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Advances of strain transfer analysis of optical fibre sensors

Huaping Wang, Zhi Zhou*

School of Civil Engineering, Dalian University of Technology, China

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Abstract

The test precision optical fibre sensors are increasingly important due to the widespread application of optical fibre sensing technology in structural health monitoring. Strain transfer analysis, which can be used to determine the action mechanism and to improve the precision of these sensors, is therefore an important issue. The earliest research started in the 1990s, and many excellent achievements have been obtained based on traditional elastic theory and stress transfer analysis of composites. A variety of strain transfer deductions appear to describe the differences in the mechanical models, assumptions and boundaries. A comprehensive discussion and brief review of representative strain transfer analyses is conducted, and some problems that urgently need to be addressed are stated. In addition, the developing trends in this subject are mentioned. The work in this article provides valuable guidance for understanding the research advances in strain transfer analysis, which will ultimately serve for the strain transfer error modification of optical fibre sensing models.

Keywords: Strain transfer analysis; Optical fibre sensors; Structure health monitoring; Error modification

Introduction

Optical fibre is the most popular sensing element, due to its excellent long-term stability, durability, good geometrical shape-versatility, corrosion resistance, electromagnetic interference resistance, low cost and high precision. It has been widely applied in the aeronautics, energy, civil engineering, and nuclear environmental fields [1,2]. Because bare optical fibre is vulnerable to harsh environments, encapsulation technologies were developed to provide protection. Therefore, the test precision of packaged optical fibre sensors has become an important issue that is studied by many scientists. For strain sensors, high-precision detection is defined as the detected strain infinitely close to the true strain of the host material. However, a part of the strain of the host material is absorbed by the middle layer (usually composed of a protective layer and an adhesive layer) in the transfer process before being recognised by the fibre core.

The strain lost is called the strain transfer error and is influenced by the materials and encapsulation technology. The strain transfer analysis is developed to establish the quantitative strain relationship of host

* Corresponding author.
E-mail addresses: wanghuaping1128@sina.cn (H. Wang), zhouzhi@dlut.edu.cn (Z. Zhou).
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material and optical fiber. The different mechanical models, assumptions and boundaries used in the deduction leads to the birth of diversiform strain transfer theories. The immethical theory indicates immature applications, which cannot aid in the design of industrial sensors [3]. Therefore, a systematic strain transfer theory that has an intimate relationship with optical fibre sensing models is required.

The earliest research on strain transfer analysis started in the 1990s and used only elastic theory to analyse simplified mechanical models [4,5]. Improved strain transfer theory [6] was formed later based on the stress transfer mechanism of composites [7]. Since then, this field has grown because of the demands of practical engineering problems.

Based on these factors, advances in strain transfer analysis using optical fibre sensing models will be discussed in this article. A comprehensive discussion and brief review using representative strain transfer analysis will be conducted. Moreover, some problems that urgently need to be addressed will also be discussed, and the developing trends in this field will be mentioned.

Basic conception of strain transfer analysis

Bare optical fibres and fibre Bragg gratings are brittle and vulnerable to harsh environments. Therefore, encapsulation is required to protect them. As a consequence, a middle layer between the sensing element and the host material is created. Strain of host material firstly makes the middle layer deformed, and then arrives at fiber core. Some of the strain is dissipated by the middle layer, the magnitude of this dissipation is greatly influenced by the materials, packaging and bonded length. Strain transfer analysis that focuses on establishing the relationship of strains of optical fibre and host material in multi-layered models is introduced. The ratio of strain sensed by the optical fibre and the strain of host material is called the strain transfer coefficient.

Brief introduction of the existing theory

Strain transfer analysis, which reflects the action mechanism and improves test precision, has received considerable attention due to the extensive use of optical fibre sensors in various engineering fields. Initial research on discussing the relationship between the measured strain of embedded optical fibre and the real values started in 1991, which was limited by special hypotheses and sizes [4]. The strain transfer relationships for embedded optical fibre sensors were determined by simplifying the model as an infinite elastic body and considering it as equivalent plane strain problem. However, these assumptions were too ideal to use in practical cases [5]. In 1998, improved strain transfer theory was achieved for the first time by introducing the stress transfer analysis of composites [6,8,9]. The following research on the strain transfer analysis of different models was extracted from real engineering problems and conducted in succession. Several outstanding strain transfer deductions will be discussed in detail in the sections below.

The parameters $\sigma_m/\varepsilon_m/\tau_m$, $\sigma_f/\varepsilon_f/\tau_f$, $\sigma_p/\varepsilon_p/\tau_p$ and $\sigma_a/\varepsilon_a/\tau_a$ stand for the normal stress/strain/shear stresses of the host material, fibre core, protective layer and adhesive layer, respectively. The letters r_m, r_f, r_p and r_a indicate the radius, and the variables u_m, u_f, u_p and u_a indicate the displacement of the host material, fibre core, protective layer and adhesive layer, respectively; 2L is the bonded length.

Theory deduced by Farhad Ansari [6]

The three-layered mechanical model used in the deduction is shown in Fig. 1. The assumptions, boundaries and primary processes of the strain transfer theory based on typical elastic mechanics are as follows.

Assumptions

1) The bonded length L is supposed to be far greater than $(r - r_f^2)$, which produces $(r^2 - r_f^2)/L \approx 0$;
2) The displacement increments relationships between the host material, fibre core and protective layer obeys the summation $u_m = u_f + u_p$;
3) By ignoring the axial variation in the radial displacement, the simplified Hooke's law for shear strain is rewritten as $\gamma_p(r, x) = du/dr$;

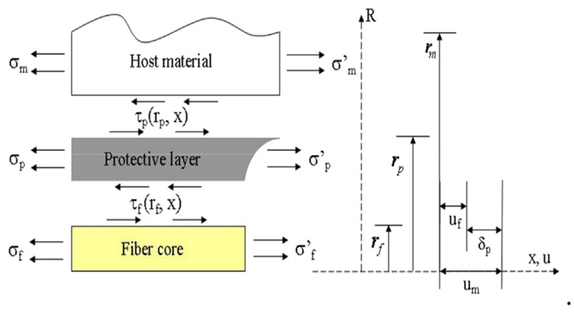

Fig. 1. The three-layered mechanical model.

4) The compatibility between the strains of the host material and the optical fibre along the axis of symmetry is introduced, which gives $\sigma_m/E_m = \sigma_f/E_f$.

Boundary conditions

1) There is no axial force at the end of the optical fibre, $T_f\ (L) = 0$;
2) Because of the symmetry of the structures, the sum of the odd functions for the shear stress should equal zero;
3) The strain in the host material is considered equivalent to the strain of the optical fibre located at the symmetric centre; that is, $\varepsilon_m\ (0) = \varepsilon_f\ (0)$.

Primary processes

Static equilibrium of the protective layer gives rise to

$$\frac{r^2 - r_f^2}{2L} \times \left(\sigma_p^{'} - \sigma_p\right) + r \times \tau_p(r,x) = r_f \times \tau_f\left(r_f,x\right) \tag{1}$$

By introducing assumption 1), the expression for the shear stress is

$$\tau_p(r,x) = r_f \bullet \tau_f\left(r_f,x\right)/r \tag{2}$$

Combining the geometric and physical equations and taking assumption 3) into account gives

$$\tau_p(r,x) = G_p \cdot \mathrm{d}u/\mathrm{d}r \tag{3}$$

Simultaneously, Equations (2) and (3) after integration with respect to r yield

$$u_p = r_f \ln\left(r_p/r_f\right) \bullet \tau_f\left(r_f,x\right)/G_p \tag{4}$$

By utilising the transformational relationships among displacement, strain and stress, two equations are obtained:

$$u_m(x) = \int_0^x \frac{\sigma_m(\xi)}{E_m} d\xi \tag{5}$$

$$u_f(x) = \frac{1}{\pi r_f^2 E_f} \times \int_0^x \left[\pi r_f^2 \sigma_f(x) - 2\pi r_f \int_0^x \tau_f\left(\xi, r_f\right) d\xi\right] d\xi \tag{6}$$

Citing assumption 2) and substituting the related expressions into it, a formula for shear strain τ_p is obtained. Introducing assumption 4) and taking second-order integration with respect to x gives

$$\frac{\mathrm{d}^2\tau_f\left(r_f,x\right)}{\mathrm{d}x^2} - k^2\tau_f\left(r_f,x\right) = 0 \tag{7}$$

Adopting the boundary conditions 1), 2) and 3), the strain transfer relationship can be written as

$$\varepsilon_f(x) = \varepsilon_m \bullet \left[1 - \sinh(kx)/\sinh(kL)\right] \tag{8}$$

Theory derived by Michel LeBlanc [10]

The two-layered mechanical model, which was used in the derivation, is shown in Fig. 2. It is thought that the host material could act as the protective layer. The general procedures are listed below.

Assumptions

1) The elastic modulus of the host material is deemed to be much smaller than that of the fibre core, which results in $(r^2 - r_f^2)\ E_m/r^2E_f << 1$;
2) The increment in the radial displacement determined by the Poisson effect is small enough that it can be neglected, which brings about $\gamma_m(r, x) = du/dr$.

Boundary conditions

1) When an optical fibre embedded in a host material is long enough, it is regarded as completely delivered, which gives, $\varepsilon_f(x \to \infty) = \varepsilon_m$;
2) The stress at the end of optical fibre is known and the stress $\sigma_f(x = \pm\mathrm{L})$ is 0.

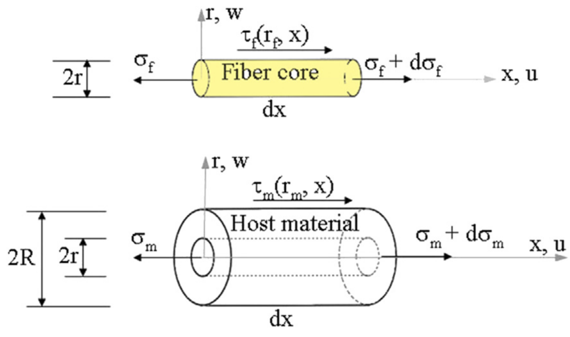

Fig. 2. The two-layered mechanical model.

Primary processes

Static equilibrium in the host material produces

$$\tau_m(r,x)=\frac{r_f}{r}\cdot\tau_f(r_f,x)-\frac{\left(r^2-r_f^2\right)}{2r}\cdot\frac{d\sigma_m}{dx} \tag{9}$$

By applying assumption 1), the equation above could be rewritten

$$\tau_m(r,x)=r_f\cdot\tau_f(r_f,x)/r \tag{10}$$

Using assumption 2) and Hooke's law gives

$$\tau_m(r,x)=G_m\cdot du/dr \tag{11}$$

Establishing simultaneous Equations (10) and (11) and performing integration with respect to r and x generates

$$\frac{d^2\sigma_f(x)}{dx^2}-\frac{n^2}{r_f^2}\sigma_f(x)=-\frac{n^2}{r_f^2}\cdot E_f\varepsilon_m \tag{12}$$

Substituting boundary conditions 1) and 2) into the general solution equation yields

$$\varepsilon_f(x)=\varepsilon_m\cdot\{1-\exp[-n(L-x)/r_f]\} \tag{13}$$

Additionally, the Fourier transform is able to solve the second-order differential equation [11], which produces an expression for the strain transfer coefficient:

$$H(k)=\frac{\varepsilon_f(k)}{\varepsilon_m(r_m,k)}=\frac{1}{(2\pi k r_f/n)^2+1} \tag{14}$$

Theory originating from Zhi Zhou [12]

A four-layered mechanical model is set up, as shown in Fig. 3. The middle layer is composed of a protective layer and an adhesive layer. The main steps in this strain transfer analysis are provided.

Assumptions

1) The elastic moduli of the protective layer and the adhesive layer are considered to be much smaller than that of the optical fibre, which creates $E_p/E_f << 1$, $E_a/E_f << 1$;
2) The relationship between the displacement increments of the four layers follows the summation $u_m = u_a + u_p + u_f$;
3) When the testing area is small enough, the normal stress is constant: σ_m = cons.

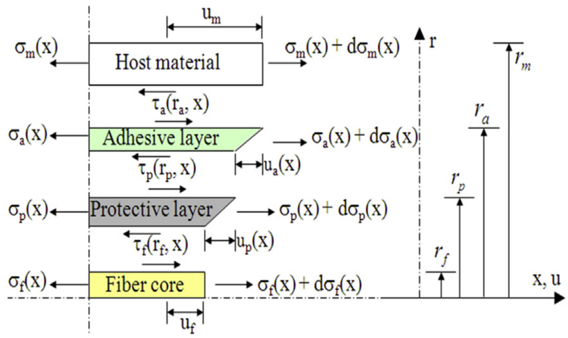

Fig. 3. The four-layered mechanical model.

Boundary conditions

1) The strain on the host material at the symmetric centre is taken as completely delivered to the fibre core, which means ε_m $(r, 0) = \varepsilon_f$ $(r, 0)$;
2) The force at the end of optical fibre is thought to be 0: T_f $(x = \pm L) = 0$;
3) Strain on the optical fibre at the centre $\sigma_f(x = 0)$ is considered to be equivalent to σ_f.

Primary processes

Based on static equilibrium of the protective and adhesive layers, the following relationships exist:

$$\frac{2[\tau_p(r,x)\cdot r-\tau_f(r_f,x)\cdot r_f]}{r_p^2-r_f^2}+\frac{d\sigma_p(x)}{dx}=0 \tag{15}$$

$$\frac{2[\tau_a(r,x)\cdot r-\tau_p(r_p,x)\cdot r_p]}{r_a^2-r_p^2}+\frac{d\sigma_a(\bar{x})}{dx}=0 \tag{16}$$

By introducing assumptions 1), the above equations can be abbreviated as

$$\tau_p(r,x)=\tau_f(r_f,x)\cdot r_f/r \tag{17}$$

$$\tau_a(r,x)=\tau_p(r_p,x)\cdot r_p/r \tag{18}$$

Using the geometric and physical equations and assumption 2), as well as performing second-order differentiation with respect to x, gives

$$\left(\frac{r_f}{G_a}\ln\frac{r_a}{r_p}+\frac{r_f}{G_p}\ln\frac{r_p}{r_f}\right)\frac{\partial^2\tau_f(r_f,x)}{\partial x^2}+\frac{1}{E_f}\frac{d\sigma_f(x)}{dx}-\frac{1}{E_m}\frac{d\sigma_m(x)}{dx}=0 \tag{19}$$

Using assumption 3), the above equation is simplified as

$$\frac{\partial^2\tau_f(r_f,x)}{\partial x^2}-\lambda^2\bullet\tau_f(r_f,x)=0 \tag{20}$$

By adopting the boundary conditions 1), 2) and 3), the strain relationship is written as

$$\varepsilon_f(x)=\varepsilon_m\bullet[1-\sinh(\lambda x)/\sinh(\lambda L)] \tag{21}$$

In addition, the case with linear viscoelasticity in the host material and the optical fibre is also discussed [13]. The main difference is that Hooke's law cannot be employed in this condition, and Stieltjes convolution integral is used to depict the relationship between the strain and the stress.

Theory deduced by Hongnan Li [14]

A three-layered mechanical model is established, as displayed in Fig. 4, and the general processes are the following. The two cases with and without shear stress τ_m (r_m, x) are discussed separately.

Assumptions

1) The elastic modulus of the protective layer is thought to be much smaller than that of the optical fibre, namely ($r^2 - r_f^2$) $E_p/r^2E_f << 1$;
2) The radial displacement increment determined by the Poisson effect is small enough to be neglected compared with the axial displacement, which brings about $\gamma_p(r, x) = du/dr$.

Boundary conditions

1) The symmetry of the optical fibre results in ε_f (L) = ε_f ($-L$);
2) The cross section of the optical fibre at the end is free, and no force is applied; that is, ε_f (L) = ε_f ($-L$) = 0.

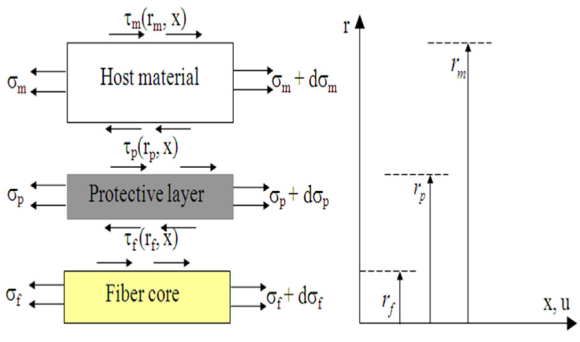

Fig. 4. The three-layered mechanical model.

Primary processes

Case 1: Shear stress τ_m (r_m, x) does not exist.

Static equilibrium between the protective layer results in

$$\tau_p(r,x)=\frac{r_f}{r}\bullet\tau_f(r_f,x)-\frac{\left(r_p^2-r_f^2\right)}{2r}\bullet\frac{d\sigma_p(x)}{dx} \tag{22}$$

After introducing assumption 1), the formula above is rewritten as

$$\tau_p(r,x)=\tau_f(r_f,x)\bullet r_f/r \tag{23}$$

Employing assumption 2) and combining the geometric and physical equations gives

$$\tau_p(x,r)=G_p\bullet du/dr \tag{24}$$

After forming simultaneous Equations (23) and (24), performing integration with respect to r and then second-order differentiation with respect to x produces

$$\frac{d\varepsilon_f^2}{dx^2}-k^2\bullet\varepsilon_f=-k^2\bullet\varepsilon_m \tag{25}$$

Taking the boundary conditions 1) and 2) into account, the relationship between the strains is solved:

$$\varepsilon_f(x)=\varepsilon_m\bullet[1-\cosh(kx)/\cosh(kL)] \tag{26}$$

Case 2: The influence of shear stress τ_m is calculated [15].

The additional assumptions that strain gradients are expected is used, which produces, $d\varepsilon_f/dx \approx d\varepsilon_p/dx$ $d\varepsilon_m/dx$.

The shear stress derived from the static equilibrium of the structure with the additional assumption is then simplified as

$$\tau_m(r,x)\approx\frac{E_f r_f^2}{2r}\left[1-\frac{r^2-r_p^2}{r_m^2-r_p^2}\right]\frac{d\varepsilon_f}{dx} \tag{28}$$

The other result is the same as case 1. The relationship between the strains in the host material and the optical fibre follows Equation (26). The influence of the calculated shear stress τ_m is imposed on parameter k, which has a different expression than the former case.

Theory deduced by Shiuhchuna Her [16]

A four-layered mechanical model was chosen, as shown in Fig. 5. The general procedures of this deduction are listed below.

Assumptions

1) Due to the low elastic modulus of the protective and adhesive layers compared with the host material and the optical fibre, pure shear deformation of the protective and adhesive layers is considered;
2) The radial displacement increment determined by the Poisson effect is small enough to be neglected compared with the axial displacement, resulting in $\gamma_p(r, x, \theta) = \partial u/\partial r$.

Boundary conditions

1) The symmetry of the structure produces $\varepsilon_f(L) = \varepsilon_f(-L)$;
2) The normal stress of the optical fibre at the end is deemed to be zero: $\sigma_f(L) = E_f\varepsilon_f(L) = 0$.

Primary processes

By introducing assumption 1), the static equilibrium of the protective layer gives

$$r_p\int_0^{\pi}\tau_p(r_p,\theta,x)d\theta\cdot dx - r_f\int_0^{2\pi}\tau_p(r_f,\theta,x)d\theta\cdot dx = 0 \tag{29}$$

The shear stress of the protective layer inverse to the radius is expressed as

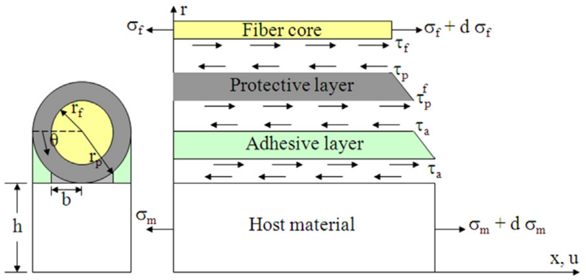

Fig. 5. The four-layered mechanical model.

$$\tau_p(r,\theta,x) = r_p\cdot\tau_p(r_p,\theta,x)/r \tag{30}$$

Employing assumption 2), the geometric equation for the protective layer is rewritten

$$\gamma_p = \partial u_p(r,\theta,x)/\partial r \tag{31}$$

Combining the above two equations, conducting integration with respect to r, and taking the interlayer continuity into account results in

$$u_a(r_p,\theta,x) = u_p(r_p,\theta,x) = \frac{r_p}{G_p}\cdot\tau_p(r_p,\theta,x)\ln(r_p/r_f) + u_f(r_f,\theta,x) + C \tag{33}$$

The shear stress of the adhesive layer is expressed as

$$\tau_a = [u_m - u_a(r_p,\theta,x)]\cdot G_a/(r_p - r_p\cdot\sin\theta) \tag{34}$$

Substituting Equation (33) into Equation (34) and taking the interlayer continuity into account gives

$$\tau_p(r_p,\theta,x) = \frac{[u_m - u_f(r_f,\theta,x)]}{r_p\cdot(1-\sin\theta)/G_a + r_p\ln(r_p/r_f)/G_p} \tag{35}$$

Replacing τ_p in Equation (29) with Equation (35) and performing the interval transform yields

$$\int_0^{2\pi}\tau_p(r_f,\theta,x)d\theta = \int_0^{\cos^{-1}\left(\frac{b}{r_p}\right)}\frac{2(u_m-u_f)}{(1-\sin\theta)r_f/G_a + r_f\ln(r_p/r_f)/G_p}d\theta \tag{36}$$

By considering the static equilibrium of the fibre core, the expression becomes

$$\int_0^{2\pi} \tau_p(r_f,\theta,x)d\theta = -\pi r_f \cdot \frac{d\sigma_f}{dx} \tag{37}$$

Substituting Formula (37) into Formula (36) leads to

$$\frac{d^2\sigma_f}{dx^2} - \lambda^2\sigma_f = -\frac{2\lambda^2 E_f h r_p}{\pi r_f^2 E_f + 2h r_p E_h} \cdot \sigma_0 \tag{38}$$

where λ is1

$$\lambda^2 = \left(\frac{1}{h r_p E_h} + \frac{2}{\pi r_f^2 E_f}\right) \times \int_0^{\cos^{-1}\left(\frac{h}{r_p}\right)} \frac{1}{(1-\sin\theta)/G_a + \ln(r_p/r_f)/G_p} d\theta.$$

After employing the boundary conditions 1) and 2), the strain transfer relation is expressed as

$$\varepsilon_f = \frac{\varepsilon_m'}{\pi r_f^2 E_f/(2h r_p E_h) + 1} \cdot \left[1 - \frac{\cosh(\lambda x)}{\cosh(\lambda L)}\right] \tag{39}$$

where $\varepsilon_m{}'$ stands for the far field strain applied to the host material.

Theory derived by Xin Feng [17]

A four-layered mechanical model simulating surface bonded optical fibre sensors was built, as shown in Fig. 6. The influence of one fixed-width crack located in the centre of the host material on the strain transfer coefficient is considered in this deduction. The general steps are shown. The variable τ_{cr} indicates the critical shear stress when plastic deformation starts.

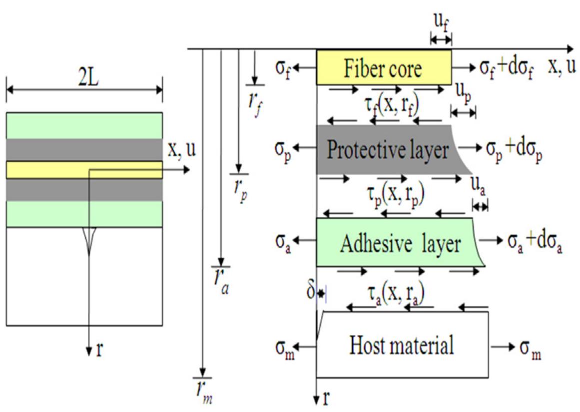

Fig. 6. The four-layered mechanical model.

Assumptions

1) The variation in the radial displacement along the axis is ignored, and Hooke's law for shear strain is simplified as $\gamma_p(r, x) = du/dr$;
2) The bonded length L is thought to be far greater than $(r - r_f^2)$, that is, $(r - r_f^2)/L \approx 0$, which produces $\tau_p(x, r) = r_f\tau_f(x, r_f)/r$;
3) The displacement increments of the protective and adhesive layers are considered equivalent in value, that is, $u_p = u_a$;
4) The displacement increment of the host material is thought to behave as $u_m(x) = \varepsilon_m x + \delta$;
5) The relationships among the displacement increments in the host material, fibre core, protective and adhesive layer obey summation, that is, $u_m = u_f + u_p + u_a$;
6) The protective layer is assumed to be an ideal elastic–plastic material, and the constitutive equation for shear stress is expressed as $\tau_p = G_p\gamma_p$ $(\gamma_p \le \tau_{cr}/G_p)$; $\tau_p = \tau_{cr}$ $(\gamma_p \ge \tau_{cr}/G_p)$.

Boundary conditions

1) Due to the symmetry of the structure, the displacement increment of the optical fibre at the centre is considered to be zero, that is, $u_f(x = 0) = 0$;
2) The strain at the end of the optical fibre is thought to be equal to the strain of the host material, that is, $\varepsilon_f(x = L) = \varepsilon_m$.

Primary processes

According to assumption 1) and 2) and the constitutive relationship, there is

$$u_p = \int_x^{x+u_p} d\xi = \int_{r_f}^{r_p} \gamma_p dr = \frac{r_f}{G_p}\int_{r_f}^{r_p} \frac{\tau_f(x, r_f)}{r} dr \tag{40}$$

Considering the static equilibrium of the fibre core, the expression for shear stress is

$$\tau_f(x, r_f) = -r_f E_f \frac{d\varepsilon_f}{dx} \tag{41}$$

By employing the simultaneous Equations (40) and (41) and then employing assumption 3), the displacement increments in the protective and adhesive layers are

$$u_p = u_a = -\frac{E_f r_f^2}{G_p}\ln\frac{r_p}{r_f}\frac{d\varepsilon_f}{dx} = -\alpha E_f r_f u_f''(x) \tag{42}$$

Assumption 4) and equation (42) list the values of related displacement increments, and introducing assumption 5) produces

$$u_f''(x) - \beta^2 u_f = -\beta^2(\varepsilon_m x + \delta) \tag{43}$$

By introducing the boundary conditions 1) and 2), the relationship between the strains in the host material and the optical fibre are

$$\varepsilon_f = \beta C_2 \exp(\beta(L+x)) - \beta C_1 \exp(\beta(L-x)) + \varepsilon_m \tag{44}$$

Constants C_1 and C_2 are

$$C_1 = -\frac{\delta\exp(2\beta L)}{\exp(\beta L)+\exp(3\beta L)},\quad C_2 = -\frac{\delta}{\exp(\beta L)+\exp(3\beta L)} \tag{45}$$

Theory proposed in this article

A three-layered mechanical model was established, as shown in Fig. 7. The state of the contact interfaces of this model are illustrated using Goodman's hypothesis. The strain transfer theory is especially appreciated for the strain transfer error modification of optical fibre sensors embedded in some structures, such as asphalt pavement [18]. The interlayer adhesion coefficients of the host material and the protective layer are denoted as k_m and k_p.

Assumptions

1) As the radial displacement influenced by Poisson effects are very small compared with the axial displacements, the radial displacement gradients $\partial w/\partial x$ could be neglected, which produces, $\tau_p = G_p\partial u/\partial r$, $\tau_m = G_m\partial u/\partial r$;
2) Goodman's hypothesis is introduced to describe the interlayer contact state of the three-layered model, which results in $\tau_m = k_m(\alpha - 1)u_m$, $\tau_p = k_p(u_m - u_p)$. Letter α means the displacement correlation coefficient.

Boundary conditions

1) Due to the asymmetry of the three-layered structure, $\varepsilon_f(x) = \varepsilon_f(-x)$;
2) No constraints or forces employed at the two ends of optical fibre results in $\sigma_f(x = \pm L) = 0$.

Primary processes

The static equilibrium of host material, protective layer and fibre core produces three equations, and the recombination gives,

$$\tau_p(r,x) = -\frac{r^2-r_f^2}{2r}\cdot\frac{d\sigma_p(x)}{dx} - \frac{r_f^2}{2r}\cdot\frac{d\sigma_f(x)}{dx} \tag{46}$$

$$\tau_m(r,x) = -\frac{r^2-r_p^2}{2r}\cdot\frac{d\sigma_m(x)}{dx} - \frac{r_p^2-r_f^2}{2r}\cdot\frac{d\sigma_p(x)}{dx} - \frac{r_f^2}{2r}\cdot\frac{d\sigma_f(x)}{dx} \tag{47}$$

Introducing assumption 1) and performing integration of the two equations above respect to r gives

$$\begin{aligned} u_m - u_f = &-\frac{1}{G_m}\left(\frac{r_m^2-r_p^2}{4} - \frac{r_p^2}{2}\ln\frac{r_m}{r_p}\right)\cdot\frac{d\sigma_m(x)}{dx} \\ &-\frac{r_f^2}{2}\left(\frac{1}{G_m}\ln\frac{r_m}{r_p} + \frac{1}{G_p}\ln\frac{r_p}{r_f}\right)\cdot\frac{d\sigma_f(x)}{dx} \\ &-\left[\frac{1}{G_m}\ln\frac{r_m}{r_p}\frac{r_p^2-r_f^2}{2} + \frac{1}{G_p}\left(\frac{r_p^2-r_f^2}{4} - \frac{r_f^2}{2}\ln\frac{r_p}{r_f}\right)\right]\cdot\frac{d\sigma_p(x)}{dx} \end{aligned} \tag{48}$$

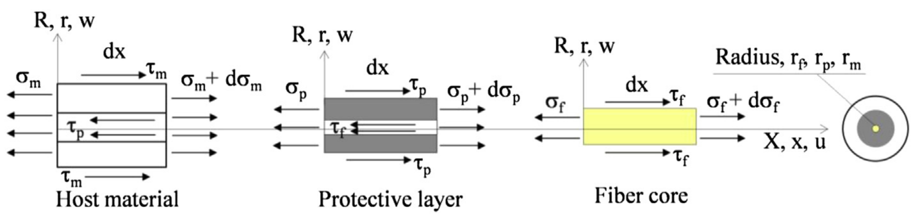

Fig. 7. The three-layered mechanical model.

By utilising assumption 2) and performing conversions to replace $d\sigma_{\mathrm{m}}/dx$ and $d\sigma_{\mathrm{p}}/dx$, a differential equation about ε_{f} is obtained,

$$\frac{\mathrm{d}^2\varepsilon_{\mathrm{f}}(x)}{\mathrm{d}x^2}-\lambda_2^2\varepsilon_{\mathrm{f}}=-\lambda_2^2\varepsilon_{\mathrm{m}}^{'} \tag{49}$$

Here, $\varepsilon_{\mathrm{m}}{}'$ stands for $\lambda_1\varepsilon_{\mathrm{m}}$, and λ_2^2 is constant.

Adopting the boundary conditions 1) and 2) to solve the general solution function yields

$$\varepsilon_{\mathrm{f}}(x)=\varepsilon_{\mathrm{m}}\lambda_1\left[1-\frac{\cosh(\lambda_2 x)}{\cosh(\lambda_2 L)}\right] \tag{50}$$

The expressions for constants λ_1 and λ_2^2 are listed below:

$$\lambda_1=1-\frac{r_m k_m(\alpha-1)\left(\ln\frac{r_m}{r_p}-\frac{E_m}{2r_p k_p(1+\nu_m)}\right)}{\frac{r_m^2-r_p^2}{4}-\frac{r_m^2}{2}\ln\frac{r_m}{r_p}+\frac{(1+\nu_m)\left(r_m^2-r_p^2\right)}{E_m r_p k_p}}\cdot\left[\frac{(1+\nu_p)\left(r_m^2-r_p^2\right)}{E_p}\left(\frac{1}{4}-\frac{r_f^2}{2\left(r_p^2-r_f^2\right)}\ln\frac{r_p}{r_f}\right)+\frac{1+\nu_m}{E_m}\left(r_m^2\ln\frac{r_m}{r_p}-\frac{r_m^2-r_p^2}{2}\right)\right.$$
$$\left.-2r_m k_m(\alpha-1)\cdot\left[\frac{1+\nu_m}{E_m}\ln\frac{r_m}{r_p}+\frac{1+\nu_p}{E_p}\left(\frac{1}{2}-\frac{r_f^2}{r_p^2-r_f^2}\ln\frac{r_p}{r_f}\right)\right]\right. \tag{51}$$

$$\lambda_2^2=\left[\frac{E_{\mathrm{f}}}{E_{\mathrm{p}}}(1+\nu_{\mathrm{p}})r_{\mathrm{f}}^2\left(\frac{r_{\mathrm{p}}^2}{r_{\mathrm{p}}^2-r_{\mathrm{f}}^2}\ln\frac{r_{\mathrm{p}}}{r_{\mathrm{f}}}-\frac{1}{2}\right)\right]^{-1} \tag{52}$$

Corresponding strain transfer functions

Seven typical strain transfer deductions have been discussed separately. The differences between the seven theories are mainly caused by the discrepancies in the assumptions and boundary conditions used in the derivations. The assumptions and boundary conditions employed are reasonable under specific conditions, which means that the application of the corresponding strain transfer error modification function is restricted. The strain relationships between the host materials and the optical fibres derived from these theories are shown in Table 1. The constants included in the strain transfer functions are also provided.

Comprehensive analysis

To determine the strain transfer relationships, assumptions are necessary to simplify the equations, and

Table 1
Various strain transfer functions.

Scholars	Strain transfer functions	Related constants
Farhad Ansari	$\varepsilon_{\mathrm{f}}(x)=\varepsilon_{\mathrm{m}}\left[1-\frac{\sinh(kx)}{\sinh(kL)}\right]$	$k^2=\frac{2G_{\mathrm{p}}}{r_{\mathrm{f}}^2E_{\mathrm{f}}\ \ln(r_{\mathrm{p}}/r_{\mathrm{f}})}$
Michel LeBlanc	$\varepsilon_{\mathrm{f}}(x)=\varepsilon_{\mathrm{m}}\left\{1-\exp\left[-\frac{n(L-x)}{r_{\mathrm{f}}}\right]\right\}$	$n^2=\frac{2G_{\mathrm{m}}}{E_{\mathrm{f}}\ \ln(r_{\mathrm{m}}/r_{\mathrm{f}})}$
Zhi Zhou	$\varepsilon_{\mathrm{f}}(x)=\varepsilon_{\mathrm{m}}\left[1-\frac{\sinh(\lambda x)}{\sinh(\lambda L)}\right]$	$\lambda^2=\frac{2}{r_{\mathrm{f}}^2E_{\mathrm{f}}[\ln(r_{\mathrm{a}}/r_{\mathrm{p}})/G_{\mathrm{a}}+\ln(r_{\mathrm{p}}/r_{\mathrm{f}})/G_{\mathrm{p}}]}$
Hongnan Li	$\varepsilon_{\mathrm{f}}(x)=\varepsilon_{\mathrm{m}}\left[1-\frac{\cosh(kx)}{\cosh(kL)}\right]$	$k^2=\frac{2G_{\mathrm{p}}}{r_{\mathrm{f}}^2E_{\mathrm{f}}\ \ln(r_{\mathrm{m}}/r_{\mathrm{f}})}$
Shiuh-Chuan Her	$\varepsilon_{\mathrm{f}}=\varepsilon'_{\mathrm{m}}\frac{1-\cosh(\lambda x)/\cosh(\lambda L)}{\pi r_{\mathrm{f}}^2E_{\mathrm{f}}/(2hr_{\mathrm{p}}E_{\mathrm{m}})+1}$	$\lambda^2=\left(\frac{1}{hr_{\mathrm{p}}E_{\mathrm{m}}}+\frac{2}{\pi r_{\mathrm{f}}^2E_{\mathrm{f}}}\right)\int_0^{\cos^{-1}\left(\frac{b}{r_{\mathrm{p}}}\right)}\frac{1}{\frac{1-\sin\theta}{G_{\mathrm{a}}}+\frac{1}{G_{\mathrm{p}}}\ln\frac{r_{\mathrm{p}}}{r_{\mathrm{f}}}}d\theta$
Xin Feng	$\varepsilon_{\mathrm{f}}=\beta C_2\exp(\beta(L+x))$ $-\beta C_1\exp(\beta(L-x))+\varepsilon_{\mathrm{m}}$	Expressions of constants C_1 and C_2 are shown in Equation (45)
Theory proposed in this article	$\varepsilon_{\mathrm{f}}(x)=\varepsilon_{\mathrm{m}}\lambda_1\left[1-\frac{\cosh(\lambda_2x)}{\cosh(\lambda_2L)}\right]$	Constants λ_1 and λ_2^2 respectively follow Equations (51) and (52)

boundary conditions are required to solve the equations. The reasonability and engineering correlation of the proposed assumptions and boundary conditions are the critical factors that influence the applicability of a strain transfer theory. A brief overview of the existing theories will be given considering the two aspects.

Four assumptions were made in the strain transfer theory deduced by Farhad Ansari et al., 1998. When half of the bonded length L is 40 mm and the radius of protective layer is less than 0.6 mm, the value $(r^2 - r_f^2)/$ L is close to zero, which means that assumption 1) is convincing. However, when the radius of the protective layer is greater than 1 mm, the value $(r^2 - r_f^2)/L$ could not be zero. Protective layer thicknesses greater than 1 mm are very common in engineering. Considering the compatibility of the axial deformation, assumption 4), i.e., $\sigma_m/E_m = \sigma_f/E_f$, is made. Differentiating with respect to x gives $\varepsilon_m(x) = \varepsilon_f(x)$, which is inconsistent with the result $\varepsilon_m(x = 0) = \varepsilon_f(x = 0)$ and $\varepsilon_m(x \neq 0) = \varepsilon_f(x)$, which indicates that assumption 4) is weak. Although the theory is not perfect, it still provides meaningful guidance for the strain transfer error modification of optical fibre sensors.

For the theory derived by Michel LeBlanc 1999, the elastic modulus of the host material, E_m, must be much smaller than that of the fibre core, E_f, i.e., assumption 1). If the host material is composed of concrete or steel, the elastic modulus is 30 GPa or 210 GPa, which is the same order of magnitude as the elastic modulus of the fibre core ($E_f = 72$ GPa), which should not be neglected. This strain transfer function is suitable for the cases when optical fibres are embedded in flexible structures.

The strain transfer analysis of multi-layered structures was first mentioned in the theory of Zhi Zhou 2003. The same assumption was made that the elastic moduli of the protective and adhesive layers are much smaller than that of the optical fibre, which requires a flexible middle layer. This strain transfer deduction could be recognised as a further continuation of that deduced by Farhad Ansari.

In the theory of Hongnan Li, the assumption that the elastic modulus of the protective layer E_p is much smaller than that of the fibre core is made, which is similar to the analysis of Michel LeBlanc. This strain transfer error modification function is appropriate in the case when the optical fibre is packaged in a flexible material.

In the analysis of Shiuh-Chuan Her, the protective and adhesive layers only bear pure shear deformation, which means that cross sections of the structure should be always parallel. In practical applications, the normal stress along the axis usually cannot be ignored. However, the deduction is still valuable and provides a special approach to achieve the strain transfer relationship.

In the strain transfer analysis of Xin Feng, the influence of the host material with a fixed-width crack was first calculated. The assumptions used by Farhad Ansari were also employed, but additional assumptions were introduced. Assumption 3) requires the displacement increment of the protective layer to be equal to that of the adhesive layer, which is inconsistent with reality because the contact forces imposed on the two layers are different. Assumption 4) that $u_m(x) = \varepsilon_m x + \delta$ ignores the influence of the crack on the axial normal strain could be improved. However, this research still has importance in explaining strain transfer relationships.

Given the analysis mentioned above, an improved strain transfer theory is put forward in this article with fewer assumptions adopted in the deduction. Moreover, most widely used Goodman's hypothesis was introduced for the first time to describe the interfacial shear stress instead of shear-lag model which assumes the middle layers bearing pure shear force. The proposed strain transfer error modification function has relatively universal application and is especially suitable for the case that interface interactions between the multi-layered structures obey Goodman's hypothesis.

Prospects

Based on the above analysis, it is well known that the consideration of various mechanical analyses, improved assumptions and multiple-phase constitutive relationships have greatly advanced the development of various strain transfer analysis. It has been expected that systematic strain transfer theory is established so as to serve for the design and error modificaition of wide-spread applied optical fiber sensors. Moreover, strain transfer analysis on cases where the host material suffered damage, such as holes, cracks, debonding, fatigue and creep deformation, should also be addressed because most damaged structures are required to serve for long times.

Conclusions

Strain transfer analysis is an efficient approach to eliminate strain transfer error and aid in the design of sensors; it has received increasing attention in recent years. A lot of research on surface bonded or embedded optical fibre sensors has been performed, resulting in some meaningful achievements. Six excellent deductions that established strain transfer relationships between host materials and optical fibres were presented and discussed in detail. In addition, an improved strain transfer theory that introduces Goodman's hypothesis to describe the interlayer mechanical state was proposed. A brief overview of the seven strain transfer deductions was given, and the most suitable conditions for each theory were also mentioned. The developing trends in strain transfer analysis were presented, based on a comprehensive analysis of existing strain transfer theories. The research in this article provides valuable and meaningful guidance for quickly perceiving the advances in strain transfer analysis.

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