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Graphical matroid for causality assignment in bond graphs

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Abstract

Graphical methods provide useful tools to study the structure of systems. The bond graph approach is used in modelling process of dynamical systems. The matroid theory offers a powerful tool if we are interested in combinatorial aspects of causality assignment. This paper is organized as follows: after an introduction, Section 2 introduces some matroid background, and Section 3 presents some matroid definitions about structural properties. In Section 4, the main result of this paper is the proposal of a procedure that constructs cycle and co-cycle graphs from graphical matroids defined in the previous section. Validity of this procedure and some examples are shown in Sections 5 and 6, respectively, and concluding remarks are given in Section 7. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Matroid theory; Bond graphs; Causality assignment; Combinatorial problems

1. Introduction

In system control analysis and design, graphical tools are often used to help the engineer in his task [4,7,10].

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The bond graph theory [7] is one of these methods in which the structural and the physical information are encoded in bond graph words. The capabilities of the bond graph concept as a pictorial representation of combinatorial structure have already been examined [2,3].

The aim of this work is to deal with the structural analysis aspect of dynamical systems modelled by bond graphs [11]. We propose by using matroid theory [13] a unified method as one general combinatorial technique.

A correspondence between graph theory and matroids defined on the bond graph causality concept is pointed out and we propose a procedure for constructing graph associated with a graphical matroid.

It is known, in matroid theory, that the greedy algorithm always gives optimal solution. We propose an algorithm of this kind to cope with large-scale bond graph models in order to obtain a base (maximal independent set) of a defined matroid.

This base will be used in the proposed procedure to construct a graph that, in turn, leads to any valid combination of causality assignment on the bond graph model.

Finally, this procedure is validated by some theoretical results.

2. Matroid background

Many applications of this theory in engineering can be found in [9]. We give some definitions that concern our study.

Definitions. A *matroid* **M** is obtained as a pair $\mathrm{M} = (\mathbf{S}, \mathbf{I})$ where **S** is a set which represents the domain on which the matroid is defined and **I** is the *independent set* of this matroid.

The independent set **I** must verify the three following axioms:

$(\mathbf{I}_1)$ $\emptyset \in \mathbf{I}$,

$(\mathbf{I}_2)$ $\mathbf{X}_1 \in \mathbf{I}, \mathbf{X}_2 \subset \mathbf{X}_1 \Rightarrow \mathbf{X}_2 \in \mathbf{I}$ (i.e. each subset of an independent set is also independent),

$(\mathbf{I}_3)$ $\mathbf{X}, \mathbf{Y} \in \mathbf{I}, |\mathbf{X}| > |\mathbf{Y}| \Rightarrow \exists\, \mathbf{x} \in \mathbf{X} \setminus \mathbf{Y}$ such that $\mathbf{Y} \cup \{\mathbf{x}\} \in \mathbf{I}$.

The sets that do not verify these axioms are called *dependent*. A *minimal dependent* set is called a *circuit*.

A *base* of a matroid is a maximal independent set, it belongs to the B set which verifies:

$(\mathbf{B}_1)$ $\mathbf{B} \neq \emptyset$,

$(\mathbf{B}_2)$ $|\mathbf{B}_1| = |\mathbf{B}_2|\ \forall\ \mathbf{B}_1, \mathbf{B}_2 \in \mathsf{B}$,

$(\mathbf{B}_3)$ $\mathbf{B}_1, \mathbf{B}_2 \in \mathsf{B}, x \in \mathbf{B}_1 \Rightarrow \exists\, y \in \mathbf{B}_2$ such that $(\mathbf{B}_1 - \{x\}) \cup \{y\} \in \mathsf{B}$.

All the bases of B set have the same cardinality: it is called the *rank* of the matroid, and is noted rg or ρ.

Examples. We retrieve the matrix theory, graph theory, etc. Below some examples of matroids defined by their S-set and I-sets are given:

S: The edges set of a graph G; I: the forests of G.

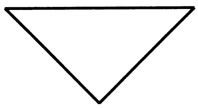
S: The columns set of a matrix; I: linearly independent sets of columns.

S: a set with cardinality n; I: subsets whose cardinality is at most k, $0 \leqslant k \leqslant n$. The latter matroid is called the uniform matroid $U_{n,\mathrm{k}}$. The matroid $U_{n,n}$ is called the free matroid, $U_{n,0}$ the trivial matroid.

A matroid is graphic if the set S is isomorphic to the set of edges of a graph G.

A base of a matroid in this case corresponds to a forest of cardinality $n - p$ where n is the number of vertices and p the number of the connected components.

$U_{3,2}$ is graphic and represented by:



$U_{4,2}$ is not graphic, i.e. cannot be depicted by a figure.

For matroids, duality is well complete and symmetrical.

Let $\mathsf{B}^* = \{X \text{ such that } S - X \in B\}$ where $B \in \mathsf{B}$ for some matroid M. Then there exists a matroid M^* called the dual of M whose bases constitute the B^* set.

One important and unique feature of matroids, unlike graphs, is the fact that any matroid possesses a unique dual whereas a graph has a dual if and only if it is planar.

Definition. A co-base of M is a base of M^* and the dual rank function, noted ρ^*, is defined by

$$\rho^*(M) = \rho(M^*) = \text{co-rg}(M).$$

Property. The function ρ^* verifies: $\rho^*(X) = |X| + \rho(S - X) - \rho(S)$.

3. Matroids for structural properties study

This section proposes a new theory that transposes all results obtained in bond graph theory into matroid theory.

It is well known in bond graph theory that, the variables related to the dynamical elements that are in derivative causality when a preferred integral causality is assigned, constitute together with the other variables a dependent set with regard to the equational model. The order of the model, i.e. the dimension of its vector space, corresponds to the number of **I** and **C** elements in integral causality. These elements then constitute an independent set.

It is known that bond graphs are suitable for generating state space equations. On these equations matroidal conditions for structural controllability can be defined, as it is shown in [8], but the design of this intermediate model is very costly.

From a structural point of view, on the digraph (or linear graph), Reinschke [10] has established the conditions for a model to be controllable by computing the number of cycles of *width n* (number of state vertices) in the graph associated with the underlying linear model.

We recall in the sequel an interpretation of the degree of freedom in the assignment causality procedure [6]. As a matter of fact, the maximal number of *I*–*C*-elements that may be assigned to integral causality (via the MSCAP procedure assignment of causality) [12] is fixed even if some *I*–*C*-elements are in derivative causality. The existence of causal paths between *I* or *C* elements and some *R*-elements may allow different possibilities in the assignment of the causal stroke. We have insisted on the consequence of these combinatorics when expressing the rank of the state matrix in matroidal terms.

In the application of the sequential causality SCAP method [7], independent states are associated with storage ports that obtain the preferred integral causality and dependent states are associated with storage ports that obtain derivative causality. In the application of SCAP in derivative mode, independent rates are associated with storage ports that obtain the preferred derivative causality while dependent rates are associated with storage ports in integral causality.

In the same manner, we can give some interpretation to the dependent and independent sets among *I*–*C* elements when causality is assigned in integral or in derivative. The correspondence between the Tables 1 and 2 summarizes this analogy.

In the same previous work [6], matroid theory has been used to express some results about structural analysis of systems modelled by bond graphs that generalize the theorem shown in [11]. This theorem states the following:

Table 1

	Assignation in integral	Assignation in derivative
I–*C* in integral	Independent states	Dependent rates
I–*C* in derivative	Dependent states	Independent rates

Table 2

	Primal matroid (M)	Dual matroid (M^*)
Independent set (base)	Tree	Co-tree
Dependent set	Circuits (or cycles)	Co-circuits (or co-cycles)

Theorem. *A dynamical system represented by bond graph is structurally controllable if and only if*:

(a) *All I–C elements in integral causality are causally connected to a source.*

(b) *All I–C elements in integral causality when integral causality is performed accept derivative causality when derivative causality is performed on bond graph.*

Definition 1. The definition set of the matroid associated with a bond graph is constituted by the set of the (R, C, I) elements, noted E. A subset S of E restricted to the dynamical elements I and C is defined. If these elements do not exist, then the system is called static and thus will not be concerned by our study.

Definition 2. A subset of dynamical elements S is *independent* if all I and C-elements of this subset are in integral causality when integral causality is performed. This subset is noted IND_1, and a matroid $\mathbf{M_1(S, IND_1)}$ is then defined.

Definition 3. A base of the matroid M_1 is a maximal independent set with regard to the cardinality of a set.

Property 1. A subset X of I–C-elements included in S is *dependent* if there exists at least one element in derivative causality belonging to this set when integral causality is performed on the bond graph. The minimal dependent sets are called *circuits* or *stigms*. They constitute the set C and must verify the following axioms:

(C1) $\emptyset \notin C$

(C2) $C_1 \in C,\ C_2 \subseteq C_1 \implies C_2 = C_1$

(C3) $C_1 \in C,\ C_2 \in C,\ a, b \in S,\ a \in C_1 \cap C_2,\ b \in C_1 - C_2$, then
$\exists\ C_3 \in C$ such that $b \in C_3 \subset C_1 \cup C_2 - \{a\}$.

Remark. A co-base of M_1 is constituted by all the I–C elements in derivative causality when integral causality is performed on the bond graph, the number of these elements is the co-rank of the matroid M_1 *which was denoted by* $\rho^*(M_1)$.

Natural notions specific to matroid theory, like Duality, have their interpretation in bond graph terms. In this work, duality is not only limited to the junction structure as shown by Birkett [2], but also extended to causality. It will be shown that integral causality and derivative causality induce dual matroids.

Definition 4. Another matroid M_2 noted $M_2(S, \text{IND}_2)$ can be defined on the same I–C elements set S. The independent set is each subset of S such that all the I–C elements are assigned in derivative causality when derivative causality is performed in the bond graph.

Recalling that the co-rank is the complement of the rank relatively to the dimension of the space or equivalently the rank of a co-base, the following property is verified.

Property 2. $\rho^*(M_2) = \rho(M_2^*)$ is the number of I and C elements remaining in integral causality when a derivative causality is performed on the bond graph. This property is helpful, if we want to express in matroid terms the theorem shown in [11], regarding the structural controllability/observability property.

4. Graphical matroids

The existence of causal paths between the I–C elements in integral causality and the I–C elements in derivative causality provides information about algebraic relations between the rows of the junction structure matrix, associated with the bond graph.

The interest in checking whether a matroid is graphic lies in the construction of the graph that visualizes this matroid. This is achieved by performing an isomorphism between the cycles of the graph and the circuits of the matroid or, equivalently, between the trees that may be obtained from the graph and the independent sets of the matroid.

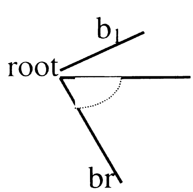
We shall show, through an example, that the co-cycles of the graph deduced from a bond graph give, in a combinatorial way, all the possible relations between the rows of the associated junction structure matrix. We propose a procedure which constructs the graph if either the M_1 or M_2 matroid is graphic.

Procedure. (1) Given an independent set on the matroid M_2, it can be associated with a tree in a graph G where the edges are some dynamical elements among I and C-elements.

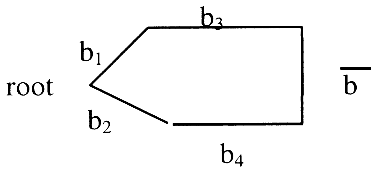
The tree is built in an iterative way:

(a) Take any valid combination of derivative causality among the n I–C elements. We may denote by b_i the base element $i = 1, \dots, r$ and $\overline{b}_k$ the co-base elements $k = 1, \dots, q$ such that $r + q = n$. The combination is then written $b_1 \dots b_r \overline{b}_1 \dots \overline{b}_q$.

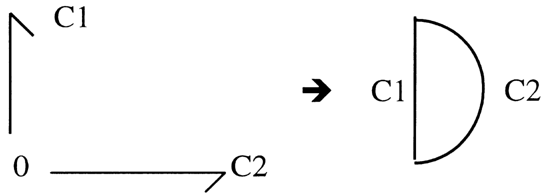
(b) Make a corresponding spanning tree so that the edges are incident at the same and unique extremity called the root as seen below:



(2) We complete the edges of the graph such that we obtain cycles as follows:
(a) For all elements in the co-base do:
If there exists a simple causal path (a causal path that does not go through any I, C, R element) between an element $\overline{b}_k$ integral causality and a subset $b_{i1}, b_{i2}, \ldots, b_{im}$, $i_m \geqslant 1$, in derivative causality, then $b_{i1}, b_{i2}, \ldots, b_{im}$, $\overline{b}_k$ constitutes a cycle.
In this step, if $i_m > 2$, then we remove the extra edges attached to the root as shown below for $i_m = 4$:



b_3 and b_4 go to the second level of the tree
(b) For the remaining co-base elements, if there exists a causal path between $\overline{b}_m$ and some b_i element, then b_i and $\overline{b}_m$ are two elements in integral causality and derivative causality, respectively and they must interchange the causality, they are kept in parallel in G.
The simplest case corresponds to two C-elements incident to 0-junction or two I-elements incident to 1-junction.



We insist on the fact that the procedure fails if the matroid is not graphic as we shall see in Example 2. Finally, we can say that this procedure can be executed by the aid of computer using *cut and paste* methods for building the graph [2].

5. Validity of the procedure

The causality assignment is subject to a combinatorial problem [12], especially when several I–C elements have to be assigned in derivative causality or when some R-elements are present, leading to a great number of possibilities which grows exponentially. This combinatorial aspect is taken into account by introducing matroids on bond graphs dealing with integral causality and/or derivative causality.

Proposition 1. *If* $\overline{b} - b_1, \ldots, \overline{b} - b_i$ *are causal paths, then in the assignment* $b_1 \ldots b_{p-1}\overline{b}_p b_{P+1} \ldots b_i b$, *we have the causal paths* $\overline{b}_p - b_1, \ldots, \overline{b}_p - b_{p-1}$, $\overline{b}_p - b_{p+1}, \ldots, \overline{b}_p - \overline{b}_i, \overline{b}_p - b$. *In other words, the knowledge of causal paths in one maximal causality combination implies the knowledge of causal paths in any other maximal combination of valid causality assignment.*

Proof. In the graph G, $b_1 \ldots b_i\overline{b}$ constitutes an elementary cycle, i.e. the deletion of one element leads to an independent subset. As the partial subgraph $b_1 \ldots b_i\overline{b}$ is connected and by virtue of a known result in matroid theory which says that for each co-base element, there exists a unique circuit which contains it and no other co-base element (this circuit is called fundamental circuit), we get the desired result. □

The same reasoning can be done if we consider the matroid M_1. By duality, for each base element there exists a unique cutset containing it and no other base element.

Proposition 2. *If, in a bond graph, we have* r *elements in derivative causality and* q *elements in integral causality, then in any dual graph* G^* *of the graph* G *generated by the procedure, we have a base of* q *co-cycles and a base of* r *cycles.*

Proof. By construction, the procedure provides q elementary cycles in graph G. Since the independent set is maximal (tree in the graph) it is a base of cycles whose dimension is given by the *nullity* $\mu(G)$ [5]. If a graph has n vertices, m edges and p components, then this nullity can be computed by

$$\mu(G) = m - n + p.$$

In our case $m = r + q$, $\mu(G) = q$ and $p = 1$.

Thus $r + q - n + 1 = q \rightarrow n = r + 1$.

If $\lambda(G)$ designs the rank of G, then $\lambda(G) = n - p = r$. □

Theorem 1. *Let* G *the set of graphs obtained by application of the procedure from the* p *possible valid assignments of causality:* $\mathsf{G} = \{G_1, \ldots, G_p\}$. *Thus* G_i *is isomorphic to* G_j, $i \neq j$.

Proof. Let us consider a graph G_k obtained by this procedure. A cycle in this graph is constituted by $b_1 \ldots b_j\overline{b}_k$ where $b_1 \ldots b_j$ correspond to elements in derivative causality and $\overline{b}_k$ to element in integral causality.

By construction, there exists a simple causal path between $\overline{b}_k$ and b_i, $1 \leqslant i \leqslant j$. Thus, we can obtain another valid causality assignment by inverting causality on b_i and $\overline{b}_k$.

The latter combination is, by virtue of Proposition 1, a partial subgraph of a graph G_i (more precisely a cycle) obtained by the procedure on the bond graph with the previous assignation.

This theorem shows that we may assign an arbitrary valid derivative causality to the bond graph, the others are generated automatically.

We shall see now what happens when two elements or more change their causality simultaneously:

1. The first case corresponds to disjoint causal paths, between two different couples of *I–C* elements, where one is in integral and the other in derivative causality. In this case if $b_i - \overline{b}_k$ and $b_j - \overline{b}_l$ contain no internal bond in common, then the simultaneous transition can occur corresponding to a partial subgraph obtained by the procedure on the bond graph with the valid causality assignment:

   $$b_1 \ldots \overline{b}_i \ldots \overline{b}_j \ldots b_k \ldots b_l \ldots$$

2. The second case is when the causal paths between the two couples are not disjoint: two possibilities can occur if the common bonds stop at 0-junction or at 1-junction as shown in Figs. 1 and 2, respectively (E_i symbolizes an element belonging to the *I–C* elements):

Let us denote the causal path $\overline{E}1E2$ and $\overline{E}1E3$ (the bar, "$\overline{}$", placed upon the element means that it is in integral causality), and by $\rightarrow$ the inversion of the causality process. It is obvious that: In a maximal derivative assignment, if we change the causality in $E2$ ($E2 \rightarrow \overline{E}2$) or $E3$ ($E3 \rightarrow \overline{E}3$) we can easily verify that

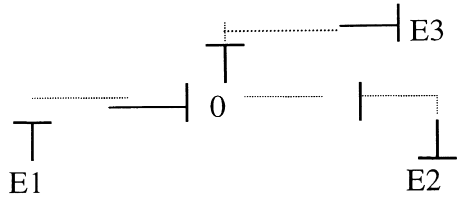


Fig. 1. Causal path in common stop at 0-junction.

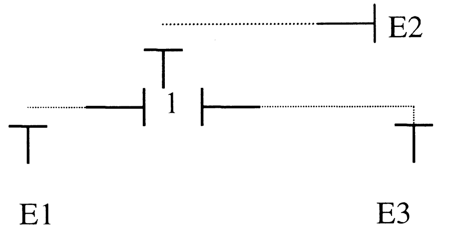


Fig. 2. Causal path in common stop at 1-junction.

the causality in $\overline{E}1$ has to change for $E1$ and thus, there exist causal paths $\overline{E}2E1$, $\overline{E}2E3$ or $\overline{E}3E1$, $\overline{E}3E2$.

If some R-elements are present, then another possible assignment can occur but it is not maximal regarding to a base and we cannot deduce information about causal paths.

The same reasoning is carried out if the common causal path stops at 1-junction as shown in Fig. 2.

We conclude that it is impossible to have three simultaneous transitions or more if the causal paths are not disjoint and no graph can be obtained from those bond graphs.

Remark that two elements may transit simultaneously but not the third.

- If two elements were initially in integral causality and they transit in derivative, it means that the maximum derivative causality was not assigned and this is in contradiction with the assumption of the M_2 matroidal structure.
- If the two elements were in derivative and they transit in integral, we loose the concept of the base.

If one element is in integral causality and the other in derivative, then we obtain a valid assignment when interchanging process.

Finally, all cases are covered by the procedure, isomorphism between different graphs being obtained by a simple transition of causality. □

The next step and more important result is the fact that any graph obtained by the procedure is a graphical instance of the matroid M_2. The next theorem highlights the power of matroid theory which is an abstraction of graph theory and bond-graph theory in the context of structural analysis study.

In what follows, we suppose that there exists at least one element in integral causality because in the opposite case, only one graph can be obtained, more precisely it is a tree.

Theorem 2. *Any graph G_i, $i = 1, \ldots, p$, is a graphical representation of the matroid M_2; in other words, the matroid M_2 is isomorphic to any cycle matroid of some graph G_i.*

Proof. To prove this theorem, it is necessary to prove that for any graph G_i, its cycle matroid is isomorphic to M_2 and vice versa.

(a) In one hand, let $b_1 \ldots b_i\overline{b}_k$, $i \leqslant r$, a cycle of a graph G_i obtained by the procedure.

This implies that $b_1 \ldots b_i$ are in derivative and $\overline{b}_k$ is in integral, thus the set $\{b_1, \ldots, b_i\}$ is an independent set in M_2.

Remark that $\{b_1, \ldots, b_i\}$ is not necessary a maximal independent set but only independent and we cannot affirm that $\{b_1, \ldots, b_i, \overline{b}_k\}$ is dependent in the matroid M_2.

Let us take the set of the n dynamical elements in the bond graph $b_1, \dots, b_i, \dots, b_r, \overline{b}_1, \dots, \overline{b}_q$ such that $r + q = n$.

From each element $\overline{b}_j, (1 \leqslant j \leqslant q)$, there exists a causal path to some element b_i.

The evident case is when the two elements are of the same type, in the opposite, we have two subcases.

(a1) b_i and $\overline{b}_j$ are of different types, and the unique way to have a causal path consists in traversing the R-elements, as shown in Fig. 3.

The latter case is in contradiction with the hypothesis of the base. Indeed, we can increase by one the number of elements in derivative causality, the R-element allows this transition.

(a2) b_i and $\overline{b}_j$ are of different types and there exists an odd number of gyrators in the causal path. An example is shown in Fig. 4.

In this case, we cannot change the causality without introducing causal conflict.

(b) In the other hand , a minimal generating set of circuits of the matroid M_2 is of dimension q, we can express this set by

$$C = \{b_1 \dots b_r \overline{b}_1, b_1 \dots b_r \overline{b}_2, \dots, b_1 \dots b_r \overline{b}_q\}.$$

Let us consider now the p cycles generated from any graph G_i obtained by application of the procedure. We have already shown by Theorem 1 the isomorphism between those graphs and thus any graph can represent this class.

Let us denote by $G\{C\}$ the cycle graph of the matroid M_2.

Both of the graphs can lead to a circuit incidence matrix. Denote them by D_1 and D_2, respectively, where the rows represent the circuit's incidence and the column set is the set edges of the graph, that is all the dynamical elements in the bond graph.

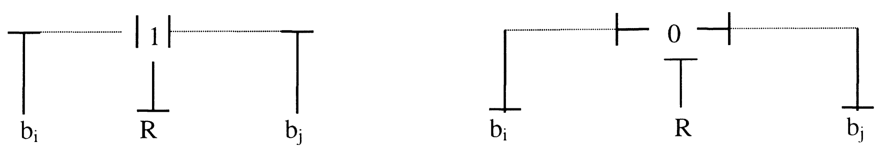


Fig. 3. b_i and b_j are different types and only one is in integral (the first case).

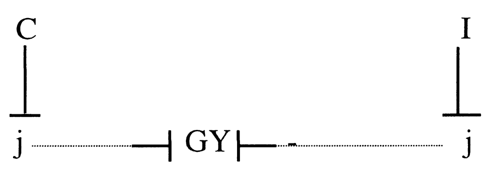


Fig. 4. b_i and b_j are different types and only one is in integral (the second case).

By applying the procedure, for each $j \in \{1, \dots, p\}$, two cases may appear:

- either we obtain the causal paths $\overline{b}_j - b_1, \dots, \overline{b}_j - b_i$ $(i > 2)$ and we obtain the cycle $b_1 \dots b_i \overline{b}_j$,
- or we have $\overline{b}_j - b_l$ and thus $\overline{b}_j - b_l$ is a loop.

Consider now the graph G_i. A tree is, by construction, in correspondence with the r-elements in derivative; each time a $\overline{b}_j$, $1 \leqslant j \leqslant p$, is added, a new cycle (a dependent set with $r + 1$ elements). As introduced, this corresponds to a circuit in M_2.

The D_1 matrix has the following form:

	b_1	b_2		br	$\overline{b_1}$			$\overline{bq}$
1	1	1	0	0	1	0	0	0
2	0	1	1	0	0	1	0	0
	....				0.......	1............		0
q	1	0		1	0	0		1

and the D_2 matrix is as follows:

b_1	b_2		br	$\overline{b_1}$			$\overline{bq}$
1	1	1	1	1	0	0	0
1	1	1	1	0	1	0	0
....				0	1...............		0
1	1	1	1	0	0	0	1

Two matrices $A_{m\times n}$ and $B_{m\times n}$ are equivalent if and only if there exist $P_{m\times m}$ and $Q_{n\times n}$ regular such that

$$B = P\ A\ Q.$$

It has been demonstrated that two matrices are equivalent if and only if they have the same rank. The latter result is easily verified on the previous D_1 and D_2 matrices, both are of the rank q and we conclude about the isomorphism between the two graphs. Fig. 5 summarizes the correspondence. □

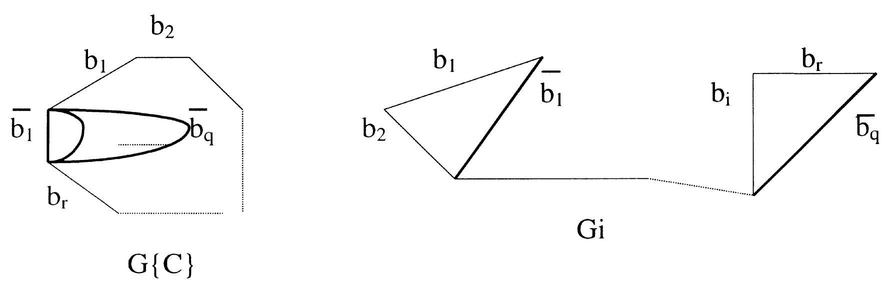
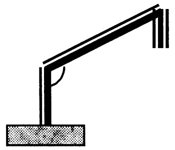


Fig. 5. Isomorphism between $G\{C\}$ and G_i.

6. Examples

Example 1. Consider the example given in [11] that models a two-axis robot, one possible preferred derivative causality assignment is displayed (Fig. 6), $I{:}I1$ and $I{:}I4$ have an integral causality.



The corresponding matroid M_2 is graphic and we obtain the graph of Fig. 7.

Consider now the partial graph composed of the I-elements and construct its dual, i.e. each cycle will correspond to a co-cycle and vice versa. We have removed the C-elements, C_1 and C_2, because they do not determine faces in the graph. Then, we obtain the following graph.

We find three co-cycles $E1^* = \{I1, I2, I3\}$, $E2^* = \{I2, I3, I4\}$, and $E3^* = \{I1, I4\}$.

Six possibilities appear for assigning a preferred derivative causality but one of them is not valid (i.e. taking $I1$ and $I4$ in derivative): we may take four IC-elements among the six to construct the base. The C-elements are always in derivative because if one of them is in integral then we loose the base.

In this example, each co-cycle corresponds to the presence of direct causal paths between elements in integral causality and those in derivative causality.

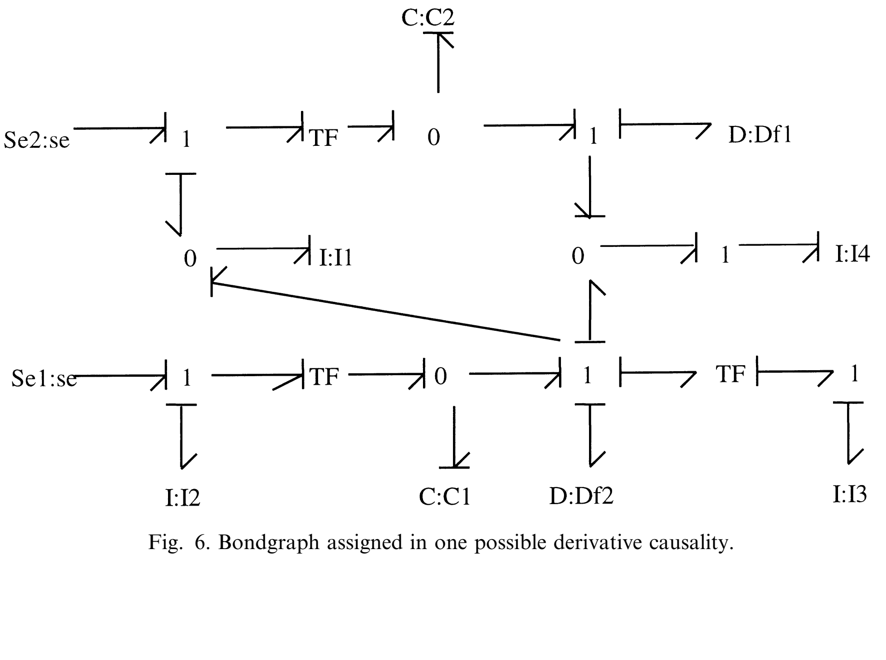


Fig. 6. Bondgraph assigned in one possible derivative causality.

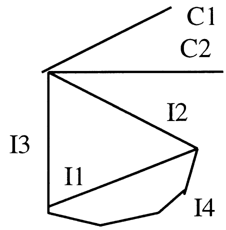


Fig. 7. The graph obtained by application of the procedure.

Each possible causality assignment gives two possible co-cycles among the three previous ones.

We can remark that the $I2$ and $I3$ elements always occur together in the co-cycles, this reveals that both of them are causally connected with $I1$ and $I4$, and they are represented in parallel in the dual graph (Fig. 8): this explains the fact that we cannot simultaneously affect $I2$ and $I3$ in integral causality when a preferred derivative causality is performed without decreasing the cardinality of the maximal independent set, namely the base. Moreover, through this example, we have shown an important feature: integral causality and derivative causality are handled by duality.

Finally, it can be noted that the interest of the proposed procedure lies in the fact that we have not to generate all the possibilities, the result can be obtained from any arbitrary valid initial derivative causality assignment, which gives always isomorphic graphs.

In order to optimize the causality assignment procedure, the combination which minimizes the interconnections between these co-cycles, $E_i^* \cap E_j^* (i \neq j)$ can be chosen.

We easily verify Proposition 2, that is $q = 2$, $n = 5$, and $\lambda(G) = r = 4$.

We may obtain two sets among $E1^*$, $E2^*$, $E3^*$, which really constitutes a co-cycle base of the dual of the graphs stemming from the procedure.

Finally, we have voluntary omitted in (Fig. 8) the edges $C1$ and $C2$ because it is known in graph theory [1] that in a planar graph, the contours of the different faces constitute a base of independent cycles.

Example 2. Consider the following bond graph (Fig. 9) isomorphic to $U_{4,2}$. It should be noticed that there is a base composed of four elements in M_1 (or M_2),

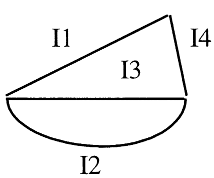


Fig. 8. The dual graph obtained by removing the C-elements.

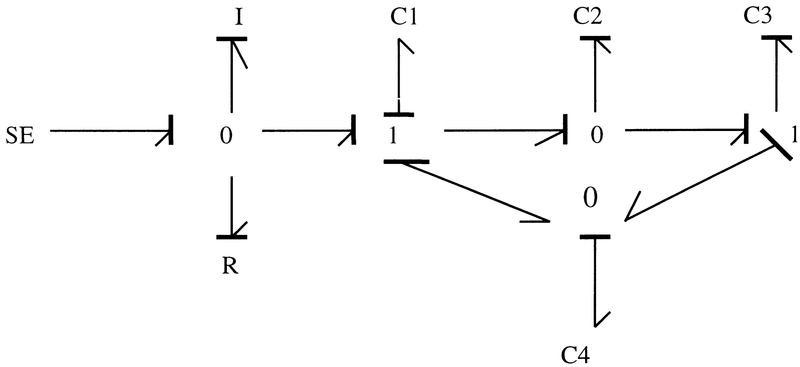


Fig. 9. Bond graph whose matroid cycle is isomorphic to $U_{4,2}$.

but the *I*-element is common to each base, meanwhile we cannot assign more than two elements in integral (or in derivative) causality among the four elements $C_i,\ i = 1, \ldots, 4$. This provides six valid combinations in the assignment process (combination of 4 elements 2 by 2).

7. Conclusion

We have proposed mathematical foundations for the causality assignment process in bond graphs based on matroids defined upon elements in integral or derivative causality.

Thus, we have constructed a combinatorial procedure visualizing graphical matroids and demonstrated theorems validating what we proposed.

Our contribution stipulates that any pictorial schema of bond graph is in fact a representation of some equivalence class which regroups bond graphs related together by existence of causal paths between the *I*–*C* elements in integral causality and the *I*–*C* elements in derivative causality.

Thus, only one pictorial representation of the class is necessary, we can generate the others by isomorphism through the graphical matroid that involves this class.

The important conclusion is that matroids on bondgraphs show, in a combinatorial way, how we can analyse structural properties and the causality assignment process in a more abstract manner.

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