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ORIGINAL ARTICLE

A Grey Theory Based Multiple Attribute Approach for R&D Project Portfolio Selection

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Abstract In this paper, the research and development (R&D) project portfolio selection problem is introduced as a multiple attribute decision making problem. Recognizing and modeling the project interdependencies provide valuable cost savings and other greater benefits to organizations. Therefore, besides conventional attributes like cost and outcome, different type of interdependencies are also considered as attributes. Since the decision makers' preferences on the project alternatives or attributes are uncertain, a grey theory based method is proposed to cope with the uncertainty. correspondingly, the preferences and ratings of the attributes are described by linguistic variables, which are further expressed as grey numbers. Consequently, a ranking order of the projects is done using grey possibility degree and is used to determine the portfolio. To explore, an illustration is done by a case study. The methodology proposed here is shown to be an efficient approach to solve the R&D project portfolio selection problem.

Keywords Multiple attribute decision making · Grey number · R&D project portfolio selection · Project interdependency · Grey possibility degree

1. Introduction

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R&D project portfolio selection is one of the most important problems for decision makers in industry as well as in academic research. In today's environment with increasing competition and limitations on financial resources, the way of selection of R&D projects that maximize some utility measure or benefit to the organization has become a critical one. The purposes of project portfolio decision are to allocate a limited set of resources to projects in a way that balances risk, reward and alignment with corporate strategy. However, portfolio decisions are complicated, because of the long lead-time for R&D and market and technology dynamics. In addition, complex project and resource interdependencies make portfolio decisions more difficult. Needless to mention, poor selection of R&D projects could have a significantly negative impact on organizations for decades.

R&D project portfolio decision deals with future events and opportunities. Most of the information required to make portfolio decisions is at best, uncertain and at worst, very unreliable. Project selection is usually described in term of constraint optimization problems. Given a set of project proposals, the goal is to select a subset of projects to maximize some objective without violating the constraints. Though some methods [1-4, 8, 12, 14-19, 25] for R&D project portfolio selection already exist, unfortunately, R&D project managers have not been able to adopt many of these mechanisms.

Interdependent projects render an increase in benefit. When interdependencies occur, the parameters associated with a particular project depend upon which other projects have been selected; so that the total cost and benefit obtained from a portfolio is not equal to the sum of the individual project costs and benefits. Not much research has been done so far in tackling interdependencies. Schmidt [20] presented a model that accounts for the combined effect of benefit, outcome and resource interactions within a single set of projects. The model also allows for the allocation of several different resources. [21] discussed a nonlinear 0 -1 goal programming model for interdependent information system project selection. There, the authors formulated benefit, resource and technical interdependencies among candidate projects. [22] considered three phases of R&D project selection: first, a score based screening process identifies proposal candidates; next, an integer linear programming model determines all efficient portfolios keeping in mind multiple objectives, project interdependencies and time; finally, an interactive procedure matches portfolios with aspired benefit and resource. [7] proposed and demonstrated a methodology to the construction and analysis of efficient, effective and balanced portfolio of R&D project with interactions. Considering outcome, technical, resource and risk interdependencies, [9] proposed a R&D project portfolio selection model based on 0 -1 nonlinear mathematic programming method.

Deng [6] introduced Grey System Theory as an interdisciplinary scientific area. The theory has become quite popular since then with its ability to deal with the systems that have partially unknown parameters. As superiority to conventional statistical models, grey models require only a limited amount of data to estimate the behavior of unknown systems. During the last 30 years, the grey system theory has developed rapidly and has caught the attention of many researchers. It has been widely and successfully applied to various systems such as social, economical, financial, scientific, technological, medical, and other systems. In particular, in economical and financial field, Wang [24] combined the grey system theory [GM (1, 1) model with adaptive step size] and fuzzification technique to predict stock prices and showed the effectiveness of the method. Huang and Jane [10] combined the moving average autoregressive exogenous (ARX) prediction model with grey predictors for time series prediction and also showed that this hybrid technique has a greater forecasting accuracy than the GM (1, 1) model. Chang and Tsai [5] introduced a support vector regression grey model (SVRGM) which combines support vector regression (SVR) learning algorithm and grey system theory to obtain a better approach to time series prediction. Li et al. [13] used a grey possibility degree to determine the ranking order of all alternatives of a supplier selection problem. Kayacan et al. [11] investigated the accuracies of different grey models such as GM (1, 1), grey Verhulst model, modified grey models using Fourier series to predict time series and showed that the modified GM (1, 1) using Fourier series in time is the best in model fitting an forecasting.

In this paper, we deal with the problem of selecting projects among many project proposals in uncertain environment by a grey-based approach developed with the help of [13]. Firstly, the weight and rating of attributes for all alternative projects are explained and described by linguistic variables that can be expressed in a grey number. Secondly, a degree of grey possibility is proposed to determine the ranking order of all alternatives. Lastly, by depending upon the available assets and by taking into consideration the ranks of the projects, the project portfolio is selected. Here in Section 2, some preliminaries are discussed. In Section 3, the grey-based project portfolio selection approach is discussed. Section 4 consists of a case study leading to the application of the proposed approach. Finally, in Section 5 some concluding remarks are specified.

2. Preliminaries

Grey theory is one of the methods used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information. It is an efficient technique used to solve uncertainty problems with discrete data and incomplete information. In the theory, according to the degree of information, if the system information is fully known, the system is called a white system; if the information is totally unknown, the system is called a black system. A system with information known partially is called a grey system. In recent years, a fuzzy based approach has been proposed to deal with uncertainty with grey system background. The advantage of grey theory over fuzzy theory is that grey theory considers the condition of the fuzziness; that is grey theory can deal flexibly with the fuzziness situation. The theory includes five major parts: grey prediction, grey relational analysis (GRA), grey decision, grey programming and grey control. Here some basic definitions of the grey system, grey set and grey number in grey theory are provided.

Definition 2.1 *A grey system is defined as a system containing uncertain information presented by a grey number and grey variables.*

Fig. 1 explains the concept of grey system.

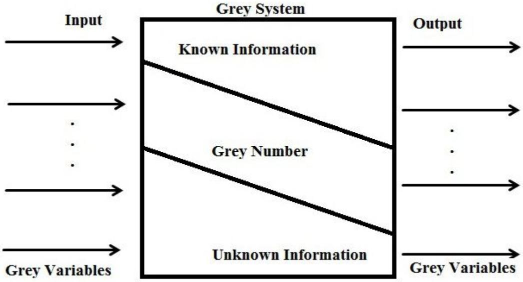

Fig. 1 Grey system

Definition 2.2 *Let X be the universal set and $X = \mathbb{R}$, the set of all real numbers. Then a grey system G of X is defined by the two mappings $\overline{\mu_G}$ and $\underline{\mu_G}$ where $\overline{\mu_G} : X \to [0, 1]$, $\underline{\mu_G} : X \to [0, 1]$, $\overline{\mu_G} \geq \underline{\mu_G}$.*

$\overline{\mu_G}$ and $\underline{\mu_G}$ are the upper and lower membership functions in G respectively.

Note: When $\overline{\mu_G} = \underline{\mu_G}$, the grey set G becomes a fuzzy set. Thus, the grey theory considers the condition of the fuzziness and can deal with the fuzziness situation.

Definition 2.3 *The grey number can be defined as a number with uncertain information. Basically, white number, black number and grey number are three classifications to distinguish the uncertainty level of information. Let $\otimes x = [\underline{x}, \overline{x}] = \{x \mid \underline{x} \leq \overline{x}; \underline{x}, \overline{x} \in \mathbb{R}\}$. Then $\otimes x$ that has two real parts $\underline{x}$ (the lower limit of $\otimes x$) and $\overline{x}$ (the upper limit of $\otimes x$) is defined as follows.*

- *If $\underline{x} \to -\infty$ and $\overline{x} \to \infty$, then $\otimes x$ is termed as the black number, which means without any meaningful information.*
- *Else if $\underline{x} = \overline{x}$, then $\otimes x$ is labeled as the white number, which means with complete information.*
- *Otherwise, $\otimes x = [\underline{x}, \overline{x}]$ is said to be the grey number, which means insufficient and uncertain information.*
- *$\otimes x = [-\infty, \overline{x}]$ is called a lower-limit grey number.*
- *$\otimes x = [\underline{x}, \infty]$ is named an upper-limit grey number.*

Definition 2.4 *Operations on grey numbers are defined on sets of intervals, not on real numbers. Let $+, -, \times, \div$ denote the operations of addition, subtraction, multiplication and division respectively. Let $\otimes x = [\underline{x}, \overline{x}]$, $\otimes y = [\underline{y}, \overline{y}]$ be two grey numbers. Then the arithmetic operations between $\otimes x, \otimes y$ are defined as follows:*

- $\otimes x + \otimes y = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$,

- $\otimes x - \otimes y = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$,
- $\otimes x \times \otimes y = [\min\{\underline{x}.\underline{y}, \underline{x}.\overline{y}, \overline{x}.\underline{y}, \overline{x}.\overline{y}\}, \max\{\underline{x}.\underline{y}, \underline{x}.\overline{y}, \overline{x}.\underline{y}, \overline{x}.\overline{y}\}]$,
- $\otimes x \div \otimes y = \otimes x \times [\frac{1}{\overline{y}}, \frac{1}{\underline{y}}], 0 \notin \otimes y$,
- $k \times \otimes x = [k\underline{x}, k\overline{x}]$.

Definition 2.5 *Let $\otimes x = [\underline{x}, \overline{x}]$, $\otimes y = [\underline{y}, \overline{y}]$ be two grey numbers. Then the possibility degree of $\otimes x \leq \otimes y$ is defined as (Shi et al.* [23]*):*

$$Pos\{\otimes x \leq \otimes y\} = \frac{\max\{0, \overline{x} + \overline{y} - \underline{x} - \underline{y} - \max\{0, \overline{x} - \underline{y}\}\}}{\overline{x} + \overline{y} - \underline{x} - \underline{y}}.$$

Remarks:

- *If $\overline{x} = \overline{y}, \underline{x} = \underline{y}$, then $\otimes x = \otimes y$ and $Pos\{\otimes x \leq \otimes y\} = 0.5$.*
- *If $\underline{y} > \overline{x}$, then $\otimes y > \otimes x$ and $Pos\{\otimes x \leq \otimes y\} = 1$.*
- *If $\overline{y} < \underline{x}$, then $\otimes y < \otimes x$ and $Pos\{\otimes x \leq \otimes y\} = 0$.*
- *If $Pos\{\otimes x \leq \otimes y\} > 0.5$, then also $\otimes y > \otimes x$.*
- *If $Pos\{\otimes x \leq \otimes y\} < 0.5$, then also $\otimes y < \otimes x$.*

3. A Grey Theory Based R&D Projects Portfolio Selection Method

In this section, we develop a grey-based ranking approach to a R&D project portfolio selection problem [13].

Let $P = \{P_1, P_2, \cdots, P_N\}$ be a discrete set of N possible R&D project alternatives.

Let $C = \{C_1, C_2, \cdots, C_N\}$ be the set representing the assets required for the candidate projects.

Let F be the total asset available such that $F < C_1 + C_2 + \cdots + C_N$. Thus, not all the candidate projects can be selected. Our aim is to select the best set of projects to distribute F among them according to their requirements.

Let $A = \{A_1, A_2, \cdots, A_M\}$ be a set of M attributes of projects. We have divided the attributes into two types: gain attributes and loss attributes. The attributes for which the greater values represent better results are called gain attributes. For example, mileage is a gain attribute while deciding to choose a car. The attributes for which the greater values represent worst results are called loss attributes. For example, cost is a loss attribute while deciding to choose a car.

Let $\otimes w = \{\otimes w_1, \otimes w_2, \cdots, \otimes w_M\}$ be the vector of attribute weights. We consider the attribute weights as linguistic variables. The linguistic variables are expressed in the form of grey numbers by the scales as shown in Table 1.

Table 1: The scale of attribute weights.

Scale	Attribute weights ($\otimes w$)
Very low (VL)	[0.0, 0.1]
Low (L)	[0.1, 0.3]
Medium low (ML)	[0.3, 0.4]
Medium (M)	[0.4, 0.6]
Medium high (MH)	[0.6, 0.7]
High (H)	[0.7, 0.9]
Very high (VH)	[0.9, 1.0]

The attribute ratings $\otimes r$ are also considered as linguistic variables. These linguistic variables are expressed in the form of grey numbers by the scales as shown in Table 2. The attribute weights in Table 1 and Table 2 are determined by using expert's opinions [from both academic and industry], which are further supported by many literatures of multiple attribute decision making [13]. However, one must agree that these weight distributions cannot be absolute and with different contexts, they may have to change.

Table 2: The scale of attribute ratings.

Scale	Attribute ratings ($\otimes r$)
Very poor (VP)	[0, 1]
Poor (P)	[1, 3]
Medium poor (MP)	[3, 4]
Fair (F)	[4, 6]
Medium good (MG)	[6, 7]
Good (G)	[7, 9]
Very good (VG)	[9, 10]

The decision makers (the committee of experts) will decide what figures are appropriate for the projects. As any single expert may not provide the exact weights, we have considered K number of experts and have taken the average of the weights given by them [see Eq. (1)]. In the case study [Section 4], B. M. Enterprise assigns five experts for the study.

In this article, we have considered seven scales for attributes. This cannot be claimed to be fixed and different scales like three or five may also be used. However, seven scales are used in many literatures of crisp, fuzzy and uncertain logic systems with great successes. That is why we have decided to use it in grey system theories.

The procedure is discussed in the following steps.

Step 1: Form a committee of experts and identify the attribute weights of projects. Let a decision group has K number of experts. Let $\otimes w_j^d$ be the attribute weight of the d^{th} decision maker for the j^{th} attribute A_j and $\otimes w_j^d = [\underline{w}_j^d, \overline{w}_j^d], d = 1, 2, 3, \cdots, K$. Then the attribute weight of attribute A_j is

$$\otimes w_j = \frac{1}{K}[\otimes w_j^1 + \otimes w_j^2 + \cdots + \otimes w_j^K]. \tag{1}$$

Step 2: For $i^{th}(i = 1, 2, \cdots, N)$ project and $j^{th}(j = 1, 2, \cdots, M)$ attribute, the attribute rating value is defined as

$$\otimes r_{ij} = \frac{1}{K}[\otimes r_{ij}^1 + \otimes r_{ij}^2 + \cdots + \otimes r_{ij}^K], \tag{2}$$

where $\otimes r_{ij}^d$ is the attribute rating value of the d^{th} decision maker and $\otimes r_{ij}^d = [\underline{r}_{ij}^d, \overline{r}_{ij}^d], (d = 1, 2, \cdots, K)$.

Step 3: Construct the grey decision matrix D as

$$D = \begin{pmatrix} \otimes r_{11} & \otimes r_{12} & \cdots & \otimes r_{1M} \\ \otimes r_{21} & \otimes r_{22} & \cdots & \otimes r_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes r_{N1} & \otimes r_{N2} & \cdots & \otimes r_{NM} \end{pmatrix}. \tag{3}$$

Step 4: If A_j is a gain attribute, let

$$\otimes r_{ij}^* = [\frac{\underline{r}_{ij}}{\max_{1\leq i\leq N} \overline{r}_{ij}}, \frac{\overline{r}_{ij}}{\max_{1\leq i\leq N} \overline{r}_{ij}}]. \tag{4}$$

If A_j is a loss attribute, let

$$\otimes r_{ij}^* = [\frac{\min_{1\leq i\leq N} \underline{r}_{ij}}{\overline{r}_{ij}}, \frac{\min_{1\leq i\leq N} \underline{r}_{ij}}{\underline{r}_{ij}}]. \tag{5}$$

The normalization method is to preserve the property that the ranges of normalized grey numbers belong to [0, 1]. The grey decision matrix D is normalized as

$$D^* = \begin{pmatrix} \otimes r_{11}^* & \otimes r_{12}^* & \cdots & \otimes r_{1M}^* \\ \otimes r_{21}^* & \otimes r_{22}^* & \cdots & \otimes r_{2M}^* \\ \vdots & \vdots & \ddots & \vdots \\ \otimes r_{N1}^* & \otimes r_{N2}^* & \cdots & \otimes r_{NM}^* \end{pmatrix}. \tag{6}$$

Step 5: Create the weighted normalized grey decision matrix. By taking into consideration of different importance in each attributes, the weighted normalized grey decision matrix V is constructed as

$$V = \begin{pmatrix} \otimes v_{11} & \otimes v_{12} & \cdots & \otimes v_{1M} \\ \otimes v_{21} & \otimes v_{22} & \cdots & \otimes v_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes v_{N1} & \otimes v_{N2} & \cdots & \otimes v_{NM} \end{pmatrix}, \tag{7}$$

where $\otimes v_{ij} = \otimes r^*_{ij} \times \otimes w_{ij}$.

Step 6: Compose the ideal alternative as a referential one. For N project alternatives set $P = \{P_1, P_2, \cdots, P_N\}$, the ideal referential project alternative P^{ideal} is obtained as

$$P^{ideal} = \{\otimes g_1, \otimes g_2, \cdots, \otimes g_M\}, \tag{8}$$

where

$$\otimes g_j = [\max_{1 \le i \le N} \underline{v}_{ij}, \max_{1 \le i \le N} \overline{v}_{ij}]. \tag{9}$$

Step 7: The grey possibility degree between candidate project alternatives set $P = \{P_1, P_2, \cdots, P_N\}$ and the ideal referential alternative P^{ideal} by

$$Pos\{P_i \le P^{ideal}\} = \frac{1}{M} \sum_{j=1}^{M} Pos\{\otimes v_{ij} \le g_{ij}\}. \tag{10}$$

Step 8: Rank the order of project alternatives according to the ascending values of grey possibility degrees. When $Pos\{P_i \le P^{ideal}\}$ is smaller, the ranking order of P_i is better. Otherwise, it is worse.

Step 9: Let after rearranging the projects according to their ranks, the set of projects become $P' = \{P'_1, P'_2, \cdots, P'_N\}$. Let $C' = \{C'_1, C'_2, \cdots, C'_N\}$ be the set representing the respective assets required for these projects. Let

$$C'_1 + C'_2 + \cdots + C'_m \le F, C'_1 + C'_2 + \cdots + C'_{m+1} > F, m < N. \tag{11}$$

Then the project portfolio will consist of the projects $P'_1, P'_2, \cdots, P'_m$.

Note: The attribute weights in Table 2 are set intentionally in 0 and 10. The reason behind choosing the scale [0, 10] in place of [0, 1] is nothing but to distinguish between the scales of attribute weights and attribute ratings. Basically, the scale does not have any impact on the final output. In place of [0, 10], if you consider [0, 1], or even [0, 100], we would get the same result/output. See the normalized matrix D^* in Eq. (6). For different scales, we will get different D [Eq. (3)]. However, after doing the normalization by Eqs. (4) and (5), when we construct the D^* [Eq. (6)], it would become independent of the scale we choose.

4. A Case Study

B. M. Enterprise is a medium-large scale organization of West Bengal, India. The R&D wing of this organization is involved in different structural works in civil, mechanical and electrical fields. Last year, the organization has short-listed five project proposals from private as well as public sectors for consideration. They are renamed as P_1, P_2, P_3, P_4 and P_5 due to privacy. All the proposals accompany data on the attributes like estimated outcome, cost and risk, required resources, sharing cost and effects of different interdependencies between projects. There are ten attributes among which five are gain attributes and five are loss attributes. The details about the attributes are specified in Table 3.

So there are five candidate projects P_i (i = 1, 2, 3, 4, 5) selected as alternatives against the ten attributes A_j ($j = 1, 2, \cdots, 10$).

B. M. Enterprise assigns five experts (decision makers) to follow the proposed approach and rename them as D_1, D_2, D_3, D_4 and D_5. The R&D projects portfolio is then obtained through the following steps. The whole structure of R&D project portfolio selection is shown in Fig. 2.

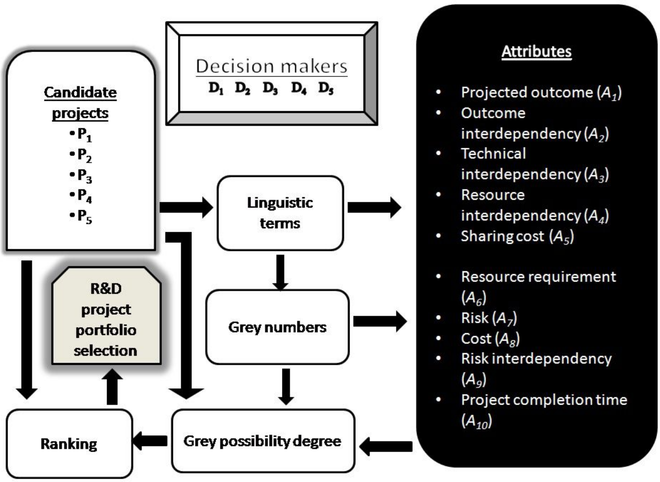

Fig. 2 R&D project portfolio section

Step 1: The decision makers are asked to assign the weights to the attributes by following Table 1. According to Eq. (1), the evaluation values of attribute weights from the five decision makers are obtained and the results are shown in Table 4.

Step 2: We request the experts to assign the ratings for the projects against the attributes by following Table 2. According to Eq. (2), the attribute rating values for the five projects are obtained and are shown in Table 5.

Table 3: Attributes & their properties.

Attribute type	Attribute name	Notation	Significance
Gain attributes	Projected outcome	A_1	Traditional attribute; this attribute implies the expected outcome of individual projects, if selected.
	Outcome interdependency	A_2	This interdependency affect the overall outcome obtained from a project portfolio. When the outcome interdependency occurs, the total value of a project portfolio is greater than the sum of the individual project values.
	Technical interdependency	A_3	Technical interdependencies result from leveraging common technology across multiple projects. When the technical interdependency occurs, the total value of a project portfolio is greater than the sum of the individual project values.
	Resource interdependency	A_4	Resource interdependencies result from sharing limited resources between different projects. The resource allocation for each project is inversely related to resources for each concurrent project, an increase in the resource level for one project would lead to a decrease level of another project. Some resources may be shared among one or more projects in such way that the implementation of one project reduces the resource consumption of interrelated projects.
	Sharing cost	A_5	It is the savings due to costs shared between projects resulting reduction in the total cost.
Loss attributes	Resource requirement	A_6	Resources required for individual projects; the less is good.
	Risk	A_7	Risk attached with the projects must be as less as possible. As the futures of all the projects are uncertain, implementation of a project may or may not yield us success. In case of failure, the decision makers may lose their money, time, and resource.
	Cost	A_8	Traditional attribute; this attribute implies the expected cost for individual projects, if selected.
	Risk interdependency	A_9	Consideration of two or more projects in a period may increase the risk of the portfolio by a large amount.
	Project completion time	A_{10}	Time required for individual projects; the less is good.

Step 3: By using Eq. (3), the grey decision table is constructed and is shown in Table 6.

Step 4: By using Eqs. (4)-(6), the normalized grey decision table is constructed and it is shown in Table 7.

Step 5: The grey weighted normalized decision matrix is formulated according to Eq. (7). It is shown in Table 8.

Step 6: Using Eqs. (8) and (9), P^{ideal}, an ideal referential alternative is constructed and is shown in Table 9.

Step 7: Next, the grey possibility degrees between the five R&D project alternatives and the ideal referential project alternative P^{ideal} are calculated by using Eq. (10). The results are shown in Table 10.

Step 8: Finally, we rank the project alternatives according to the ascending values of their grey possibility grades. Since P_3 has the minimum grey possibility grade, it is ranked 1; the next minimum possibility grade is of P_4 that is ranked 2 and so on. The result of the ranking order is as follows:

$$P_3 > P_4 > P_2 > P_5 > P_1.$$

Step 9: By following our approach and according to the assets available, using Eq. (11), B. M. Enterprise selects the projects P_3, P_4 and P_2.

Table 4: Attribute weights.

	D_1	D_2	D_3	D_4	D_5	$\otimes w_j$
A_1	H	VH	H	VH	H	[0.78, 0.94]
A_2	H	H	MH	H	MH	[0.66, 0.82]
A_3	MH	MH	M	H	H	[0.60, 0.76]
A_4	H	VH	H	MH	H	[0.72, 0.88]
A_5	VH	H	MH	MH	VH	[0.74, 0.86]
A_6	H	MH	MH	H	H	[0.66, 0.82]
A_7	H	H	H	VH	VH	[0.78, 0.94]
A_8	H	M	MH	H	H	[0.66, 0.82]
A_9	VH	VH	H	H	H	[0.74, 0.92]
A_{10}	M	M	MH	MH	MH	[0.52, 0.66]

Table 5: Attributes rating values for candidate projects.

A_j	P_i	D_1	D_2	D_3	D_4	D_5	$\otimes r_{ij}$
A_1	P_1	F	MG	F	G	MG	[5.4, 7.0]
	P_2	G	MG	G	G	G	[6.8, 8.6]
	P_3	G	VG	G	VG	VG	[8.2, 9.6]
	P_4	VG	MG	MG	G	G	[7.0, 8.4]
	P_5	MG	G	G	MG	F	[6.0, 7.6]
A_2	P_1	F	MG	MG	MG	F	[5.2, 6.6]
	P_2	F	F	MG	MG	G	[5.4, 7.0]
	P_3	VG	G	VG	VG	VG	[8.6, 9.8]
	P_4	G	G	G	G	G	[7.0, 9.0]
	P_5	F	F	MG	MG	MP	[4.6, 6.0]
A_3	P_1	MP	MP	F	MG	F	[4.0, 5.4]
	P_2	F	F	F	MG	F	[4.4, 6.2]
	P_3	G	G	MG	G	MG	[6.6, 8.2]
	P_4	G	VG	G	VG	G	[7.8, 9.4]
	P_5	G	MG	MG	MG	F	[5.8, 7.2]
A_4	P_1	G	MG	G	MG	G	[6.6, 8.2]
	P_2	MP	F	F	MP	MP	[3.4, 4.8]
	P_3	F	F	F	MG	G	[5.0, 6.8]
	P_4	MG	MG	G	MG	G	[6.4, 7.8]
	P_5	VG	G	VG	MG	G	[7.6, 9.0]
A_5	P_1	MG	F	MG	G	F	[5.4, 7.0]
	P_2	G	VG	VG	VG	VG	[8.6, 9.8]
	P_3	F	F	F	F	F	[4.0, 6.0]
	P_4	G	MG	G	MG	G	[6.6, 8.2]
	P_5	MG	F	G	G	G	[6.2, 8.0]
A_6	P_1	G	G	MG	MG	G	[6.6, 8.2]
	P_2	G	VG	MG	G	G	[7.2, 8.8]
	P_3	F	F	MG	MG	G	[5.4, 7.0]
	P_4	MG	MG	MG	G	G	[6.4, 7.8]
	P_5	G	F	G	F	MP	[5.0, 6.8]
A_7	P_1	G	G	G	G	G	[7.0, 9.0]
	P_2	MG	G	F	MG	F	[5.4, 7.0]
	P_3	MP	MP	F	MP	F	[3.4, 4.8]
	P_4	F	MP	MG	F	MG	[4.6, 6.0]
	P_5	G	VG	MG	G	G	[7.2, 8.8]
A_8	P_1	G	VG	VG	G	VG	[8.2, 9.6]
	P_2	MP	F	MP	F	F	[3.6, 5.2]
	P_3	F	F	MG	F	F	[4.4, 6.2]
	P_4	G	F	VG	G	G	[6.8, 8.6]
	P_5	G	G	MG	MG	MG	[6.4, 7.8]
A_9	P_1	MG	G	G	G	G	[6.8, 8.6]
	P_2	G	MG	MG	G	F	[6.0, 7.6]
	P_3	F	F	MP	F	F	[3.8, 5.6]
	P_4	MP	F	F	MP	MP	[3.4, 4.8]
	P_5	VG	G	G	MG	G	[7.2, 8.8]
A_{10}	P_1	G	VG	MG	G	G	[7.2, 8.8]
	P_2	VG	VG	G	G	MG	[7.6, 9.0]
	P_3	F	F	F	MP	MP	[3.6, 5.2]
	P_4	MG	MG	G	F	MG	[5.8, 7.2]
	P_5	VG	G	MG	G	VG	[7.6, 9.0]

Table 6: Grey decision table (D).

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
P_1	[5.4, 7.0]	[5.2, 6.6]	[4.0, 5.4]	[6.6, 8.2]	[5.4, 7.0]	[6.6, 8.2]	[7.0, 9.0]	[8.2, 9.6]	[6.8, 8.6]	[7.2, 8.8]
P_2	[6.8, 8.6]	[5.4, 7.0]	[4.4, 6.2]	[3.4, 4.8]	[8.6, 9.8]	[7.2, 8.8]	[5.4, 7.0]	[3.6, 5.2]	[6.0, 7.6]	[7.6, 9.0]
P_3	[8.2, 9.6]	[8.6, 9.8]	[6.6, 8.2]	[5.0, 6.8]	[4.0, 6.0]	[5.4, 7.0]	[3.4, 4.8]	[4.4, 6.2]	[3.8, 5.6]	[3.6, 5.2]
P_4	[7.0, 8.4]	[7.0, 9.0]	[7.8, 9.4]	[6.4, 7.8]	[6.6, 8.2]	[6.4, 7.8]	[4.6, 6.0]	[6.8, 8.6]	[3.4, 4.8]	[5.8, 7.2]
P_5	[6.0, 7.6]	[4.6, 6.0]	[5.8, 7.2]	[7.6, 9.0]	[6.2, 8.0]	[5.0, 6.8]	[7.2, 8.8]	[6.4, 7.8]	[7.2, 8.8]	[7.6, 9.0]

Table 7: Normalized grey decision table (D^*).

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
P_1	[0.563, 0.729]	[0.531, 0.673]	[0.426, 0.574]	[0.733, 0.911]	[0.551, 0.714]	[0.610, 0.758]	[0.378, 0.486]	[0.375, 0.439]	[0.395, 0.500]	[0.409, 0.500]
P_2	[0.708, 0.896]	[0.551, 0.714]	[0.468, 0.660]	[0.378, 0.533]	[0.878, 1.000]	[0.568, 0.694]	[0.486, 0.630]	[0.692, 1.000]	[0.447, 0.567]	[0.400, 0.474]
P_3	[0.854, 1.000]	[0.878, 1.000]	[0.702, 0.872]	[0.556, 0.756]	[0.408, 0.612]	[0.714, 0.926]	[0.708, 1.000]	[0.581, 0.818]	[0.607, 0.895]	[0.692, 1.000]
P_4	[0.729, 0.875]	[0.714, 0.918]	[0.830, 1.000]	[0.711, 0.867]	[0.673, 0.837]	[0.641, 0.781]	[0.567, 0.739]	[0.419, 0.529]	[0.708, 1.000]	[0.500, 0.621]
P_5	[0.625, 0.792]	[0.469, 0.612]	[0.617, 0.766]	[0.844, 1.000]	[0.633, 0.816]	[0.735, 1.000]	[0.386, 0.472]	[0.462, 0.563]	[0.386, 0.472]	[0.400, 0.474]

Table 8: Weighted normalized grey decision table (V).

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
P_1	[0.439, 0.685]	[0.350, 0.552]	[0.256, 0.436]	[0.528, 0.802]	[0.408, 0.614]	[0.403, 0.622]	[0.295, 0.457]	[0.248, 0.360]	[0.292, 0.460]	[0.213, 0.330]
P_2	[0.552, 0.842]	[0.364, 0.585]	[0.281, 0.502]	[0.272, 0.469]	[0.650, 0.860]	[0.375, 0.569]	[0.379, 0.592]	[0.457, 0.820]	[0.331, 0.522]	[0.208, 0.313]
P_3	[0.666, 0.940]	[0.579, 0.820]	[0.421, 0.663]	[0.400, 0.665]	[0.302, 0.526]	[0.471, 0.759]	[0.552, 0.940]	[0.383, 0.671]	[0.449, 0.823]	[0.360, 0.660]
P_4	[0.569, 0.823]	[0.471, 0.753]	[0.498, 0.760]	[0.512, 0.763]	[0.498, 0.720]	[0.423, 0.640]	[0.442, 0.695]	[0.277, 0.434]	[0.524, 0.920]	[0.260, 0.410]
P_5	[0.488, 0.744]	[0.310, 0.502]	[0.370, 0.582]	[0.608, 0.880]	[0.468, 0.702]	[0.485, 0.820]	[0.301, 0.444]	[0.305, 0.462]	[0.286, 0.434]	[0.208, 0.313]

Table 9: Ideal referential alternative (P^{ideal}).

$\otimes g_1$	$\otimes g_2$	$\otimes g_3$	$\otimes g_4$	$\otimes g_5$	$\otimes g_6$	$\otimes g_7$	$\otimes g_8$	$\otimes g_9$	$\otimes g_{10}$
[0.666, 0.940]	[0.579, 0.820]	[0.498, 0.760]	[0.608, 0.880]	[0.650, 0.860]	[0.485, 0.820]	[0.552, 0.940]	[0.457, 0.820]	[0.524, 0.920]	[0.360, 0.660]

Table 10: Grey possibility degree.

	$Pos\{\otimes v_{ij} \leq \otimes g_j\}$										$Pos\{P_i$
$\downarrow j \mid i \rightarrow$	1	2	3	4	5	6	7	8	9	10	$\leq P^{ideal}\}$
1	0.963	1.000	1.000	0.645	1.000	0.754	1.000	1.000	1.000	1.000	0.936
2	0.688	0.987	0.992	1.000	0.500	0.841	0.932	0.500	1.000	1.000	0.844
3	0.500	0.500	0.673	0.894	1.000	0.560	0.500	0.671	0.612	0.500	0.641
4	0.816	0.667	0.500	0.704	0.838	0.719	0.777	1.000	0.500	0.889	0.741
5	0.853	1.000	0.823	0.500	0.883	0.500	1.000	0.990	1.000	1.000	0.855

5. Conclusion

In this paper, we consider the R&D project portfolio selection problem as an MADM problem. A grey theory based approach is proposed for it in a grey uncertain environment. In a conventional MADM problem, the ratings and weights of the attributes are considered as exact numeric values. Since the decision-makers' judgments are uncertain and hence cannot be expressed precisely in many situations, thereby expressing weights and rankings of the attributes for projects as an exact numerical value is not realistic. Therefore, traditional MADM is not appropriate to deal with this problem. System analysis can be treated from the point of view of the degree of information on hand. The advantage of grey theory over fuzzy set theory is that it can deal flexibly with the fuzzy situation. Here R&D project portfolio selection is viewed as a grey system process. Moreover, the ratings and weights are expressed by linguistic terms with their grey number representation. Subsequently, the concept of grey possibility degree is used to rank the candidate projects for selection. Finally, an empirical case study of R&D project portfolio selection is done to illustrate the method. The experimental result shows that the proposed approach is consistent and sensible.

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