simulation). However, examples of scalar theories incorporating asymptotic freedom at the expense of the non-locality of the action have been treated in the literature [17,19–21].

In order to establish the equivalence between the theory of the monopole loops and the scalar field theory, we consider the general form of the partition function of a complex scalar field

$$Z[M] = \int \mathcal{D}\phi \, \mathcal{D}\phi^\dagger \exp\left\{ - \int d^4x \, \phi^\dagger(x) \left[-\partial^2 + m^2 + M(x) \right] \phi(x) + V\left(\phi^\dagger \phi\right) \right\}. \tag{1}$$

Here m is the usual mass term, and $M(x)$ is an external source, which we will specify later. $V(\phi^\dagger \phi)$ describes the interaction of the scalar field. The only restriction which we impose here is that we assume the potential term $V(\phi^\dagger \phi)$ to admit a Taylor expansion so that (1) can be written as

$$Z[M] = \exp\left\{ - \int d^4x \, V\left(\frac{\delta}{\delta M(x)} \right) \right\} Z_0[M], \tag{2}$$

$$Z_0[M] = \mathrm{Det}^{-1}\left[-\partial^2 + m^2 + M(x) \right]. \tag{3}$$

Using the proper-time representation of the functional determinant in (3), i.e.,

$$\Gamma_0[M] = -\ln Z_0[M] = \int\limits_0^\infty \frac{dT}{T} e^{-m^2 T} \, \mathrm{tr}\exp\left\{ -\tau\left(-\partial^2 + M(x) \right) \right\}, \tag{4}$$

the emerging heat kernel can be interpreted as the time evolution operator of a point particle, for which the usual Feynman path integral representation holds (we refer to [22] for a recent review of the world line formalism)

$$\Gamma_0[M] = \int\limits_0^\infty \frac{dT}{T} e^{-m^2 T} \int d^4x_0 \int\limits_{x(T)=x(0)} \mathcal{D}x(\tau) \, e^{-\int_0^T d\tau\left(\frac{\dot{x}^2}{4} + M(x(\tau)) \right)}. \tag{5}$$

Here we have split off the integral over the zero-modes of the path integral, $\int d^4x_0$, where x_0, the so-called loop center of mass, corresponds to the average position of the loop: $x_0^\mu := (1/T) \int_0^T d\tau\, x^\mu(\tau)$, i.e.,

$$\int \mathcal{D}x(\tau) \to \int \prod_\tau dx^\mu(\tau)\, \delta^{(4)}\left[x_0^\mu - (1/T) \int_0^T d\tau\, x^\mu(\tau) \right].$$

In order to relate the functional integral over the world lines $x(\tau)$ in (5) to the expectation values over loop clouds, we normalize it with respect to the free theory ($M = 0$) and introduce

$$\langle \mathcal{O}(x) \rangle_x = \mathcal{N}^{-1} \int_{x(T)=x(0)} \mathcal{D}x(\tau)\, e^{-\int_0^T d\tau\, \frac{\dot{x}^2}{4}} \mathcal{O}(x(\tau)), \tag{6}$$

where

$$\mathcal{N} = \int \mathcal{D}x(\tau)\, e^{-\int_0^T d\tau\, \frac{\dot{x}^2}{4}} = \int \frac{d^4p}{(2\pi)^4} e^{-p^2 T} = \frac{1}{(4\pi T)^2}. \tag{7}$$

Eq. (6) defines the expectation value of an observable $\mathcal{O}$ evaluated over an ensemble of closed loops $x(\tau)$; the loops are centered at a common average position x_0 ("center of mass") and are distributed according to the Gaussian weight $\exp[-\int_0^T d\tau\, \frac{\dot{x}^2}{4}]$. These definitions lead us to the compact formula

$$\Gamma_0[M] = \frac{1}{(4\pi)^2} \int d^4x_0 \int_0^\infty \frac{dT}{T^3} e^{-m^2 T} \left\langle \exp\left\{ -\int M(x(\tau))\, d\tau \right\} \right\rangle_x. \tag{8}$$

The world line representation of the interacting scalar field theory (1) is obtained by inserting (8) into (2). Thereby, the interaction of the scalar field $V(\phi^\dagger \phi)$ gives rise to an effective interaction of the loops $\widetilde{V}(x(\tau))$, i.e.,

$$\Gamma[M] = \frac{1}{(4\pi)^2} \int d^4x_0 \int_0^\infty \frac{dT}{T^3} e^{-m^2 T} \left\langle e^{-\widetilde{V}(x(\tau))} \exp\left\{ -\int M(x(\tau))\, d\tau \right\} \right\rangle_x. \tag{9}$$

Note, that once a particular choice of the scalar interactions $V(\phi^\dagger \phi)$ is made, the determination of $\widetilde{V}(x(\tau))$ in (9) is, in principle, straightforward.

In this Letter, we propose to determine the closed monopole loops of the Yang–Mills vacuum using lattice gauge simulations (see below for details). Employing the above illustrated equivalence between a world line ensemble and a scalar field theory, the effective scalar field theory underlying the dual superconductor of the Yang–Mills vacuum can be, in principle, extracted as suggested in [18]. It was recently observed that Monte Carlo calculations of the loop averages (such as those in (8) and (9)) are feasible [23,24].

In order to connect properties of the monopole loops with expectation values of the scalar field theory, we study different choices of the external current $M(x)$ in the context of the scalar field theory (2), (3) and in the context of the world line formalism (9), respectively.

To shorten the presentation, we introduce the shorthand notation

$$\langle\!\langle \mathcal{O}(x) \rangle\!\rangle = \frac{1}{(4\pi)^2} \int d^4x_0 \int_0^\infty \frac{dT}{T^3} e^{-m^2 T} \langle e^{-\widetilde{V}(x(\tau))} \mathcal{O}(x(\tau)) \rangle_x. \tag{10}$$

Firstly, we choose

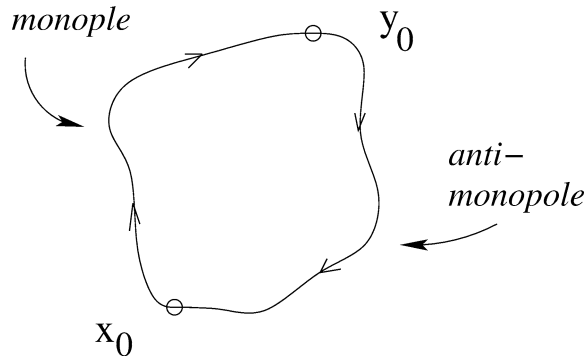$$M(x) = j\delta^4(x - x_0) \tag{11}$$

Fig. 1. Closed world line contributing to the scalar correlation function $C(x_0 - y_0)$.

and insert this ansatz into (1) yielding

$$\frac{d\Gamma[M]}{dj} = -\frac{d}{dj}\ln Z[M] = \langle\phi^\dagger\phi(x_0)\rangle. \tag{12}$$

On the other hand, it is clear from (9) that $d\Gamma[M]/dj$ counts the number times a monopole loop passes through the specified point x_0,

$$\rho(x_0) = \left\langle\!\!\left\langle \int d\tau\, \delta^4\big(x(\tau) - x_0\big)\right\rangle\!\!\right\rangle, \tag{13}$$

and, hence, corresponds to the probability of finding a monopole or antimonopole (depending on the orientation of the trajectory) at x_0. We will call this quantity monopole density. Comparing (13) and (12), one identifies

$$\rho(x_0) = \langle\phi^\dagger(x_0)\phi(x_0)\rangle. \tag{14}$$

In order to get a first insight into the propagators of the full interacting scalar theory (1), we investigate the particular choice of the source

$$M(x) = j_1\delta^4(x - x_0) + j_2\delta^4(x - y_0). \tag{15}$$

Inserting (15) into (1), taking the derivative with respect to the currents j_1 and j_2, respectively, we obtain the connected Green function

$$C(x_0 - y_0) = \frac{d^2}{dj_1\,dj_2}\ln Z[M] = \langle\phi^\dagger\phi(x_0)\phi^\dagger\phi(y_0)\rangle - \langle\phi^\dagger\phi(x_0)\rangle\langle\phi^\dagger\phi(y_0)\rangle, \tag{16}$$

which in view of (13) can be interpreted as the correlation function of the monopole density. The long distance behavior of this correlation function determines the screening mass of the scalar–antiscalar excitation.

Taking the derivatives with respect to the currents j_1 and j_2 of the effective action (9) in the world line formulation yields the loop representation of the above correlation function:

$$C(x_0 - y_0) = \left\langle\!\!\left\langle \int d\tau\, \delta^4\big(x(\tau) - x_0\big) \int d\tau'\, \delta^4\big(x(\tau') - y_0\big)\right\rangle\!\!\right\rangle - \rho(x_0)\rho(y_0). \tag{17}$$

The latter equation has a simple interpretation: the correlation function $C(x_0 - y_0)$ is obtained by taking the average over all closed loops which pass through points x_0 and y_0, respectively (see Fig. 1). Once the monopole world lines are at our disposal, we are able to calculate the full propagator (16) of the corresponding scalar field theory without specifying the scalar interactions $V(\phi^\dagger\phi)$.

3. Lattice results

In order to determine the closed world lines of the magnetic monopoles we performed simulations on a $N_s^3 \times N_t$, $N_s = 12$ and $N_t = 24$, lattice using the Wilson action. In order to express the size of the lattice spacing in physical units, we extract the string tension σa^2 in units of the lattice spacing a from a calculation of the quark–antiquark potential $V(r)$ for the above lattice size. For this purpose, we calculated the Polyakov loop correlation function $P(n)$ as function of the quark–antiquark distance $r = na$. For this task, we used the 2-level Lüscher–Weisz method [25]. The averages at level 1 were performed using 50 iterations while 10 evaluations were employed for the averages at level 2. 600 independent 2-level measurements were performed.

The raw data were fitted to

$$P(n) \propto \exp\{-N_t V(n)a(\beta)\} + \exp\{-N_t V(N_s - n)a(\beta)\}, \tag{18}$$

thereby taking into account the periodicity of the lattice. For the potential we used the standard form

$$V(n)a = -\frac{\alpha}{n} + \sigma a^2 n. \tag{19}$$

The results are shown in Table 1.

The values are in reasonable agreement with the known values for symmetric lattices. In the following, we will use a string tension $\sigma = 440$ MeV as reference scale.

The Maximal Abelian Gauge (MAG) condition, i.e.,

$$\sum_{\{x\},\mu} \mathrm{tr}\{U_\mu^\Omega(x)\tau^3 U^{\Omega\dagger}(x)\tau^3\} \overset{\Omega}{\to} \max, \tag{20}$$

where $U_\mu^\Omega(x) = \Omega(x)U_\mu(x)\Omega^\dagger(x + \mu)$ are the gauge transformed link variables, is implemented by employing a standard iteration over-relaxation algorithm. We do not expect that our numerical procedure locates the *global* maximum of the non-linear functional (20). Choosing different sets of *local* maxima of (20) implies that different gauge conditions are implemented (see, e.g., [26] for a more detailed study of the issue of gauge fixing ambiguities). We stress that the properties of the monopoles corresponding to these different gauges might turn out to be different. Rather than pursuing a detailed study of the effects of these so-called Gribov ambiguities, the aim of the present Letter is to show that a monopole–antimonopole bound state exists at least for a specific choice of the gauge (MAG, using an iteration over-relaxation algorithm to install the gauge condition). Once the MAG is implemented, the $SU(2)$ gauge theory is projected onto a compact $U(1)$ gauge theory by the usual prescription

$$U_\mu^\Omega(x) = \exp\{i\theta^a\tau^a\} \to \exp\{i\theta^3\tau^3\}. \tag{21}$$

Once the compact $U(1)$ gauge theory is at our disposal, we use the standard method of DeGrand and Toussaint [14] to extract the closed monopole trajectories.

In order to obtain the screening mass m of the scalar bound state, instead of (17) we consider the related correlation function

$$C(t) = \langle \psi(t)\psi(0) \rangle - \langle \psi(t) \rangle \langle \psi(0) \rangle \propto \exp\{-mt\}, \tag{22}$$

Table 1
String tension in units of the lattice spacing on the $12^3 \times N_t$ lattice

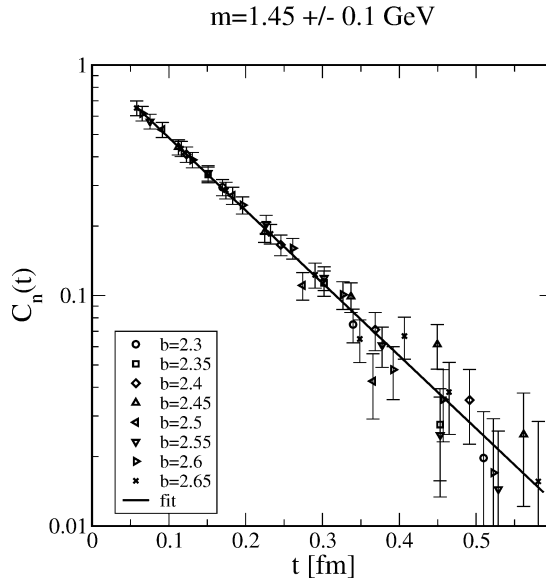| β	2.3	2.35	2.4	2.45
σa^2	0.1396 ± 0.002	0.110 ± 0.002	0.073 ± 0.002	0.061 ± 0.002
β	2.5	2.55	2.6	2.65
σa^2	0.040 ± 0.002	0.027 ± 0.002	0.0206 ± 0.001	0.016 ± 0.001

m=1.45 +/- 0.1 GeV

Fig. 2. Monopole–antimonopole, i.e., $\phi^\dagger\phi$, correlation function.

where

$$\psi(t) := \sum_{\{\vec{x}\}} \phi^\dagger(t, \vec{x})\phi(t, \vec{x}). \tag{23}$$

The correlation function $C(t)$ is obtained from the monopole trajectories as follows: the number n_t of monopoles is counted for a given time slice. The correlation function

$$C_{\text{dis}}(t) = \langle n_t n_0 \rangle \tag{24}$$

provides the disconnected counterpart of the Green function (22). The function can be well represented by the fit function

$$C_{\text{dis}}(t) = \rho^2 + \alpha \exp\{-mt\}, \tag{25}$$

where we used the fact that the disconnected correlation function asymptotically $(t \gg 1/m)$ approaches the monopole density squared.

After thermalization, we performed 100 measurements which were separated by 15 dummy sweeps to reduce the auto-correlations. For each β, we extract ma from a fit of (25) to the numerical data for $C_{\text{dis}}(t)$ (24). Fig. 2 shows the normalized connected correlation function

$$C_n(t) = \left(C_{\text{dis}}(t) - \rho^2\right)/\alpha = \exp\{-mt\}, \tag{26}$$

where the value of t (in physical units), $t = a(\beta)N_t$, is obtained by using the measured string tension (see Table 1). We observe that the data points obtained from several β values fall on top of the same curve. This signals that the screening mass extrapolates to the continuum limit. Using a string tension $\sigma = (440\text{ MeV})^2$, we find a mass $m \approx 1.45 \pm 0.1$ GeV, which is of order of the mass of the low lying glueballs.

Acknowledgements

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