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# A Mathematical Model of Coal-gas Flow Conveying In the Process of Coal and Gas Outburst and Its Application

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## Abstract

Taking the coal-gas flow in the process of coal and gas outburst as the study object, the coal particles movement process is divided into acceleration process and balance process according to the solid particle suspension mechanism in the gas flow and the energy conservation law. A mathematical model for outburst coal conveying in the roadway in one-dimensional case is established, and its reasonableness is proved through resolving an on-site example. The mathematical model which is a new method for engineering calculation of outburst two-phase flow can both estimate the coal particles conveying distance according to the initial state of coal seam and inverse the initial energy of coal-gas flow according to the coal particles conveying distance record on spot.

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# 1. Introduction

Coal and gas outburst is one of the worst mine disasters. The emitted large quantity of coal and gas pours into roadway and forms the coal-gas fluid while the coal and gas outburst happens. In a super-huge outburst, a million tons of coal and several million cubic meters of gas can be ejected out, and the two-

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phase coal-gas flow can move against the underground ventilation by thousands of meters. After outburst, the accumulative coal has an obvious separation phenomenon that the large coal locates at the bottom of roadway near the outburst mouth, and the granular coal is next, and the powdery coal locates at the top coal dump and more distant place<sup>[1][2]</sup>.

The characteristics of coal and gas outburst determine the particularity of the coal-gas two-phase flow. Currently, there are many research achievements on mathematical models of two-phase flow at home and abroad, such as the slip model, the VOF model, the two-fluid model, the discrete particle model, and so on, but documents about the conveying rules of outburst two-phase flow in roadway space are so few that there is no complete theory for reference. The conveying of coal-gas flow in the process of outburst is a complex problem related with many factors, including physical properties of coal seam, gas pressure, gas content, particle size, roadway environment, etc. If solving the problem from a micro perspective, we should study the move process of particles group in detail first in order to establish the continuity equation, the energy equation and the species equation, but the derivation of this method is too complicated to meet the actual field needs. Therefore, setting up the simplified model of outburst two-phase flow conveying in terms of the engineering application is beneficial to reveal the principle of outburst coal and rock accumulation phenomenon, and has important theoretical and practical significance.

## 2. Motion equations of coal particles

Because of complexity and particularity, the interacting mechanism of multi-phase flow is still not very clear, on this basis, all the mathematical models are built with subjectivity. Under this circumstances, the problem may be well simplified to start with force analysis of a single particle and minimize the uncertain impact factors according to the characteristics of coal and gas outburst and two-phase flow theory.

In the one-dimensional case, the motion equation of the coal particle with unit weight  $dG_s$  in the horizontal roadway is mainly determined by three forces shown in Fig.1, where  $df_s$ ,  $df_R$ ,  $df_g$  designate the resistance caused by the friction and collision between wall and particles, the viscous resistance between gas flow and particles, and the resistance caused by gravity respectively<sup>[3]</sup>.



Fig. 1 Stress analysis of coal particle

According to stress analysis, the suspended motion equation of coal particles is:

$$df_{R} - df_{g} - df_{s} = \left(\frac{v - u}{v_{t}}\right)^{2 - i} \cdot dG_{s} - \frac{v_{t}}{v} \cdot dG_{s} - \frac{\lambda_{s}}{D_{e}} \cdot \frac{u^{2}}{2g} \cdot dG_{s} = \frac{u \cdot dG_{s}}{g} \cdot \frac{du}{dt}$$
(1)

Where, v is gas velocity, u particle velocity, and  $v_t$  particle suspension velocity. The equation (1) can also be written as:

$$\left(\frac{v-u}{v_t}\right)^{2-i} - \frac{v_t}{v} - \frac{\lambda_s}{D_e} \cdot \frac{u^2}{2g} = \frac{u}{g} \cdot \frac{du}{dL}$$
(2)

Where, the index *i* is assigned to 0 or 1 according to flow patterns.

#### 3. A Mathematical model of coal particles conveying in the roadway

The coal bed is at static state before outburst. While outburst, the coal particles are accelerated by the gas flow constantly, and the pushing force will be equal to the resistance at a time point. After that, coal particles will keep the force equilibrium state in the horizontal direction, and its velocity will decrease constantly as the gas flow velocity decreased. Finally, when the gas flow velocity is less than the particles suspension speed, the particles stop moving. Therefore, the conveying process of coal particles can be divided into the acceleration process and the balance process, and two processes can be discussed and analyzed respectively.

## 3.1. Conveying distance La of particles acceleration process

When gas velocity is constant, the acceleration distance of particles can be obtained from the integral of equation (3):

$$L_{a} = \int dL = \int \frac{u du}{\left(\frac{v-u}{v_{t}}\right)^{2-i} \cdot g - \frac{v_{t}}{v} \cdot g - \frac{\lambda_{s} \cdot u^{2}}{2De}}$$
(3)

In the process of coal and gas outburst, the gas velocity is ever-changing with move distance increase, so the solution of equation (3) should be combined with the energy conservation equation, and then be solved by the gradual integration method.

According to the energy conservation equation and the pressure loss formula of two-phase flow conveying in pipeline, if the gas velocity decreases from  $v_c$  to v and the coal particles velocity changes from  $u_c$  to u when the move distance of coal-gas flow increases from L to  $L+\Delta L$ , the following equation can be written as:

$$\frac{1}{2}G_g \cdot v_c^2 + \frac{1}{2}G_s \cdot u_c^2 = \frac{1}{2}G_g \cdot v^2 + \frac{1}{2}G_s \cdot u^2 + \lambda_g \cdot G_g \cdot \frac{\Delta L}{De} \cdot \frac{v_c^2}{2} + \lambda_s \cdot G_s \cdot \frac{\Delta L}{De} \cdot \frac{u_c^2}{2} \tag{4}$$

Where,  $G_g$  is gas mass flow; and  $G_s$  is mass flow of coal particles.

Owing to solid-gas ratio  $n = G_s / G_g$ , the equation (4) can be simplified as:

$$v_c^2 + n \cdot u_c^2 = v^2 + n \cdot u^2 + \lambda_g \cdot \frac{\Delta L}{De} \cdot v_c^2 + \lambda_s \cdot n \cdot \frac{\Delta L}{De} \cdot u_c^2$$
<sup>(5)</sup>

The gas flow linear resistance coefficient  $\lambda_g$  can be calculated by <sup>[4]</sup>:

$$\lambda_g = 0.067 \cdot \left(2\Delta/De\right)^{0.2} \tag{6}$$

Where,  $\lambda_g$  is gas flow resistance coefficient in roadway,  $D_e$  roadway equivalent diameter of equal resistance, and  $\Delta$  roadway equivalent roughness.

The resistance coefficient  $\lambda_s$  is related with pipe shape, physical property and particle size of solid phase, gas velocity and other factors, and there is no mature theory currently. Stegmaier's empirical equation takes account of the influence of various factors on resistance and can be used as a reference. The empirical equation is <sup>[5]</sup>:

$$\lambda_s = 2.1 \cdot n^{-0.3} \cdot \left(\frac{v_t}{\sqrt{gd_s}}\right)^{0.5} \cdot \left(\frac{v}{\sqrt{gDe}}\right)^{-1.21} \cdot \left(\frac{De}{d_s}\right)^{-0.1}$$
(7)

#### 3.2. Conveying distance Lb of particles balance process

In the acceleration process, the stress of particles starts the equilibrium state in the horizontal direction while coal particles velocity u increases to a certain speed  $u_b$  which is called the initial equilibrium velocity. While udu/dL=0, the equation (2) yields:

$$\left(\frac{v}{v_t}\right)^{2-i} \left[1 - \left(\frac{u}{v}\right)^{2-i}\right] - \frac{v_t}{v} - \frac{\lambda_s}{2} \cdot \frac{v^2}{gD_e} \left(\frac{u}{v}\right)^2 = 0$$
<sup>(8)</sup>

Then the equation (5) yields:

$$\Delta L_{b} = \frac{\left[ (v_{c}^{2} - v^{2}) + n \cdot (u_{c}^{2} - u^{2}) \right] \cdot De}{\lambda_{g} \cdot v_{c}^{2} + \lambda_{s} \cdot n \cdot u_{c}^{2}}$$
(9)

First the initial equilibrium velocity  $u_b$  of gas flow should be calculated by the equation (8), and then the conveying distance  $L_b$  of particles balance process is solved by the numerical method.

## 3.3. The mathematical model

According to the solid particle suspension equation, the energy conservation law in two-phase flow and the balance mechanism of gas and particles in the case of one-dimensional, the equation (2), (3), (5), (7), (8), and (9) are combined and organized to create a mathematical model of coal-gas flow conveying through discussing the particles move process separately, as shown in equation (10).

$$\begin{aligned} \lambda_{g} &= 0.067 \cdot \left( 2\Delta/De \right)^{0.2}, \ \lambda_{s} &= 2.1 \cdot n^{-0.3} \cdot \left( \frac{v_{t}}{\sqrt{gd_{s}}} \right)^{0.5} \cdot \left( \frac{v}{\sqrt{gDe}} \right)^{-1.21} \cdot \left( \frac{De}{d_{s}} \right)^{-0.1} \\ L_{a} &\left\{ \frac{(v-u)}{v_{t}} \right)^{2-i} - \frac{v_{t}}{v} - \frac{\lambda_{s}}{D_{e}} \cdot \frac{u^{2}}{2g} = \frac{u}{g} \cdot \frac{du}{dL} (du > 0) \\ v_{c}^{2} + n \cdot u_{c}^{2} &= v^{2} + n \cdot u^{2} + \lambda_{g} \cdot \frac{\Delta L}{De} \cdot v_{c}^{2} + \lambda_{s} \cdot n \cdot \frac{\Delta L}{De} \cdot u_{c}^{2} \\ L_{b} &\left\{ \frac{(v)}{v_{t}} \right)^{2-i} \left[ 1 - \left( \frac{u}{v} \right)^{2-i} \right] - \frac{v_{t}}{v} - \frac{\lambda_{s}}{2} \cdot \frac{v^{2}}{gD_{e}} \left( \frac{u}{v} \right)^{2} = 0 \\ \Delta L_{bi} &= \left[ \Delta(v^{2}) + n \cdot \Delta(u^{2}) \right] \cdot De / (\lambda_{g} \cdot v_{c}^{2} + \lambda_{s} \cdot n \cdot u_{c}^{2}) \\ L &= L_{a} + L_{b} \end{aligned}$$
(10)

#### 4. Model calculations

The mathematical model can both estimate the coal particles conveying distance according to the initial state of coal seam and inverse the initial energy of coal-gas flow according to the coal particles conveying distance record on spot. The model calculation uses the stepwise integral feedback control method. Assuming that the coal particles start the stress balance state at the moment while coal particles velocity is  $u_b$  and gas flow velocity is  $v_b$ , the calculation equations are shown as:

Acceleration process distance  $L_a$ :

$$\begin{aligned} X_{0} &= \frac{g}{v_{i}^{2}} - \frac{\lambda_{s}}{2D_{e}}, \ Y_{0} &= -\frac{2vg}{v_{i}^{2}}, \ Z_{0} = \frac{v^{2}g}{v_{i}^{2}} - \frac{v_{i}g}{v}, \ X_{1} = -\frac{\lambda_{s}}{2De}, \ Y_{1} = -\frac{g}{v_{i}}, \ Z_{1} = \frac{v \cdot g}{v_{i}} - \frac{v_{i} \cdot g}{v} \\ u_{a} &= j \cdot h, \ u_{ac} = (j-1) \cdot h; \ L_{a} = (L_{ai})_{j}, \ L_{ac} = (L_{ai})_{j-1}; \ (v_{a})_{1} = v_{0} \ (h \text{ is calculation step length}) \\ L_{ai} &= \frac{1}{2X_{i}} \cdot \ln \left| X_{i} \cdot u^{2} + Y_{i} \cdot u + Z_{i} \right| + \frac{Y_{i}}{2 \cdot X_{i} \cdot B_{i}} \cdot \ln \left| \frac{2X_{i} \cdot u + Y_{i} - B_{i}}{2X_{i} \cdot u + Y_{i} + B_{i}} \right| + C_{i} \ (i = 0, 1) \\ B_{i} &= \sqrt{Y_{i}^{2} - 4 \cdot X_{i} \cdot Z_{i}}, \ C_{i} &= -\frac{1}{2X_{i}} \cdot \ln \left| Z_{i} \right| - \frac{Y_{i}}{2X_{i} \cdot B_{i}} \cdot \ln \left| \frac{Y_{i} - B_{i}}{Y_{i} + B_{i}} \right| \ (j = 1, 2, 3, \dots) \\ v_{a(j+1)}^{2} &= v_{aj}^{2} - n \cdot (u_{a}^{2} - u_{ac}^{2}) - \lambda_{g} \cdot (L_{a} - L_{ac}) \cdot v_{aj}^{2} / De - \lambda_{s} \cdot n \cdot (L_{a} - L_{ac}) \cdot u_{a}^{2} / De \end{aligned}$$

Balance process distance  $L_b$ :

$$\begin{cases} v_{bj} = v_b - j \cdot h, \ u_{b0} = u_b, \ L_{b0} = L_a \ (j = 1, 2, 3....) \\ (u_{bj})_i = (-Y_i - B_i)/(2 \cdot X_i) \ (i = 0, 1) \\ L_{bj} = L_{b(j-1)} + De \cdot \left[ v_{b(j-1)}^2 - v_{bj}^2 + n \cdot (u_{b(j-1)}^2 - u_{bj}^2) \right] / (\lambda_g \cdot v_{b(j-1)}^2 + \lambda_s \cdot n \cdot u_{b(j-1)}^2) \end{cases}$$
(12)

In the course of solving equations, the calculation value  $L_a$  at each node is used to make a feedback control of the gas velocity v value, thus the calculation errors caused by the step length is reduced. Solving nonlinear equations with this method, if the right step length is set, the solution is very accurate. The solving process of equation (10) is shown as follows:





## 5. An application example

An accident of coal and gas outburst occurred in a mine, bursting out 169529 cubic meters gas and 2382.37 tons coal. The roadway is crammed about 300 meters long by the outburst coal. The cross-section form of the roadway is oblique rectangular, width 4.4m, and middle height 2.8m.

The roadway equivalent diameter *De* is calculated to be 3.42m according to the field data, but in view of the change of effective cross-section area in process of outburst, De=2m is used in the model calculation. According to the support form and conditions of roadway, taking  $\Delta = 0.1$ m, then calculating the gas flow resistance coefficient  $\lambda_g \approx 0.03797$  from the equation (6).

the gas flow resistance coefficient  $\lambda_g \approx 0.03797$  from the equation (6). The total solid-gas ratio *n* 17.2, the gravity acceleration g 9.81kg.m/s, the coal density  $\rho_s$  1460kg/m<sup>3</sup>, gas density  $\rho_g$  0.816 kg/m<sup>3</sup>, and the gas dynamic viscosity coefficient  $\mu$ =18.0×10<sup>-6</sup>*Pa.s* when t=29 °C, so the particle size range of the Newton resistance equation is  $d_s > 20.4 \times [\mu^2/\rho_g(\rho_s - \rho_g)]^{1/3} = 1.3219 \text{mm}^{[6]}$ .

The pneumatic transmission branch of lifting machinery teaching and research group in Shanghai Maritime University had measured the suspension velocity of many kinds of materials in laboratory experiments in 1970s<sup>[7]</sup>, some of which can be used as the calculation basis of the model after data fitting, as shown in Table 1.

Table 1 Suspension velocity of coal

Diameters (mm)	0.2	1	10	20	30	50	100	200
Suspension velocity (m/s)	2	3.46	10.60	14.84	18.08	23.17	32.45	45.46

The average solid-gas ratio is certainly larger than the total solid-gas ratio 17.2 in process of coal-gas flow conveying because the gas quantity for outburst is less than total outburst quantity. The solid-gas ratio should be adjusted many times for solving model in order to make the calculation value consistent with the field data. If setting the solid-gas ratio n=50 in the Newton resistance area and n=10 in the Stokes resistance area, the model calculation results are shown in Table 2 and Fig. 3.

Table 2 Calculation results of different initial energy particles conveying distance L<sub>sum</sub>

$d_s$	<i>v<sub>0</sub></i> =50 m/s	v <sub>0</sub> =100	v <sub>0</sub> =200	v <sub>0</sub> =300	v <sub>0</sub> =400	v <sub>0</sub> =500	v <sub>0</sub> =600	v <sub>0</sub> =700	v <sub>0</sub> =800
0.2	13.22	29.78	65.52	87.40	121.55	151.57	197.31	212.57	258.51
1	21.90	47.21	81.10	145.55	170.96	222.61	260.41	307.52	353.98
10	25.01	43.89	75.54	128.88	196.92	344.24	414.45	474.95	542.12
30	34.86	51.68	87.31	128.95	176.52	229.54	287.55	350.10	417.07
50	39.36	55.01	90.68	130.34	174.78	223.66	276.67	333.46	393.77
100	40.10	58.49	95.80	134.08	175.49	220.14	267.83	318.38	357.37
200	20.01	58.65	99.73	116.85	177.21	218.62	262.10	307.57	354.89

As shown in Fig.3, when the solid-gas ratio n is constant, the movement distance of coal particles increases with the gas initial energy increment. But when the initial gas velocity  $v_0$  is constant, the  $L_{sum}$  value increases at Stokes resistance area and increases first and decreases afterward at Newton resistance area with the particle diameters increment. The Stokes resistance area trends is mainly due to the larger size particles making the resistance coefficient  $\lambda_s$  value smaller; At the Newton resistance area, when the particle diameter is smaller than the critical value  $d_{sl}$ , the resistance coefficient  $\lambda_s$  decreases with the particle size increase, but when the particle diameter is larger than the critical value  $d_{sl}$ , the resistance coefficient  $\lambda_s$  increases with the particle size increase.



Fig. 3 Relation between initial velocity and conveying distance

# 6. Conclusions

In summary, the mathematical model has considered the impact of various factors on the coal particles conveying distance, and provides a new method for the engineering calculation of outburst coalgas two-phase flow, but the model still have many limitations, such as:

(1) The model has mainly considered the interaction between gas and particles, but it is short of study on the interaction among different size particles.

(2) Because of no measured data, several equations in the model are empirical equations whose applicability needs experimental certification.

(3) The model doesn't analyze the influence of coal bed property on gas energy and solid-gas ratio.

Therefore, the mathematical model still needs to update and replenish through actual application. In process of application, the comparative analysis of calculation results and measured data in outburst field or laboratory can optimize the model parameters and summarize the relation of different parameters with the inductive method, thereby getting a more perfect mathematical model.

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