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Reduction Method Based on a New Fuzzy Rough Set in Fuzzy Information System and Its Applications to Scheduling Problems

MIN LIU*
Department of Automation, Tsinghua University
Beijing, 100084, P.R. China
lium@mail.tsinghua.edu.cn

DEGANG CHEN
Department of Mathematics and Physics, North China Electric Power University
Beijing, 102206, P.R. China

CHENG WU
Department of Automation, Tsinghua University
Beijing, 100084, P.R. China

HONGXING LI
Department of Mathematics, Normal University
Beijing, 100875, P.R. China

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Abstract—In this paper, we present the concept of fuzzy information granule based on a relatively weaker fuzzy similarity relation called fuzzy T_L-similarity relation for the first time. Then, according to the fuzzy information granule, we define the lower and upper approximations of fuzzy sets and a corresponding new fuzzy rough set. Furthermore, we construct a kind of new fuzzy information system based on the fuzzy T_L-similarity relation and study its reduction using the fuzzy rough set. At last, we apply the reduction method based on the defined fuzzy rough set in the above fuzzy information system to the reduction of the redundant multiple fuzzy rule in the scheduling problems, and numerical computational results show that the reduction method based on the new fuzzy rough set is more suitable for the reduction of multiple fuzzy rules in the scheduling problems compared with the reduction methods based on the existing fuzzy rough set.

Keywords—Fuzzy rough set, Fuzzy information granule, Fuzzy information system, Reduction, Scheduling.

1. INTRODUCTION

As a tool for processing uncertain and incomplete information, the rough set theory was originally proposed by Pawlak [1]. This theory deals with the approximations of an arbitrary subset of a

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*Author to whom all correspondence should be addressed.

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universe by two definable subsets called lower and upper approximations, respectively. Additionally, the fuzzy set theory [2] for processing vagueness and uncertainty is also a generalization of the crisp set theory. It is generally accepted that these two theories are related, but distinct and complementary [3–6]. Many attempts have been made to combine these two theories. Dubois and Prade [4] first studied the fuzzification problem of rough sets. Furthermore, they defined the upper and lower approximations of the fuzzy sets with respect to a fuzzy min-similarity relation [4]. Additionally, the above definitions of the fuzzy rough set were generalized to a more general case in [7]. At present, the fuzzy rough set has been applied to practical problems. For example, the fuzzy rough set theory was used to extract the fuzzy decision rules from the fuzzy information systems [8,9] and to reduce the redundant condition attributes in information systems [10,11].

Up to now, the existing fuzzy rough sets are all defined with respect to fuzzy min-similarity relation having the following characteristics. First, the lower and upper approximations of a fuzzy set are defined just by its membership function, and they can not be expressed as the unions of some elementary information granules so that they have not some properties as the crisp rough sets do. Second, the lower and upper approximations of a definable fuzzy set may not always be definable. Third, the condition needed for the fuzzy min-transitivity in the fuzzy min-similarity relation R defined seems too strict so that the similar degree between two objects in U cannot be measured effectively by means of the fuzzy min-similarity relation. Due to these characteristics, the application fields of the existing fuzzy rough sets are limited.

In this paper, we use a relatively weaker fuzzy similarity relation called fuzzy T_L-similarity relation [12,13] to measure the similar degree between two objects in universe U. Based on the fuzzy T_L-similarity relation, we present the concept of fuzzy information granule for the first time. Then, on the basis of the fuzzy information granule, we define the lower and upper approximations of fuzzy sets and a corresponding new fuzzy rough set. Furthermore, we construct a kind of new fuzzy information system based on the fuzzy T_L-similarity relation and study its reduction using the new fuzzy rough set. At last, we apply the reduction method based on the defined fuzzy rough set in the new fuzzy information system to the scheduling problems. Numerical computational results show that the reduction method based on the new rough set is more suitable for the reduction of the redundant multiple fuzzy rules in the scheduling problems compared with the one based on the existing fuzzy rough set.

This paper is organized as follows. In Section 2, some basic notions of the crisp rough sets are given. In Section 3, we define a new fuzzy rough set. In Section 4, we construct a kind of new fuzzy information system and study its reduction. In Section 5, we apply the proposed reduction method to the scheduling problems and give the numerical computational results. At last, some conclusions are presented in Section 6.

2. LOWER AND UPPER APPROXIMATIONS AND CRISP ROUGH SETS

Let U be a finite and nonempty universe, and let $R \subseteq U \times U$ be an equivalence relation on U, i.e., R is reflexive, symmetric, and transitive. Then, the equivalence relation R partitions the universe U into disjoint subsets, each of which is an equivalence class defined by R. Elements in the same equivalence class are indistinguishable. Additionally, equivalence classes are also called elementary sets or information granules and any union of elementary sets is called a definable set [1]. Also, (U, R) is called an approximation space.

Given an arbitrary set $X \subseteq U$, we can characterize X by a pair of lower and upper approximations. The lower approximation $\underline{\mathrm{apr}}_R X$ is the greatest definable set contained in X and the upper approximation $\overline{\mathrm{apr}}_R X$ is the least definable set containing X. They can be obtained by the following two formulas respectively,

$$\underline{\mathrm{apr}}_R X = \{x \mid [x]_R \subseteq X\} \qquad \text{or} \qquad \underline{\mathrm{apr}}_R X = \cup \{[x]_R \mid [x]_R \subseteq X\},$$
$$\overline{\mathrm{apr}}_R X = \{x \mid [x]_R \cap X \neq \phi\} \qquad \text{or} \qquad \overline{\mathrm{apr}}_R X = \cup \{[x]_R \mid [x]_R \cap X \neq \phi\}.$$

The lower and upper approximation operators $\underline{\mathrm{apr}}_R$ and $\overline{\mathrm{apr}}_R$ have the following properties.

(1) $\underline{\mathrm{apr}}_R U = \overline{\mathrm{apr}}_R U = U$, $\overline{\mathrm{apr}}_R \phi = \underline{\mathrm{apr}}_R \phi = \phi$;
(2) $\underline{\mathrm{apr}}_R (X \cap Y) = \underline{\mathrm{apr}}_R X \cap \underline{\mathrm{apr}}_R Y$, $\overline{\mathrm{apr}}_R (X \cup Y) = \overline{\mathrm{apr}}_R X \cup \overline{\mathrm{apr}}_R Y$;
(3) $\underline{\mathrm{apr}}_R X^c = (\overline{\mathrm{apr}}_R X)^c$, $\overline{\mathrm{apr}}_R X^c = (\underline{\mathrm{apr}}_R X)^c$;
(4) $\underline{\mathrm{apr}}_R X \subseteq X$, $X \subseteq \overline{\mathrm{apr}}_R X$;
(5) $X \subseteq \underline{\mathrm{apr}}_R (\overline{\mathrm{apr}}_R X)$, $\overline{\mathrm{apr}}_R (\underline{\mathrm{apr}}_R X) \subseteq X$;
(6) $\underline{\mathrm{apr}}_R X \subseteq \underline{\mathrm{apr}}_R (\underline{\mathrm{apr}}_R X)$, $\overline{\mathrm{apr}}_R (\overline{\mathrm{apr}}_R X) \subseteq \overline{\mathrm{apr}}_R X$.

Let $\mathbf{R}$ be a family of equivalence relations on U, then $\cap \mathbf{R} = \{\cap R \mid R \in \mathbf{R}\}$ is also an equivalence relation on U, where $\cap \mathbf{R}$ is called indiscernibility relation on $\mathbf{R}$ and is denoted as $\mathrm{ind}\,(\mathbf{R})$. It is clear that $[x]_{\mathrm{ind}\,(\mathbf{R})} = \cap \{[x]_R \mid R \in \mathbf{R}\}$. Additionally, $(U, \mathbf{R})$ is called an information system.

Suppose that $R \in \mathbf{R}$, if $\mathrm{ind}(\mathbf{R}) = \mathrm{ind}\,(\mathbf{R} - \{R\})$, R is called unnecessary in $\mathbf{R}$, otherwise R is called necessary in $\mathbf{R}$. If every $R \in \mathbf{R}$ is necessary in $\mathbf{R}$, $\mathbf{R}$ is called independent, otherwise $\mathbf{R}$ is called dependent. Suppose that $\mathbf{P} \subseteq \mathbf{R}$, if $\mathbf{P}$ is independent and $\mathrm{ind}\,(\mathbf{P}) = \mathrm{ind}\,(\mathbf{R})$, $\mathbf{P}$ is called a reduction of $\mathbf{R}$. It is clear that $\mathbf{R}$ may have more than one reduction. The intersection of all reductions of $\mathbf{R}$ is called core of $\mathbf{R}$ which is denoted as $\mathrm{core}(\mathbf{R})$. If $\mathrm{core}(\mathbf{R}) \neq \phi$, we have $\mathrm{core}(\mathbf{R}) = \cap \mathrm{red}(\mathbf{R})$, where $\mathrm{red}(\mathbf{R})$ is the set of all reductions of $\mathbf{R}$.

3. FUZZY INFORMATION GRANULE AND LOWER AND UPPER APPROXIMATIONS OF FUZZY SETS

In the above section, we give some basic notions of crisp rough sets. In this section, crisp rough sets are generalized to fuzzy rough sets.

The existing fuzzy rough sets are all defined on the basis of the fuzzy min-similarity relation. Let U be a nonempty universe. For any $x, y \in U$, let $R(x, y)$ denote the similar degree between two objects x and y. Then, a fuzzy binary relation R on U is called a fuzzy min-similarity relation iff R is

(1) reflexive: $R(x, x) = 1$ for any object $x \in U$;
(2) symmetric: $R(x, y) = R(y, x)$ for any $x, y \in U$;
(3) min-transitive: $R(x, y) \geq \sup \min_{z \in U} \{R(x, z), R(z, y)\}$ for any $x, y, z \in U$.

According to the above fuzzy min-transitivity, we have that

$$\begin{aligned} R(x, z) &= R(y, z), \qquad \text{when } R(x, y) > R(y, z), \\ R(x, z) &= R(x, y), \qquad \text{when } R(x, y) < R(y, z), \end{aligned}$$

for any $x, y, z \in U$. It is obvious that at least two of three values $R(x, y)$, $R(y, z)$, and $R(x, z)$ are equal to each other. The above condition needed for the fuzzy min-transitivity in the fuzzy min-similarity relation R defined seems too strict so that the similar degree between two objects in U cannot be measured effectively by means of the fuzzy min-similarity relation. In this section, we define a new fuzzy rough set based on a relatively weaker transitive relation called fuzzy T_L-similarity relation by the well-known Lukasiewicz t-norm T_L which is defined as $T_L(a, b) = \max\{0, a + b - 1\}$ [7]. A fuzzy T_L-similarity relation R on U is

(1) reflexive: $R(x, x) = 1$ for any object $x \in U$;
(2) symmetric: $R(x, y) = R(y, x)$ for any $x, y \in U$;
(3) T_L-transitive: $R(x, y) \geq T_L\{R(x, z), R(z, y)\}$ for any $x, y, z \in U$.

For any three objects $x, y, z \in U$, if $R(x, z)$ and $R(y, z)$ are both close to 1, $R(x, y)$ should be close to but may not equal $R(x, z)$ or $R(y, z)$. For example, if we take that $R(x, z) = 0.8$, $R(z, y) = 0.9$, and $R(x, y) = 0.75$, it is obvious that $R(x, y)$, $R(y, z)$, and $R(x, z)$ satisfy the T_L-transitivity and are not equal to each other. From the above example, we can see that the condition needed for the T_L-transitivity in the fuzzy T_L-similarity relation is relatively weaker than that needed for the fuzzy min-transitivity.

Let R be a fuzzy min-similarity relation on U, then, for any fuzzy set $A \in F(U)$, the fuzzy rough set is a pair of fuzzy sets $(R_*(A), R^*(A))$ in [4], where $R_*(A)(x) = \inf_{y\in U} \max\{1 - R(x,y), A(y)\}$ and $R^*(A)(x) = \sup_{y\in U} \min\{R(x,y), A(y)\}$ for any $x \in U$. The lower and upper approximation operators R_* and R^* above satisfy that $R^*(A^c) = (R_*(A))^c$ and $R_*(A^c) = (R^*(A))^c$, but do not satisfy that $R^*(R_*(A)) = R_*(A)$ and $R_*(R^*(A)) = R^*(A)$, that is, the lower and upper approximations of a definable fuzzy set may be undefinable. So, the lower and upper approximations of a definable fuzzy set have not the corresponding characteristics of the definable set in the crisp rough set theory. Also, from the above definitions of R_* and R^*, there do not exist the elementary sets for constructing R_* and R^*. For generalizing reasonably the concepts of the lower and upper approximations of a definable set in the crisp rough set theory to fuzzy cases, we define the fuzzy information granule in Definition 3.1.

First, we introduce the definition of fuzzy point. For any object $x \in U$ and $\lambda \in (0,1]$, fuzzy point x_λ is a fuzzy set defined as

$$x_\lambda(y) = \begin{cases} \lambda, & y = x, \\ 0, & y \neq x. \end{cases}$$

It is obvious that any fuzzy set A on U can be expressed as a union of some fuzzy points, and every fuzzy point can always be expressed as a union of some smaller elements (fuzzy points) in the fuzzy set theory. In contrast, every point is the smallest element and cannot be expressed as a union of other elements in the crisp set theory.

Then, based on the above fuzzy point, we define the fuzzy information granule in Definition 3.1.

DEFINITION 3.1. *Let R be a fuzzy T_L-similarity relation on U, and let x_λ be a fuzzy point, then fuzzy set $[x_\lambda]_R$ defined by $[x_\lambda]_R(y) = T_L(\lambda, R(x,y))$ is called the fuzzy information granule with respect to x_λ.*

It is obvious that fuzzy information granule $[x_\lambda]_R(y)$ can be constructed by means of fuzzy point x_λ and the corresponding fuzzy T_L-similarity relation R.

The fuzzy information granule has the following properties in Lemma 3.2.

LEMMA 3.2. *For any $x, y \in U$, $\lambda, \mu \in (0,1]$, we have the following.*

(1) $[x_\lambda]_R(x) = \lambda = \sup_{y\in U}[x_\lambda]_R(y)$.
(2) *If* $y_\mu \subseteq [x_\lambda]_R$, *we have that* $[y_\mu]_R \subseteq [x_\lambda]_R$.
(3) $[x_\lambda]_R = \bigcup_{y_\mu \subseteq [x_\lambda]_R} [y_\mu]_R$.

PROOF.

(1) $[x_\lambda]_R(x) = T_L(\lambda, R(x,x)) = T_L(\lambda, 1) = \max(0, \lambda+1-1) = \lambda$. Since $T_L(a,b)$ is a monotone increasing function with respect to b, we can infer that

$$\sup_{y\in U}[x_\lambda]_R(y) = \sup_{y\in U} T_L(\lambda, R(x,y)) = T_L(\lambda, R(x,x)) = \lambda.$$

(2) If $y_\mu \subseteq [x_\lambda]_R$, we have $\mu \le T_L(\lambda, R(x,y))$. For any $z \in U$,

$$[y_\mu]_R(z) = T_L(\mu, R(y,z)) \le T_L(T_L(\lambda, R(x,y)),\ R(y,z)).$$

Due to $T_L(a, T_L(b,c)) = T_L(T_L(a,b), c)$ [14], we have that

$$T_L(T_L(\lambda, R(x,y)), R(y,z)) = T_L(\lambda, T_L(R(x,y), R(y,z))) \le T_L(\lambda, R(x,z)) = [x_\lambda]_R(z).$$

Thus, $[y_\mu]_R(z) \le [x_\lambda]_R(z)$, that is, $[y_\mu]_R \subseteq [x_\lambda]_R$ if $y_\mu \subseteq [x_\lambda]_R$.

(3) According to (2), we have that $[x_\lambda]_R \supseteq \bigcup_{y_\mu \subseteq [x_\lambda]_R}[y_\mu]_R$. And, since $x_\lambda \subseteq [x_\lambda]_R$, it is obvious that $[x_\lambda]_R \subseteq \bigcup_{y_\mu \subseteq [x_\lambda]_R}[y_\mu]_R$. Thus, $[x_\lambda]_R = \bigcup_{y_\mu \subseteq [x_\lambda]_R}[y_\mu]_R$ holds.

According to the above definition of the fuzzy information granule, for any fuzzy set $A \in F(U)$, we can define the lower approximation $\underline{R}A$ and the upper approximation $\overline{R}A$ of A in Definition 3.3.

DEFINITION 3.3. *Let R be a fuzzy T_L-similarity relation on U. Then, for any fuzzy set $A \in F(U)$, based on the fuzzy information granule, the lower approximation $\underline{R}A$ and the upper approximation $\overline{R}A$ of A with respect to R are defined as $\underline{R}A = \cup\{[x_\lambda]_R \mid [x_\lambda]_R \subseteq A\}$ and $\overline{R}A = \cup[x_{A(x)}]_R = \cup\{[x_\lambda]_R \mid \lambda \leq A(x)\}$, respectively.*

If R is a crisp equivalence relation, we have that

(1) $\lambda = 1$, x_λ is a crisp point, and A is a crisp set;
(2) $[x_\lambda]_R$ is just the equivalence class $[x]_R$;
(3) $\underline{R}A = \cup\{[x]_R \mid [x]_R \subseteq A\}$ and $\overline{R}A = \cup\{[x]_R \mid [x]_R \cap A \neq \phi\}$, that is, the crisp lower and upper approximation operators $\underline{\mathrm{apr}}_R$ and $\overline{\mathrm{apr}}_R$ are the special cases of $\underline{R}$ and $\overline{R}$, respectively.

Hence, the definitions of the lower and upper approximation operators $\underline{R}$ and $\overline{R}$ given in this paper are a generalization of the corresponding lower and upper approximation operators $\underline{\mathrm{apr}}_R$ and $\overline{\mathrm{apr}}_R$ in the crisp rough set theory, in which $\underline{R}$ and $\overline{R}$ have the following properties in Theorem 3.4.

THEOREM 3.4. *Suppose R is a fuzzy T_L-similarity relation on U and $A, B \in F(U)$. Let $\underline{R}$ and $\overline{R}$ be the lower and upper approximation operators with respect to R, respectively, then we have*

(1) $\underline{R}A \subseteq A \subseteq \overline{R}A$;
(2) $\overline{R}x_\lambda = [x_\lambda]_R$;
(3) $(\overline{R}(A^c))^c = \underline{R}A$, $(\underline{R}(A^c))^c = \overline{R}A$;
(4) $\underline{R}(S_L|A\bar{\bar{\alpha}}|) = S_L|\underline{R}A, \bar{\bar{\alpha}}|)$, $\overline{R}(T_L|A, \bar{\bar{\alpha}}|) = T_L|\overline{R}A, \bar{\bar{\alpha}}|$;
(5) $A \subseteq B \Rightarrow \underline{R}A \subseteq \underline{R}B$, $\overline{R}A \subseteq \overline{R}B$;
(6) $\underline{R}(A \cap B) = \underline{R}A \cap \underline{R}B$, $\overline{R}(A \cup B) = \overline{R}A \cup \overline{R}B$;
(7) $\underline{R}(A \cup B) \supseteq \underline{R}A \cup \underline{R}B$, $\overline{R}(A \cap B) \subseteq \overline{R}A \cup \overline{R}B$;
(8) $\overline{R}(\underline{R}A) = \underline{R}(\underline{R}A) = \underline{R}A$, $\overline{R}(\overline{R}A) = \underline{R}(\overline{R}A) = \overline{R}A$;
(9) $\overline{R}U = \underline{R}U = U$, $\overline{R}\phi = \underline{R}\phi = \phi$;
(10) $\underline{R}(U - \{y\})(x) = \underline{R}(U - \{x\})(y)$, $\overline{R}x_1(y) = \overline{R}y_1(x) = R(x, y)$;
(11) $\overline{R}A = A \Leftrightarrow \underline{R}A = A$.

Where $x, y \in U$, x_1 and y_1 are two fuzzy points, S_L is the dual t-conorm of T_L defined as $S_L(a, b) = \min\{1, a+b\}$, $S_L|A, B|$ is defined as $S_L|A, B|(x) = S_L(A(x), B(x))$, $T_L|A, B|$ is defined as $T_L|A, B|(x) = T_L(A(x), B(x))$, and $\bar{\bar{\alpha}}$ is defined as $\bar{\bar{\alpha}}(x) = \alpha$, $\forall\, x \in U$.

PROOF. By the definitions of $\underline{R}$ and $\overline{R}$, (1) and (2) are clear.

(3) According to Lemma 3.2, we have that $[y_\mu]_R \subseteq [x_\lambda]_R$ when $y_\mu \subseteq [x_\lambda]_R$. Thus, $\overline{R}A = \cup\{[x_\lambda]_R \mid \lambda \leq A(x)\} = \cup\{[x_\lambda]_R \mid \lambda = A(x)\}$. Then,

$$\overline{R}A\,(y) = \sup_{x \in U,\ \lambda = A(x)} [x_\lambda]_R\,(y) = \sup_{x \in U,\ \lambda = A(x)} T_L\,(\lambda, R\,(x, y)) = \sup_{x \in U} T_L\,(A\,(x)\,, R\,(x, y))\,.$$

Furthermore, for any $A \in F(U)$ and $y \in U$, we have

$$\begin{aligned}
\left(\overline{R}\,(A^c)\right)^c(y) &= 1 - \left(\overline{R}\,(A^c)\right)(y) \\
&= 1 - \sup_{x \in U} T_L\,(A^c\,(x)\,, R\,(x, y)) \\
&= 1 - \sup_{x \in U} T_L(1 - A(x), R(x, y)) \\
&= 1 - \sup_{x \in U} \max\{0, R(x, y) - A(x)\} \\
&= \inf_{x \in U} \min\{1, 1 - R(x, y) + A(x)\}.
\end{aligned}$$

Let $\Gamma_L(a, b) = \min\{1, 1 - a + b\}$ is the residuation implicator of $T_L(a, b)$ [7]. Then, we have that $\inf_{x \in U} \min\{1, 1 - R(x, y) + A(x)\} = \inf_{x \in U} \Gamma_L(R(x, y), A(x))$. Thus, $(\overline{R}(A^c))^c(y) = \inf_{x \in U} \Gamma_L(R(x, y), A(x))$.

Let $\underline{R}^{\times}A(y) = \inf_{x\in U}\Gamma_L(R(x,y),A(x))$, then, for any $y\in U$, we have

$$\underline{R}^{\times}\left(\overline{R}A\right)(y) = \inf_{x\in U}\Gamma_L\left(R(x,y),\overline{R}A(x)\right).$$

Since $\overline{R}A(x) = \sup_{z\in U}T_L(R(x,z),A(z))$, $T_L(a,b)$ is a monotone increasing function with respect to a and $\Gamma_L(a,b)$ is a monotone increasing function with respect to b, we have that

$$\begin{aligned}\underline{R}^{\times}\left(\overline{R}A\right)(y) &= \inf_{x\in U}\Gamma_L\left(R(x,y),\sup_{z\in U}T_L(R(x,z),A(z))\right)\\ &\geq \inf_{x\in U}\sup_{z\in U}\Gamma_L\left(R(x,y),T_L(R(x,z),A(z))\right)\\ &\geq \inf_{x\in U}\sup_{z\in U}\Gamma_L\left(R(x,y),T_L\left(T_L(R(x,y),R(y,z)),A(z)\right)\right).\end{aligned}$$

Due to $T_L(a,T_L(b,c)) = T_L(T_L(a,b),c)$ [14],

$$\begin{aligned}\underline{R}^{\times}\left(\overline{R}A\right)(y) &\geq \inf_{x\in U}\sup_{z\in U}\Gamma_L\left(R(x,y),T_L\left(R(x,y),T_L(R(y,z),A(z))\right)\right)\\ &= \inf_{x\in U}\sup_{z\in U}\min\left\{1,1-R(x,y)+\max\left\{0,R(x,y)+T_L(R(y,z),A(z))-1\right\}\right\}.\end{aligned}$$

When $R(x,y)+T_L(R(y,z),A(z))-1\geq 0$,

$$\begin{aligned}\underline{R}^{\times}\left(\overline{R}A\right)(y) &\geq \inf_{x\in U}\sup_{z\in U}\min\left\{1,1-R(x,y)+R(x,y)+T_L(R(y,z),A(z))-1\right\}\\ &= \inf_{x\in U}\sup_{z\in U}\min\left\{1,T_L(R(y,z),A(z))\right\}\\ &= \inf_{x\in U}\sup_{z\in U}T_L(R(y,z),A(z))\\ &= \sup_{z\in U}T_L(R(y,z),A(z)).\end{aligned}$$

And, when $R(x,y)+T_L(R(y,z),(A(z))-1<0$, we have that

$$\begin{aligned}\underline{R}^{\times}\left(\overline{R}A\right)(y) &\geq \inf_{x\in U}\sup_{z\in U}\min\left\{1,1-R(x,y)\right\}\\ &= \inf_{x\in U}\sup_{z\in U}\left(1-R(x,y)\right)\\ &> \inf_{x\in U}\sup_{z\in U}T_L(R(y,z),A(z))\\ &= \sup_{z\in U}T_L(R(y,z),A(z)).\end{aligned}$$

Thus, $\underline{R}^{\times}(\overline{R}A)(y)\geq \sup_{z\in U}T_L(R(y,z),A(z)) = \overline{R}A(y)$.

Also, since $\underline{R}^{\times}A(y) = \inf_{x\in U}\Gamma_L(R(x,y),A(x))\leq \Gamma_L(R(y,y),A(y)) = A(y)$, we have that $\underline{R}^{\times}(\overline{R}A)(y)\geq \overline{R}A(y)$.

Therefore, $\underline{R}^{\times}(\overline{R}A) = \overline{R}A$.

Additionally, we have that

$$\begin{aligned}\overline{R}\left(\underline{R}^{\times}A\right)(y) &\leq \sup_{x\in U}T_L\left(R(x,y),\underline{R}^{\times}A(x)\right)\\ &= \sup_{x\in U}T_L\left(R(x,y)\inf_{z\in U}\Gamma_L(R(x,z),A(z))\right)\\ &\leq \sup_{x\in U}\inf_{z\in U}T_L\left(R(x,y),\Gamma_L(R(x,z),A(z))\right).\end{aligned}$$

Since $T_L(a,b)$ is a monotone increasing function with respect to b and $\Gamma_L(a,b)$ is a monotone decreasing function with respect to a, we have that

$$\overline{R}\left(\underline{R}^{\times}A\right)(y) \leq \sup_{x\in U}\inf_{z\in U}T_L\left(R(x,y),\Gamma_L\left(T_L(R(x,y),R(y,z)),A(z)\right)\right).$$

Additionally, since $\Gamma_L(T_L(a,b),c) = \Gamma_L(a,\Gamma_L(b,c))$ [14],

$$\begin{aligned}\overline{R}\left(\underline{R}^{\times}A\right)(y) &= \sup_{x\in U}\inf_{z\in U} T_L\left(R\left(x,y\right)\Gamma_L\left(R\left(x,y\right),\Gamma_L\left(R\left(y,z\right),A\left(z\right)\right)\right)\right) \\ &= \sup_{x\in U}\inf_{z\in U} \max\left\{0, R\left(x,y\right) + \min\left\{1, 1 - R\left(x,y\right) + \Gamma_L\left(R\left(y,z\right)A\left(z\right)\right)\right\} - 1\right\}.\end{aligned}$$

When $1 - R(x,y) + \Gamma_L(R(y,z),A(z)) \leq 1$,

$$\begin{aligned}\overline{R}\left(\underline{R}^{\times}A\right)(y) &\leq \sup_{x\in U}\inf_{z\in U} \max\left\{0, R\left(x,y\right) + 1 - R\left(x,y\right) + \Gamma_L\left(R\left(y,z\right),A\left(z\right)\right) - 1\right\} \\ &= \sup_{x\in U}\inf_{z\in U} \max\left\{0, \Gamma_L\left(R\left(y,z\right),A\left(z\right)\right)\right\} \\ &= \sup_{x\in U}\inf_{z\in U} \left(\Gamma_L\left(R\left(y,z\right),A\left(z\right)\right)\right) \\ &= \inf_{z\in U} \left(\Gamma_L\left(R\left(y,z\right),A\left(z\right)\right)\right).\end{aligned}$$

And, when $1 - R(x,y) + \Gamma_L(R(y,z),A(z)) > 1$, we have

$$\begin{aligned}\overline{R}\left(\underline{R}^{\times}A\right)(y) &\leq \sup_{x\in U}\inf_{z\in U} \max\left\{0, R\left(x,y\right) + 1 - 1\right\} \\ &= \sup_{x\in U}\inf_{z\in U} R\left(x,y\right) \\ &< \sup_{x\in U}\inf_{z\in U} \Gamma_L\left(R\left(y,z\right),A\left(z\right)\right) \\ &= \inf_{z\in U} \Gamma_L\left(R\left(y,z\right),A\left(z\right)\right).\end{aligned}$$

Thus, $\overline{R}(\underline{R}^{\times}A)(y) \leq \inf_{z\in U}\Gamma_L(R(y,z),A(z)) = \underline{R}^{\times}A(y)$, that is, $\overline{R}(\underline{R}^{\times}A) = \underline{R}^{\times}A$.

Then, $(\overline{R}A^c)^c = \underline{R}^{\times}A = \overline{R}(\underline{R}^{\times}A) = \cup\{[x_\lambda]_R \mid x_\lambda \subseteq \underline{R}^{\times}A\}$. Since $\overline{R}(\underline{R}^{\times}A) = \underline{R}^{\times}A$, we have that $[x_\lambda]_R \subseteq \underline{R}^{\times}A$ when $x_\lambda \subseteq \underline{R}^{\times}A$. Furthermore, since $\underline{R}^{\times}A(y) \leq A(y)$, we can infer that $(\overline{R}A^c)^c = \cup\{[x_\lambda]_R \mid [x_\lambda]_R \subseteq \underline{R}^{\times}A\} \subseteq \cup\{[x_\lambda]_R \mid [x_\lambda]_R \subseteq A\} = \underline{R}A$.

When $\overline{R}x_\lambda = [x_\lambda]_R \subseteq A$, since $\Gamma_L(a,b)$ is a monotone increasing function with respect to b, we have $\underline{R}^{\times}A(y) = \inf_{z\in U}\Gamma_L(R(z,y),A(z)) \geq \inf_{z\in U}\Gamma_L(R(z,y),[x_\lambda]_R(z)) = \underline{R}^{\times}[x_\lambda]_R(y)$, that is, $\underline{R}^{\times}\overline{R}x_\lambda \subseteq \underline{R}^{\times}A$. So, we can infer that $\underline{R}^{\times}\overline{R}x_\lambda = \overline{R}x_\lambda = [x_\lambda]_R \subseteq \underline{R}^{\times}A = (\overline{R}A^c)^c$ when $\overline{R}x_\lambda = [x_\lambda]_R \subseteq A$. Then, $\underline{R}A = \cup\{[x_\lambda]_R \mid [x_\lambda]_R \subseteq A\} \subseteq (\overline{R}A^c)^c$.

Since $(\overline{R}A^c)^c \subseteq \underline{R}A$ and $(\overline{R}A^c)^c \supseteq \underline{R}A$ have been proved above, $(\overline{R}A^c)^c = \underline{R}A$ holds.

Similarly, we can infer that $(\underline{R}(A^c))^c = \overline{R}A$.

(4) For any $x \in U$, from the proof of (3), we can infer that

$$\underline{R}A\left(x\right) = \left(\underline{R}\left(A^c\right)\right)^c = \inf_{y\in U}\min\left\{1, 1 - R\left(x,y\right) + A\left(y\right)\right\} = \inf_{y\in U} S_L\left(1 - R\left(x,y\right), A\left(y\right)\right).$$

So,

$$\underline{R}\left(S_L\left|A,\bar{\bar{\alpha}}\right|\right)(x) = \inf_{y\in U} S_L\left(1 - R\left(x,y\right), S_L\left(A\left(y\right),\alpha\right)\right).$$

Since $S_L(a, S_L(b,c)) = S_L(S_L(a,b),c)$ [14], we have

$$\begin{aligned}\underline{R}\left(S_L\left|A,\bar{\bar{\alpha}}\right|\right)(x) &= \inf_{y\in U} S_L\left(S_L\left(1 - R\left(x,y\right), A\left(y\right)\right),\alpha\right) \\ &= S_L\left(\inf_{y\in U} S_L\left(1 - R\left(x,y\right), A\left(y\right)\right),\alpha\right) \\ &= S_L\left|\underline{R}A,\bar{\bar{\alpha}}\right|(x).\end{aligned}$$

That is, $\underline{R}(S_L|A,\bar{\bar{\alpha}}|) = S_L|\underline{R}A,\bar{\bar{\alpha}}|$.

Similarly, we can prove that $\overline{R}(T_L|A,\bar{\bar{\alpha}}|) = T_L|\overline{R}A,\bar{\bar{\alpha}}|$.

(5) According to the definitions of $\underline{R}$ and $\overline{R}$, it is clear that $\underline{R}A \subseteq \underline{R}B$ and $\overline{R}A \subseteq \overline{R}B$ when $A \subseteq B$.

(6) From the proof of (3), $\overline{R}A(y) = \sup_{x\in U} T_L(R(x,y),A(x))$. Then, for each $x \in U$, since $T_L(a,b)$ is a monotone increasing function with respect to b, we have that

$$\begin{aligned}\overline{R}(A\cup B)(x) &= \sup_{y\in U} T_L\left(R(x,y),\max\{A(y),B(y)\}\right)\\ &= \sup_{y\in U} \max\{T_L(R(x,y),A(y)),T_L(R(x,y),B(y))\}\\ &= \max\left\{\sup_{y\in U} T_L(R(x,y),A(y)),\sup_{y\in U} T_L(R(x,y),B(y))\right\}\\ &= \max\{\overline{R}A(x),\overline{R}B(x)\}\\ &= \left(\overline{R}A\cup\overline{R}B\right)(x).\end{aligned}$$

That is, $\overline{R}(A\cup B) = \overline{R}A \cup \overline{R}B$ holds.

Similarly, we can prove that $\underline{R}(A\cap B) = \underline{R}A \cap \underline{R}B$.

(7) According to (5), since $A \subseteq A\cup B$, we have that $\underline{R}A \subseteq \underline{R}(A\cup B)$. Similarly, $\underline{R}B \subseteq \underline{R}(A\cup B)$. Thus, $\underline{R}(A\cup B) \supseteq \underline{R}A\cup\underline{R}B$ holds. Additionally, according to the definitions of $\underline{R}$ and $\overline{R}$, it is obvious that $\overline{R}(A\cap B) \subseteq \overline{R}A\cap\overline{R}B$ holds.

(8) According to the definitions of $\underline{R}$ and $\overline{R}$, due to (2) in Lemma 3.2, we can infer easily that $\overline{R}(\underline{R}A) = \underline{R}(\underline{R}A) = \underline{R}A$ and $\overline{R}(\overline{R}A) = \underline{R}(\overline{R}A) = \overline{R}A$.

(9) According to (1), it is obvious that $\overline{R}U = \underline{R}U = U$ and $\overline{R}\phi = \underline{R}\phi = \phi$.

(10) According to the crisp rough set theory, it is obvious that $\underline{R}(U-\{y\})(x) = \underline{R}(U-\{x\})(y)$. Additionally, we have that $\overline{R}x_1(y) = [x_\lambda]_R(y) = T_L(1,R(x,y)) = R(x,y)$. Similarly, we can infer that $\overline{R}y_1(x) = R(x,y)$. Thus, $\overline{R}x_1(y) = \overline{R}y_1(x) = R(x,y)$ holds.

(11) If $\underline{R}A = A$, we have that $\overline{R}A = \overline{R}(\underline{R}A) = \underline{R}A = A$. And, if $\overline{R}A = A$, $\underline{R}A = \underline{R}(\overline{R}A) = \overline{R}A = A$ holds.

According to Theorem 3.4, the lower and upper approximations $\underline{R}A$ and $\overline{R}A$ of a definable fuzzy set $A \in F(U)$, which satisfies that $\underline{R}A = A = \overline{R}A$, are also definable, while the lower and upper approximation operators R_* and R^* of a definable fuzzy set may be undefinable [4].

Based on the above approximation operators $\underline{R}$ and $\overline{R}$, we give the definition of a new fuzzy rough set. For any $A \in F(U)$, the pair $(\underline{R}A,\overline{R}A)$ is called a T_L-fuzzy rough set. According to the proof of Theorem 3.4, the membership functions of $\underline{R}A$ and $\overline{R}A$ are $\underline{R}A(x) = \inf_{y\in U} S_L(1-R(x,y),A(y))$ and $\overline{R}A(x) = \sup_{y\in U} T_L(R(x,y),A(y))$, respectively.

Furthermore, in Theorem 3.5, we give the axiomatic characteristics of $\underline{R}A$ and $\overline{R}A$.

THEOREM 3.5. *Suppose $A,B \in F(U)$. Let $L : F(U) \to F(U)$ and $H : F(U) \to F(U)$ be a pair of fuzzy set operators, then there exists a fuzzy T_L-similarity relation R such that $L = \underline{R}$ and $H = \overline{R}$ iff L and H satisfy the following axioms.*

(1) $(L(A^c))^c = HA$, $(H(A^c))^c = LA$;

(2) $L(S_L|A,\bar{\bar{\alpha}}|) = S_L|LA,\bar{\bar{\alpha}}|$, $H(T_L|A,\bar{\bar{\alpha}}|) = T_L|HA,\bar{\bar{\alpha}}|$;

(3) $L(A\cap B) = L(A)\cap L(B)$, $H(A\cup B) = H(A)\cup H(B)$;

(4) $LA \subseteq A$, $A \subseteq HA$;

(5) $LA \subseteq L(LA)$, $H(HA) \subseteq HA$;

(6) $L(U-\{y\})(x) = L(U-\{x\})(y)$, $Hx_1(y) = Hy_1(x)$.

PROOF. Due to the limited length of the paper, we omit the procedure of proof. In the above part, we define a new fuzzy rough set based on the fuzzy information granule and give some properties of the lower and upper approximations of the fuzzy sets. In the following section, we will define a new fuzzy information system based on the fuzzy T_L-similarity relation and study its reduction using the new fuzzy rough set.

4. REDUCTION METHOD BASED ON THE NEW FUZZY ROUGH SET IN FUZZY INFORMATION SYSTEM

First, we define the fuzzy approximation space based on the fuzzy T_L-similarity relation. Let U be a nonempty universe, and let R be a fuzzy T_L-similarity relation on U, then (U, R) is called a fuzzy approximation space. We denote the collection of all the definable sets on U with respect to R as $F_R(U)$, i.e., $F_R(U) = \{A \in F(U) \mid \overline{R}A = A = \underline{R}A\}$, in which definable set A is the union of some fuzzy information granules. By (8) in Theorem 3.4, we have that $F_R(U) = \{\overline{R}A \mid A \in F(U)\} = \{\underline{R}A \mid A \in F(U)\}$.

Then, we analyze the property of the fuzzy approximation space based on the fuzzy T_L-similarity relation in Lemma 4.1.

LEMMA 4.1. *Suppose (U, R) is a fuzzy approximation space and $A \in F_R(U)$. If $A(x) = \lambda$, we have that $[x_\lambda]_R \subseteq A$.*

PROOF. Since $A \in F_R(U)$, $\overline{R}A = A$. According to $A(x) = \lambda$, we have $x_\lambda \subseteq A$, thus $[x_\lambda]_R = \overline{R}x_\lambda \subseteq \overline{R}A = A$.

Furthermore, based on the above property of the fuzzy approximation space in Lemma 4.1, we give the relation between two fuzzy approximation spaces in Theorem 4.2.

THEOREM 4.2. *Suppose (U, R_1) and (U, R_2) are two fuzzy approximation spaces, then the following three statements are equivalent.*

(1) $F_{R_1}(U) \subseteq F_{R_2}(U)$;
(2) $R_2 \subseteq R_1$;
(3) *For any* $A \in F(U)$, $\overline{R}_2A \subseteq \overline{R}_1A$ *and* $\underline{R}_2A \supseteq \underline{R}_1A$.

PROOF.

(1) $\Rightarrow$ (2) Suppose that $F_{R_1}(U) \subseteq F_{R_2}(U)$. For any $x \in U$, we have $[x_1]_{R_1} \in F_{R_1}(U)$. Thus, $[x_1]_{R_1} \in F_{R_2}(U)$. According to Lemma 4.1, since $[x_1]_{R_1}(x) = 1$, we have that $[x_1]_{R_2} \subseteq [x_1]_{R_1}$. Also, for any $x \in U$, $R_1(x, y) = T_L(1, R_1(x, y)) = [x_1]_{R_1}(y)$, and $R_2(x, y) = T_L(1, R_2(x, y)) = [x_1]_{R_2}(y)$. Then, $R_1(x, y) \geq R_2(x, y)$, that is, $R_2 \subseteq R_1$ holds.

(2) $\Rightarrow$ (1) According to the proof of Theorem 3.4, for any $A \in F_{R_1}(U)$, we have that $A(x) = \overline{R}_1A(x) = \sup_{y \in U} T_L(R_1(x, y), A(y))$. Then, $A(x) \geq T_L(R_1(x, y), A(y))$ for any $y \in U$. Additionally, when $R_2 \subseteq R_1$, $R_1(x, y) \geq R_2(x, y)$. Furthermore, since $T_L(a, b)$ is a monotone increasing function with respect to a, we have that $A(x) \geq T_L(R_2(x, y), A(y))$ for any $y \in U$. So, $A(x) \geq \sup_{y \in U} T_L(R_2(x, y), A(y))$. Additionally, $T_L(R_2(x, x), A(x)) = A(x)$. Thus, $A(x) = \sup_{y \in U} T_L(R_2(x, y), A(y)) = \overline{R}_2A(x)$. Therefore, $A \in F_{R_2}(U)$. So, $F_{R_1}(U) \subseteq F_{R_2}(U)$ holds.

(2) $\Rightarrow$ (3) From the definitions of $\underline{R}$ and $\overline{R}$, (2) $\Rightarrow$ (3) is clear.

(3) $\Rightarrow$ (2) According to (3), $\overline{R}_2A \subseteq \overline{R}_1A$ for any $A \in F(U)$. Since $x_1 \in F(U)$, it is obvious that $\overline{R}_2x_1 \subseteq \overline{R}_1x_1$. Also, $\overline{R}_2x_1(y) = [x_1]_{R_2}(y) = T_L(1, R_2(x, y)) = R_2(x, y)$. Similarly, we can obtain that $\overline{R}_1x_1(y) = R_1(x, y)$. Thus, $R_2(x, y) \leq R_1(x, y)$. So, $R_2 \subseteq R_1$ holds.

For two fuzzy approximation spaces (U, R_1) and (U, R_2), if every fuzzy information granule on (U, R_1) is a definable set on (U, R_2), we have that $F_{R_1}(U) \subseteq F_{R_2}(U)$. Then, according to Theorem 4.2, for any $A \in F(U)$, we have $\overline{R}_2A \subseteq \overline{R}_1A$ and $\underline{R}_2A \supseteq \underline{R}_1A$, that is, fuzzy set A on U can be characterized more precisely by means of the corresponding lower and upper approximations in (U, R_2) than in (U, R_1). Theorem 4.2 can offer a significant theoretical foundation for studying further the suitable fuzzy T_L-similarity relations facing the special applications.

Subsequently, on the basis of fuzzy approximation space (U, R), we define a kind of new fuzzy information system in Definition 4.3.

DEFINITION 4.3. *Suppose that* $\mathbf{R}$ *is a family of fuzzy* T_L*-similarity relations on* U*, then* $(U, \mathbf{R})$ *is called a fuzzy information system, in which all objects in* U *can be characterized by some attributes, and every attribute corresponds to one fuzzy* T_L*-similarity relation* R*, which can be obtained according to the attribute values of the corresponding attribute.*

Suppose $R \in \mathbf{R}$, if $\mathrm{ind}(\mathbf{R}) = \mathrm{ind}(\mathbf{R} - \{R\})$, R is called unnecessary in $\mathbf{R}$, otherwise R is called necessary in $\mathbf{R}$. If every $R \in \mathbf{R}$ is necessary in $\mathbf{R}$, $\mathbf{R}$ is called independent, otherwise $\mathbf{R}$ is called dependent. Suppose $\mathbf{P} \subseteq \mathbf{R}$, if $\mathbf{P}$ is independent and $\mathrm{ind}(\mathbf{P}) = \mathrm{ind}(\mathbf{R})$, $\mathbf{P}$ is called a reduction of $\mathbf{R}$ and accordingly, $(U, \mathbf{P})$ is called a reduction of $(U, \mathbf{R})$. The objective of the reduction of $(U, \mathbf{R})$ is to find a minimal set $\mathbf{P}$ (a reduction of $\mathbf{R}$) of the fuzzy T_L-similarity relation R in order that $\mathrm{ind}(\mathbf{P}) = \mathrm{ind}(\mathbf{R})$ holds, in which $\mathbf{R}$ and $(U, \mathbf{R})$ can have more than one reduction, respectively. Additionally, it can be proved that $\mathrm{core}(\mathbf{R}) = \cap \mathrm{red}(\mathbf{R})$, where $\mathrm{core}\mathbf{R}$ is the set of all the necessary fuzzy T_L-similarity relations in $\mathbf{R}$ and $\mathrm{red}(\mathbf{R})$ is the set of all the reductions of $\mathbf{R}$.

In order to reduce the fuzzy information system $(U, \mathbf{R})$ based on the fuzzy T_L-similarity relation, we construct $(U, \mathbf{R})$ according to the following procedure. First, aiming at every attribute of objects in U, we obtain one fuzzy T_L-similarity relation R on U based on the corresponding grades of membership functions of the fuzzy attribute values according to the following procedure. Let x_p and x_q be any two objects in U. For every object in U, let μ_{ik}, $i = 1, 2, \dots, I$, $k = 1, 2, \dots, K_i$ be the membership functions of the attribute values of attributes i, $i = 1, 2, \dots, I$, then the similar degree between x_p and x_q can be obtained by $R(x_p, x_q) = \inf_{k=1}^{K_i}(1 - |\mu_{ik}(x_p) - \mu_{ik}(x_q)|)$, in which I is the total number of the attributes of objects in U and K_i is the total number of the attribute values of attribute i $(i = 1, 2, \dots, I)$.

Then, let $\mathbf{R}$ be the set of all fuzzy T_L-similarity relations R.

Consequently, we can construct a fuzzy information system $(U, \mathbf{R})$ based on the fuzzy T_L-similarity relation using U and $\mathbf{R}$ obtained above.

5. APPLICATION

In this section, we apply the reduction method based on the new fuzzy rough set to the reduction of the redundant premises of the multiple fuzzy rules used in the larger scale resource constrained project scheduling problem (RCPSP) [15–17], which can be described as follows. Given some activities (no activity preemption allowed) and some renewable resources, where each resource has a fixed capacity. Every activity has a given processing time (duration) and needs a certain units of several resources. Additionally, there exist precedence relations between some activities. The objective is to find a feasible schedule so as to make the makespan minimized.

5.1. The Multiple Fuzzy Rules with Multiple Premises for Solving the Larger Scale RCPSP

The multiple fuzzy rules with multiple premises for solving the larger scale RCPSP can be given as follows.

R_1 : if A_1 is low, A_3 is low, ..., A_{20} is low, then B is very high;
R_2 : if A_1 is low, A_5 is high, ..., A_{19} is middle, then B is high;

$\vdots \qquad \vdots$

R_{20} : if A_2 is middle, A_3 is low, ..., A_{20} is middle, then B is middle;

$\vdots \qquad \vdots$

R_{49} : if A_5 is high, A_7 is middle, ..., A_{18} is high, then B is low;
R_{50} : if A_1 is low, A_5 is high, ..., A_{20} is high, then B is very low.

The above fuzzy rules have multiple different premises and the same consequence. A_i, $i = 1, 2, \dots, 20$ as Table 1 [17] are linguistic variables of all premises representing the condition attributes, which are also the premises of multiple fuzzy rules. B is a linguistic variable representing the decision attribute (scheduling priority), which is also the consequence of multiple fuzzy rules.

Table 1. Premises of the multiple fuzzy rules above.

Linguistic Variable	Rule	Meaning
A_1	Spt	Shortest processing time
A_2	Est	Earliest start time
A_3	Eft	Earliest finish time
A_4	Mslk	Minimum slack
A_5	Lis	Least immediate successors
A_6	Lts	Least total successors
A_7	Estd	Dynamic earliest start time
A_8	Eftd	Dynamic earliest finish time
A_9	Mslkd	Dynamic minimum slack
A_{10}	Irsm	Improved resource scheduling method
A_{11}	Wcs	Worst case slack
A_{12}	Grpw	Greatest rank positional weight of the activity considered and its immediate successors
A_{13}	Grpwa	Greatest rank positional weight of the activity considered and its total successors
A_{14}	Grd	Greatest resource demand
A_{15}	Gcumrd	Greatest cumulative resource demand
A_{16}	Wrup	Weighted resource utilization and precedence
A_{17}	Red	Resource equivalent duration
A_{18}	Cumred	Cumulative resource equivalent duration
A_{19}	Comp1	Composite priority rules of Spt and Grd
A_{20}	Comp2	Composite priority rules of Eft and Red

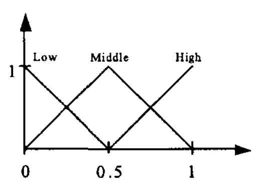

Figure 1. Membership function of A_i.

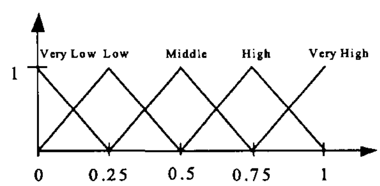

Figure 2. Membership function of B.

Each linguistic variable A_i $(i = 1, 2, \ldots, 20)$ has three attribute values, whose membership functions are given in Figure 1, and linguistic variable B has five attribute values, whose membership functions are given in Figure 2.

5.2. The Reduction of the Multiple Fuzzy Rules

In this paper, we use the above multiple fuzzy rules to solve the RCPSP. In order to apply the reduction method based on the new fuzzy rough set to the reduction of the redundant premises of the multiple fuzzy rules, we need to construct dynamically the fuzzy information system $(U, \mathbf{R})$ in the scheduling process. Furthermore, we transform the reduction of the redundant premises of multiple fuzzy rules into the reduction of $(U, \mathbf{R})$. Then, by the reduction of $(U, \mathbf{R})$, we remove the redundant premises. We construct $(U, \mathbf{R})$ according to the following procedure.

At every decision point, we adopt the above multiple fuzzy rules with multiple premises to generate dynamically the corresponding activity priority list and take an activity in the activity priority list as an object in U, in which the start time of the corresponding unscheduled activities are determined in the order of the activities in the above priority list. In this way, we can obtain universe U when the scheduling process ends. Additionally, since the multiple fuzzy rules used in this paper include 20 premises, every object has 20 condition attributes and a decision attribute. Also, every condition attribute A_i $(i = 1, 2, \ldots, 20)$ has three fuzzy attribute values Low, Middle and High as Figure 1, and the decision attribute B has five fuzzy attribute values Very Low, Low, Middle, High and Very High as Figure 2. Let C_k^i, $k = 1, 2, 3$ denote the grades of membership

Table 2. Grades of membership functions of every object in universe.

U	A_1			A_2			...	...	A_{20}			B				
	C_1^1	C_2^1	C_3^1	C_1^2	C_2^2	C_3^2	...	...	C_1^{20}	C_2^{20}	C_3^{20}	D_1	D_2	D_3	D_4	D_5
1	0	0.4	0.6	0.5	0.5	0			0	0.3	0.7	0	0	1	0	0
2	0.8	0.2	2	0	0.2	0.8			0	0.4	0.6	0	0.5	0.5	0	0
⋮		⋮			⋮		...	...		⋮				⋮		
⋮		⋮			⋮		...	...		⋮				⋮		

functions of the three fuzzy attribute values of A_i respectively, and let D_l, $l = 1, 2, \ldots, 5$ denote the grades of membership functions of the five fuzzy attribute values of B, then we give the grades of membership functions of every object in universe U in Table 2.

According to every three grades of membership functions C_k^i, $k = 1, 2, 3$ of the fuzzy attribute values of A_i ($i = 1, 2, \ldots, 20$), we can construct one fuzzy T_L-similarity relation. Let E_k^{ip}, $i = 1, 2, \ldots, 20$, $k = 1, 2, 3$ denote the grades of membership functions C_k^i, $i = 1, 2, \ldots, 20$, $k = 1, 2, 3$ for object p. Suppose that p and q are two objects in U, then we can obtain one fuzzy T_L-similarity relation R by $R(p, q) = \inf_{k=1}^{3}(1 - |E_k^{ip} - E_k^{iq}|)$, which is the similarity degree between objects p and q.

Similarly, according to the grades of membership functions D_l, $l = 1, 2, \ldots, 5$ of the five fuzzy attribute values of B, we can also obtain one fuzzy T_L-similarity relation R by $R(p, q) = \inf_{l=1}^{5}(1 - |F_l^p - F_l^q|)$, where F_l^p, $l = 1, 2, \ldots, 5$ denote the grades of membership functions D_l, $l = 1, 2, \ldots, 5$ for object p.

Then, the above 21 fuzzy T_L-similarity relations including 20 fuzzy T_L-similarity relations corresponding to condition attributes A_i, $i = 1, 2, \ldots, 20$ and one fuzzy T_L-similarity relation corresponding to decision attribute B constitute the fuzzy T_L-similarity relation family $\mathbf{R}$. Consequently, we obtain a fuzzy information system $(U, \mathbf{R})$ based on the fuzzy T_L-similarity relation.

After constructing $(U, \mathbf{R})$, by the reduction method based on the new fuzzy rough set in Section 4, we reduce only the redundant condition attributes of the objects in $(U, \mathbf{R})$. Consequently, the corresponding redundant premises of the above multiple fuzzy rules can be removed.

5.3. Numerical Computation

In this paper, aiming at benchmark problems in the PSPLIB [18], we compare the performances of the reduction method based on the new fuzzy rough set defined with those of the reduction method based on the existing fuzzy rough set.

First, we construct $(U, \mathbf{R})$ based on the above two kinds of fuzzy rough sets according to the above method for constructing the fuzzy information systems respectively, in which the fuzzy min-similarity relation used in the existing fuzzy rough set can be obtained by the transitive closure of the fuzzy T_L-similarity relation above [19]. Then, the redundant condition attributes in $(U, \mathbf{R})$ are reduced by means of the above two kinds of fuzzy rough sets, and the corresponding premises of the above multiple fuzzy rules can be removed. Consequently, we can obtain the reduced multiple fuzzy rules. Then, based on the initial multiple fuzzy rules and the reduced multiple fuzzy rules respectively, we can get the corresponding sequences of all activities needed to be scheduled.

On the basis of the above two sequences, we can judge the correctness of the corresponding reduction, that is, if the above two sequences are consistent, the reduction is thought to be right, otherwise it is wrong. For example, when the sequences obtained by the former and the latter are (1, 2, 3, 4) and (1, 2, 4, 3) respectively, the reduction is wrong whether the scheduling objectives obtained corresponding to the above two sequences are same or different.

Additionally, considering that the defuzzification method has influences on fuzzy reasoning consequences, we compare the performance of the reduction methods corresponding to different defuzzification methods.

Table 3. Numerical computational results.

Problem Scale	M_1		M_2		M_3		M_4		M_5	
	RN (%)	RE (%)	RN (%)	RE (%)	RN (%)	RE (%)	RN (%)	RE (%)	RN (%)	RE (%)
30	44	0	41	0	42	0	18	0	14	0
60	65	1	65	0	81	0	62	0	61	0
90	66	0	70	0	82	0	77	0	82	0
120	74	1	76	1	86	0	81	0	84	1

In Table 2, aiming at different scale benchmark scheduling problems, different defuzzification methods and different reduction methods of the fuzzy information systems, we list the percentage of the scheduling problems in which the reduction result is right. Where RN and RE denote the correctness of the reductions when we adopt the reduction method based on the new rough set and the one based on the existing rough set respectively, and M_i, $i = 1, 2, \ldots, 5$ denote the following defuzzification methods respectivley: the centroid of area method, the bisector of area method, the mean of maximum method, the smallest of maximum method and the largest of maximum method.

From Table 3, it can be seen that the reduction method based on the new rough set is obvious advantageous over the reduction method based on the existing rough set for the reduction of redundant premises of multiple fuzzy rules in RCPSP.

Aiming at the problems with 30, 60, 90, and 120 activities, by the defuzzification method M_1 in the fuzzy reasoning, the percentage of the right reductions by the reduction method based on the new rough set is 44%, 65%, 66%, and 74%, respectively, while the percentage of the right reductions by the reduction method based on the existing rough set is almost zero. Furthermore, the above conclusion is also correct when defuzzification methods M_2, M_3, M_4, and M_5 are used, respectively. These show that the reduction method based on the new rough set is more suitable for the reduction of the redundant premises of the multiple rules for solving RCPSP compared with the reduction method based on the existing rough set for different defuzzification methods.

Additionally, with different defuzzification methods, the percentages of the right reductions by the reduction method based on the new rough sets are different, in which we can obtain the best reduction effect comparatively for larger scale scheduling problems by the defuzzification method M_3.

Furthermore, for different scale scheduling problems, the reduction effects of the reduction method based on the new rough set have also distinction. This may be explained as follows. With the scale of the scheduling problem increasing, the total number of the objects in U becomes larger, which leads to more classifications. Then, the total number of the premises which can be removed decreases. Thus, for larger scale scheduling problems, the percentage of the right reduction is relatively larger. For example, for the scheduling problems with 120 activities, by the defuzzification method M_3, the percentage of the right reductions has achieved 86%.

5.4. Analysis

From the numerical computational results, we can see that the reduction method based on the new rough set is more suitable for the reduction of the redundant premises of the multiple fuzzy rules for solving RCPSP compared with the reduction method based on the existing rough set. This can be analyzed as follows.

Compared with the fuzzy T_L-similarity relation, the condition needed for the fuzzy min-transitivity in the fuzzy min-similarity relation is too strict so that the similar degree between two objects in U cannot be measured effectively by means of the fuzzy min-similarity relation. In fact, a fuzzy similarity relation R given directly can often not satisfy the fuzzy min-transitivity and fuzzy min-similarity relation R is usually gotten by the following method. First, a fuzzy similarity relation R' satisfying reflexivity and symmetry is constructed. Then, we can obtain the fuzzy min-similarity relation R by the transitive closure of R' [19]. In the procedure for solving the transitive closure of R', a great of data is assimilated, that is, a lot of significant information is lost. Thus, the assimilation of data must lead to that some efficient attributes may be reduced in the reduction of the fuzzy information system by the reduction method based on the existing fuzzy rough set.

6. CONCLUSION

In this paper, we present the concept of fuzzy information granule based on the fuzzy T_L-similarity relation for the first time. Also, according to the fuzzy information granule, we define the lower and upper approximations of fuzzy sets with the fuzzy information granules and a corresponding new fuzzy rough set. Furthermore, based on the fuzzy T_L-similarity relation, we construct a new fuzzy information system and study its reduction using the new fuzzy rough set. Additionally, we apply the reduction method based on the new fuzzy rough set to the scheduling problems. Numerical computational results show that the reduction method based on the new rough set is more suitable for the reduction of the redundant premises of the multiple fuzzy rules used for scheduling problems compared with the reduction method based on the existing rough set. Our future work will focus on the study of more effective reduction methods based on rough sets for scheduling problems.

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