Study on TMTD Statistical Model of Arch Dam Deformation Monitoring

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Abstract

The traditional monitoring statistical model of arch dam deformation is established based on early average temperature as the temperature factors, but its extension forecast function is poorer. Based on the analysis of the shortcomings and causes of the traditional monitoring statistical model, this paper has studied a new deformation statistical model (called TMTD statistical model). In TMTD statistical model, the temperature factors use mean temperature $T_m$ and linear temperature difference $T_d$ in the direction of the thickness of horizontal sections on arch dam. $T_m$ and $T_d$ are obtained by theoretical calculation, so it can better reflect and forecast changes of arch dam temperature field. The contrast of project cases in terms of forecast effect between two models has been presented, and the contrast shows, TMTD statistical model has significant superiority in terms of extension forecast of arch dam deformation.

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1. Introduction

The statistical model of dam safety monitoring, based on mathematical statistical analysis, is a mathematical equation established to quantitatively describe the relationship between the effect-variables...
and influencing factors. According to existing knowledge and experience of dam engineering, there are three kinds of factors causing dam deformation: water pressure, temperature and time effect. The structural form of dam deformation can be expressed as follows [1]:

\[
\hat{\delta}(t) = \hat{\delta}_t(t) + \hat{\delta}_w(t) + \hat{\delta}_h(t)
\]

(1)

Here: \( \hat{\delta}(t) \) is the regression value of displacement at the time of “t”, \( \hat{\delta}_t(t) \) is the water pressure of displacement at the time of “t”, \( \hat{\delta}_w(t) \) is the temperature component of displacement at the time of “t”, \( \hat{\delta}_h(t) \) is the time effect component of displacement at the time of “t”.

In Formula (1), the change law of temperature field fails to be reflected in case of insufficient temperature monitoring points on dam. We generally adopt mean value \( \bar{T}_{e-s}(t) \) between the \( e \)th day and the \( s \)th day before the day of displacement observation as temperature factors. Then the temperature component \( \hat{\delta}_t(t) \) of the statistical model has a structural form as follows.

\[
\hat{\delta}_t(t) = c_0 + \sum_{i=1}^{m} c_i \bar{T}_{e-s}(t)
\]

Here: \( c_0 \) refers to regression constant, and \( c_i \) refers to regression coefficient.

The statistical model of dam monitoring is based on the monitoring data obtained under load condition that has already happened. When the future load is within the range of previous load, the statistical model will be provided with fine fitting precision and forecast effect. Once the future load exceeds previous load, by a large margin in particular, the forecast effect of the model will decline greatly, or even lose its forecast function. Thus the extension forecast effect of statistical model is poor [2][3].

Let’s adopt the arch dam data for case study. Before 1992, the arch dam was running at a low water level. In May-July 1992, the water level rose by 20-odd meters, which was the first time to have reached close to the normal water level. We obtained the observed data of the dam plumb lines during May-August 1992, and inspect the forecast effect of statistical model which adopted the early average temperature as the temperature factor. Based on the observed data before May 1992, we established the statistical model with the early average temperature as the temperature factor, and “forecast” the radial horizontal displacement of monitoring points during May-August 1992 according to the observed water level and temperature. See Formula (1) for the general expression of the forecast model, and see Formula (3) for the forecast effect inspection model.

\[
\eta(t) = \hat{\delta}(t) - \delta(t)
\]

(3)

In Formula (3), \( \eta_i(t) \) is the difference between the observed value and the forecast value \( \hat{\delta}(t) \) at the \( i \)th time, referred to as the residuals. If \( \eta_i(t) > 3S \) (S is residual standard deviation), the forecast effect proves poor. As the result shows, the forecast effect is unsatisfactory; the residuals for the most of the times exceed or far exceed 3S. It is mainly for the reason that environmental variables (water level, temperature, etc.) in forecast period change far beyond the range of environmental variables change in modeling period.

Temperature deformation component of dam is caused by the change of temperature field. The change of water temperature is mainly determined by the change of air temperature and upstream water level. When the upstream water level changes in similar range and margin, we can just use temperature factor to easily describe the change law of temperature displacement (i.e. temperature component of displacement). When water level in the two periods changes in different ranges, the temperature will change differently under the effect of temperature conditions at the upstream boundary. In this case, a better temperature factor is the one able to generalize the change of dam temperature field, e.g. the observed temperature of monitoring point on dam, see Formula (4); the average observed temperature \( T_\tau(t) \) of several horizontal sections of dam and the observed temperature gradient \( \psi_\tau(t) \) on these sections, see Formula (5).
\[
\hat{\delta}_i(t) = c_0 + \sum_{i=1}^{m} c_i T_i(t) \quad (4)
\]
\[
\hat{\delta}_j(t) = c_0 + \sum_{i=1}^{m} c_i T_j(t) + \sum_{i=1}^{m} c_i \Psi_j(t) \quad (5)
\]

Generally, \( T_i(t) \) in Formula (4) is directly derived according to the observed value. The mean temperature \( \overline{T}_i(t) \) in Formula (5) and temperature gradient \( \Psi_i(t) \) are derived according to the observed temperature. In case of insufficient data on the observed temperature of dam, they can as well be derived by theoretical calculation.

When we use statistical model to carry out extension forecast, it is hard for \( \overline{T}_i(t) \) and \( \Psi_i(t) \) to accurately reflect the actual \( \overline{T}_i(t) \) and \( \Psi_i(t) \) in extension condition according to the previous observed data. The paper hereby adopts theoretical calculation to establish statistical model by using the mean temperature \( T_m \) and linear temperature difference \( T_d \) of dam temperature in the direction of horizontal section thickness as temperature factors \( \overline{T}_i(t) \) and \( \Psi_i(t) \), so as to carry out arch dam monitoring data analysis and safety monitoring forecast.

2. Calculation Method of Mean Temperature and Equivalent Linear Temperature Difference

Arch dam has three characteristic temperature fields: arch-closure temperature field, annual average temperature field and temperature change field. In arch dam monitoring data analysis, temperature deformation is relative to the observed reference value. Arch-closure temperature field and annual average temperature field will not change with the running time of dam, thus the temperature deformation of dam is mainly determined by the change of temperature change field \([4]\).

When the arch dam is at normal water level and the cement hydration heat is dispersed, the dam temperature is almost in quasi-stationary temperature field. Then the dam temperature field is mainly affected by ambient temperature (e.g. water temperature, air temperature), and temperature deformation is mainly affected by the temperature change field. If the effect of arch dam surface curvature is neglected, we can adopt plane plate to calculate temperature field. See Figure 1. The temperature in the dam can be divided into three parts: mean temperature \( T_m \) in the direction of section thickness \( L \), equivalent temperature difference \( T_d \) and non-linear temperature difference \( T_n \). \( T_n \) has only slight effect on dam, so we only consider \( T_m \) and \( T_d \) under normal circumstances.

\[
T_m = \frac{1}{L} \int_{-L/2}^{L/2} T(x)dx \quad (6)
\]
\[
T_d = \frac{12}{L} \int_{-L/2}^{L/2} T(x)dx \quad (7)
\]

\( L \) is dam thickness, \( T(x) \) is temperature function.

Formula (6) and (7) are general theoretical expressions. Under the effect of water temperature and air temperature change, \( T_m \) and \( T_d \) of dam temperature change field can be calculated according to the formulas:

\[
T_m = \frac{\rho_1}{2} [A_{si} \cos \omega(\tau - \theta_1 - \tau_0) + A_s \cos \omega(\tau - \epsilon - \theta_1 - \tau_0)] \quad (8)
\]
\[
T_d = \rho_2 [A_{si} \cos \omega(\tau - \theta_2 - \tau_0) - A_s \cos \omega(\tau - \epsilon - \theta_2 - \tau_0)] \quad (9)
\]

\( A_{si} \) is temperature amplitude of downstream surface; \( A_s \) is temperature amplitude of upstream surface; \( \epsilon \) is phase difference.
In Figure 1: $T$- temperature; $T(x)$- temperature curve; $L$- dam thickness; $T_m$- mean temperature; $T_d$- linear temperature difference

For the sections above water level, both upstream and downstream dam surfaces have contact with air, so we can calculate $T_m$ and $T_d$ according to the following formula:

$$T_m = \rho_i A_i \cos \omega (\tau - \theta_i - \tau_0), \quad T_d = 0$$

For sections below water level, we can calculate the annual amplitude $A_s$ of water temperature and the phase difference $\epsilon$ at the depth of “$y$” according to the following formula:

$$A_s(y) = A_{s0} e^{-0.018y}, \quad \epsilon(y) = 2.15 - 1.30 e^{-0.085y}$$

If the arch dam is divided into “$k$” layers (horizontal sections), we can calculate the mean temperature $T_{m0}, T_{m1}, \ldots, T_{mk}$ and linear temperature difference $T_{d0}, T_{d1}, \ldots, T_{dk}$ of different layers with the changing of time according to Formula (8) and (9), serving as temperature data for statistical modeling. When we select water depth “$y$” in calculating annual temperature load, if the annual amplitude of water level is too small, we can approximately adopt annual average water level; if the annual amplitude of water level is excessively large, we should divide the water level and calculate one by one to mitigate error.

In TMTD statistical model, the structural form of temperature component can be expressed as follows:

$$\delta_T(t) = c_0 + \sum_{i=1}^{k} [c_{i1} T_{m_i}(t) + c_{i2} T_{d_i}(t)]$$

3. Composition of TMTD Statistical Model

In TMTD statistical model, the structural form of water pressure component and time effect component still adopts the form of conventional statistical models. See Formula (13) for water pressure component, and see Formula (14) for time effect component.

$$\delta_H(t) = b_0 + \sum_{i=1}^{n} b_i H'(t)$$

$$I_1 = \ln(t + 1), \quad I_2 = 1 - e^{-t}, \quad I_3 = t/(t + 1), \quad I_4 = t, \quad I_5 = t^2, \quad I_6 = t^{0.5}, \quad I_7 = t^{-0.5}, \quad I_8 = 1/(1 + e^{-t})$$

$$\delta_H(t) = d_0 + \sum_{i=1}^{k} d_i I_i(t)$$

TMTD statistical model is thus made available.

4. Application Examples

There is a concrete arch dam described as double-curvature thin arch dam with variable center and variable radius, its crest elevation is 294.0m, maximum height is 157.0m, normal water level is 285.0m.
The reservoir is a multi-year regulation reservoir.

The arch dam is set with totally five groups of plumb lines to monitor dam safety. Each group consists of one inverted plumb line and one direct plumb line, making up 17 deflection (horizontal displacement) points. Radial and tangential horizontal displacement of each monitoring point is observed.

The dam body is divided into ten horizontal sections. According to the previous calculation method, we can respectively calculate $T_m$ and $T_d$ of each section at different observing time. Codes for section elevations, $T_m$ and $T_d$ are as follows: 294.0m ($T_m^{294}, T_d^{294}$), 275m ($T_m^{275}, T_d^{275}$), 255m ($T_m^{255}, T_d^{255}$), 235m ($T_m^{235}, T_d^{235}$), 216m ($T_m^{216}, T_d^{216}$), 197m ($T_m^{197}, T_d^{197}$), 178m ($T_m^{178}, T_d^{178}$), 159m ($T_m^{159}, T_d^{159}$), 145m ($T_m^{145}, T_d^{145}$) and 137m ($T_m^{137}, T_d^{137}$). In fact, $T_d^{294}=0$, so there are 19 TMTD temperature factors in total.

The arch dam was running at a low water level before 1992. In May-July 1992, the water level rose by 20-odd meters, which was the first time to have reached close to normal water level. So January-April 1992 is taken as the fitting period, and TMTD statistical model of the radial horizontal displacement of monitoring points of plumb lines is established. May-August 1992 is taken as the forecast period to check the forecast effect of TMTD statistical model available. As a model, Formula (16) presents TMTD statistical model of the monitoring point of L5 plumb line with 291m elevation at the arch crown section. Table 1 presents the forecast condition of TMTD statistical model of the monitoring point of L3 plumb line with 291m elevation at the arch crown section and L5 plumb line with 291m, 250m and 205m elevation. For contrast, Table 1 presents the forecast condition of the statistical model by taking the early average temperature as the temperature factor.

$$
\delta = 1332.265 - 1866.203(1 - e^{-t}) + 1199.26\left[\frac{t}{(t + 1)}\right] - 1.890354 t^2 + 1.3 \times 10^{-7} H^4
- 2.428972 T_M^{294} - 23.29135 T_M^{255} - 4.864928 T_D^{255} - 27.51144 T_D^{235}
+ 99.74121 T_D^{178} + 47.48906 T_D^{159} - 124.1887 T_D^{137}
$$

**Table 1 Analysis on TMTD Statistical Model of Plumb Line Radial Horizontal Displacement of a Dam (L3, L5 plumb lines)**

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Point elevation</th>
<th>Forecast times</th>
<th>TMTD statistical model</th>
<th>Statistical model with early average temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Measuring times</td>
<td>Measuring times</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>with residuals</td>
<td>with residuals</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>larger than 2S</td>
<td>larger than 3S</td>
</tr>
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<td>291</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>L3</td>
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<td>250</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>L3</td>
<td>205</td>
<td>205</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>L5</td>
<td>291</td>
<td>291</td>
<td>4</td>
<td>2</td>
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<tr>
<td>L5</td>
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<td>250</td>
<td>6</td>
<td>3</td>
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<td></td>
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<td></td>
<td>Measuring times with residuals larger than 3S</td>
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</tr>
<tr>
<td>L5</td>
<td>205</td>
<td>205</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

According to Table 1:

(1) The upstream water level rises significantly in the forecast period, so we adopt the model which uses the early average temperature as the temperature factor. It is poor in extension forecast effect, almost devoid of any forecast function.

(2) Adopt TMTD statistical model established by using mean temperature $T_m$ and equivalent temperature difference $T_d$ as temperature factors based on theoretical calculation. Its extension forecast effect is evidently better than the model which adopts the early average temperature as the temperature factor.
factor. Most of the monitoring points can be well forecast.

(3) In case water level rises significantly in the forecast period, it is inappropriate to adopt the statistical model which uses early average temperature as the temperature factor to serve as the mathematical model for safety monitoring. Instead, we shall figure out a proper mathematical model able to reflect the said objective conditions. TMTD statistical model serves as a practical and effective approach to address the problem; it is significantly superior to the early temperature factor statistical model in terms of extension forecast.

(4) In Table 1, the forecast effect of TMTD statistical model is significantly improved, but there remain quite a few measuring times with the residuals exceeding 3S. This is mainly because TMTD statistical model only improves the form of temperature factor, but the water pressure factor still adopts the structural form of conventional statistical models. To achieve sound forecast effect, we shall improve the water pressure factor.

5. Conclusion

When the range of environmental variable change in forecast period significantly exceeds that in the fitting period, we adopt the early average temperature as temperature factor and establish the statistical model of dam deformation. The model is poor in terms of extension forecast. The paper hereby has obtained the mean temperature $T_m$ and linear temperature difference $T_d$ in the direction of horizontal section thickness upon theoretical calculation, and established a new-type statistical model by taking $T_m$ and $T_d$ as temperature factors, i.e. TMTD statistical model. As the study result shows, TMTD statistical model established herein is significantly superior to the early temperature factor statistical model in terms of forecast function.

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References