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Decay constants of P-wave mesons

Guo-Li Wang

Department of Physics, Harbin Institute of Technology, Harbin 150001, China

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Abstract

Decay constants of P-wave mesons are computed in the framework of instantaneous Bethe–Salpeter method (Salpeter method). By analyzing the parity and possible charge conjugation parity, we give the relativistic configurations of wave functions with definite parity and possible charge conjugation parity. With these wave functions as input, the full Salpeter equations for different P-wave states are solved, and the mass spectra as well as the numerical values of wave functions are obtained. Finally we compute the leptonic decay constants of heavy–heavy and heavy–light $^3P_0$, $^3P_1$ and $^1P_1$ states.

1. Introduction

The quantities of meson decay constants are important, since they play important role in many aspects, and the studies of them have become hot topics in recent years [1–12]. But most of these investigations are focused on the estimating decay constants for S-wave mesons, and we lack the knowledge of decay constants for P-wave mesons, there are only a few of papers available [13–18]. We present a careful study of the decay constants for heavy $^3P_0$, $^3P_1$ and $^1P_1$ states including the relativistic corrections.

In previous letters [19], the decay constants of heavy–heavy and heavy–light pseudoscalar ($^1S_0$) and vector ($^3S_1$) mesons are calculated in the framework of relativistic instantaneous Bethe–Salpeter method [20] (also called Salpeter method [21]), good agreement of our predictions with recent lattice, QCD sum rule, other relativistic model calculations as well as available experimental data is found.

In this Letter, we extend our previous analysis to include P-wave mesons, present the calculations of decay constants for heavy P-wave states in the framework of full Salpeter equation which is a relativistic method. Based on the S–L coupling scheme, we analyze the parity and possible charge conjugation of $^3P_0$, $^3P_1$ and $^1P_1$ bound states, and give general formula for the wave functions which are in relativistic form with definite parity and charge conjugation symmetry (0^{++}, 1^{++}, 1^{+-} for equal mass $^3P_0$, $^3P_1$ and $^1P_1$ bound states, and 0^+, 1^+, 1^+ for unequal-mass bound states). Then with these forms of wave functions as input, we solve the full Salpeter equations, obtain the mass spectra and numerical values of wave functions for different P-wave states. Finally, we compute the leptonic decay constants for heavy $^3P_0$, $^3P_1$ and $^1P_1$ states.

This Letter is organized as following, in Section 2, we introduce the relativistic Bethe–Salpeter equation and Salpeter equation. In Section 3, we give the formula of relativistic wave functions and decay constants for P-wave states. We solve the full Salpeter equations, obtain the mass spectra and wave functions of P-wave mesons. Finally, we use these relativistic wave functions to calculate the decay constants of heavy P-wave mesons and show the numerical results and discussion in Section 4.

E-mail address: gl_wang@hit.edu.cn.

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2. Instantaneous Bethe–Salpeter equation

In this section, we briefly review the instantaneous Bethe–Salpeter equation and introduce our notations, interested reader can find the details in Ref. [22].

The Bethe–Salpeter (BS) equation is read as [20]:

$$(\not{p}_1 - m_1)\chi(q)(\not{p}_2 + m_2) = i\int \frac{d^4k}{(2\pi)^4} V(P,k,q)\chi(k), \tag{1}$$

where $\chi(q)$ is the BS wave function, $V(P,k,q)$ is the interaction kernel between the quark and anti-quark, and p_1, p_2 are the momenta of the quark 1 and anti-quark 2. The total momentum P and the relative momentum q are defined as:

$$p_1 = \alpha_1 P + q, \quad \alpha_1 = \frac{m_1}{m_1+m_2},$$
$$p_2 = \alpha_2 P - q, \quad \alpha_2 = \frac{m_2}{m_1+m_2}.$$

We divide the relative momentum q into two parts, $q_\parallel$ and $q_\perp$,

$$q^\mu = q^\mu_\parallel + q^\mu_\perp, \quad q^\mu_\parallel \equiv (P\cdot q/M^2)P^\mu,\ q^\mu_\perp \equiv q^\mu - q^\mu_\parallel.$$

Correspondingly, we have two Lorentz invariant variables:

$$q_P = \frac{(P\cdot q)}{M}, \qquad q_T = \sqrt{q_P^2 - q^2} = \sqrt{-q_\perp^2}.$$

When $\vec{P} = 0$, they turn to the usual component q_0 and $|\vec{q}|$, respectively.

After instantaneous approach, the kernel $V(P,k,q)$ takes the simple form:

$$V(P,k,q) \Rightarrow V(k_\perp, q_\perp).$$

Let us introduce the notations $\varphi_P(q^\mu_\perp)$ and $\eta(q^\mu_\perp)$ for three-dimensional wave function as follows:

$$\varphi_P(q^\mu_\perp) \equiv i\int \frac{dq_P}{2\pi}\chi(q^\mu_\parallel, q^\mu_\perp), \qquad \eta(q^\mu_\perp) \equiv \int \frac{dk_\perp}{(2\pi)^3} V(k_\perp, q_\perp)\varphi_P(k^\mu_\perp). \tag{2}$$

Then the BS equation can be rewritten as:

$$\chi(q_\parallel, q_\perp) = S_1(p_1)\eta(q_\perp)S_2(p_2). \tag{3}$$

The propagators of the two constituents can be decomposed as:

$$S_i(p_i) = \frac{\Lambda^+_{ip}(q_\perp)}{J(i)q_P + \alpha_i M - \omega_i + i\epsilon} + \frac{\Lambda^-_{ip}(q_\perp)}{J(i)q_P + \alpha_i M + \omega_i - i\epsilon}, \tag{4}$$

with

$$\omega_i = \sqrt{m_i^2 + q_T^2}, \qquad \Lambda^\pm_{ip}(q_\perp) = \frac{1}{2\omega_{ip}}\left[\frac{\not{P}}{M}\omega_i \pm J(i)(m_i + \not{q}_\perp)\right], \tag{5}$$

where $i = 1, 2$ for quark and anti-quark, respectively, and $J(i) = (-1)^{i+1}$. Here $\Lambda^\pm_{ip}(q_\perp)$ satisfy the relations:

$$\Lambda^+_{ip}(q_\perp) + \Lambda^-_{ip}(q_\perp) = \frac{\not{P}}{M}, \qquad \Lambda^\pm_{ip}(q_\perp)\frac{\not{P}}{M}\Lambda^\pm_{ip}(q_\perp) = \Lambda^\pm_{ip}(q_\perp), \qquad \Lambda^\pm_{ip}(q_\perp)\frac{\not{P}}{M}\Lambda^\mp_{ip}(q_\perp) = 0. \tag{6}$$

Introducing the notations $\varphi^{\pm\pm}_p(q_\perp)$ as:

$$\varphi^{\pm\pm}_p(q_\perp) \equiv \Lambda^\pm_{1p}(q_\perp)\frac{\not{P}}{M}\varphi_p(q_\perp)\frac{\not{P}}{M}\Lambda^\pm_{2p}(q_\perp), \tag{7}$$

we have

$$\varphi_p(q_\perp) = \varphi^{++}_p(q_\perp) + \varphi^{+-}_p(q_\perp) + \varphi^{-+}_p(q_\perp) + \varphi^{--}_p(q_\perp).$$

With contour integration over q_P on both sides of Eq. (3), we obtain:

$$\varphi_p(q_\perp) = \frac{\Lambda^+_{1p}(q_\perp)\eta_p(q_\perp)\Lambda^+_{2p}(q_\perp)}{(M-\omega_1-\omega_2)} - \frac{\Lambda^-_{1p}(q_\perp)\eta_p(q_\perp)\Lambda^-_{2p}(q_\perp)}{(M+\omega_1+\omega_2)},$$

and the full Salpeter equation:

$$(M-\omega_1-\omega_2)\varphi_p^{++}(q_\perp)=\Lambda_{1p}^{+}(q_\perp)\eta_p(q_\perp)\Lambda_{2p}^{+}(q_\perp),$$
$$(M+\omega_1+\omega_2)\varphi_p^{--}(q_\perp)=-\Lambda_{1p}^{-}(q_\perp)\eta_p(q_\perp)\Lambda_{2p}^{-}(q_\perp),$$
$$\varphi_p^{+-}(q_\perp)=\varphi_p^{-+}(q_\perp)=0. \tag{8}$$

The normalization condition for BS wave function is:

$$\int\frac{q_T^2\,dq_T}{2\pi^2}\operatorname{Tr}\left[\bar\varphi^{++}\frac{\not P}{M}\varphi^{++}\frac{\not P}{M}-\bar\varphi^{--}\frac{\not P}{M}\varphi^{--}\frac{\not P}{M}\right]=2P_0. \tag{9}$$

In our model, Cornell potential, a linear scalar interaction plus a vector interaction is chosen as the instantaneous interaction kernel V [22]:

$$V(\vec q)=V_s(\vec q)+\gamma_0\otimes\gamma^0V_v(\vec q),$$
$$V_s(\vec q)=-\left(\frac{\lambda}{\alpha}+V_0\right)\delta^3(\vec q)+\frac{\lambda}{\pi^2}\frac{1}{(\vec q^2+\alpha^2)^2},\qquad V_v(\vec q)=-\frac{2}{3\pi^2}\frac{\alpha_s(\vec q)}{(\vec q^2+\alpha^2)}, \tag{10}$$

where the coupling constant $\alpha_s(\vec q)$ is running:

$$\alpha_s(\vec q)=\frac{12\pi}{27}\frac{1}{\log(a+\frac{\vec q^2}{\Lambda_{\mathrm{QCD}}^2})},$$

and the constants λ, α, a, V_0 and Λ_{QCD} are the parameters that characterize the potential.

3. Relativistic wave functions and decay constants

In this section, by analyzing the parity and possible charge conjugation parity of corresponding bound state, we give a formula for the wave function that is in a relativistic form with definite parity and possible charge conjugation parity symmetry.

3.1. Wave function for $^3P_0$ state

The general form for the relativistic Salpeter wave function of $^3P_0$ state, which $J^P=0^+$ (or $J^{PC}=0^{++}$ for equal mass system), can be written as:

$$\varphi_{0^+}(q_\perp)=f_1(q_\perp)\not q_\perp+f_2(q_\perp)\frac{\not P\not q_\perp}{M}+f_3(q_\perp)M+f_4(q_\perp)\not P. \tag{11}$$

The equations

$$\varphi_{0^+}^{+-}(q_\perp)=\varphi_{0^+}^{-+}(q_\perp)=0 \tag{12}$$

give the constraints on the components of the wave function, so we have the relations

$$f_3(q_\perp)=\frac{f_1(q_\perp)q_\perp^2(m_1+m_2)}{M(\omega_1\omega_2+m_1m_2+q_\perp^2)},\qquad f_4(q_\perp)=\frac{f_2(q_\perp)q_\perp^2(\omega_1-\omega_2)}{M(m_1\omega_2+m_2\omega_1)}.$$

Then there are only two independent wave functions $f_1(q_\perp)$ and $f_2(q_\perp)$ been left, from Eq. (8), we obtain two coupled integral equations, by solving them we obtain the numerical results of mass spectra and wave functions, interesting reader can find the details of this method in Ref. [22] or Ref. [23].

In our calculation, we choose the center-of-mass system of the corresponding state, so $q_\parallel$ and $q_\perp$ turn to the usual components $(q_0,\vec 0)$ and $(0,\vec q)$, $\omega_1=(m_1^2+\vec q^2)^{1/2}$ and $\omega_2=(m_2^2+\vec q^2)^{1/2}$. The normalization condition for the $^3P_0$ wave function is:

$$\int\frac{d\vec q}{(2\pi)^3}\frac{16f_1f_2\omega_1\omega_2\vec q^2}{m_1\omega_2+m_2\omega_1}=2M. \tag{13}$$

3.2. Decay constant of $^3P_0$ state

The decay constant $F_{^3P_0}$ of scalar $^3P_0$ meson is defined as

$$\langle 0|\bar q_1\gamma_\mu(1-\gamma_5)q_2|^3P_0\rangle\equiv F_{^3P_0}P_\mu, \tag{14}$$

which can be written in the language of the Salpeter wave functions as:

$$\begin{aligned}\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|^3P_0\rangle &= \sqrt{N_c}\int \mathrm{Tr}\big[\varphi_{0^+}(\vec{q})\gamma_\mu(1-\gamma_5)\big]\frac{d\vec{q}}{(2\pi)^3}\\ &= \sqrt{N_c}\int\frac{d\vec{q}}{(2\pi)^3}\mathrm{Tr}\bigg[\bigg(f_1\not{q}+f_2\frac{\not{P}\not{q}_\perp}{M}-\frac{f_1\vec{q}^2(m_1+m_2)}{\omega_1\omega_2+m_1m_2-\vec{q}^2}-\frac{f_2\vec{q}^2(\omega_1-\omega_2)\not{P}}{M(m_1\omega_2+m_2\omega_1)}\bigg)\gamma_\mu\bigg]\\ &= 4\sqrt{N_c}\int\frac{d\vec{q}}{(2\pi)^3}\bigg(-\frac{f_2\vec{q}^2(\omega_1-\omega_2)}{M(m_1\omega_2+m_2\omega_1)}\bigg)P_\mu. \end{aligned}\tag{15}$$

Therefore, we have

$$F_{^3P_0}=\frac{4\sqrt{N_c}}{M}\int\frac{d\vec{q}}{(2\pi)^3}\frac{f_2\vec{q}^2(\omega_2-\omega_1)}{(m_1\omega_2+m_2\omega_1)}.\tag{16}$$

3.3. Wave function for $^3P_1$ state

The general form for the Salpeter wave function of $^3P_1$ state, which $J^P=1^+$ (or $J^{PC}=1^{++}$ for equal mass system), can be written as:

$$\varphi_{1^+}(q_\perp)=i\varepsilon_{\mu\nu\alpha\beta}P^\nu q_\perp^\alpha\epsilon^\beta\big[f_1M\gamma^\mu+f_2\not{P}\gamma^\mu+f_3\not{q}_\perp\gamma^\mu+if_4\varepsilon^{\mu\rho\sigma\delta}q_{\perp\rho}P_\sigma\gamma_\delta\gamma_5/M\big]/M^2.\tag{17}$$

The equations

$$\varphi_{1^+}^{+-}(q_\perp)=\varphi_{1^+}^{-+}(q_\perp)=0\tag{18}$$

give the constraints on the components of the wave function

$$f_3(q_\perp)=\frac{f_1(q_\perp)M(m_1\omega_2-m_2\omega_1)}{q_\perp^2(\omega_1+\omega_2)},\qquad f_4(q_\perp)=\frac{f_2(q_\perp)M(-\omega_1\omega_2+m_1m_2+q_\perp^2)}{q_\perp^2(m_1+m_2)}.$$

The normalization condition for the $^3P_1$ wave function is:

$$\int\frac{d\vec{q}}{(2\pi)^3}\frac{32f_1f_2\omega_1\omega_2(\omega_1\omega_2-m_1m_2+\vec{q}^2)}{3(m_1+m_2)(\omega_1+\omega_2)}=2M.\tag{19}$$

3.4. Decay constant of $^3P_1$ state

The decay constant $F_{^3P_1}$ is defined as

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|^3P_1,\epsilon\rangle\equiv F_{^3P_1}M\epsilon_\mu^\lambda,\tag{20}$$

and can be formulated using the Salpeter wave function as:

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|^3P_1,\epsilon\rangle=\sqrt{N_c}\int\mathrm{Tr}\big[\varphi_{1^+}(\vec{q})\gamma_\mu(1-\gamma_5)\big]\frac{d\vec{q}}{(2\pi)^3},\tag{21}$$

then we have

$$F_{^3P_1}=\frac{8\sqrt{N_c}}{3M}\int\frac{d\vec{q}}{(2\pi)^3}\frac{f_2(\omega_1\omega_2-m_1m_2+\vec{q}^2)}{(m_1+m_2)}.\tag{22}$$

3.5. Wave function for $^1P_1$ state

The general form for the Salpeter wave function of $^1P_1$ state, which $J^P=1^+$ (or $J^{PC}=1^{+-}$ for equal mass system), can be written as:

$$\varphi_{1^+}(q_\perp)=q_\perp\cdot\epsilon_\perp^\lambda\bigg[f_1(q_\perp)+f_2(q_\perp)\frac{\not{P}}{M}+f_3(q_\perp)\frac{\not{q}_\perp}{M}+f_4(q_\perp)\frac{\not{P}\not{q}_\perp}{M^2}\bigg]\gamma_5.\tag{23}$$

The equations

$$\varphi_{1^+}^{+-}(q_\perp)=\varphi_{1^+}^{-+}(q_\perp)=0\tag{24}$$

give the constraints on the components of the wave function

$$f_3(q_\perp) = -\frac{f_1(q_\perp)M(m_1-m_2)}{(\omega_1\omega_2+m_1m_2-q_\perp^2)}, \qquad f_4(q_\perp) = -\frac{f_2(q_\perp)M(\omega_1+\omega_2)}{(m_1\omega_2+m_2\omega_1)}.$$

The normalization condition for the $^1P_1$ wave function is:

$$\int \frac{d\vec{q}}{(2\pi)^3}\frac{16 f_1 f_2 \omega_1\omega_2 \vec{q}^2}{3(m_1\omega_2+m_2\omega_1)} = 2M. \tag{25}$$

3.6. Decay constant of $^1P_1$ state

The decay constant $F_{^1P_1}$ is defined as

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2\big|^1P_1,\epsilon\rangle \equiv F_{^1P_1} M\epsilon_\mu^\lambda, \tag{26}$$

which can be formulated as:

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2\big|^1P_1,\epsilon\rangle = \sqrt{N_c}\int \mathrm{Tr}\big[\varphi_{1^+}(\vec{q})\gamma_\mu(1-\gamma_5)\big]\frac{d\vec{q}}{(2\pi)^3}, \tag{27}$$

finally, we obtain

$$F_{^1P_1} = \frac{4\sqrt{N_c}}{3M}\int \frac{d\vec{q}}{(2\pi)^3}\frac{f_1(m_1-m_2)\vec{q}^2}{(\omega_1\omega_2+m_1m_2+\vec{q}^2)}. \tag{28}$$

4. Numerical results and discussion

In our method, there are some input parameters appearing in the potential, we need to fix them when solving the full Salpeter equations. Usually, we fixed the parameters by fitting the experimental mass spectra for mesons, but for P-wave states, we lack experimental data, so we adopt almost the same parameters as in the 0^- states Ref. [22], and only vary the parameter V_0 by fitting the ground P-wave $c\bar{c}$ states, χ_{c0}, χ_{c1} and h_c. In previous letter [19], we found if we choose same parameters set, the mass predictions of our model cannot agree very well with experimental data for pseudoscalar and vector mesons, we find the same thing happens to the different P-wave states, so we vary the only possible different parameter V_0 to fit the data. For $^3P_0$ states, we choose $V_0 = -0.566$ GeV, for $^3P_1$ states, $V_0 = -0.452$ GeV, and for $^1P_1$ states, $V_0 = -0.437$ GeV. The values for other parameters are same as in the 0^- states Ref. [22]:

$$a = e = 2.7183, \quad \alpha = 0.06\ \text{GeV}, \quad \lambda = 0.20\ \text{GeV}^2, \quad \Lambda_{\text{QCD}} = 0.26\ \text{GeV} \quad \text{and}$$
$$m_b = 5.224\ \text{GeV}, \quad m_c = 1.7553\ \text{GeV}, \quad m_s = 0.487\ \text{GeV}, \quad m_d = 0.311\ \text{GeV}, \quad m_u = 0.305\ \text{GeV}. \tag{29}$$

We show our theoretical predictions of mass spectra for $c\bar{c}$ states up to the $4P$ states as well as the experimental data in Table 1. One can see that, our predictions for mass splitting are, $2P-1P \simeq 410$ MeV, $3P-2P \simeq 300$ MeV, $4P-3P \simeq 235$ MeV; the predicted mass for state 2^3P_1 is 3923 MeV, this is a little smaller but consist with the traditional prediction of potential model, which is about 50 MeV higher than the one of new state $X(3872)$. We show the predicted mass spectra for other states in Table 2,

Table 1
Mass spectra in unit of MeV for $c\bar{c}$ and $b\bar{b}$ P-wave states

	Ex($c\bar{c}$)	$c\bar{c}$	Ex($b\bar{b}$)	$b\bar{b}$
$1\,^3P_0$	3415.2	3415.9	9859.9	9860.1
$2\,^3P_0$		3831.1	10 232.1	10 223.9
$3\,^3P_0$		4132.4		10 497.0
$4\,^3P_0$		4369.4		10 719.2
$1\,^3P_1$	3510.6	3510.9	9892.7	9892.1
$2\,^3P_1$		3923.1	10 255.2	10 255.0
$3\,^3P_1$		4222.0		10 527.4
$4\,^3P_1$		4456.9		10 750.0
$1\,^1P_1$	3524.4	3524.4		9900.4
$2\,^1P_1$		3935.8		10 262.6
$3\,^1P_1$		4234.2		10 534.6
$4\,^1P_1$		4468.7		10 757.1

Table 2
Mass spectra in unit of MeV for heavy P-wave states

	$1\,^3P_0$	$2\,^3P_0$	$1\,^3P_1$	$2\,^3P_1$	$1\,^1P_1$	$2\,^1P_1$
$c\bar{b}$	6728.7	7127.8	6829.5	7225.3	6845.1	7239.6
$s\bar{b}$	5767.2	6130.3	5830.9	6192.2	5836.4	6197.3
$d\bar{b}$	5667.9	5998.9	5711.7	6042.8	5709.2	6041.4
$u\bar{b}$	5664.8	5994.1	5707.6	6037.2	5704.7	6035.5
$s\bar{c}$	2386.5	2767.4	2447.8	2827.3	2449.8	2830.4
$d\bar{c}$	2273.1	2619.2	2314.1	2661.3	2307.6	2657.7
$u\bar{c}$	2269.3	2613.7	2309.5	2655.0	2302.5	2651.1

Table 3
Decay constants in unit of MeV for P-wave $c\bar{c}$ and $b\bar{b}$ states

	$n\,^3P_0$	$n\,^1P_1$	$1\,^3P_1$	$2\,^3P_1$	$3\,^3P_1$	$4\,^3P_1$
$c\bar{c}$	0	0	206	−207	199	−189
$b\bar{b}$	0	0	129	−131	126	−121

Table 4
Decay constants in unit of MeV for heavy P-wave states

	$1\,^3P_0$	$2\,^3P_0$	$1\,^3P_1$	$2\,^3P_1$	$1\,^1P_1$	$2\,^1P_1$
$c\bar{b}$	88	−85	160	−165	50	−49
$s\bar{b}$	140	−130	157	−156	76	−71
$d\bar{b}$	145	−129	150	−144	76	−70
$u\bar{b}$	145	−128	150	−143	76	−70
$s\bar{c}$	112	−91	219	−204	62	−50
$d\bar{c}$	132	−102	212	−190	72	−56
$u\bar{c}$	133	−102	211	−189	72	−56

the interesting quantity is also the mass splitting between the first radial excited state and ground state, for all the $^3P_0$, $^3P_1$ and $^1P_1$ states, $2P-1P\simeq 330$ MeV for $u\bar{b}$ and $2P-1P\simeq 345$ MeV for $u\bar{c}$, $2P-1P\simeq 362$ MeV for $s\bar{b}$ and $2P-1P\simeq 381$ MeV for $s\bar{c}$.

We also calculate the mass spectra for P-wave $b\bar{b}$ system, as argued in Ref. [24], there are double heavy b quarks, and the flavor $N_f=4$, so we have to choose a new set of parameters as well as smaller value of coupling constant. We change the previous scale parameters to $\Lambda_{\mathrm{QCD}}=0.20$ GeV, $m_b=5.13$ GeV which have been adopted in Ref. [24], choose $V_0=-0.553$ GeV for $^3P_0$ states, $V_0=-0.521$ GeV for $^3P_1$ states, $V_0=-0.514$ GeV for $^1P_1$ states, and other parameters are not changed. With this set of parameters, the coupling constant at the scale of bottom quark mass is $\alpha_s(m_b)=0.23$. We also show the numerical results and experimental data in Table 1. One can see that our predictions, $2P-1P\simeq 363$ MeV, can fit the experimental data very well, and our mass splitting prediction, $3P-2P\simeq 273$ MeV, $4P-3P\simeq 223$ MeV.

Besides the mass spectra, we also obtained the relativistic wave functions for heavy mesons when solving the full Salpeter equation. With these wave functions, we calculated the decay constants for heavy–heavy and heavy–light P-wave mesons. In Table 3, we show our estimates of decay constants for $c\bar{c}$ and $b\bar{b}$ $^3P_1$-wave systems up to third radial excitation states, for $^3P_0$ and $^1P_1$ equal-mass states, the decay constants vanish. Our predictions show that, the decline of the numerical value of decay constant for higher excited state is not evident comparing with the lower excited state, for example, $F_{3P}-F_{1P}=7$ MeV for $^3P_1 c\bar{c}$ system, and $F_{3P}-F_{1P}=3$ MeV for $^3P_1$ $b\bar{b}$ system.

In Table 4, we show our estimates of decay constants for unequal-mass P-wave ground and first radial excited states. It is observed that the decay constants of $^3P_1$ states are much larger than those of the corresponding $^1P_1$ states, while the corresponding values for $^3P_0$ is between them.

For comparison, we show our predictions for decay constants and other theoretical predictions [13,14] in Table 5. We have changed results in Ref. [14] from the j–j coupling scheme to S–L coupling scheme by using the following equations [13,25]

$$\left|{}^1P_1\right\rangle=\sqrt{\frac{2}{3}}\left|P_1^{3/2}\right\rangle-\frac{1}{\sqrt{3}}\left|P_1^{1/2}\right\rangle,\qquad \left|{}^3P_1\right\rangle=\frac{1}{\sqrt{3}}\left|P_1^{3/2}\right\rangle+\sqrt{\frac{2}{3}}\left|P_1^{1/2}\right\rangle. \tag{30}$$

Rough agreement can be found between the values of decay constants estimated by different methods, this means we need more effort for the knowledge of P-wave decay constants.

In conclusion, we estimated the decay constants for heavy P-wave $^3P_0$, $^3P_1$ and $^1P_1$ mesons in the framework of the relativistic Bethe–Salpeter method.

Table 5
Decay constants in unit of MeV for heavy P-wave states in different models

	$1\,^3P_0$			$1\,^3P_1$			$1\,^1P_1$		
	Ours	[13]	[14]	Ours	[13]	[14]	Ours	[13]	[14]
$s\bar{b}$	140		146	157		181	76		84
$u\bar{b}$	145	112	162	150	123	187	76	68	93
$s\bar{c}$	112	71	110	219	121	240	62	38	63
$u\bar{c}$	133	86	139	211	127	249	72	45	82

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